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An informational rationale for action over disclosure ☆

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Abstract

The past two decades have seen a considerable increase in the amount of public information provided by policymakers. Are such disclosures desirable? Or is it instead preferable to use such information to condition a policy instrument, such a tax or an interest rate? This paper studies the relative merits of each means to use a policymaker's information in a flexible class of economies that feature dispersed information, and payoff and learning externalities. I provide conditions for when the exclusive use of a policy instrument or disclosure is optimal. I then relate these to differences in the equilibrium and socially optimal use of information. I conclude with a series of applications that show how my results apply to common beauty-contest models, competitive economies, and a broad class of macroeconomic models, among others.

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1. Introduction

The past two decades have witnessed a considerable increase in the amount of public information provided by monetary and fiscal authorities about the state of the economy. A simple estimate based on data from wire-services points to over a two-fold increase for the US.[1]

Despite the prevalence of such policymaker releases, and the considerable resources devoted to their production, the benefits of such statements are open for debate. Should we be concerned that releasing such noisy information may cause confusion and lower welfare? Do such disclosures help people make better choices? Or would it be preferable to instead use such information to aptly set a policy instrument, such as a tax or an interest rate?

A substantial debate since Morris and Shin's (2002) influential contribution has attempted to provide exact conditions for the social value of public information releases. Despite frequent mentions of policy maker communication, much of this debate has, however, abstracted from the presence of policy instruments. This abstraction is not without cost. As Angeletos and Pavan (2009) and Angeletos et al. (2016) show, a tax based on ex-post information about realized fundamentals or aggregate activity can modify people's use of public and private information, and thus profoundly shape the social value of additional public releases.

In this paper, I sharpen the focus onto the best use of a policymaker's own information. I study the combined, optimal choice between the two means that exist to exploit a policymaker's news: to either disclose it, or to condition a policy instrument upon its realization. To do so, I embed a policymaker with noisy private information and access to a policy instrument into a flexible class of quadratic beauty-contest economies. The economies feature dispersed information, and payoff and learning externalities. In the model, agents observe noisy private and public information, including from the policymaker about her own information. Agents then use this information to choose their actions in response to their expectations about an unobserved fundamental and the policymaker's instrument.

My main contribution is to provide a set conditions under which the conditional use of a policy instrument or public disclosure is the sole best means to exploit a policymaker's information. Importantly, the conditions that I provide are not limited to any specific application. Instead, my objective is to provide a set of interpretable conditions to guide normative analysis across different environments. To this end, I show how my results apply from popular beauty-contest games to a broad class of macroeconomic models, among others.

At the center of my results lies a tension between the respective advantages that a policy instrument and disclosure have in exploiting a policymaker's information. On the one hand, I show that the conditional use of a policy instrument is a better tool to adjust people's use of information, compared to (potentially noisy) disclosure of the policymaker's news. On the other hand, disclosure has the advantage that people always change their actions in response to new information about the fundamental state of the economy, even in cases where the policy instrument has little bite on individual choices. Disclosure gets into all the cracks.

The logic behind these advantages is simple. First, consider the advantage that the conditional use of a policy instrument has in exploiting a policymaker's information. Suppose the presence of payoff or learning externalities causes full disclosure not to achieve the efficient outcome. Imagine, for example, that it causes agents to erroneously over-emphasize the policymaker's

[1] This is based on Bloomberg news data. The precise number of releases about the state of the US economy is 453 in 2015 and 213 in 1995. The count includes speeches, comments, and public documents by the President, Federal Reserve Presidents, US Treasury officials, and members of the CEA and the CBOE.

information, to the detriment of welfare. With the anticipated, conditional use of the policy instrument, the policymaker can alter the incentives that people face to use their information. The more people expect the policymaker to use her information to counter the fundamental, the less they will respond to any information about it, including that which comes from the policymaker. By contrast, with disclosure, the policymaker can only incentivize agents to decrease their use of her information by releasing an obfuscated, noisier version of her news. This warping of a signal that enters agents' decision-making yet further distorts their actions, a welfare cost which the policy instrument completely avoids.

Second, consider now instead the advantage disclosure provides in exploiting a policymaker's information. Suppose that we consider an economy in which the policymaker needs to rely on people's efficient responses to her information, to better align people's actions with the fundamental state of the economy. The central advantage that disclosure then provides is that it gets into all the cracks. Disclosure allows people to always respond to information about the economy's state. This is especially beneficial in cases where the policy instrument is constrained in its ability to move the economy in response to the policymaker's information.

The contribution of this paper is to provide a clear structure of when these advantages create an informational rationale for the use of policy instruments or disclosure. I show that economies in which (i) people's use of information is inefficient, and (ii) the policy instrument can perfectly substitute for agents' use of the policymaker's information, the conditional use of a policy instrument is preferable. Indeed, in such cases, any disclosures by the policymaker merely result in lower welfare. This holds true irrespective of whether the economy uses the policymaker's information too much or too little under full disclosure, or the underlying reason for the failure of full disclosure to achieve the efficient outcome; factors that elsewhere have been shown to be critical for the optimal disclosure of public information (e.g. Angeletos and Pavan, 2007). By contrast, economies in which the policy instrument acts as a poor substitute for agents' use of the policymaker's information, such as economies in which the policy instrument has little bite on individual actions, tend to favor disclosure.

I map these conditions into properties of agents' payoff functions and discuss how they are affected (i) by the extent and efficiency of the strategic complementarity that people perceive, (ii) by the amount and efficiency of social learning, and last (iii) by economies' underlying aversion to dispersion and the volatility of actions. I also demonstrate how the conditions that I identify extend to more complex instrument rules.[2]

I conclude the paper with a series of examples that not only show how my results guide normative analysis in specific applications, but also illustrate some limitations of my results. I show that in a competitive island-economy with an efficient use of information, a complete separation between instrument and communication policy arises. Optimally, the policymaker should fully disclosure her information, to best alleviate the information friction, while the conditional use of the policy instrument is set to its full-information, flexible-price value.

In contrast, in models in which the equilibrium use of information is inefficient, the sole use of a policy instrument can be optimal. For example, in the model studied by James and Lawler (2011), who embed a policymaker into the beauty-contest model proposed by Morris and Shin (2002), the exclusive use of a policy instrument is optimal. This is because the policy instrument can perfectly substitute for agents' use of the policymaker's information. The advantage that the

[2] Specifically, I consider cases in which the policy instrument is conditioned both on the policymaker's own noisy information and on her estimate of the average action in the economy. Angeletos and Pavan (2009) show that the latter is important for an accurate assessment of the social value of public information.

policy instrument has in adjusting agent' use of information then makes it the unique best means to use a policymaker's information.

Importantly, I show that the same conditions hold for a prominent class of business cycle models with monopolistic competition, in which firms' pre-set prices under dispersed information (e.g. Woodford, 2002). This shows that the benefits of central bank disclosures within this class of models, discussed for example in Hellwig (2005) and Roca (2010), rest crucially on the central bank not conditioning its policy instrument on its own information.

My last two applications, by contrast, provide examples of the benefits of disclosure. Specifically, I consider a simple prediction game with endogenous public information that embeds a policymaker into a reduced-form version of the model proposed by Amador and Weill (2010). I also consider a different prediction game in which agents' attempt to align their actions with individual-specific fundamentals. In both cases, the sole use of (potentially noisy) disclosure is optimal, because it allows the policymaker to better adjust agents' use of information. The policy instrument cannot, in these cases, substitute sufficiently well for individual actions.

Finally, one wider implication of my results is worth noting: My analysis leaves the determination of the policy instruments available to the policymaker unexplored. Yet, to a certain extent, this decision is also a choice for the policymaker. This opens the prospect that my results can be used as inputs into the study of the optimal choice of policy instruments.

Related literature In addition to the literature cited above, this paper is related to three strands of research. I review these below, starting with those that pertain to the social value of public information and ending with those on the informational effects of policy instruments.

First, this paper is related to the debate about the social value of public information started by Morris and Shin's (2002) influential contribution. Hellwig (2005) and Roca (2010) question the relevance of the negative welfare effects in Morris and Shin's analysis by showing that the converse is true in macroeconomic models with monopolistic competition.[3] Angeletos and Pavan (2007) extend Morris and Shin's analysis to a class of quadratic games with exogenous information and payoff externalities, and show that the social value of public information depends crucially on any wedge between the equilibrium and socially optimal use of information. This clarifies and organizes the previous conflicting results.

Closely related, Morris and Shin (2005) and Amador and Weill (2010, 2012) illustrate that social welfare can decrease with additional public information in models in which people observe endogenous signals. The presence of endogenous information, and the associated learning externalities (e.g. Vives, 1993, 2017), slow down social learning to the detriment of social welfare. I below depart from the class of quadratic games with payoff externalities studied by Angeletos and Pavan (2007) but allow for both the presence of exogenous and endogenous information in addition to the conditional use of a policy instrument. I then use this flexible framework to show that the social value of policymaker information differs importantly from other sources of public news. Through this lens, my contribution is to derive a set of interpretable conditions that show how the social value of public information changes when a policy instrument can also be conditioned on that information.

Second, this paper is related to the literature that studies the informational effects of policy instruments. King (1982) and Weiss (1982) provide early important contributions that show how the conditional use of a policy instrument can influence the weight on private and public informa-

[3] See also inter alia Svensson (2006), Morris et al. (2006), and the overview in Angeletos and Lian (2016).

tion. More recently, Lorenzoni (2010), Wiederholt (2017), Angeletos and La'O (2019), Kohlhas (2019) analyze how to optimally set policy instruments in settings with market-based information. Angeletos and Pavan (2009), importantly, show that the set of conditioning variables is central for policy instruments' capacity to render agents' use of private and public information efficient. But as with the earlier contributions, they do not consider the dual use of instrument and communication policy that provides the basic premise behind my analysis.

Complementary to this paper, James and Lawler (2011) have independently studied how the presence of a policy instrument affects the social value of public information. Yet their contribution focuses exclusively on the relative weight accorded to public and private information within Morris and Shin's (2002) beauty-contest framework. I show how their results are a special case of those derived below, and that their focus is somewhat misplaced; it abstracts from the true, principal mechanism and assumptions that underlie their main results.

Finally, a distinct mechanism to those I examine below has been suggested by Walsh (2007) and Baeriswyl and Cornand (2010). Their respective contributions show that the observation of a central bank's policy instrument can modify firms' beliefs about the mix of (efficient vs. inefficient) shocks to the economy, and thus alter the social value of public information. I demonstrate below how my results extend to environments in which people also in part learn about the policymaker's information from the current stance of her policy instrument.

Organization The plan for the rest of this paper is as follows. I introduce the class of economies studied in the next section. Section 3 derives the equilibrium and socially optimal use of information, and explores the effects that policy has on the former. The crux of the paper is in Sections 4 and 5, which provide a taxonomy for the optimal use of a policymaker's information depending on the precise structure of agents' payoffs. I conclude in Section 6 with a series of applications. Additional extensions and all proofs are in the Appendix.

2. A baseline framework

2.1. Actions and payoffs

The economy is comprised of a continuum of measure one of private sector agents, indexed by $i \in [0, 1]$. Each agent chooses an action $a_i \in \mathbb{R}$ to maximize his payoff $u_i \in \mathbb{R}$, which depends upon his own action, the actions of others in the economy, an exogenous fundamental $\theta \in \mathbb{R}$, in addition to a policymaker's policy instrument $m \in \mathbb{R}$:

$$u_i = \mathcal{U}\left(a_i, \bar{a}, \sigma_a, \theta, m\right), \tag{2.1}$$

where $\bar{a} \equiv \int a_i dF(a_i)$ and $\sigma_a \equiv \left[\int (a_i - \bar{a})^2 dF(a_i)\right]^{1/2}$ denote the cross-sectional mean and standard deviation of actions, respectively, and F the cumulative distribution function for actions a_i in the cross-section of the population. The payoff function $\mathcal{U} : \mathbb{R}^2 \times \mathbb{R}_+ \times \mathbb{R}^2 \to \mathbb{R}$ is a quadratic polynomial with derivatives that satisfy $\mathcal{U}_{aa} < 0$ and $-\mathcal{U}_{a\bar{a}}/\mathcal{U}_{\bar{a}\bar{a}} < 1$.[4] I further assume that $\mathcal{U}_\sigma = \mathcal{U}_{\sigma\sigma}\sigma$ and $\mathcal{U}_{a\theta} \neq 0$, which rules out any strategic effects of dispersion as well as uninteresting cases in which agents' actions do not depend upon the fundamental. To ensure

[4] These assumptions ensure that the resulting equilibrium choices are unique and bounded under full information; they impose concavity at the individual level ($\mathcal{U}_{aa} < 0$) and limit the extent of strategic complementarity between actions ($-\mathcal{U}_{a\bar{a}}/\mathcal{U}_{\bar{a}\bar{a}} < 1$) (see the "Online Supplement" to Angeletos and Pavan, 2007).

that an appropriately defined first-best outcome is also unique and bounded, I also assume that $\mathcal{U}_{aa} + \mathcal{U}_{\sigma\sigma} < 0$ and $\mathcal{U}_{aa} + 2\mathcal{U}_{a\bar{a}} + \mathcal{U}_{\bar{a}\bar{a}} < 0$. Lastly, the policymaker sets her policy instrument in accordance with

$$m = \phi z, \tag{2.2}$$

where $\phi \in \mathbb{R}$ denotes the publicly known level of policy activism and z the policymaker's own private signal about the fundamental (defined below). I thus characterize instrument policy in terms of a commitment to a linear rule. Because of the quadratic nature of payoffs, this assumption by itself does not prevent policy from achieving the first-best outcome. Section 5 discusses how my results extend to other cases in which the policymaker conditions her policy instrument on more variables, such as estimates of the average action $\bar{a}$.

Besides these restrictions, the framework is quite flexible. It allows for either strategic complementarity or substitutability between actions ($\mathcal{U}_{a\bar{a}} \gtrless 0$), for payoff externalities with respect to the mean ($\mathcal{U}_{\bar{a}} \neq 0$), as well as welfare effects of dispersion ($\mathcal{U}_{\sigma} \neq 0$). Importantly, the framework allows the policymaker's instrument to enter agents' payoffs in a flexible manner. The policy instrument can affect agents' payoffs both through their individual actions ($\mathcal{U}_{am} \neq 0$), the average action in the economy ($\mathcal{U}_{\bar{a}m} \neq 0$), and by having a direct effect on individual payoffs ($\mathcal{U}_{m} \neq 0$ when $a = \bar{a} = 0$). Accounting for all three possibilities will be important for the optimal use of the policy instrument. Nevertheless, despite this flexibility, the framework remains tractable. The first-order conditions that characterize agents' behavior are linear in $(a, \bar{a}, \theta, m)$ and independent of σ_a. This ensures that equilibrium outcomes can be derived analytically, even in the presence of endogenous information.

2.2. *Information structure*

To complete the description of the economy, it is necessary to specify the information structure. I assume that all agents observe a combination of private and public information. Before agents choose their actions, nature draws the fundamental θ from a Normal distribution with mean μ and precision τ_θ. The realization of θ is unobserved by agents and the policymaker.[5]

Agent i's private information is summarized by two private signals. First, a noisy *exogenous* signal $x_{i\theta}$ of the unobserved fundamental,

$$x_{i\theta} = \theta + \epsilon^i_{x\theta}, \quad \epsilon^i_{x\theta} \sim \mathcal{N}\left(0,\ 1/\tau_{x\theta}\right), \tag{2.3}$$

where $\tau_{x\theta}$ denotes the common precision of the type-specific signal and $\epsilon^i_{x\theta}$ is independent of all other disturbances with $\mathbb{C}\text{ov}\left[\epsilon^i_{x\theta}, \epsilon^j_{x\theta}\right] = 0$ for all $i \neq j$. Second, a noisy *endogenous* signal $x_{i\bar{a}}$ of the average action in the economy,

$$x_{i\bar{a}} = \bar{a} + \epsilon^i_{x\bar{a}}, \quad \epsilon^i_{x\bar{a}} \sim \mathcal{N}\left(0,\ 1/\tau_{x\bar{a}}\right), \tag{2.4}$$

where $\tau_{x\bar{a}}$ denotes the precision of the private signal and $\epsilon^i_{x\bar{a}}$ is independent of all other shocks with $\mathbb{C}\text{ov}\left[\epsilon^i_{x\bar{a}}, \epsilon^j_{x\bar{a}}\right] = 0$ for $i \neq j$. I follow convention and assume that $\int \epsilon^i_{x\theta} dF\left(\epsilon^i_{x\theta}\right) = 0$ and $\int \epsilon^i_{x\bar{a}} dF\left(\epsilon^i_{x\bar{a}}\right) = 0$ almost surely (a.s.). The exogenous private signal in (2.3) summarizes agents'

[5] While several of my results below extend to other distributional assumptions, the assumption of Gaussianity is key for maintaining tractability in the presence of endogenous signals. This is because it ensures that individual actions (and hence the endogenous signals themselves) are linear in equilibrium (e.g. Vives, 2010).

private information about the fundamental. By contrast, the endogenous private signal in (2.4) captures agents' private learning about aggregate outcomes through, for example, social interactions. Both can in a reduced-form manner be explained by agents' limited attention (e.g. Sims, 2003; Wiederholt, 2010; and Vives and Yang, 2017).

In addition to their private information, agents observe two distinct public signals: (i) a noisy signal of the average action $\bar{a}$; and (ii) a (potentially) noisy signal of the policymaker's information z. The public signal of the average action, my stand-in for public, market-based information, such as financial prices or macroeconomic data releases, equals

$$y = \bar{a} + \epsilon_y, \quad \epsilon_y \sim \mathcal{N}\left(0,\ 1/\tau_y\right), \tag{2.5}$$

where ϵ_y is assumed independent of all other random disturbances with precision τ_y.[6] The signal of the policymaker's own information is, in turn,

$$\omega = z + \epsilon_\omega, \quad \epsilon_\omega \sim \mathcal{N}(0,\ 1/\tau_\omega), \tag{2.6}$$

where

$$z = \theta + \epsilon_z, \quad \epsilon_z \sim \mathcal{N}(0,\ 1/\tau_z), \tag{2.7}$$

and the error terms ϵ_ω and ϵ_z are independent of each other, θ, ϵ_y, and $\epsilon^i_{x\theta}$ and $\epsilon^i_{x\bar{a}}$ for all i. The case of full disclosure corresponds to the limit in which $\tau_\omega \to \infty$, while complete opacity is equivalent to the situation where the policymaker's communication contains no valuable information, $\tau_\omega \to 0$. Partial disclosure refers to the interim case, $\tau_\omega \in \mathbb{R}_+$.[7] Neither the policymaker nor private sector agents observe other agents' actions. We can summarize the information structure by the following information sets: $\Omega_i = \{x_{i\theta}, x_{i\bar{a}},\ y,\ \omega\}$ for $i \in [0, 1]$.

2.3. Timeline

There are three stages. In the first stage, the policymaker announces her communication and instrument policy (τ_ω and ϕ), respectively. After the policy choices are sunk, the economy transitions to the second stage, where agents observe their information and choose their actions a_i to maximize their expectations of realized utility u_i. In the final stage, the policy instrument m is set, the fundamental θ is revealed, payoffs are realized, and the game ends.

2.4. Discussion of environment

The above environment includes two central features that differentiate it from previous work. First, in contrast to the related setup studied in Angeletos and Pavan (2009), the policymaker can use her private information to set *both* instrument and communication policy. As I argue below, accounting for both means to use a policymaker's information is important. This is because

[6] The reason for the introduction of the shock ϵ_y in (2.5) is purely technical: the important role it plays is to limit agents' ability to infer the true value of θ from the observation of y. The use of "non-invertibility" shocks like ϵ_y to maintain imperfect information is common in the noisy rational expectations literature (e.g. Hellwig, 1980).

[7] One advantage of this approach to model communication policy is that it allows for a meaningful discussion of different levels of partial disclosure (e.g. Cukierman and Meltzer, 1986; Walsh, 2007). This advantage, of course, rests on the ability of the policymaker to commit to a disclosure rule, such as (2.6). Without this commitment, the policymaker could announce anything following the realization of her information, and the only values that would be consistent with equilibrium would be the limits of full disclosure and complete opacity. I demonstrate below how several of my main results remain valid in such a case.

plausible conditions exist under which the exclusive use of either policy tool is preferable, and because their optimal uses are intimately linked. Furthermore, and also in contrast to previous work, the policy instrument enters directly into agents' utility u_i. As a result, the conditional use of the policy instrument can potentially improve welfare, even under full information. In turn, this allows me to capture, in a reduced form manner, the implications of non-information frictions (and the corrective role that policy instruments can play for these) for the optimal mix of instrument and communication policy.

Second, the environment allows for both payoff and learning externalities (cf. Angeletos and Pavan, 2007), the latter through the observation of the two endogenous signals $x_{i\bar{a}}$ and y. As discussed in, for instance, Vives (2010), the presence of both types of externalities is important for an accurate picture of the social value of public information. For example, in a prominent contribution, Amador and Weill (2010) show that additional public information may reduce welfare by decreasing the informativeness of endogenous market signals. However, this argument rests importantly on a positive social value of additional information in the first place, which may not be the case once we also allow for payoff externalities.

Finally, a stark feature of the information structure is that agents' choices are pre-set and made before the realization of the policy instrument. In reality, however, both prospective and current policy matter for agents choices, and changes to current instruments often provide an indicator of the policymaker's information. Yet, as I emphasize in Section 6, the observation of current instruments (nor the noisy errors that may arise in the setting of these) does not meaningfully alter my results. All that is necessary is that (i) agents actions also in part depend upon expectations of the policy instrument; and that (ii) the policymaker's disclosure provides additional information about her information beyond what could be learned from the observation of the current instrument (see Blinder et al., 2008 for empirical evidence in support of this assumption). Combined, these characteristics ensure that both communication and instrument policy alter agents' uncertainty, and hence their equilibrium use of information.

3. Equilibrium and socially optimal use of information

I now proceed to characterize agents' equilibrium and socially optimal use of information. In this section, I also study how the two dimensions of policy – disclosure and the conditional use of the policy instrument – affect agents' equilibrium use of information. This will be important later, to characterize the policymaker's optimal use of her own information.

3.1. Equilibrium use of information

Each individual chooses his action a_i to maximize his own expectation of realized utility $\mathbb{E}\left[u_i \mid \Omega_i\right]$. Because of the assumptions made on the payoff function $\mathcal{U}$, the first-order conditions for an agent's optimal action delivers his best-response function. The fixed-point of this function provides us with the equilibrium choices in the economy. In accordance with the pertinent literature, I restrict myself to symmetric linear Bayesian equilibria.

Definition 1. A *symmetric linear (Bayesian) equilibrium* is a linear strategy $a : \mathbb{R}^4 \to \mathbb{R}$ such that, for all $i \in [0, 1]$ and for all realization of $(x_\theta, x_{\bar{a}}, y, \omega)$,

$$a\left(x_\theta, x_{\bar{a}}, y, \omega\right) = \arg\max_{a'} \mathbb{E}\left[\mathcal{U}\left(a', \bar{a}, \sigma_a, \theta, m\right) \mid x_\theta, x_{\bar{a}}, y, \omega\right], \tag{3.1}$$

where $\bar{a}\left(\theta, y, \omega\right) = \int a\left(x_\theta, x_{\bar{a}}, y, \omega\right) dF\left(a\right)$ and $\sigma_a\left(\theta, y, \omega\right) = \left[\int \left(a - \bar{a}\right)^2 dF\left(a\right)\right]^{1/2}$.

Proposition 1 provides a characterization of agents' equilibrium actions. It does so by extending similar results to those in Angeletos and Pavan (2007) and Angeletos and Pavan (2009) to the current setting of this paper with a policymaker.

Proposition 1. *Let* $\alpha \equiv -\mathcal{U}_{a\bar{a}}/\mathcal{U}_{aa}$ *and let* $\hat{a}(\theta, m) \equiv \eta_0 + \eta_1(\theta + \eta_2 m)$ *denote agents' equilibrium actions under full information.*[8]

(i) Then, a strategy $a : \mathbb{R}^4 \to \mathbb{R}$ *is an equilibrium if and only if*

$$a(x_\theta, x_{\bar{a}}, y, \omega) = \mathbb{E}\left[(1-\alpha)\hat{a}(\theta, m) + \alpha\bar{a}(\theta, y, \omega) \mid x_\theta, x_{\bar{a}}, y, \omega\right]. \tag{3.2}$$

(ii) The set of symmetric linear equilibria is non-empty, and is comprised of actions,

$$a(x_\theta, x_{\bar{a}}, y, \omega) = \eta_0 + \eta_1(k_0 x_\theta + k_1 x_{\bar{a}} + k_2 y + k_3\omega + k_4\mu), \tag{3.3}$$

where $k_j(\kappa) \in \mathbb{R}$ *for* $j = \{0, 1, 2, 3, 4\}$, *and* $\kappa \equiv \frac{\eta_1 k_0(\kappa)}{1-\eta_1 k_1(\kappa)} \in \mathbb{R}$ *solves*

$$\kappa = \eta_1(1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)\frac{1 + \eta_2\phi\frac{\tau_z}{\tau_\omega+\tau_z}}{\tau_\theta + \tau_y\kappa^2 + (1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right) + \frac{\tau_\omega\tau_z}{\tau_\omega+\tau_z}}. \tag{3.4}$$

The first part of Proposition 1 characterizes agents' equilibrium actions in terms of a weighted average of their expectation of the common, full information choice $\hat{a}$ and the average action in the economy $\bar{a}$ (with weights $1-\alpha$ and α, respectively). In this sense, the coefficient $\alpha = -\mathcal{U}_{a\bar{a}}/\mathcal{U}_{aa}$ measures the *equilibrium amount of payoff coordination* in the economy (Angeletos and Pavan, 2007). Lastly, notice that $\alpha \neq 0$ only because of payoff externalities ($\mathcal{U}_{a\bar{a}} \neq 0$). Learning externalities do not alter individual agents' private incentives.

The second part of Proposition 1, in turn, establishes the existence of a linear equilibrium directly in terms of the signals that agents observe. However, because of the presence of both endogenous public and private information, the economy can admit multiple equilibria (either one or three solutions exist for κ in (3.4), see Appendix A). This multiplicity introduces a well-known impediment to any welfare analysis. One has to decide on which equilibrium agents coordinate, and what the comparative statics are in each case. I circumvent this problem in Appendix A by focusing on the highest welfare equilibrium, in line with Harsanyi and Selten's (1988) "Payoff Dominance Argument", and thus abstract from any possible coordination failures (see also Amador and Weill, 2010; Amador and Weill, 2012). Appendix A shows how none of my results depend crucially on the exact equilibrium selection device used. All hold in areas of the parameter space where the equilibrium is unique.

A central coefficient in Proposition 1, and for my analysis of how to best use a policymaker's information, is $\kappa = \frac{\eta_1 k_0}{1-\eta_1 k_1}$. This variable namely determines the informativeness of the signals of the average action $x_{i\bar{a}}$ and y, and hence how much weight agents accord to the various elements of their information sets. To see this, consider the endogenous public signal y. After solving for $\bar{a} = \int a_i dF(a_i)$ from (3.3), combining terms, and subtracting μ, ω, and y from

$$y = \bar{a} + \epsilon_y = \frac{\eta_0}{1-\eta_1 k_1} + \frac{\eta_1}{1-\eta_1 k_1}(k_0\theta + k_2 y + k_3\omega + k_4\mu) + \epsilon_y,$$

it follows that the observation of y is equivalent to the observation of $\frac{\eta_1 k_0}{1-\eta_1 k_1}\theta + \epsilon_y$ or $s_y \equiv \theta + \frac{1}{\kappa}\epsilon_y$. When κ is large in absolute value, this signal is very informative about the fundamental,

[8] The full information coefficients are: $\eta_0 = -\frac{\mathcal{U}_a(0,0,0,0,0)}{\mathcal{U}_{aa}+\mathcal{U}_{a\bar{a}}}$, $\eta_1 = -\frac{\mathcal{U}_{a\theta}}{\mathcal{U}_{aa}+\mathcal{U}_{a\bar{a}}}$, and $\eta_2 = \frac{\mathcal{U}_{am}}{\mathcal{U}_{a\theta}}\eta_1$.

and vice versa when κ is small. (Similar steps show that the observation of the endogenous private signal $x_{i\bar{a}}$ is equivalent to the observation of $s_{i\bar{a}} \equiv \theta + \frac{1}{\kappa}\epsilon^{i}_{x\bar{a}}$.)

Corollary 1. *The informativeness of the endogenous signals $x_{\bar{a}}$ and y is determined by the solution to the fixed-point condition for $\kappa \equiv \frac{\eta_1 k_0(\kappa)}{1-\eta_1 k_1(\kappa)}$ in (3.4).*

The fixed-point condition (3.4) has a natural interpretation. It describes the equilibrium link between the weight that agents attach to their private information and the informativeness of the signals of the average action. In essence, all that agents can hope to learn from the observation of signals of the average action is the sum of agents' private information; that is the only truly new information contained in the average action. This, in turn, helps explain why the weight attached to the private signals $x_{i\theta}$ and $x_{i\bar{a}}$ (k_0 and k_1) ultimately determines the informativeness of endogenous signals ($\kappa = \frac{\eta_1 k_0}{1-\eta_1 k_1}$).[9] Yet, it also demonstrates the fixed-point problem inherent to my analysis. The informativeness of the signals of the average action depend on the equilibrium weight that agents attach to their private information, but that weight in turn depends critically on the informativeness of the average action.[10]

Corollary 2. *Agents' equilibrium actions can alternatively be written as*

$$a\left(\chi, \gamma, \omega\right) = \eta_0 + \eta_1 \left(\eta_1^{-1} \kappa \chi + \kappa_1 \gamma + \kappa_2 \omega\right), \tag{3.5}$$

where $\chi \equiv \mathbb{E}\left[\theta \mid x_\theta, s_{\bar{a}}\right]$, $\gamma \equiv \mathbb{E}\left[\theta \mid s_y\right]$, and $(\kappa, \kappa_1(\kappa), \kappa_2(\kappa)) \in \mathbb{R}^3$.

Corollary 2 simplifies the characterization of the equilibrium by condensing the information in private and public signals into two summary variables. First, $\chi_i = \mathbb{E}\left[\theta \mid x_{i\theta}, s_{i\bar{a}}\right]$, which describes the summary information about the fundamental θ contained in the two private signals $x_{i\theta}$ and $s_{i\bar{a}}$. The latter once more denotes the signal backed-out from the observation of $x_{i\bar{a}}$. And second, $\gamma \equiv \mathbb{E}\left[\theta \mid s_y\right]$, which details the public signal about the fundamental inferred from the observation of y and the common prior. This simplification also has the advantage of stating individual actions directly in terms of the key coefficient κ, which here equals the weight placed on the summary private signal. Finally, notice that, unlike the other public signals, the policymaker's disclosure ω is kept separate from γ. This will be helpful later for clarifying the conditions under which the use of communication policy is optimal.[11] In what follows, I will often identify private information directly with χ_i (rather than $x_{i\theta}$ and $x_{i\bar{a}}$), and similarly public, non-policymaker information with γ (rather than y and the prior).

3.2. Policy and the equilibrium use of information

An important consequence of communication and instrument policy within our framework is that both modify agents' use of private and public information, and hence the informativeness of the average action. Depending on whether such changes are socially desirable or not, this

[9] The link between the weight on private information and the informativeness of endogenous public signals has been studied elsewhere. See, for example, Vives (2010) or Amador and Weill (2010).

[10] For ease of exposition, I will from now on mainly refer to "the informativeness of the average action". By that, I always mean "the informativeness of *the signals* of the average action".

[11] Furthermore, notice that condensing ω into the summary public measure γ is less straightforward than one would perhaps expect. This is because ω covaries closer with z, and hence with m, than θ.

can provide an added reason for the use of either policy tool. Proposition 2 and Corollary 3 summarize these effects, while Appendix A contains the full set of comparative statics.

Proposition 2. *Consider the summary weight on private information κ.*

(i) Increases in the precision of policymaker disclosure $\tau_\omega \in \mathbb{R}_+$ either decrease or increase κ, $\frac{d\kappa}{d\tau_\omega} \lesseqgtr 0$. This depends on $\phi \lesseqgtr \tilde{\phi} \equiv -\eta_2^{-1} \frac{\tau_z}{\tau_\theta+\tau_y\kappa^2+(1-\alpha)(\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2)+\tau_z}$.

(ii) Increases in the use of the policymaker's instrument $\phi \in \mathbb{R}$ likewise either decrease or increase κ, $\frac{d\kappa}{d\phi} \lesseqgtr 0$. This depends on whether $\eta_1\eta_2 \lesseqgtr 0$.

(iii) All values of κ achievable with communication policy can also be obtained by instrument policy, but not conversely, $\kappa\,(0,\, \mathbb{R}_+) \subset \kappa\,(\mathbb{R},\, \tau_\omega \to 0)$.

Corollary 3. *Suppose $\eta_1 > 0$, $\phi \in \left[0, -\eta_2^{-1}\right)$, and $\tau_{x\bar{a}} \to 0$.*

Then, $\kappa > 0$, $\frac{d\kappa}{d\tau_\omega} \lesseqgtr 0$ if and only if $\phi \lesseqgtr \tilde{\phi}$, while $\frac{d\kappa}{d\phi} \lesseqgtr 0$ if and only if $\eta_2 \lesseqgtr 0$.

Proposition 2 focuses on the summary weight on private information κ. As we have seen, this coefficient also determines the informativeness of the public signal γ. As a result, it also determines the weight attached to γ, as well as to the policymaker's disclosure ω (the remaining signal in Corollary 2). In this sense, changes to the weight on private information κ summarizes the overall effects of policy on the equilibrium use of information.[12]

Proposition 2 follows from the total differentiation of the fixed-point condition (3.4). The mechanics of how increases in the precision of public information, or in the responsiveness of instrument policy, change the weight on private information are well-known and have been discussed *inter alia* by King (1982), Vives (1997), Amador and Weill (2010), Angeletos and Pavan (2009), and Kohlhas (2019). The above results summarize much of this work, but also extend it to consider the interaction between communication and instrument policy.

For example, consider the case of additional disclosure when instrument policy is mute ($\phi = 0$). This is, in effect, the case considered by Amador and Weill (2010). When the policymaker's disclosure becomes more precise, she directly decreases agents' uncertainty about the fundamental. However, this decrease in uncertainty also causes agents to place less weight on private information when updating their beliefs. The added use of the policymaker's information crowds-out how much agents have to rely on their own private signals. This, in turn, makes the aggregate action reflect less the combined, independent private information, the truly new information that agents could learn from one another, and as a result the information content of signals of the average action falls ($|\,\kappa\,|$ decreases).

The use of the policy instrument has similar effects to those of disclosure. For instance, consider the case in which $\eta_1 > 0$ and $\eta_2 < 0$ (e.g. King, 1982), but suppose, to start, the policymaker does not condition her policy instrument on her own information ($\phi = 0$). Increases in the conditioning of the policy instrument ϕ then initially stabilize agents' optimal choice under full information $\hat{a} = \eta_0 + \eta_1\,(\theta + \eta_2\phi z)$, and thereby also decreases agents' uncertainty about their optimal choice under imperfect information (Proposition 1). This, in turn, decreases how much

[12] Appendix A shows that $\kappa_1 = \frac{\tau_\theta+\tau_y\kappa^2}{\tau_{x\bar{a}}+\tau_y\kappa^2}\frac{\kappa}{1-\alpha\eta_1}$ while $\kappa_2 = \eta_1\,(1+\eta_2\phi) - \kappa_1 - \kappa$. Thus, more weight on private information κ either increases or decreases the weight on the summary public signal κ_1. This depends on whether the increase in the informativeness of the endogenous public signal outweighs the increased weight on private information. The weight on the policymaker's disclosure κ_2, by contrast, always decreases in κ.

agents update their expectations in response to private information, and thus once more causes the informativeness of the average action to fall. By contrast, if $\eta_2 > 0$ increases in instrument policy increase agents' ex-ante uncertainty, and thereby all else equal increase the extent to which agents' update their expectations with private information.

The perhaps more surprising result in Proposition 2 is that additional disclosure can also increase the weight on private information, because of its interaction with instrument policy. This occurs in Corollary 3 when $\eta_2 < 0$ and instrument policy is sufficiently forceful ($\phi > \hat{\phi} > 0$). As discussed in Kohlhas (2019), the reason for this is that the weight on private information κ is comprised of two components, both of which decrease with disclosure.

Consider agents' full information choice $\hat{a} = \eta_0 + \eta_1 (\theta + \eta_2 \phi z)$. Because the expectation of $\hat{a}$ is what matters for agents' equilibrium actions, the total weight on private information is comprised of two components: It reflects *both* the weight agents place on private information in their expectation of the fundamental θ, and the weight placed on private information in their expectation of the policymaker's information z. Additional disclosure decreases both.

Furthermore, when $\phi \in \left(0, -\eta_2^{-1}\right)$ while $\eta_1 > 0$ and $\eta_2 < 0$, the parameter restrictions ensure that the total weight on private information reflects the difference between these two weights, themselves weighted by the use of the policy instrument ϕ.[13] Combined this shows that if the policy instrument is used sufficiently forcefully (ϕ is sufficiently large), the total weight on private information, and hence the information content of the average action, can increase with policymaker disclosure. The decrease in the weight agents place on private information in their expectation of the fundamental is more than offset by the decline in the weight placed on private information in agents' expectation of the policymaker's information.

Finally, point (iii) of Proposition 2 shows that the exclusive use of instrument policy can span a greater set of weights on private (and hence public) information than communication policy. This gives rise to a potential equivalence between the two policies, usefully demonstrated by the full disclosure case.

Example 1. *The full disclosure case and* κ*:* With full disclosure $\tau_\omega \to \infty$, κ equals

$$\kappa = \eta_1 (1-\alpha) \frac{\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2}{\tau_\theta + \tau_y \kappa^2 + \tau_{x\theta} + \tau_{x\bar{a}}\kappa^2 + \tau_z} \tag{3.6}$$

(see Proposition 1). But this weight on private information can also be obtained under complete opacity with $\phi = \tilde{\phi}$, $\kappa\left(\tilde{\phi},\ \tau_\omega \to 0\right) = \kappa\left(\phi,\ \tau_\omega \to \infty\right)$. It follows that a coefficient of instrument policy equal to $\tilde{\phi}$ under complete opacity replicates the weight on private and endogenous public information achieved under full disclosure.

3.3. Socially optimal use of information

When the equilibrium use of information is also socially optimal, the best means to use the policymaker's information is simple. Because agents' optimally use all available information, the policymaker can do no better than to perfectly disclose her information, and to then set the policy instrument to its optimal, full disclosure value. However, the above environment contains

[13] Indeed, as the Proof of Corollary 2 shows, we can always write $\kappa = \eta_1 \left(w_\chi - \phi\eta_2 v_\chi\right)$, where w_χ and v_χ denote the weight on the summary private signal in agents' expectations of θ and z, respectively.

two sets of externalities, payoff and learning externalities, both of which skew the equilibrium use of information away from the societal best.

To speak meaningfully about any discrepancy between the equilibrium and socially optimal use of information, we nevertheless first need to establish an appropriate welfare benchmark. Such a benchmark is provided by the *team solution*: The social planner problem in which agents internalize collective welfare but must still rely on their own information sets when making their own choices (Radner, 1979 and Vives, 1988).

Definition 2. An *efficient ("team solution")* allocation is a linear strategy $a : \mathbb{R}^3 \to \mathbb{R}$, which maximizes ex-ante social welfare

$$\mathbb{E}[\mathcal{W}] = \mathbb{E}\int \mathcal{U}(a(\chi_i, \gamma, \omega), \bar{a}(\theta, \gamma, \omega), \sigma_a(\theta, \gamma, \omega), \theta, m)\, dF(a). \tag{3.7}$$

Because the team solution restricts agents to rely on their own information, it identifies the best that the society could do by adjusting how agents' use their information, without communicating with each other. Comparing equilibrium outcomes to this welfare benchmark thus isolates the welfare losses that arise solely from the equilibrium use of information.

To characterize this efficient allocation, it is helpful to first decompose utilitarian welfare $\mathcal{W}$ in (3.7) into its constituent components. Although our ultimate focus is on ex-ante welfare $\mathbb{E}\mathcal{W}$, such a decomposition is useful because it helps reveal the forces that drive welfare losses.

Lemma 1. *Let $\hat{a}^\star(\theta, m) = \eta_0^\star + \eta_1^\star\left(\theta + \eta_2^\star m\right)$ denote agents' optimal actions under full information, given ϕ.*[14] *Then, utilitarian welfare $\mathcal{W}$ can be decomposed into*

$$\mathcal{W}(\bar{a}, \sigma_a, \theta, m) = \mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right) + \frac{1}{2}\mathcal{W}_{\sigma\sigma}\sigma_a^2 + \frac{1}{2}\mathcal{W}_{\bar{a}\bar{a}}\left(\bar{a} - \hat{a}^\star\right)^2, \tag{3.8}$$

where $\mathcal{W}_{\bar{a}\bar{a}} = \mathcal{U}_{aa} + \mathcal{U}_{\bar{a}\bar{a}} + 2\mathcal{U}_{a\bar{a}}$ and $\mathcal{W}_{\sigma\sigma} = \mathcal{U}_{\sigma\sigma} + \mathcal{U}_{aa}$.

Lemma 1 follows Angeletos and Pavan (2007) and decomposes utilitarian welfare into three components. First, a component that reflects welfare in the efficient full information case, in which $a_i = \hat{a}^\star$ for all i. Second, a component that depends upon the dispersion of individual actions σ_a^2. And last, a component that depends upon the distance between the average action in the economy and its optimal full information counterpart $(\bar{a} - a^\star)^2$. Consistent with this decomposition, $\mathcal{W}_{\sigma\sigma}$ and $\mathcal{W}_{\bar{a}\bar{a}}$ reflect society's aversion to dispersion and volatility, respectively. Because of payoff externalities, these differ from $\mathcal{U}_{\sigma\sigma}$ and $\mathcal{U}_{aa}$.

However, Lemma 1 also differentiates itself from previous work along an important dimension. The full information baseline $\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right)$, and the associated efficient action $\hat{a}^\star$, are only *constrained optimal*. That is, only optimal conditional on m, and hence the use of the policy instrument ϕ. Thus, differences between $\mathcal{W}(\bar{a}, \sigma_a, \theta, m)$ and $\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right)$ only reflect the welfare losses that arise from inefficient use of information *given the policy instrument*. By contrast, the welfare losses that arise from a suboptimal use of the policy instrument under the efficient, full information choice are reflected in $\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right)$ being lower than its maximum value. With the help of Lemma 1, we can characterize the team solution.

[14] The optimal full information coefficients are: $\eta_0^\star = -\frac{\mathcal{W}_{\bar{a}}(0,0,0,0)}{\mathcal{W}_{\bar{a}\bar{a}}}$, $\eta_1^\star = -\frac{\mathcal{W}_{\bar{a}\theta}}{\mathcal{W}_{\bar{a}\bar{a}}}$, and $\eta_2^\star = \frac{\mathcal{W}_{\bar{a}m}}{\mathcal{W}_{\bar{a}\theta}}\eta_1^\star$.

Proposition 3. *Let $a^\star(\chi, \gamma, \omega)$ denote the action that results from efficient use of information.*

(i) Then, there exist coefficients $(c, c_1, c_2) \in \mathbb{R}^3$ such that $a^\star(\chi, \gamma, \omega)$ satisfies

$$a^\star(\chi, \gamma, \omega) = \eta_0^\star + \eta_1^\star \left(\eta_1^{\star-1} c\chi + c_1\gamma + c_2\omega \right), \tag{3.9}$$

where c_1 and c_2 depend only on c and the parameters of the model.

(ii) Given the coefficients (c, c_1, c_2), there exists a unique $\alpha^\star \in \mathbb{R}$ such that

$$a^\star(\chi, \gamma, \omega) = \mathbb{E}\left[\left(1 - \alpha^\star\right) \hat{a}^\star(\theta, m) + \alpha^\star \bar{a}^\star(\theta, \gamma, \omega) \mid \chi, \gamma, \omega \right], \tag{3.10}$$

where $\bar{a}^\star(\theta, \gamma, \omega) = \int a^\star(\chi, \gamma, \omega)\, dF(a^\star)$.

As with the equilibrium allocation, the presence of endogenous public and private information can lead to multiple solutions to the first-order conditions that characterize the team solution. The efficient outcome maximizes welfare within this set of solutions (Appendix A).

The first part of the proposition characterizes the efficient use of information in terms of the different weights on private and public information. Payoff externalities, learning externalities, and the fact that the equilibrium action under full information $\hat{a}$ may not coincide with the efficient action $\hat{a}^\star$ all contribute to the efficient weights being different from their equilibrium counterparts. For example, consider the efficient weight on private information.

Corollary 4. *The efficient weight on the summary private signal χ is given by*

$$c = \eta_1^\star \left(1 - \alpha^\dagger\right) \frac{\left(\tau_{x\theta} + \tau_{x\bar{a}} c^2\right)\left(1 + \eta_2^\star \phi \frac{\tau_z}{\tau_\omega + \tau_z}\right)}{\left(1 - \alpha^\dagger\right)\left(\tau_{x\theta} + \tau_{x\bar{a}} c^2\right) + \left(\tau_\theta + \tau_y c^2 + \frac{\tau_\omega \tau_z}{\tau_\omega + \tau_z}\right)\Delta_1 - \Delta_0}, \tag{3.11}$$

where $\alpha^\dagger \equiv 1 - \frac{\mathcal{W}_{\bar{a}\bar{a}}}{\mathcal{W}_{\sigma\sigma}}$, $\Delta_1 \equiv \frac{\tau_{x\theta}}{\tau_{x\theta} + \tau_{x\bar{a}} c^2} \in (0, 1)$, and

$$\Delta_0 \equiv \left(1 - \alpha^\dagger\right)\left(\tau_{x\theta} + \tau_{x\bar{a}} c^2\right) \frac{\left(1 - c + \eta_2^\star \phi \frac{\tau_z}{\tau_\omega + \tau_z}\right)^2}{\tau_\theta + \tau_y c^2 + \frac{\tau_\omega \tau_z}{\tau_\omega + \tau_z}} \tau_y \left(\tau_\theta + \tau_y c^2\right) > 0.$$

As in the equilibrium case, the efficient weight on private information determines the optimal informativeness of the public signal γ, and hence also the efficient weights that agents attach to public information (c_1 and c_2). Indeed, when the equilibrium weight κ coincides with the efficient weight c (and the full information action is efficient), the equilibrium use of information equals that in the team solution case ($\kappa = c$, $\kappa_1 = c_1$, $\kappa_2 = c_2$).

The effect of payoff externalities on the efficient weight on private information c is captured by $\alpha^\dagger$ being different from α in (3.11). To see this, suppose $\tau_{x\bar{a}} = \tau_y \to 0$ and $\eta \equiv \begin{pmatrix} \eta_1 & \eta_2 \end{pmatrix} = \begin{pmatrix} \eta_1^\star & \eta_2^\star \end{pmatrix} \equiv \eta^\star$. Then, neither the presence of endogenous information, nor that the full information outcome is inefficient, matter for any discrepancy between the equilibrium and socially optimal use of information. In fact, the only difference between the efficient weight c and its equilibrium counterpart κ in (3.4) then arises because the equilibrium amount of payoff coordination α is different from $\alpha^\dagger$.

The effect of learning externalities are, by contrast, captured by $\Delta_0 > 0$ and $\Delta_1 < 1$. Suppose that $\alpha = \alpha^\dagger = 0$, while maintaining that $\eta = \eta^\star$. Then, the efficient weight c always exceeds its equilibrium value κ in absolute value. (This is under the additional assumption that Δ_0 is not large enough to change the sign of c.)[15] The first term $\Delta_0 > 0$ measures the additional weight on private information required to internalize the learning externality that arises due to the *endogenous public signal* y. The second term $\Delta_1 < 1$, in contrast, captures the added weight necessary to internalize the learning externality that arises because of the *endogenous private signal* $x_{\bar{a}}$. Lastly, the effects of any inefficiencies in the equilibrium action under full information are captured by $\eta \neq \eta^\star$ in (3.4) and (3.11).

Let me now briefly return to Proposition 3, and especially its second part. This part characterizes the team solution action as the fixed-point of a best response function. This best-response function closely resembles our characterization of the equilibrium action in Proposition 1. The central difference being that the *socially optimal amount of coordination* $\alpha^\star$ (Angeletos and Pavan, 2009) generically differs from its equilibrium counterpart α.[16]

Consistent with learning externalities increasing the socially optimal weight on private information, I find that $\alpha^\star < \alpha^\dagger$, where $\alpha^\dagger$ equals the value of $\alpha^\star$ when there is only exogenous information. Internalizing learning externalities is similar to agents' perceiving a lower degree of strategic complementarity between their actions than what they would perceive only in the presence of exogenous information. In turn, this lower perceived strategic complementarity causes a larger optimal weight on private information ($| c |>| \kappa |$, since public information is a relatively better predictor of others action than private information.[17]

I conclude this section with a brief comment about the conditional use of the policy instrument, which I also highlighted in Section 2. Because the policy instrument enters directly into agents' payoff function $\mathcal{U}$, the conditional use of the policy instrument has a dual role in my analysis. On the one hand, it can be used to adjust agents' equilibrium use of information, to equate it with its socially optimal counterpart in Proposition 3. On the other hand, even with an efficient use of information, the conditional use of the policy instrument can improve welfare. Indeed, non-zero values of $\mathcal{U}_{m\theta}$, for example, offer the potential for instrument policy to improve welfare in such cases. The tension between these distinct roles is crucial for the optimal use of the policy instrument.

4. A classification for optimal policy

In this section, I turn to a classification of different economies based on what they entail for the best means to use a policymaker's information. To do so, I first study the benchmark case, in which an economy's use of information is efficient. I then provide a set of conditions on the payoff function for the inefficient case under which the sole use of either instrument or communication policy is optimal. Lastly, I end with some general lessons for policy.

[15] All of my results extend to the case in which Δ_0 is so large as to change the sign of c. But as this case would substantially complicate the exposition below, I choose to invoke this assumption from now on.

[16] Notice that, as the proof of Proposition 3 in Appendix A shows, the value $\alpha^\star$ is independent of ϕ.

[17] Although the efficient use of information responds more to private information than in the absence of endogenous signals, it is not necessarily the case that the efficient use of information is overall more sensitive to private information than in equilibrium. This is because payoff externalities also skew the optimal use of information (Corollary 4). As a result, and unlike the findings in, for example, Amador and Weill (2010), we cannot conclude that our economy always features too little social learning.

4.1. Efficient economies

Consider economies in which the amount of strategic complementarity that agents perceive and the full information solution are efficient ($\alpha = \alpha^\star$ and $\eta = \eta^\star$). Proposition 1 and 3 show that such economies are characterized by an efficient use of information. The policymaker can for these economies do no better than to fully disclose her information, and to then set the policy instrument to the value that maximizes welfare under full information.

Proposition 4. *Suppose $\alpha = \alpha^\star$ and $\eta = \eta^\star$. Then:*
(i) Full disclosure $\tau_\omega^\star \to \infty$ is optimal.
(ii) ϕ should be set to its efficient, full information value $\phi^\star = \hat{\phi}^\star$.

The optimality of full disclosure in Proposition 4 follows from a standard Blackwell-like argument (Blackwell, 1953; Perez-Richet, 2017), because of the efficient use of information.

A more surprising feature of the proposition is that an efficient economy separates the optimal use of the policy instrument from the information friction. The policy instrument should be set to its optimal value in the efficient, full information case ($\phi^\star = \hat{\phi}^\star$). The intuition behind this result is simple: Under full disclosure, changes in the responses of the policy instrument towards the policymaker's information do not alter agents' uncertainty. Agent's fully know the realization of the prospective policy instrument under full disclosure. As a result, changes in its use do not alter agents' weight on private or public information (Proposition 1 and Corollary 1).[18] It is this separation of instrument policy from the equilibrium use of information that causes its optimal value to equal that under full information.

In effect, Proposition 4 extends the results of Svensson and Woodford (2004) to economies in which agents have dispersed information and observe endogenous signals. It shows that the optimal value of instrument policy equals that in the efficient, full information case when the information the policy instrument is conditioned on is included in agents' information sets. Only in such cases do changes in the responsiveness of the policy instrument not alter agents' weights on private and public information, and hence skew the optimal use of the policy instrument away from its full information value.

4.2. Inefficient economies: the general case

When either $\alpha \neq \alpha^\star$ or $\eta \neq \eta^\star$, the economy's use of information is inefficient. Agents either perceive a wrong amount of strategic complementarity or the economy features a full information solution that is not efficient. Both contribute to the equilibrium weight on private and public information being different from their efficient counterparts, and hence create a desire to use policy to adjust agents' use of information. Depending on the precise details of the economy in question, this in turn makes all combinations of instrument and communication policy possible as optimal outcomes. I show this result by construction in Appendix B, while Appendix D explores the policymaker's problem in this general case.

Proposition 5. *Suppose $\alpha \neq \alpha^\star$ or $\eta \neq \eta^\star$. Then, all combinations of communication policy $\tau_\omega \in \mathbb{R}_+$ and instrument policy $\phi \in \mathbb{R}$ can arise as optimal outcomes.*

[18] Notice that L'Hôpital's rule implies that κ in (3.4) becomes independent of ϕ when $\tau_\omega \to \infty$.

At its heart, the reason that any combination of policy options can be optimal is that each means to use a policymaker's information has its own distinct advantages. In the rest of this section, I turn to a useful classification of economies that provides conditions under which the exclusive use of either instrument or communication policy is optimal. This will have the benefit of making the advantages of each means to use a policymaker's information clear. I then return to the general case at the end of this section.

4.3. Inefficient economies: the use of a policy instrument

I start with an important class of inefficient economies for which the exclusive use of instrument policy is optimal. This class features the property that the policy instrument can *perfectly substitute* for agents' use of the policymaker's information.

Lemma 2. *Consider economies in which $\eta_1^\star = \delta \frac{\mathcal{U}_{m\theta}}{\mathcal{U}_{mm}}$ and $\eta_2 = \eta_2^\star = -\frac{\mathcal{U}_{mm}}{\mathcal{U}_{am}+\mathcal{U}_{\bar{a}m}} = \delta$ for $\delta \in \mathbb{R}$.*[19] *Then, $\mathcal{U}(a, \bar{a}, \sigma_a, \theta, m) = U(a - \delta m, \bar{a} - \delta m, \sigma_a, \theta)$ for some quadratic polynomial U.*

Lemma 2 formalizes the conditions on the payoff function under which agents' payoffs depend only upon the difference between their actions and the policy instrument, rather than separately on both. Since individual actions are linear in the policymaker's information (Corollary 2), this ensures that the policy instrument can *perfectly substitute* for agents' use of the policymaker's information. At first pass, the conditions in Lemma 2 may seem to lie on a knife-edge. However, as I argue in Section 6, they indeed cover several workhorse models in the applied theory literature, especially in macroeconomics.

To better appreciate the conditions, suppose the policymaker could set the value of the policy instrument m under full information. She would set m to solve the first-order condition:

$$\begin{aligned}\mathcal{W}_m &= \partial \mathcal{U}(\bar{a}, \bar{a}, 0, \theta, m)/\partial m \\ &= (\mathcal{U}_{am} + \mathcal{U}_{\bar{a}m})\,\bar{a} + \mathcal{U}_{\theta m}\theta + \mathcal{U}_{mm} m = 0, \end{aligned} \tag{4.1}$$

where I have used the expression for utilitarian welfare in (3.8). Now, notice that the conditions in Lemma 2 ensure that the efficient full information action $\hat{a}^\star$, which is common for all i, also solves (4.1). Because both the efficient full information action and the optimal full information policy instrument solve the same first-order condition, the policy instrument must for these economies enter linearly with individual actions in agents' payoffs.

The implications of the conditions in Lemma 2 are stark. Despite the fact that additional disclosure always decreases agents' uncertainty, and helps them better coordinate their actions, complete opacity is uniquely optimal if the economy's use of information is inefficient.

Proposition 6. *Suppose $\alpha \neq \alpha^\star$ or $\eta \neq \eta^\star$, and suppose further that $\eta_1^\star = \delta \frac{\mathcal{U}_{m\theta}}{\mathcal{U}_{mm}}$ and $\eta_2 = \eta_2^\star = -\frac{\mathcal{U}_{mm}}{\mathcal{U}_{am}+\mathcal{U}_{\bar{a}m}} = \delta$ for $\delta \in \mathbb{R}$. Then:*

(i) Complete opacity $\tau_\omega^\star \to 0$ is uniquely optimal.

(ii) $\phi^\star$ optimally solves $\delta\phi^\star = \eta_1 \left(1 - \frac{\kappa(\phi=\phi^\star, \tau_\omega=\tau_\omega^\star)}{\kappa(\phi=0, \tau_\omega \to 0)}\right) \neq \tilde{\phi}$, where $\kappa^\star \neq \kappa(\phi, \tau_\omega \to \infty)$.

[19] I here abstract from an uninteresting restriction on constant terms (see Appendix B).

Proposition 6 comprises one of the main results of this paper. It shows that economies in which the policy instrument can perfectly substitute for agents' use of information, the best means to exploit a policymaker's information is instrument policy. This is true whenever the equilibrium use information is inefficient. The optimal value of the policy instrument should, in this case, simply reflect the ratio between the optimal weight on private information $\kappa^\star$ and its "no-policy" counterpart κ ($\phi = 0$, $\tau_\omega \to 0$). Disclosures, by contrast, should be mute.

The sharpness of Proposition 6 follows from two inherent advantages that instrument policy has over communication policy in adjusting agents' use of information. Let me turn to how these advantages combine in the above class of models to lead to Proposition 6.

4.3.1. The advantages of instrument policy

When the equilibrium use of information is inefficient ($\alpha \neq \alpha^\star$ or $\eta \neq \eta^\star$), the full disclosure outcome is suboptimal. At full disclosure, the benefit of a modified use of private and public information, and hence also a different level of informativeness for the endogenous signals, outweighs the (second-order) loss from imperfect use of the policymaker's signal. But why is the policy instrument above the better means to adjust agents' use of information?

Below, I show that instrument policy has two advantages over communication policy. The first is that it is able to modify the weight on private and public information without adding noise to the information structure. This is unlike the alternative of partial disclosure. The second is that it can create even more (or even less) coordination between agents' actions than that achieved with full disclosure (or completely opacity). These advantages combine in the above cases, where the policy instrument can perfectly substitute for agents' use of the policymaker's information, to make the sole use of instrument policy optimal.

Advantage I of Instrument Policy Lemma 2 shows that agents' payoffs for this class of economies is determined by the difference between agents' actions and the policy instrument, $a_i - \delta m$. When this expression is equated for all realizations of shocks across different policies, so too is the difference between the average action and the policy instrument $\bar{a} - \delta m$, and hence social welfare in economy.

But consider now the difference between an agent's action and the policy instrument $a_i - \delta m$ in two specific cases. First, in the case in which the policymaker fully discloses her information, but does not use the policy instrument ($\phi = 0$, $\tau_\omega \to \infty$),

$$\begin{aligned}\Delta^a \equiv a_i - \delta m^a &= \mathbb{E}\left[(1-\alpha)\,\hat{a}(\theta, 0) + \alpha\bar{a} \mid \chi_i, \gamma, z\right] \\ &= \mathbb{E}\left[(1-\alpha)\,(\eta_0 + \eta_1\theta) + \alpha\bar{a} \mid \chi_i, \gamma\right] + \beta_{az}\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right),\end{aligned} \tag{4.2}$$

where I have used that $\mathbb{E}\left[q \mid \chi_i, \gamma, z\right] = \mathbb{E}\left[q \mid \chi_i, \gamma\right] + \beta_{qz}\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right)$ for any Gaussian variable q.[20] And second, in the case in which the policymaker sets her policy instrument, but does not disclose any information ($\phi \in \mathbb{R}$, $\tau_\omega \to 0$),

$$\begin{aligned}\Delta^b \equiv a_i - \delta m^b &= \mathbb{E}\left[(1-\alpha)\,\hat{a}(\theta, m) + \alpha\bar{a} \mid \chi_i, \gamma\right] - \delta m^b \\ &= \mathbb{E}\left[(1-\alpha)\,(\eta_0 + \eta_1\theta) + \alpha\bar{a} \mid \chi_i, \gamma\right] - \delta\phi^b\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right),\end{aligned} \tag{4.3}$$

[20] Specifically, q is here equal to $q = (1-\alpha)\,\hat{a}(\theta, 0) + \alpha\bar{a}$.

where I have used that $\eta_2 = \delta$ for this class of economies (Lemma 2). Comparing (4.2) with (4.3) shows that $\phi^b = -\beta_{az}/\delta$ equates Δ^a with Δ^b, and thus welfare in the two cases.[21]

Next, consider a small decrease in policymaker disclosure τ_ω in case (a), and a change in ϕ in case (b) that pushes weight onto private information. Such a change would, for example, be beneficial in cases in which learning externalities cause too little weight on private information. This results in a difference between agents' actions and the policy instrument, the term that matters for welfare, equal to, in the first case,

$$\Delta^a = \mathbb{E}\left[(1-\alpha)(\eta_0 + \eta_1\theta) + \alpha\bar{a} \mid \chi_i, \omega\right] + \beta_{a\omega}\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right) + \beta_{a\omega}\epsilon_\omega, \tag{4.4}$$

where $\beta_{a\omega} \neq \beta_{az}$, while the second-case remains unchanged except with a different ϕ^b,

$$\Delta^b = \mathbb{E}\left[(1-\alpha)(\eta_0 + \eta_1\theta) + \alpha\bar{a} \mid \chi_i, \gamma\right] - \delta\phi^b\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right). \tag{4.5}$$

Equations (4.4) and (4.5) are central to Proposition 6. They show that if we set $\phi^b = -\beta_{a\omega}/\delta$, then the *only* difference between the two policies is the noise term $\beta_{a\omega}\epsilon_\omega$ from the policymaker's partial disclosure ($\omega = z + \epsilon_\omega$).[22] Communication and instrument policy are equivalent only up to a noise term. Squaring this term and taking ex-ante expectations illustrates the additional welfare cost that communication policy entails – added noise.[23]

When the policymaker changes the conditioning of her policy instrument to increase agents' ex-ante uncertainty, she causes them to update their expectations more with their own private information. By contrast, when the policymaker chooses to use communication policy, she can only induce agents to attach more weight on private information by releasing a nosier signal of her own information. That is the only mechanism by which communication policy can increase the weight on private information. However, this additional noise comes at an added welfare cost; the cost of making the policymaker's signal worse. This warping of a signal that agents use to base their decisions on further distorts their actions, a welfare cost which the conditional use of the policy instrument completely avoids.

We can summarize this discussion in the following Lemma, which I also use to provide the proof of Proposition 6. The Lemma also shows that the identified distinction between the two means to use a policymaker's information naturally extends to any policy that also mixes the conditional use of a policy instrument with partial disclosure.

[21] Notice that the weight attached to $\mathbb{E}\left[\theta \mid \chi_i, \gamma\right]$, $\mathbb{E}\left[z \mid \chi_i, \gamma\right]$, and $\mathbb{E}\left[\bar{a}(\theta,\gamma) \mid \chi_i, \gamma\right]$ in (4.3) when $\phi^b = -\beta_{az}/\delta$ is equal to the full disclosure values in (4.2). Thus, the weight on private information in agents' actions κ is equal to its full disclosure value in both cases, and we conclude that the informativeness of endogenous signals is also the same across the two cases. Alternatively, inserting the equilibrium expression for the average action from Corollary 2 into the first line of (4.2), and using the expressions for the equilibrium coefficients, shows that $\beta_{az} = \alpha\kappa_2 + w_z\left[(1-\alpha)\eta_1 + \alpha\kappa\right] = \tilde{\phi}$, where $w_z = \frac{\tau_z}{\tau_\theta + \tau_{x\theta} + \tau_{x\bar{a}}\kappa^2 + \tau_y\kappa^2 + \tau_z}$. Example 1 then shows that $\kappa(\phi = 0, \tau_\omega \to \infty) = \kappa(\phi = \tilde{\phi}, \tau_\omega \to 0)$.

[22] The weight accorded to private information κ, and hence the informativeness of the endogenous signals, is once more equal in the two cases. This again follows from the weight attached to $\mathbb{E}\left[\theta \mid \chi_i, \gamma\right]$, $\mathbb{E}\left[z \mid \chi_i, \gamma\right]$, and $\mathbb{E}\left[\bar{a}(\theta,\gamma) \mid \chi_i, \gamma\right]$ being equal in (4.4) and (4.5) when $\phi^b = -\beta_{a\omega}/\delta$.

[23] The central decomposition used in (4.2) and (4.4) extends beyond the linear-normal paradigm. For example, Ericson (1969), DeGroot (2005), and Vives (2010) illustrate how the linearity of conditional expectations, and hence Proposition 6 for the class of economies covered by Lemma 2, extends beyond the linear-normal case. Indeed, the decomposition holds for many of the most commonly used distributions when combined with natural priors. Furthermore, the decomposition also extends, for any information structure, to the important case where we restrict agents to only construct linear-best predictors (Brockwell et al., 1991).

Lemma 3. *Suppose $\eta_1^\star = \delta \frac{\mathcal{U}_{m\theta}}{\mathcal{U}_{mm}}$ and $\eta_2 = \eta_2^\star = -\frac{\mathcal{U}_{mm}}{\mathcal{U}_{am}+\mathcal{U}_{\bar{a}m}} = \delta$ for $\delta \in \mathbb{R}$, and consider any partial disclosure (p) policy $\tau_\omega^p \in \mathbb{R}_+$ with instrument policy $\phi^p \in \mathbb{R}$.*

Then, there exists a complete opacity policy (o), where $\tau_\omega^o \to 0$ and $\phi^o \neq \phi^p$, that has the same weight on policymaker information in social welfare $\mathbb{E}\mathcal{W}$, but with a welfare benefit proportional to the variance of the noise in the policymaker's partial disclosure τ_ω^{-1}.

Proposition 6 and the related Lemma 3, in essence, show that the exclusive use of a policy instrument can be optimal because it allows the policymaker to *herself* modify the use of her own information. With communication policy, by contrast, the policymaker has to modify how much *other agents* use her information, and she can only do so by warping the signal that she sends of her own information at an added welfare cost.

Advantage II of Instrument Policy The second advantage that instrument policy has is visible from Proposition 2. Point (iii) shows that the equilibrium weight on private information spanned by instrument policy exceeds that of communication policy, $\kappa\,(\phi = 0, \tau_\omega \in \mathbb{R}_+) \subset \kappa\,(\phi \in \mathbb{R}, \tau_\omega \to 0)$. Put differently, instrument policy can create more (or less) coordination between agents' actions than that achieved with full disclosure (or completely opacity). Clearly, this second advantage is of importance only when the efficient weight on private information is below (or above) the equilibrium weight attainable under full disclosure (or complete opacity). But, for these cases, the broader scope of weights on private and public information afforded by instrument policy provides an argument for its use.

The intuition behind this second advantage can be seen from Proposition 1. Notice that increases in ϕ can arbitrarily increase the loading on the policymaker's information in the full information action $\hat{a} = \eta_1\,(\theta + \eta_2 \phi z)$. Because agents' actions reflect their expectations of this full information choice, instrument policy can arbitrarily increase agents' ex-ante uncertainty. This, in turn, causes the policymaker to be able to induce a broader scope of weights on private and public information than possible with communication policy.

Perhaps surprisingly, full disclosure combined with the conditional use of instrument policy does not achieve the same benefit. As already noted, under full disclosure, the policymaker cannot modify the equilibrium use of information with instrument policy. The policy instrument is fully known under full disclosure. Only under partial disclosure or complete opacity can the policymaker alter the equilibrium use of information. Thus, only under complete opacity can the policymaker use instrument policy to alter the use of information beyond that achievable with full disclosure and also avoid the added noise from communication policy. This shows how the second advantage interacts and builds on-top of the first one.

Summary The class of economies covered by Lemma 2 provide an attractive illustration of the advantages that instrument policy has in adjusting agents' use of information. Because the policy instrument can perfectly substitute for agents' use of the policymaker's information, instrument policy does not come at the cost often attributed to its use. Namely, that the use of a policy instrument is "cruder" than that of communication policy. By "crudeness", I here mean that the policy instrument cannot move the economy fully in response to the policymaker's information, and hence must count on the private sector to also respond efficiently to her information. To address such cases, I now turn to a class of economies in which the policy instrument's capacity to substitute for agents' use of information is severely limited.

4.4. Inefficient economies: the use of disclosures

The previous subsection discussed a class of economies for which the exclusive use of instrument policy is optimal. Conversely, a different set of economies exists for which the sole use of communication policy is desirable to adjust agents' use of information.

Proposition 7. *Consider economies where $\eta_2 = \eta_2^\star = \mathcal{U}_{m\theta} = 0$ but $\mathcal{U}_{mm} < 0$. Then, the policy instrument is optimally unused $\phi^\star = 0$. Further, suppose $\eta = \eta^\star$ but $\alpha \neq \alpha^\star$. Then, if $\alpha^\star \geq \alpha \geq 0$, full disclosure $\tau_\omega^\star \to \infty$ is uniquely optimal.*

Intuitively, the economies covered by Proposition 7 are those in which the policy instrument is of little relevance. The policy instrument does not affect individual actions, either in equilibrium or in the efficient solution ($\eta_2 = \eta_2^\star = 0$). Hence, it does not affect agents' use of information. The policy instrument also does not interact with the fundamental ($\mathcal{U}_{m\theta} = 0$), and thus has no effects on the economy except the potential to create undesired fluctuations ($\mathcal{U}_{mm} < 0$). As a result, the prescriptions that guide optimal policy are simple: The policy instrument should optimally be unused ($\phi = 0$), while a sufficient condition for full disclosure follows from well-known results by Angeletos and Pavan (2007), extended to the case with endogenous private and public information ($\alpha^\star \geq \alpha \geq 0$).

The conditions listed in Proposition 7 are stronger than those necessary for communication policy being the only tool used to adjust any discrepancy between the equilibrium and socially optimal use of information. Suppose $\eta_2 = \eta_2^\star = 0$, so that the policy instrument affects neither the equilibrium nor the socially optimal use of information. Then, the only role for the policy instrument is that which it has under full information.

Corollary 5. *Consider economies in which $\eta_2 = \eta_2^\star = 0$. Then, the policy instrument should be set to its optimal value in the efficient, full information case:*

$$\phi^\star = \hat{\phi}^\star = -\frac{\tau_z}{\tau_\theta + \tau_z}\frac{\mathcal{U}_{\theta m}}{\mathcal{U}_{mm}} \neq 0.$$

Next, consider those economies in which $\eta_2 = 0$ but $\eta_2^\star \neq 0$. In these economies, the policy instrument cannot adjust agents' equilibrium use of information. Instead communication policy must at the same time provide information to agents about the fundamental, respond to any discrepancy between the equilibrium and socially optimal use of information, and account for any interactions between the average action in the economy and the policy instrument ($\mathcal{U}_{\bar{a}m} \neq 0$), The latter is the reason why agents' efficient, full information response to the policy instrument here differs from zero ($\eta_2^\star \neq 0$).

The policy instrument in the class economies studied in this subsection has exhibited an extreme form for "crudeness". Consider the last example in which $\eta_2 = 0$ but $\eta_2^\star \neq 0$. In such economies, individual actions are unaffected by the policy instrument. The policy instrument only affects payoffs through its interactions with the average action in the economy. In this sense the policy instrument is "crude" in adjusting the economy's response to the policymaker's information, at least when compared to communication policy. Importantly, this crudeness shows one of the great virtues of communication policy. Because the policymaker's disclosure is publicly observable, communication policy can *always* adjust agents' individual responses to the policymaker's information. I return to this topic in Section 6.

4.5. Inefficient economies: the general case revisited

The analysis in this section has provided several lessons about the relative advantages of communication and instrument policy in environments with an inefficient use of information. As such, it has cast light on the forces that lead to all combinations of the two policy tools potentially being optimal in Proposition 5:

- On the one hand, the conditional use of a policy instrument affords the policymaker a better means of modifying agents' equilibrium use of information. This is because it avoids warping individual actions with additional noise, unlike the alternative of partial disclosure. Thus, in environments with an inefficient use of information and in which the policy instrument can substitute for individual agents' actions, the conditional use of a policy instrument tends to, all else equal, be preferable.
- On the other hand, however, in environments in which the policymaker needs to count on the private sector to respond efficiently to the policymaker's own information, the use of communication policy is advantageous. The central benefit that communication policy provides is that "gets into all the cracks" left over by the conditional use of instrument policy. Thus, it is all else equal a superior means to exploit a policymaker's information when the policy instrument cannot easily substitute for individual actions.

In this section, I have formalized conditions under which each of these advantages dominates the other. I have done so in terms of agents' full information responses (η, $\eta^{\star}$). When (i) agents' full information responses to the policy instrument are efficient ($\eta_2 = \eta_2^{\star}$), and (ii) agents' efficient, full information responses are similar to those of the policy instrument ($\eta_1^{\star} = \delta \frac{\mathcal{U}_{m\theta}}{\mathcal{U}_{mm}}$, $\eta_2^{\star} = -\frac{\mathcal{U}_{mm}}{\mathcal{U}_{am}+\mathcal{U}_{\bar{a}m}}$), instrument policy dominates. By contrast, economies in which the policy instrument is severely constrained in its capacity to alter agents' equilibrium use of information (e.g. $\eta_2 = 0$) favor communication policy. Appendix D uses the dual approach to explore the set of necessary and sufficient conditions for the sole use of either policy tool.

Where any specific economy falls in spectrum between the exclusive use of instrument or communication policy depends on a complete description of the economy. One needs to condense the economy's preferences, technologies, and market interactions into the reduced-form payoff function $\mathcal{U}$. Once this objective is achieved, however, one can interpret the conditions in Proposition 6 and 7 in terms of the economy's primitives. Section 6 undertakes this task and provides several examples of economies that fall within the classes of environments studied in this section. Yet, before turning to these applications, the next section shows that increasing the set of conditioning variables that the policy instrument responds to do not affect the main insights from this section.

5. Extended instrument rules

An important determinant of my results so far has been the existence of any discrepancy between the equilibrium and efficient use of information. When the policy instrument is conditioned only on the policymaker's information about the fundamental, instrument policy is, nevertheless, limited in its capacity to modify the equilibrium use of private and public information. More extended instrument rules, by allowing the policy instrument to also respond to estimates of the average action, can push the economy closer to the efficient benchmark (e.g. Angeletos and Pavan, 2009). In this section, I analyze the implications of such extended instrument rules for my results, and show that the main insights from the previous section continue to hold.

5.1. Conditioning on the fundamental and the average action

Consider an economy with an inefficient use of information ($\alpha \neq \alpha^{\star}$ or $\eta \neq \eta^{\star}$), and suppose that we change the conditioning of the policy instrument in (2.2). Specifically, suppose that the policy instrument responds directly to the fundamental and the average action:

$$m = \psi_{\theta}\theta + \psi_{\bar{a}}\bar{a}, \tag{5.1}$$

where $\psi_{\theta} \in \mathbb{R}$ and $\psi_{\bar{a}} \in \mathbb{R}$. I later turn to the case in which the policymaker responds to her own noisy information about the fundamental and the average action, and discuss why (5.1) provides an instructive example. With (5.1), the policymaker can always equate the equilibrium use of information to the socially optimal one by setting ψ_{θ} and $\psi_{\bar{a}}$ appropriately. Lemma 4 extends the results of Angeletos and Pavan (2009) to the current environment.

Lemma 4. *Consider agent i's equilibrium action a_i under (5.1). Then, there exists values of $\{\psi_{\theta}, \psi_{\bar{a}}\} \in \mathbb{R}^2$ in (5.1) such that $a = a^{\star}$ for all realizations of (χ, γ, ω).*

The result in Lemma 4 is important, because it shows that an extended instrument rule exists that can always render the equilibrium use of information efficient. As such, it shows that an extended instrument rule exists for which full disclosure of the policymaker's signal z is always optimal. The "dual-conditioning" on both the fundamental and the average action is key to this result.[24] To see why, consider agents' best-response function under the new instrument rule. Similar steps to those that lead to Proposition 1 show that

$$a_i = \mathbb{E}\left[(1 - \alpha(\psi_{\bar{a}}))\,\hat{a} + \alpha(\psi_{\bar{a}})\,\bar{a} \mid \chi_i, \gamma, \omega\right], \tag{5.2}$$

where agents' full information action is now characterized by $\hat{a} = \eta_1(\psi_{\theta}, \psi_{\bar{a}})\,\theta$.[25] The key feature of (5.2) is that both α and η are functions of the coefficients in the policy rule (5.1). To equate the equilibrium best-response function to its efficient counterpart under (5.1), we need to ensure both that the full information action is efficient ($\eta = \eta^{\star}$), and that the equilibrium and socially optimal amount of coordination are equal ($\alpha = \alpha^{\star}$). Under the new rule, both can however be achieved: $\psi_{\bar{a}}$ can be set to equate α with $\alpha^{\star}$, while, conditional on this value, ψ_{θ} can be set to equate η with $\eta^{\star}$. Combined, this shows that an extended policy rule exists that can alleviate any inefficiency in the equilibrium use of information.

However, despite this potential for the policy instrument to equate the equilibrium and socially optimal use of information, the policymaker would never (generically) choose to follow the rule in Lemma 4. This is because the policymaker would instead use her knowledge about the fundamental and the average action, embedded in the conditioning in (5.1), to move the economy closer to its full information, first-best response.

[24] Further, notice how Lemma 4 contrasts the class of dispersed information economies analyzed in this paper with their full information limits. In the latter case, there is no difference between the set of welfare outcomes attainable by conditioning on the fundamental, or the fundamental and the average action.

[25] Specifically, under (5.1) we have that $\alpha(\psi_{\bar{a}}) = -\frac{\mathcal{U}_{a\bar{a}} + \mathcal{U}_{am}\psi_{\bar{a}}}{\mathcal{U}_{aa}}$ while $\eta_1 = -\frac{\mathcal{U}_{a\theta} + \mathcal{U}_{am}\psi_{\theta}}{\mathcal{U}_{aa} + \mathcal{U}_{a\bar{a}} + \mathcal{U}_{am}\psi_{\bar{a}}}$. I also here once more abstract from an irrelevant constant term η_0.

Consider the social welfare function in Lemma 1, and suppose that we start in the full information case in which agents' actions equal their efficient value: $a_i = \hat{a}^\star$ for all $i \in [0.1]$. In this case, the latter two terms of the ex-ante welfare function ($\mathbb{E}\left[\bar{a} - \hat{a}^\star\right]$ and $\mathbb{E}\left[a - \bar{a}\right]^2$), which reflect welfare losses due to dispersed information, equal zero. The policy instrument thus optimally solves the efficient, full information problem: $\max_\psi \mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right)$, which results in values of ψ_θ and $\psi_{\bar{a}}$ that are (generically) different from those in Lemma 4.

We conclude that the optimal use of the policy instrument under dispersed information will trade-off two values; that which eliminates any inefficiency in the use of information, and that which is optimal under full information and efficient actions. Starting from the values of ψ_θ and $\psi_{\bar{a}}$ in Lemma 4, the policymaker twists the optimal use of the policy instrument away from that which replicates the efficient use of information, and towards that which moves the economy closer to its full information, first-best response.

Thus, because of the policy instrument's role under full information and efficient actions, the optimal conditioning of the policy instrument in (5.1) turns one economy with an inefficient use of information into another.[26]

Proposition 8. *Consider an economy under (5.1). Suppose this economy exhibits an inefficient use of information ($\alpha \neq \alpha^\star$ or $\eta \neq \eta^\star$) when $\psi_\theta = \psi_{\bar{a}} = 0$. Then, the economy still (generically) exhibits inefficient use of information under the optimal values $\psi_\theta^\star$ and $\psi_{\bar{a}}^\star$.*

5.2. Conditioning on noisy information

The policy rule in (5.1) rests on two stark assumptions. First, the policymaker perfectly knows the realization of the fundamental at the time of decision-making. All her uncertainty about the fundamental at the start of the period disappears before the end. Second, the policymaker also knows the average action in the economy with similar certainty. Given the sizable revisions even after several years to aggregate statistics, such certainty seems stark. Instead, it seems more natural within our framework to assume that the policymaker conditions her policy instrument on *noisy information* about the unobserved fundamental and the average action,

$$m = \phi z + \psi_{\bar{a}} \tilde{a}, \tag{5.3}$$

where $\tilde{a} = \bar{a} + \epsilon_a$ and $\epsilon_a \sim \mathcal{N}\left(0, \tau_a^{-1}\right)$. Equation (5.3) extends the policy rule in (2.2) with a noisy estimate of the average action. Importantly, though, this estimate differs from that observed by the private sector ($\tilde{a} \neq y$). If, by contrast, the two signals were the same ($\tilde{a} = y$), then the portion of the fluctuations of the policy instrument caused by the signal of the average action would be completely anticipated. Changes in $\psi_{\bar{a}}$ would then not alter the equilibrium use of information, and we would to a large extent return to the previous analysis.

The extended policy rule in (5.3) inherits several properties of its noiseless version in (5.1). Similar to the policy rule in (5.1), it can equate the equilibrium and efficient use of information. In fact, the same values as those that apply in Lemma 4 would also here render the equilibrium use of information efficient (when we equate ϕ with ψ_θ). There are, nevertheless, two main differences between the policy rule in (5.3) and its noiseless version in (5.1). First, because the policymaker conditions her policy instrument on the same information that she considers to disclose, the use of the policy instrument in (5.3) can directly substitute for the policymaker's disclosure. Second,

[26] This holds true for all $\tau_\omega \in \mathbb{R}$, and not only for the optimal level of disclosure $\tau_\omega^\star$.

because of the noise in the signals that the policy instrument is conditioned on, the policymaker will also temper her responses, compared to an economy in which she can condition her policy instrument directly on the average action, for example.

Proposition 9 mirrors the results in Proposition 8.

Proposition 9. *Consider an economy under (5.3). Suppose this economy exhibits an inefficient use of information ($\alpha \neq \alpha^\star$ or $\eta \neq \eta^\star$) when $\phi = \psi_{\bar{a}} = 0$ and $\tau_\omega \to 0$. Then, the economy still (generically) exhibits inefficient use of information under the optimal values $\phi^\star$ and $\psi_{\bar{a}}^\star$.*

Proposition 9 shows that, conditional on the optimal instrument rule, the economy retains any inefficiency in its use of information. One important consequence of this maintained inefficiency is that we can apply the results from the previous section, specifically Proposition 4, 5, 6, and 7, to the case in which the policymaker also sets her policy instrument based on her own noisy estimate of the average action. The only difference is that we also need to condition our previous results on the optimal use of $\psi_{\bar{a}} = \psi_{\bar{a}}^\star$. As a result, any combination of instrument and communication policy can still be socially optimal.

To see why, notice that we can always write agents' expected payoffs as[27]:

$$\mathbb{E}\left[\mathcal{U}\left(a_i, \bar{a}, \sigma, \theta, m\right) \mid \chi_i, \gamma, \omega\right] = \mathbb{E}\left[U\left(a_i, \bar{a}, \sigma, \theta, \tilde{m}\right) \mid \chi_i, \gamma, \omega\right] + \frac{1}{4}\mathcal{U}_{mm}^2 \psi_{\bar{a}}^2 \tau_a^{-1},$$

for some quadratic polynomial U that embeds the part of the policy instrument that depends on the average action ($\psi_{\bar{a}}\bar{a}$), and in which $\tilde{m} = \phi z$, as in Section 4. This shows that, with the exception of a moderating force arising from the policymaker responding to a noisy signal of the average action ($\mathcal{U}_{mm}^2 \psi_{\bar{a}}^2 \tau_a^{-1}$), the same conditions as those we derived in Section 4 extend to this more general case. The only difference is that we also need to condition our previous results on the chosen value of $\psi_{\bar{a}}$. The main insights from the previous section thus carry over to environments in which the policy instrument also responds to the average action.

The discussion in this subsection allows us to summarize the main trade-offs faced by the policymaker as follows: Suppose that we start from the case in which the policymaker with (5.3), under complete opacity, eliminates any inefficiencies in the equilibrium use of information with the appropriate, conditional use of her policy instrument. Once this aim is accomplished, the question becomes whether to (i) disclose more information, to potentially better align agents' actions with the fundamental, but at the possible cost of adding noise to the information structure; or to (ii) alter the conditioning of her policy instrument, to potentially better align agents' actions with the fundamental, but at the cost of breaking the efficient use of information. Through this lens, the results from the previous section can be seen to pin-down conditions on the relative strengths of these trade-offs.

[27] This follows from a quadratic approximation of the payoff function $\mathcal{U}$ around $m = 0$,

$$\begin{aligned}\mathcal{U}\left(a_i, \bar{a}, \sigma, \theta, m\right) &= \mathcal{U}\left(a_i, \bar{a}, \sigma, \theta, 0\right) + \mathcal{U}_{m_{|m=0}} m + \frac{1}{2}\mathcal{U}_{mm} m^2 \\ &= U\left(a_i, \bar{a}, \sigma, \theta, \tilde{m}\right) + \mathcal{U}_{m_{|m=0}} \psi_{\bar{a}} \epsilon_a + \frac{1}{2}\mathcal{U}_{mm}\left(\psi_{\bar{a}} \epsilon_a\right)^2,\end{aligned}$$

where $\mathbb{E}\left[\mathcal{U}_{m_{|m=0}} \psi_{\bar{a}} \epsilon_a \mid \chi_i, \gamma, \omega\right] = 0$. Notice also that agents' equilibrium actions will be independent of ϵ_a.

5.3. A noisy policy rule

Finally, consider the case, in which, unlike in (2.2), (5.1), and (5.3), the policy instrument is itself noisy. Such policy rules are frequently adopted in the applied macroeconomics literature, to account for the presence of "policymaker shocks". Specifically, suppose that the policy instrument follows the linear relationship:

$$m = \phi z + \epsilon_m, \tag{5.4}$$

where $\epsilon_m \sim \mathcal{N}\left(0, \tau_m^{-1}\right)$. Unlike the policy rule in (5.3), the welfare costs from ϵ_m do not affect the welfare consequences of changes to ϕ. As such, the presence of the added disturbance ϵ_m does not influence the characteristics of the combined, optimal use of the policymaker's information. Policymaker shocks do not affect the best means to use a policymaker's information.

6. Applications and extensions

The previous sections have shown how any inefficiency in the equilibrium use of information interacts with properties of the payoff function to shed light on the best means to use a policymaker's information. I now demonstrate how my results can guide normative analysis in specific applications. I conclude this section by discussing some extensions of my results.

6.1. An efficient Island economy

I start with a competitive economy in which production choices are made under imperfect information about future demand. The economy closely resembles the continuum-firm limit of that studied in Vives (1988) and Angeletos and Pavan (2007). The economy is delineated into a continuum of measure one of islands. On each island, a representative consumer receives utility from consuming a consumption good that is traded across all islands as well as a numéraire,

$$u_i = \theta a_i - \chi a_i^2/2 + e_i,$$

where a_i denotes his consumption of the homogeneous consumption good, e_i his chosen holdings of the numéraire, and $\chi > 0$. The consumer maximizes his utility subject to the budget constraint: $pa_i + e_i = \bar{e} + \pi_i$, where p denotes the relative price of the consumption good, $\bar{e}$ the consumer's endowment of the numéraire, and π_i the profits of the representative firm on island i. Turning to the firm on island i, it chooses its production to maximize profits: $\pi_i = a_i\,(p - m) - \frac{1}{2}a_i^2$, where m denotes a value-added tax levied on the firm. Finally, the time-line of events is as follows. First, firms set output choices under imperfect information about the demand shifter θ. For simplicity, I assume that $\tau_{x\bar{a}} = \tau_y \to 0$, so that firms do not observe any endogenous signals. Then, the demand shifter realizes, consumers choose their consumption bundles, and the economy-wide market for the consumption good clears.

A few simple derivations show that the above economy is nested within our framework. Solving a consumer's problem shows that his demand for the consumption good equals $a_i = \frac{1}{\chi}(\theta - p)$. Since all consumers face the same problem, and hence choose identical demand schedules $a_i = \bar{a}$, it follows that the inverse demand function is $p = \theta - \chi\bar{a}$. Thus, we can state the payoff function for the firm on island i as

$$\mathcal{U}\left(a_i, \bar{a}, \sigma, \theta, m\right) = \left(\theta - \chi\bar{a} - m\right) a_i - \frac{1}{2}a_i^2 + \frac{1}{2}\chi\bar{a}^2 + \bar{e}. \tag{6.1}$$

Equation (6.1) implies that $\hat{a}_i = \frac{1}{1+\chi}(\theta - m) = \hat{a}_i^\star$ and $\alpha = \alpha^\dagger = \alpha^\star = -\chi$,[28] where the latter equivalence between $\alpha^\dagger$ and $\alpha^\star$ follow from the lack of endogenous information. We conclude that the economy's use of information is efficient. Proposition 4 then shows that:

Corollary 6. *In the efficient island economy, full disclosure ($\tau_\omega^\star \to \infty$) combined with the policy instrument being set to its efficient, full information value ($\phi^\star = 0$) is optimal.*

Clearly, the above efficient island economy is a quite special case. While the First Welfare Theorem ensures that the full information economy is always efficient, the presence of endogenous information would in general make the dispersed information counterpart inefficient. This, in turn, could open up a role for the active use of the value added tax $\phi \neq 0$, to adjust firms' use of information.

6.2. An economy with inefficient payoff externalities

James and Lawler (2011) extend Morris and Shin's (2002) influential contribution by including a policymaker, who conditions her policy instrument on her own information, into their analysis. Surprisingly, James and Lawler show that, unlike in Morris and Shin (2002), additional public disclosures are *always* detrimental for social welfare, instead of only for certain parameter values. Complete opacity is uniquely optimal for the policymaker.

The analysis conducted by James and Lawler (2011) is nested within our framework. Specifically, James and Lawler consider the beauty-contest model in which payoffs equal

$$u_i = -(1-r)(a_i + m - \theta)^2 - r\left(L_i - \bar{L}\right), \tag{6.2}$$

where $r \in (0,1)$, $L_i \equiv \int (a_j - a_i)^2 di = (a_i - \bar{a})^2 + \sigma_a^2$, and $\bar{L} \equiv \int L_i di = 2\sigma_a^2$. The information structure is similarly simple: Private sector agents only observe exogenous information, $\tau_{x\bar{a}} = \tau_y \to 0$. Lastly, the policy instrument m in (6.2) is set so that $m = \phi z$.

This example is a special case of our framework with

$$\mathcal{U}(a_i, \bar{a}, \sigma_a, \theta, m) = -(1-r)(a_i + m - \theta)^2 - r(a_i - \bar{a})^2 + r\sigma_a^2.$$

It follows that $\hat{a} = \theta - m = \hat{a}^\star$, while $\alpha = \alpha^\dagger = r > 0 = \alpha^\star$.[29] Thus, we conclude that the equilibrium use of information is inefficient ($\eta = \eta^\star$ but $\alpha > \alpha^\star$), because of inefficient payoff externalities. But note that, since $a_i - \bar{a} = a_i + m - (\bar{a} + m)$ and $\sigma_a^2 = \int (a_i - \bar{a})^2 di = \int (a_i + m - (\bar{a} + m))^2 di$, the policy instrument also enters agents' payoffs as a *perfect substitute* for individual actions. (Alternatively, the payoff function satisfies: $\eta_1^\star = \frac{\mathcal{U}_{m\theta}}{\mathcal{U}_{mm}} = -1$ and $\eta_2 = \eta_2^\star = -\frac{\mathcal{U}_{mm}}{\mathcal{U}_{am} + \mathcal{U}_{\bar{a}m}} = 1$.[30]) The following is then a consequence of Proposition 6.

Corollary 7. *In the Morris and Shin (2002) beauty-contest model, where the policy instrument enters as a perfect substitute for individual actions ($\exists \delta \in \mathbb{R}$ such that $\eta_1^\star = \delta \frac{\mathcal{U}_{m\theta}}{\mathcal{U}_{mm}}$ and $\eta_2 = \eta_2^\star = -\frac{\mathcal{U}_{mm}}{\mathcal{U}_{am} + \mathcal{U}_{\bar{a}m}} = \delta$), complete opacity is uniquely optimal ($\tau_\omega^\star \to 0$, $\phi^\star \neq 0$).*

28 The relevant derivatives are: $\mathcal{U}_{aa} = -1, \mathcal{U}_{a\bar{a}} = -\chi, \mathcal{U}_{\bar{a}\bar{a}} = \chi$, and $\mathcal{U}_{\sigma\sigma} = 0$.

29 The derivates are: $\mathcal{U}_{aa} = -2, \mathcal{U}_{a\bar{a}} = 2r, \mathcal{U}_{\bar{a}\bar{a}} = -2r$, and $\mathcal{U}_{\sigma\sigma} = 2r$.

30 We further have that: $\mathcal{U}_{m\theta} = 2(1-r), \mathcal{U}_{mm} = -2(1-r), \mathcal{U}_{ma} = -2(1-r)$, and $\mathcal{U}_{m\bar{a}} = 0$.

James and Lawler (2011) couch their contribution in terms of how "the policymaker [via the policy instrument can] determine the relative weights accorded to alternative information sources" (p.1,570). Yet, a few simple derivations show that the socially optimal weight on private and public information in their analysis is also attainable with communication policy.[31] Rather, the above results show that what underlies their main findings is not instrument policy's ability to achieve a better weight on private and public information. But instead, it is policy instrument's ability to modify the relative weight on both without the introduction of noise to the information structure that is important. James and Lawler's contribution can thus be seen as one important example of how the basic mechanism from Section 4.3.1, which pushes towards the use of instrument policy, can make complete opacity uniquely optimal.

6.3. A business cycle extension

At first pass, the conditions for Proposition 6 to apply, also used in the previous application, may seem to lie on a knife-edge. However, the conditions do indeed carry over to an important class of micro-founded business cycle models with monopolistic competition, in which firms pre-set prices under dispersed information (e.g. Woodford, 2002; Hellwig, 2005; Roca, 2010; Lorenzoni, 2010; Angeletos et al., 2016). This application is also particularly pertinent, because it allows me to capture the important case in which we identify the policymaker with the central bank, the most prominent provider of policymaker information. Common among these models is that firm prices are set in accordance with

$$a_i = \mathbb{E}\left[\xi \bar{a} + (1-\xi)(m-\theta) \mid \Omega_i\right], \tag{6.3}$$

where a_i denotes the logarithm of the price set by firm i, θ unobserved firm productivity, $\xi \in (0, 1)$, and the policy instrument is set as in (2.2).[32] Social welfare is further equal to

$$\mathbb{E}\left[\mathcal{W}\right] = -\mathbb{E}\left[a_i - m + \theta\right]^2 - \chi \mathbb{E}\left[a_i - \bar{a}\right]^2, \tag{6.4}$$

where χ is a positive constant. Equations (6.3) and (6.4) are nested within our framework. They are both the outcome of the payoff function:

$$\mathcal{U}\left(a_i, \bar{a}, \sigma_a, \theta, m\right) = -\left(a_i - m + \theta\right)^2 - \chi\left(a_i - \bar{a}\right)^2 - 2\left(a_i - \bar{a}\right)\gamma\left(\bar{a} - m + \theta\right), \tag{6.5}$$

where γ solves $\xi = \frac{\chi-\gamma}{1+\chi}$. With (6.5), individual actions follow (6.3), while social welfare $\mathbb{E}\left[\mathcal{W}\right]$ is described by (6.4). It follows that $\eta = \eta^\star$ but $\alpha^\dagger > \alpha > 0$. The above class of economies features too little payoff coordination in equilibrium ($\alpha^\dagger > \alpha$), as also shown in Hellwig (2005). Hence, the equilibrium use of information is generically inefficient ($\alpha^\star \neq \alpha$), except for the special case in which learning externalities exactly offset the shortfall in payoff coordination.

The results in Proposition 6 apply directly to this class of economies. Because we can always write $a_i - \bar{a} = a_i - m - (\bar{a} - m)$ in (6.5), the policy instrument once more perfectly substitutes for individual agents' actions. Proposition 6 then shows that:

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31 Specifically, it follows from Equations (12) and (18) in James and Lawler (2011) that $c \in (\kappa(\phi = 0, \tau_\omega \to \infty), \kappa(\phi = 0, \tau_\omega \to 0))$. The efficient weight is attainable purely with communication policy. The same holds true for the efficient weight on public information.

32 See, for example, the previous version of this paper (Kohlhas, 2017).

Corollary 8. *In the class of business cycle models characterized by (6.3) and (6.4), central bank opacity is generically optimal ($\tau_\omega^\star \to 0$). Instead, the central bank should condition its policy instrument on its own information about the fundamentals of the economy* ($\phi^\star \neq 0$).

Corollary 8 provides an important example of how the conditioning of a policy instrument alters the social value of public information. Hellwig (2005) and Roca (2010) find that welfare necessarily increases with public information in the same class of models as those considered here when there is only exogenous information. This is because the optimal degree of payoff coordination exceeds its equilibrium value ($\alpha^\dagger > \alpha > 0$). Yet, unlike in the above setup, the central bank in their respective contributions does not condition its policy instrument on its own information. The central bank has no alternative means to use its information than to disclose it. This explains why the results in Hellwig (2005) and Roca (2010) differ so strongly from those in Corollary 8.

6.4. A prediction game with endogenous information

The previous two applications have shown that in several models the best means to use a policymaker's information is the exclusive use of a policy instrument. Let me now instead turn to a simple example of the benefits of communication policy. The example resembles a reduced-form version of the business cycle model studied in Amador and Weill (2010), extended to include a policymaker who conditions her policy instrument on her own noisy information. In the model, a continuum of measure one of agents choose their actions a_i to maximize,

$$\mathcal{U}\left(a_i, \bar{a}, \sigma_a, \theta, m\right) = -\frac{1}{2}\left(a_i - \theta\right)^2 - \tau, \tag{6.6}$$

where τ denotes a state-dependent transfer equal to $\tau \equiv -\frac{1}{2}\left(m - \theta\right)^2$. It follows that $\hat{a} = \hat{a}^\star = \theta$, while $\alpha = \alpha^\dagger = 0$. However, because agents observe the partially endogenous information structure from Section 2, we conclude that $\alpha^\star < \alpha$. Agents perceive too much strategic complementarity between their actions relative to what is socially desirable.

Because $\eta_2^\star = \eta_{\hat{2}}^\star = 0$, Corollary 5 shows that the policy instrument should be set to its optimal value in the efficient, full information case: $\phi^\star = \hat{\phi}^\star = 1$. The policy cannot in this case adjust agents' use of information. Characterizing the optimal amount of disclosure is similarly simple. Notice that under the optimal instrument policy, social welfare equals $\mathbb{E}\mathcal{W} = -\frac{1}{2}\left(\tau_\theta + \tau_{x\theta} + \tau_{x\bar{a}}\kappa^2 + \tau_y\kappa^2 + \frac{\tau_\omega\tau_z}{\tau_\omega + \tau_z}\right)^{-1} - \frac{1}{2}\tau_z^{-1}$.[33] But using Proposition 1, we can alternatively write this expression as $\mathbb{E}\mathcal{W} = -\frac{1}{2}\frac{\kappa}{\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2} - \frac{1}{2}\tau_z^{-1}$. Corollary 9 then follows from the realization that under (6.6) additional disclosure always decreases agents' equilibrium weight on private information ($\partial\kappa/\partial\tau_\omega < 0$ where $\kappa \in (0, 1)$; Proposition 2).

Corollary 9. *Consider the prediction game with endogenous private and public information in (6.6). Because $\eta_2^\star = \eta_{\hat{2}}^\star = 0$, the policy instrument is optimally set to its value in the efficient, full information case $\phi^\star = 1$. Because of learning externalities, either complete opacity ($\tau_\omega^\star \to 0$) or full disclosure ($\tau_\omega^\star \to \infty$) is optimal, depending on parameter values.*

[33] This follows from agents' equilibrium actions being equal to $a_i = \mathbb{E}\left[\theta \mid \chi_i, \gamma, \omega\right]$. Thus, the first term in social welfare simply reflects the mean-squared error of households' expectation of the fundamental.

The above example provides insight into the implications that different types of endogenous information have on a policymaker's optimal disclosure. When agents only observe endogenous public information ($\tau_{x\bar{a}} \to 0$), social welfare always increases with the policymaker's disclosure ($\tau_\omega^\star \to \infty$).[34] The additional information provided by the policymaker dominates the decrease in the information content of the endogenous public signal. By contrast, when agents observe only endogenous private information ($\tau_{x\theta} \to 0$), social welfare invariably decreases with the policymaker's disclosure ($\tau_\omega^\star \to 0$). The learning externality is, in this case, forceful enough to render complete opacity optimal.

Finally, notice that in this example, in which the policy instrument cannot modify agents' use of information, communication policy is still able to. Indeed, that is why complete opacity can be optimal in Corollary 9. This illustrates the great virtue of communication policy: It *always* allows the policymaker to adjust agents' equilibrium use of information.

6.5. Heterogeneous fundamentals

One important feature of the previous applications has been that all agents' optimal full information action is the same. Below, I provide two examples that demonstrate the potential implications of differences in agents' optimal full information actions for the best means to use a policymaker's information.

Individual-specific fundamentals (I) Consider the business cycle application from Section 6.3, but suppose that each agent wants to align his action to his own individual-specific fundamental θ_i, where $\theta_i = \theta + \epsilon_\theta^i$ with $\epsilon_\theta^i \sim \mathcal{N}\left(0, \bar{\tau}_\theta^{-1}\right)$,

$$\mathcal{U}\left(a_i, \bar{a}, \sigma_a, \theta, m\right) = -\left(a_i - m + \theta_i\right)^2, \tag{6.7}$$

where I, for simplicity, have ruled out any payoff externalities by setting $\chi = \gamma = 0$ in (6.5). The shock and information structure is as follows: I assume that each agents' exogenous private information pertains to his own fundamental, $x_{i\theta} = \theta_i + \epsilon_{x\theta}^i$. The policymaker's disclosure, by contrast, is about her own private information about the average of fundamentals, $z = \int \theta_i dF(\theta_i) + \epsilon_z = \theta + \epsilon_z$, which is also what the policy instrument is conditioned on. Lastly, I assume that agents observe both endogenous signals, $x_{i\bar{a}}$ and y, so that the economy's use of information is inefficient due to learning externalities.

Although the payoff structure in (6.7) is reduced-form it speaks to extensions of the business cycle framework in Section 6.3 that allow for firm-specific productivity shocks and firm-specific information about such disturbances.

The key feature of this extension is that, although agents have different optimal, full information actions ($\hat{a}_i^\star = \theta_i + m$), the policy instrument can still *perfectly substitute* for individual agents' use of the policymaker's information. This, in turn, is because all agents' use the policymaker's disclosure in the same manner. With (6.7) each agent attaches the same weight to the policymaker's release ω, because the individual-specific shocks are all drawn from the same

[34] Both this result and the directly below one follow from taking limits of $\mathbb{E}\mathcal{W} = -\frac{1}{2}\frac{\kappa}{\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2} - \frac{1}{2}\tau_z^{-1}$ with respect to $\tau_{x\theta}$ and $\tau_{x\bar{a}}$, respectively, combined with $\partial\kappa/\partial\tau_\omega < 0$.

distribution $\epsilon_\theta^i \sim \mathcal{N}\left(0, \bar{\tau}_\theta^{-1}\right)$. A simple extension of Proposition 6 shows that complete opacity therefore remains optimal.[35]

Corollary 10. *Consider the payoff function in (6.7), and suppose that agents observe both endogenous private and public information ($x_{i\bar{a}}$ and y). Then, despite that each agents' optimal, full information action is different, complete opacity remains optimal ($\tau_\omega^\star \to 0$, $\phi^\star > 0$).*

Individual-specific fundamentals (II) Instead, suppose that $\epsilon_\theta^i \sim \mathcal{N}\left(0, \bar{\tau}_{\theta i}^{-1}\right)$, so that each individual-specific shock is drawn from its own distribution. Each agent will then use the policymaker's disclosure differently. As a result, the common, conditional use of the policy instrument cannot perfectly substitute for individual agents' actions.

Corollary 11. *Consider payoffs in (6.7) when $\epsilon_\theta^i \sim \mathcal{N}\left(0, \bar{\tau}_{\theta i}^{-1}\right)$. Then, there exists values of $\tau_{x\bar{a}}$ and τ_y such that full disclosure and no-instrument policy is optimal ($\tau_\omega^\star \to \infty$, $\phi^\star = 0$).*[36]

In sum, Corollary 10 and 11 stress the importance of the policy instrument's ability to substitute for agents' use of the policymaker's information for the overall benefits of instrument policy. In doing so, the examples have also shown that, in models with heterogeneous fundamentals but only one policy instrument, communication policy may have the additional advantage that it allows an economy to differentially respond to the policymaker's information.

6.6. The effect of policy signaling

Finally, I briefly turn to how any direct signals of the policy instrument affect my results. A way to explore the effect of such signals within our framework is to extend the baseline environment with the additional public signal

$$s_m = m + \epsilon_m = \phi z + \epsilon_m, \tag{6.8}$$

where $\epsilon_m \sim \mathcal{N}\left(0, \tau_m^{-1}\right)$. The signal s_m provides a crude summary measure of all the direct signals of the prospective policy instruments that agents' might observe. When ϕ is relatively large (in absolute value), s_m is very informative about the policymaker's information z, and conversely when ϕ is small. However, because of the noise term ϵ_s, the signal never perfectly reveals the policymaker's information. This is important; it allows communication policy to still provide additional information above that conveyed by the summary signal.

Although the additional signal provides further information about the policymaker's signal, it does not meaningfully alter my results. For example, the exclusive use of instrument and communication policy are still optimal under the same conditions as those listed in Proposition 6 and 7, respectively. This is because, while s_m provides a lower-bound on the knowledge about the policymaker's signal, it does not alter the relative benefits of either policy tool. Suppose, for instance, that learning externalities cause the equilibrium weight on private information to be smaller than the efficient weight. Changes in the responsiveness of the policy instrument can then

[35] The same decomposition used in (4.4) and (4.5) allow us to extend Lemma 3 to this case. The dominance of instrument policy then follows from the same steps as those in the proof of Proposition 6.

[36] A simple example is the case in which $\tau_{x\bar{a}} = \tau_y \to 0$. Full disclosure here attains the efficient outcome.

still alleviate the lack of weight on private information, without the noise associated with partial disclosure. Increases in τ_s do, of course, diminish the effect of changes to the policy instrument. But for a finite τ_s, the relative benefit of the use of instrument policy remains. A similar argument holds for the advantages that communication policy provides.

7. Concluding remarks

Unlike a statistical office or a news service, a policymaker has two means to exploit any information about the economy: She can either disclose it, or she can use it to condition a policy instrument upon its realization. In this paper, I have studied the relative merits of each means to use a policymaker's information. To do so, I have focused on a broad class of models that feature dispersed information, and payoff and learning externalities.

At the center of my results have been a trade-off between the respective advantages that the two means to exploit a policymaker's information entail. On the one hand, I have shown that the conditional use of a policy instrument is a better mean to modify agent's use of information than communication policy. With instrument policy, a policymaker can directly control the influence of her own information on market outcomes. With communication policy, by contrast, a policymaker can only alter other agents' reliance on her information by obfuscating her news, at an added welfare cost. On the other hand, however, communication policy has the advantage that it allows agents to always respond to the policymaker's information. This is unlike the conditional use of a policy instrument, which can be constrained in its capacity to move an economy in response to the policymaker's news. In this sense, communication policy has the advantage over instrument policy that it gets into all the cracks.

My results in this paper have provided conditions under which each of these advantages dominates the other, in addition to the set of outcomes that can arise from their interaction. Although for tractability purposes, I have limited my analysis to the study of quadratic games with Gaussian information structures, neither of these assumptions appear central for the main insights of my analysis. I therefore conjecture that my results extend beyond the specific class of models studied in this paper. As such, I would like to view this paper as a step towards a more comprehensive picture of optimal policy under imperfect information. Specifically, one that underscores that policymaker information differs from other sources of public information, precisely because a policymaker can always use her information to condition a policy instrument instead.

Appendix A. On the optimal use of information

This Appendix details the proofs of the Propositions and Corollaries in Sections 2 and 3.

Proof of Proposition 1. The Proposition has two parts.

Part (i): The proof of the first part follows the steps outlined in Angeletos and Pavan (2007). The sufficient first-order condition for agents' optimal action under full information is

$$\mathcal{U}_a = \mathcal{U}_a(0,0,0,0,0) + \mathcal{U}_{aa}a + \mathcal{U}_{a\bar{a}}\bar{a} + \mathcal{U}_{a\theta}\theta + \mathcal{U}_{am}m = 0. \tag{A.1}$$

As a result, $\hat{a} = \eta_0 + \eta_1(\theta + \eta_2 m)$, where $\eta_0 = -\frac{\mathcal{U}_a(0,0,0,0,0)}{\mathcal{U}_{aa}+\mathcal{U}_{a\bar{a}}}$, $\eta_1 = -\frac{\mathcal{U}_{a\theta}}{\mathcal{U}_{aa}+\mathcal{U}_{a\bar{a}}}$, and $\eta_2 = \frac{\mathcal{U}_{am}}{\mathcal{U}_{a\theta}}\eta_1$.

The equivalent first-order condition under imperfect information is

$$\mathbb{E}[\mathcal{U}_a] = \mathbb{E}[\mathcal{U}_a(0,0,0,0,0) + \mathcal{U}_{aa}a + \mathcal{U}_{a\bar{a}}\bar{a} + \mathcal{U}_{a\theta}\theta + \mathcal{U}_{am}m \mid \Omega_i] = 0. \tag{A.2}$$

Deducting (A.1) from (A.2), where the former is evaluated at the full-information solution $\hat{a}$, and rearranging terms then immediately delivers:

$$a_i = \mathbb{E}_i\left[(1-\alpha)\hat{a} + \alpha\bar{a} \mid \Omega_i\right], \quad \alpha \equiv -\frac{\mathcal{U}_{a\bar{a}}}{\mathcal{U}_{aa}}. \tag{A.3}$$

Part (ii): The proof of the second part also follows well-known steps. To solve for the linear rational expectations equilibrium, I first conjecture that a_i satisfies

$$a_i = \beta + \beta_0 x_{i\theta} + \beta_1 s_i + \beta_2 s_y + \beta_3\omega + \beta_4\mu, \tag{A.4}$$

where $\beta_j \in \mathbb{R}$ for $j = \{0, 1, 2, 3, 4\}$ and

$$s_y \equiv \frac{1}{\kappa}\left(y - \beta - \beta_2 s_y - \beta_3\omega - \beta_4\mu\right) = \theta + \frac{1}{\kappa}\epsilon_y \tag{A.5}$$

$$s_i \equiv \frac{1}{\kappa}\left(x_{i\bar{a}} - \beta - \beta_2 s_y - \beta_3\omega - \beta_4\mu\right) = \theta + \frac{1}{\kappa}\epsilon^i_{x\bar{a}}, \tag{A.6}$$

denotes the signals contained in the observation of $x_{i\bar{a}}$ and y, respectively, conditional on the other signals in Ω_i. We have that $\kappa = \beta_0 + \beta_1$. To verify that (A.4) solves (A.3), we need to compute expressions for agents' expectations of the fundamental θ and the policymaker's information z.

Due to our linear-Gaussian information structure, these expectations equal

$$\mathbb{E}\left[\theta \mid \Omega_i\right] = w_{x\theta}x_i + w_{x\bar{a}}s_{i\bar{a}} + w_y s_y + w_\omega\omega + \left(1 - w_{x\theta} - w_{x\bar{a}} - w_y - w_\omega\right)\mu, \tag{A.7}$$

where the weights are implicitly defined with, for example, $w_{x\theta} \equiv \frac{\tau_x(\tau_z+\tau_\omega)}{(\tau_z+\tau_\omega)\left(\tau_\theta+\tau_{x\theta}+\left(\tau_y+\tau_{x\bar{a}}\right)\kappa^2\right)+\tau_z\tau_\omega}$, and

$$\mathbb{E}\left[z \mid \Omega_i\right] = v_{x\theta}x_i + v_{x\bar{a}}s_{i\bar{a}} + v_y s_y + v_\omega\omega + \left(1 - v_{x\theta} - v_{x\bar{a}} - v_y - v_\omega\right)\mu, \tag{A.8}$$

in which, for example, $v_{x\theta} \equiv \frac{\tau_x\tau_z}{(\tau_z+\tau_\omega)\left(\tau_\theta+\tau_{x\theta}+\left(\tau_y+\tau_{x\bar{a}}\right)\kappa^2\right)+\tau_z\tau_\omega}$.

Inserting (A.4) into (A.3), using that $\bar{a} = \int a_i dF(a_i)$ we find that

$$\begin{aligned} a_i = {} & \left[(1-\alpha)\eta_0 + \alpha\beta\right] + \alpha\left[\beta_2 y + \beta_3\omega + \beta_4\mu\right] \\ & + \left[(1-\alpha)\eta_1 + \alpha\kappa\right]\mathbb{E}\left[\theta \mid \Omega_i\right] + (1-\alpha)\eta_2\eta_1\phi\mathbb{E}\left[z \mid \Omega_i\right] \end{aligned} \tag{A.9}$$

which verifies our conjecture when combined with (A.7) and (A.8) *iff.* there exists a solution to

$$\beta_0 = \left[(1-\alpha)\eta_1 + \alpha\kappa\right]w_{x\theta} + (1-\alpha)\eta_2\eta_1\phi v_{x\theta} \tag{A.10}$$

$$\beta_1 = \left[(1-\alpha)\eta_1 + \alpha\kappa\right]w_{x\bar{a}} + (1-\alpha)\eta_2\eta_1\phi v_{x\bar{a}} \tag{A.11}$$

$$\beta_2 = \left[(1-\alpha)\eta_1 + \alpha\kappa\right]w_y + (1-\alpha)\eta_2\eta_1\phi v_y + \alpha\beta_2 \tag{A.12}$$

$$\beta_3 = \left[(1-\alpha)\eta_1 + \alpha\kappa\right]w_\omega + (1-\alpha)\eta_2\eta_1\phi v_\omega + \alpha\beta_3 \tag{A.13}$$

$$\beta_4 = \left[(1-\alpha)\eta_1 + \alpha\kappa\right]w_\mu + (1-\alpha)\eta_2\eta_1\phi v_\mu + \alpha\beta_4, \tag{A.14}$$

where $w_\mu \equiv 1 - w_{x\theta} - w_{x\bar{a}} - w_y - w_\omega$, $v_\mu \equiv 1 - v_{x\theta} - v_{x\bar{a}} - v_y - v_\omega$, and $\beta = \eta_0$. Because all coefficient equations depend only on κ, all that however needs to be established is that the equation for κ has a solution. The equation determining κ, which also determines the informativeness of s_i and s_y, is

$$\kappa = \beta_0 + \beta_1 \tag{A.15}$$
$$= \eta_1 (1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right) \frac{\tau_\omega + \tau_z + \eta_2 \phi \tau_z}{(\tau_\omega + \tau_z)\left(\tau_\theta + \tau_y \kappa^2 + (1-\alpha)\left(\tau_{x\bar{a}}\kappa^2 + \tau_{x\theta}\right)\right) + \tau_\omega \tau_z},$$

which we can also state as

$$Q(\kappa) \equiv \left(\tau_y + \tau_{x\bar{a}}(1-\alpha)\right)\kappa^3 - \eta_1\left(1 + \eta_2 \phi \frac{\tau_z}{\tau_\omega + \tau_z}\right)(1-\alpha)\tau_{x\bar{a}}\kappa^2 \tag{A.16}$$
$$+ \left(\tau_\theta + \frac{\tau_\omega \tau_z}{\tau_\omega + \tau_z} + (1-\alpha)\tau_x\right)\kappa - \eta_1\left(1 + \eta_2 \phi \frac{\tau_z}{\tau_\omega + \tau_z}\right)\tau_x(1-\alpha) = 0.$$

A simple application of Descartes' "Rules of Signs" then shows that $Q(\kappa) = 0$ has either *one* or *three* solutions, depending on parameter values. This shows that either one or three equilibria exists.

Finally, it follows from (A.5) that $s_y = \frac{1}{\beta_2 + \kappa}(y - \beta - \gamma_3 \omega - \gamma_4 \mu)$. Inserting this expression, in addition to (A.6), into (A.4), and collecting terms, demonstrates that

$$k_0 = \eta_1^{-1}\beta_0, \quad k_1 = \eta_1^{-1}\frac{\beta_1}{\kappa}, \quad k_2 = \eta_1^{-1}\frac{\beta_2}{\kappa + \beta_2}\left(1 - \frac{\beta_1}{\kappa}\right), \tag{A.17}$$
$$k_3 = \eta_1^{-1}\beta_3\left(1 - \frac{\beta_2}{\kappa + \beta_2}\right)\left(1 - \frac{\beta_1}{\kappa}\right), \quad k_4 = \eta_1^{-1}\beta_4\left(1 - \frac{\beta_2}{\kappa + \beta_2}\right). \quad \square \tag{A.18}$$

Proof of Corollary 1. Follows from (A.15). □

Proof of Corollary 2. The proof follows similar steps to that of Proposition 1. Suppose

$$a_i = \eta_0 + \eta_1\left(\eta_1^{-1}\kappa \chi_i + \kappa_1 \gamma + \kappa_2 \omega\right), \tag{A.19}$$

where $\chi_i \equiv \mathbb{E}[\theta \mid x_{i\theta}, s_i]$ and $\gamma = \mathbb{E}\left[\theta \mid s_y\right]$.

Consider now the expectation of the fundamental in (A.7) and of the policymaker's information in (A.8). These expectations can alternatively be written as:

$$\mathbb{E}[\theta \mid \Omega_i] = w_\chi \chi_i + w_\gamma \gamma + \left(1 - w_\chi - w_\gamma\right)\omega \tag{A.20}$$
$$\mathbb{E}[z \mid \Omega_i] = v_\chi \chi_i + v_\gamma \gamma + \left(1 - v_\chi - v_\gamma\right)\omega, \tag{A.21}$$

where

$$w_\chi = \frac{\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)(\tau_z + \tau_\omega)}{(\tau_z + \tau_\omega)\left(\tau_\theta + \tau_{x\theta} + \left(\tau_y + \tau_{x\bar{a}}\right)\kappa^2\right) + \tau_z \tau_\omega},$$
$$w_\gamma = \frac{\left(\tau_\theta + \tau_y \kappa^2\right)(\tau_z + \tau_\omega)}{(\tau_z + \tau_\omega)\left(\tau_\theta + \tau_{x\theta} + \left(\tau_y + \tau_{x\bar{a}}\right)\kappa^2\right) + \tau_z \tau_\omega},$$
$$v_\chi = \frac{\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)\tau_z}{(\tau_z + \tau_\omega)\left(\tau_\theta + \tau_{x\theta} + \left(\tau_y + \tau_{x\bar{a}}\right)\kappa^2\right) + \tau_z \tau_\omega},$$
$$v_\gamma = \frac{\left(\tau_\theta + \tau_y \kappa^2\right)\tau_z}{(\tau_z + \tau_\omega)\left(\tau_\theta + \tau_{x\theta} + \left(\tau_y + \tau_{x\bar{a}}\right)\kappa^2\right) + \tau_z \tau_\omega}.$$

Using (A.20) and (A.21) in (A.3), and applying the conjectured solution in (A.19), then shows that

$$a_i = [(1-\alpha)\eta_0 + \alpha\eta_0] + \alpha\eta_1 \left[\kappa_1\gamma + \kappa_2\omega\right] \\ + [(1-\alpha)\eta_1 + \alpha\kappa]\,\mathbb{E}[\theta \mid \Omega_i] + (1-\alpha)\eta_2\eta_1\phi\,\mathbb{E}[z \mid \Omega_i]. \tag{A.22}$$

Matching terms then establishes the corollary. □

Proof of Proposition 2. The precision of the signals s_i and s_y conditional on the fundamental equals $\tau_{x\bar{a}}\kappa^2$ and $\tau_y\kappa^2$, respectively. A smaller value of $\mid \kappa \mid$ thus entails less informative signals.

(i) Totally differentiation of (A.15) with respect to ϕ shows that

$$\frac{d\kappa}{d\phi} = \left(1 - \frac{\partial\kappa^{RHS}}{\partial\kappa}\right)^{-1} \frac{\partial\kappa^{RHS}}{\partial\phi}, \tag{A.23}$$

where κ^{RHS} denotes *the right-hand side* of (A.15). The derivatives in (A.23) are:

$$\frac{\partial\kappa^{RHS}}{\partial\kappa} = \eta_1(1-\alpha)\left(1 + \eta_2\phi\frac{\tau_z}{\tau_\omega + \tau_z}\right) \frac{\tau_{x\bar{a}}\left(\tau_\theta + \frac{\tau_z\tau_\omega}{\tau_z+\tau_\omega}\right) - \tau_y\tau_{x\theta}}{\left[\tau_\theta + \tau_y\kappa^2 + (1-\alpha)\left(\tau_{x\bar{a}}\kappa^2 + \tau_{x\theta}\right) + \frac{\tau_z\tau_\omega}{\tau_z+\tau_\omega}\right]^2}, \tag{A.24}$$

and

$$\frac{\partial\kappa^{RHS}}{\partial\phi} = \eta_1\eta_2(1-\alpha)\frac{\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)\frac{\tau_z}{\tau_\omega+\tau_z}}{\tau_\theta + \tau_y\kappa^2 + (1-\alpha)\left(\tau_{x\bar{a}}\kappa^2 + \tau_{x\theta}\right) + \frac{\tau_z\tau_\omega}{\tau_z+\tau_\omega}}. \tag{A.25}$$

Thus, with the exception of knife-edge cases in which $\eta_2 = 0$, $\frac{d\kappa}{d\phi} \neq 0$.[37]

Finally, notice that whether $\frac{\partial\kappa^{RHS}}{\partial\phi} \lesseqgtr 0$ always depends *only* on whether $\eta_1\eta_2 \lesseqgtr 0$, and that the sign of $\frac{d\kappa}{d\phi}$ depends *only* on the sign of $\frac{\partial\kappa^{RHS}}{\partial\phi}$ when $\eta_1 > 0$, $\phi \in \left(0, -\eta_2^{-1}\right)$, and $\tau_{x\bar{a}} \to 0$.

(ii) Total differentiation of (A.15) with respect to τ_ω yields

$$\frac{d\kappa}{d\tau_\omega} = \left(1 - \frac{\partial\kappa^{RHS}}{\partial\kappa}\right)^{-1} \frac{\partial\kappa^{RHS}}{\partial\tau_\omega}, \tag{A.26}$$

where

$$\frac{\partial\kappa^{RHS}}{\partial\tau_\omega} = -\eta_1(1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)\tau_z \\ \times \frac{\tau_z + \eta_2\phi\left(\tau_\theta + \left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)(1-\alpha) + \tau_y\kappa^2 + \tau_z\right)}{\left[(\tau_\omega + \tau_z)\left(\tau_\theta + \tau_y\kappa^2 + (1-\alpha)\left(\tau_{x\bar{a}}\kappa^2 + \tau_{x\theta}\right)\right) + \tau_\omega\tau_z\right]^2}. \tag{A.27}$$

This latter expression (and hence $\frac{d\kappa}{d\tau_\omega}$) is equal to zero *iff.* $\tilde{\phi}$ solves

$$\tilde{\phi} = -\eta_2^{-1}\frac{\tau_z}{\tau_\theta + \left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa\left(\tilde{\phi}, \tau_\omega\right)^2\right)(1-\alpha) + \tau_y\kappa\left(\tilde{\phi}, \tau_\omega\right)^2 + \tau_z}. \tag{A.28}$$

But such a value always exists since $\tilde{\phi}$ solves (A.28) from (A.15) *iff.* $\tilde{\phi}$ solves

[37] I here and in the immediately below abstract from the knife-edge case in which $\frac{\partial\kappa^{RHS}}{\partial\kappa} = 1$.

$$\tilde{\phi} = -\eta_2^{-1} \frac{\tau_z}{\eta_1 (1-\alpha) \left(\tau_{x\theta} + \tau_{x\bar{a}} \kappa \left(\tilde{\phi}, \tau_\omega \to \infty\right)^2\right)} \kappa \left(\tilde{\phi}, \tau_\omega \to \infty\right), \tag{A.29}$$

where I have used that the value of κ is independent of τ_ω when $\phi = \tilde{\phi}$ from (A.15), in addition to (A.15) itself. Furthermore, notice that $\kappa \left(\tilde{\phi}, \tau_\omega \to \infty\right)$, and thus the right-hand side of (A.29), is also independent of $\tilde{\phi}$. This ensures that a solution to (A.29) exists.

Thus, with the exception of the case in which $\phi = \tilde{\phi}$, $\frac{d\kappa}{d\tau_\omega} \neq 0$. Lastly, it follows from (A.24) and (A.27) that when $\eta_1 > 0$, $\phi \in \left(0, -\eta_2^{-1}\right)$, and $\tau_{x\bar{a}} \to 0$, $\frac{d\kappa}{d\tau_\omega} \lesseqgtr 0$ if and only if $\phi \lesseqgtr \tilde{\phi}$.

(iii) It remains to show that $\kappa (0, \mathbb{R}_+) \subset \kappa (\mathbb{R}, \tau_\omega \to 0)$. This, however, follows almost immediately since $\lim_{\tau_\omega \to \infty} \kappa (0, \tau_\omega) = \kappa \left(\tilde{\phi}, \tilde{\tau}_\omega\right)$ $\forall \tilde{\tau}_\omega \in \mathbb{R}_+$, including when $\tilde{\tau}_\omega \to 0$. Combined with that κ is continuous and either always increasing or decreasing in τ_ω and ϕ, this demonstrates the last part. □

Proof of Corollary 3. See the Proof of Proposition 2. □

Proof of Lemma 1. The proof follows similar steps to those in Angeletos and Pavan (2007). To start, consider a second-order Taylor expansion of $\mathcal{U} (a, \bar{a}, \sigma_a, \theta, m)$ around $a_i = \bar{a}$:

$$\mathcal{U} (a, \bar{a}, \sigma_a, \theta, m) = \mathcal{U} (\bar{a}, \bar{a}, \sigma_a, \theta, m) + \mathcal{U}_a (a - \bar{a}) + \frac{1}{2} \mathcal{U}_{aa} (a - \bar{a})^2 .$$

Thus,

$$\mathcal{W} (\bar{a}, \sigma_a, \theta, m) = \int u_i dF (a_i) = \mathcal{U} (\bar{a}, \bar{a}, \sigma_a, \theta, m) + \frac{1}{2} \mathcal{U}_{aa} \sigma_a^2. \tag{A.30}$$

Suppose now that $\bar{a} = \hat{a}^\star$ and $\sigma_a = 0$ indeed maximize $\mathcal{W}$. Then, a second-order Taylor expansion of $\mathcal{W}$ around this point provides us with

$$\mathcal{W} (\bar{a}, \sigma_a, \theta, m) = \mathcal{W} \left(\hat{a}^\star, 0, \theta, m\right) + \frac{1}{2} \mathcal{W}_{\bar{a}\bar{a}} \left(\bar{a} - \hat{a}^\star\right)^2 + \frac{1}{2} \mathcal{W}_{\sigma\sigma} \sigma_a^2.$$

It still remains to characterize $\hat{a}^\star$, and to show that $\sigma_a = 0$ maximizes $\mathcal{W}$. The latter, however, follows immediately from (A.30), combined with the assumption that $\mathcal{U}_\sigma = \mathcal{U}_{\sigma\sigma} \sigma_a^2$. By contrast, the former follows from the first-order condition of $\mathcal{W}$ with respect to $\bar{a}$. This shows that

$$\hat{a}^\star = \eta_0^\star + \eta_1^\star \left(\theta + \eta_2^\star m\right),$$

where $\eta_0^\star = -\frac{\mathcal{W}_{\bar{a}}(0,0,0,0)}{\mathcal{W}_{\bar{a}\bar{a}}}$, $\eta_1^\star = -\frac{\mathcal{W}_{\bar{a}\theta}}{\mathcal{W}_{\bar{a}\bar{a}}}$, and $\eta_2^\star = \frac{\mathcal{W}_{\bar{a}m}}{\mathcal{W}_{\bar{a}\theta}} \eta_1^\star$, indeed maximizes $\mathcal{W}$. □

Proof of Proposition 3. The Proposition has two parts.

Part (i): The results in Vives (1993) and Angeletos and Pavan (2009) show that the efficient action within the quadratic-class of games studied is linear in the sufficient statistics $\chi_i = \mathbb{E} [\theta \mid x_{i\theta}, s_i]$, $\gamma = \mathbb{E} \left[\theta \mid s_y\right]$, and ω. As a result, the efficient use of information is characterized by an action of the form:

$$a_i^\star = \eta_0^\star + \eta_1^\star \left(\eta_1^{\star -1} c \chi_i + c_1 \gamma + c_2 \omega\right), \tag{A.31}$$

where $\eta_1^{\star-1}c + c_1 + c_2 = 1 + \eta_2^\star\phi$.[38] All that remains is to characterize the set of coefficients.

To start with this characterization, consider the social welfare function

$$\mathbb{E}[\mathcal{W}] = \mathbb{E}\left[\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right)\right] + \frac{1}{2}\mathcal{W}_{\bar{a}\bar{a}}\mathbb{E}\left[\left(\bar{a} - \hat{a}^\star\right)^2\right] + \frac{1}{2}\mathcal{W}_{\sigma\sigma}\mathbb{E}\left[(a_i - \bar{a})^2\right].$$

Since the first term in this equation is independent of the coefficients (c, c_1, c_2), the maximization of $\mathbb{E}[\mathcal{W}]$ with respect to (c, c_1, c_2) is equivalent to the minimization of the loss function

$$\mathcal{L} = \mathbb{E}\left[\left(\bar{a} - \hat{a}^\star\right)^2\right] + \left(1 - \alpha^\dagger\right)\mathbb{E}\left[(a_i - \bar{a})^2\right], \quad \alpha^\dagger \equiv 1 - \frac{\mathcal{W}_{\bar{a}\bar{a}}}{\mathcal{W}_{\sigma\sigma}}.$$

It now follows from (A.31) and the definition of the sufficient statistics χ_i and γ that

$$\begin{aligned}
a_i - \bar{a} &= c\xi_{i\chi} \\
\bar{a} - \hat{a}^\star &= \eta_1^\star\Big[\left(\eta_1^{\star-1}c + c_1 + c_2 - 1 - \eta_2^\star\phi\right)\theta + c_1\xi_\gamma \\
&\quad + \left(1 - \eta_1^{\star-1}c - c_1\right)(\epsilon_z + \epsilon_\omega) + \eta_2^\star\phi\epsilon_\omega\Big],
\end{aligned}$$

where $\xi_{i\chi} \sim \mathcal{N}\left(0, \left[\tau_{x\theta} + \tau_{x\bar{a}}c^2\right]^{-1}\right)$ and $\xi_\gamma \sim \mathcal{N}\left(0, \left[\tau_\theta + \tau_y c^2\right]^{-1}\right)$.

Thus, the planner's problem can be stated as

$$\begin{aligned}
\min_{(c,c_1)\in\mathbb{R}^2} \mathcal{L} = \frac{1}{\tau_{x\theta} + \tau_{x\bar{a}}c^2}c^2 + \left(1 - \alpha^\dagger\right)\eta_1^{\star 2}\Bigg\{\frac{1}{\tau_\theta + \tau_y c^2}c_1^2 \\
+ \left(1 - \eta_1^{\star-1}c - c_1\right)^2\frac{1}{\tau_z} + \left(1 + \eta_2^\star\phi - \eta_1^{\star-1}c - c_1\right)^2\frac{1}{\tau_\omega}\Bigg\}.
\end{aligned} \tag{A.32}$$

Taking first-order conditions of (A.33), and re-arranging terms, shows that the optimal c is

$$c = \eta_1^\star\left(1 + \eta_2^\star\phi\frac{\tau_z}{\tau_{\omega+\tau_z}}\right)\frac{\left(1 - \alpha^\dagger\right)\left(\tau_{x\theta} + \tau_{x\bar{a}}c^2\right)}{\left(1 - \alpha^\dagger\right)\left(\tau_{x\theta} + \tau_{x\bar{a}}c^2\right) + \left(\tau_\theta + \tau_y c^2 + \frac{\tau_\omega\tau_z}{\tau_\omega+\tau_z}\right)\Delta_1 - \Delta_0}, \tag{A.33}$$

where $\Delta_0 = \left(1 - \alpha^\dagger\right)\left(\tau_{x\theta} + \tau_{x\bar{a}}c^2\right)\left(1 - \eta_1^{\star-1}c + \eta_2^\star\phi\frac{\tau_z}{\tau_{\omega+\tau_z}}\right)^2\tau_y\frac{\tau_\theta+\tau_y c^2}{\tau_\theta+\tau_y c^2+\frac{\tau_\omega\tau_z}{\tau_\omega+\tau_z}} > 0$ and $\Delta_1 = \frac{\tau_{x\theta}}{\tau_{x\theta}+\tau_{x\bar{a}}c^2} \in (0, 1)$, while the optimal values of c_1 and c_2, respectively, are

$$c_1 = \frac{\tau_\theta + \tau_y c^2}{\tau_\theta + \tau_y c^2 + \frac{\tau_\omega\tau_z}{\tau_\omega+\tau_z}}\left(1 - \eta_1^{\star-1}c + \eta_2^\star\phi\frac{\tau_z}{\tau_{\omega+\tau_z}}\right), \quad c_2 = 1 + \eta_2^\star\phi - \eta_1^{\star-1}c - c_1. \tag{A.34}$$

Part (ii): The proof of the second part follows from a simple "guess-and-verify" procedure. Suppose that we can write the team-solution action as a best-response function of the form:

$$a^\star = \mathbb{E}\left[\left(1 - \alpha^\star\right)\hat{a}^\star(\theta, m) + \alpha^\star\bar{a}^\star(\theta, \gamma, \omega) \mid \chi, \gamma, \omega\right], \quad \alpha^\star \in \mathbb{R}. \tag{A.35}$$

Then, identical steps to those in the proof of Corollary 2 show that $a^\star$ satisfies

$$a^\star = \eta_0^\star + \eta_1^\star\left\{\eta_1^{\star-1}h\chi + h_1\gamma + h_2\omega\right\}, \tag{A.36}$$

[38] This ensures that weights sum to those in the efficient full-information case.

where

$$h=\eta_1^\star\left(1+\eta_2^\star\phi\frac{\tau_z}{\tau_\omega+\tau_z}\right)\frac{(1-\alpha^\star)\left(\tau_{x\theta}+\tau_{x\bar{a}}h^2\right)}{\tau_\theta+\tau_y h^2+(1-\alpha^\star)\left(\tau_{x\bar{a}}h^2+\tau_{x\theta}\right)+\frac{\tau_\omega\tau_z}{\tau_\omega+\tau_z}}, \tag{A.37}$$

and

$$h_1=\left(\tau_\theta+\tau_y h^2\right)/\left[\left(1-\alpha^\star\right)\left(\tau_{x\theta}+\tau_{x\bar{a}}h^2\right)\right] \quad \text{and} \quad h_2=1+\eta_2^\star\phi-h-h_1. \tag{A.38}$$

We can now choose $\alpha^\star$ to equate h in (A.37) with c in (A.33). Notice that $\alpha^\star$ will be independent of ϕ. It further follows from (A.34) and (A.38) that $\alpha^\star$ chosen in this manner also ensures that $h_1=c_1$ and $h_2=c_2$. This completes the proof. □

Proof of Corollary 4. Follows from (A.33). □

Appendix B. Optimal policy characterizations

Proof of Proposition 4. The first part of the Proposition follows from a Blackwell-style argument (Blackwell, 1953; Perez-Richet, 2017).[39] The second part then follows from Corollary 1 and Lemma 1. To see the latter, notice that κ is independent of ϕ when $\tau_\omega\to\infty$. Thus, so too is κ_1 and κ_2, and hence the expressions $\mathbb{E}\left(\bar{a}-\hat{a}^\star\right)^2$ and $\mathbb{E}\left(a_i-\bar{a}\right)^2$ in $\mathbb{E}\mathcal{W}$ (see also A43). It follows that ϕ should optimally be set to maximize the efficient, full information term: $\mathbb{E}\mathcal{W}\left(\hat{a}^\star,0,\theta,m\right)$. □

Proof of Proposition 5. The proof follows from a series of examples:

1a) Combined use of Instrument and Disclosure Policy ($\alpha\neq\alpha^\star$): Suppose $\mathcal{U}=-(a_i-r\bar{a}-(1-r)\theta)^2-(m-\theta)^2$, where $r\in(0,1)$, $\tau_z\to\infty$, and $\tau_y=\tau_{x\bar{a}}\to 0$. It follows that $\alpha=r<\alpha^\star=\alpha^\dagger=(2-r)r$ while $\eta=\eta^\star$. Because of the separation between individual actions and the policy instrument in $\mathcal{U}$, we can use the results in Angeletos and Pavan (2007) to show that full disclosure is optimal $\tau_\omega^\star\to\infty$. In fact, full disclosure combined with $\phi^\star=1$ here achieves the unconstrained first best outcome, $\mathcal{W}^\star=0$.

1b) Combined use of Instrument and Disclosure Policy ($\eta\neq\eta^\star$): Suppose instead that $\mathcal{U}=-\frac{1}{2}(a_i-\theta)^2-\frac{1}{2}(\bar{a}-\lambda\theta)^2-(m-\theta)^2$, where $\lambda\in(-1,0)$, $\tau_z\to\infty$, and $\tau_y=\tau_{x\bar{a}}\to 0$. Then, $\alpha=\alpha^\star=\alpha^\dagger=0$, but $\hat{a}_i=\theta$ while $\hat{a}_i{}^\star=\frac{1+\lambda}{2}\theta$. Agents underreact to the fundamental under full information compared to what is optimal $\left(\eta_1<\eta_1^\star\right)$. The separation between individual actions and the policy instrument in $\mathcal{U}$, and the results in Angeletos and Pavan (2007), then once more show that $\tau_\omega^\star\to\infty$ and $\phi^\star=1$.

2) Exclusive use of Instrument Policy: See the Proof of Proposition 6.

3) Exclusive use of Communication Policy: See the Proof of Proposition 7.

4) Use of Neither Instrument nor Communication Policy ($\alpha\neq\alpha^\star$, $\eta\neq\eta^\star$): Consider the cases studied in 1a) and 1b), but now instead suppose that $r\in(-1,0)$, $\lambda\in(0,1)$, and that m only

[39] It can also be obtained directly from the first-order conditions to the optimal policy problem (Appendix D).

enters in $\mathcal{U}$ via the term m^2.[40] Then, similar steps to those in 1a) and 1b) show that $\tau_\omega^\star \to 0$ and $\phi^\star = 0$.[41] □

Proof of Lemma 2. The assumptions made in Section 2 allow us to write:

$$\mathcal{U}(a, \bar{a}, \sigma_a, \theta, m) = V(a, \bar{a}, \theta, m) + \frac{1}{2}\mathcal{U}_{\sigma\sigma}\sigma_a^2,$$

where V is a quadratic polynomial.

A second-order approximation of V around $a = \bar{a} = m = 0$ provides us with

$$\begin{aligned} V(a, \bar{a}, \theta, m) = {} & V(0, 0, \theta, 0) + \mathcal{U}_a a + \mathcal{U}_m m + \mathcal{U}_{\bar{a}}\bar{a} \\ & + \frac{1}{2}\mathcal{U}_{aa}a^2 + \frac{1}{2}\mathcal{U}_{mm}m^2 + \frac{1}{2}\mathcal{U}_{\bar{a}\bar{a}}\bar{a}^2 + \mathcal{U}_{am}am + \mathcal{U}_{a\bar{a}}a\bar{a} + \mathcal{U}_{\bar{a}m}\bar{a}m. \end{aligned} \tag{B.39}$$

The objective is to find values for the derivatives in (B.39) for which V can be stated as

$$\begin{aligned} V(a, \bar{a}, \theta, m) = {} & x_0(a - \delta m)^2 + x_1(\bar{a} - \delta m)^2 + x_2(a - \delta m)(\bar{a} - \delta m) \\ & + x_3(a - \delta m) + x_4(\bar{a} - \delta m). \end{aligned} \tag{B.40}$$

for some $\delta \in \mathbb{R}$ and some coefficient vector $\mathbf{x} = \begin{bmatrix} x_0 & x_1 & x_2 & x_3 & x_4 \end{bmatrix} \in \mathbb{R}^5$.

Expanding terms in (B.40), and matching terms with (B.39), shows that this is case *iff.*

- $\mathcal{U}_{aa} + \mathcal{U}_{\bar{a}\bar{a}} + 2\mathcal{U}_{a\bar{a}} = -\delta^{-2}\mathcal{U}_{mm}$
- $\mathcal{U}_{aa} + \mathcal{U}_{a\bar{a}} = -\delta^{-1}\mathcal{U}_{am}$
- $\mathcal{U}_{\bar{a}\bar{a}} + \mathcal{U}_{a\bar{a}} = -\delta^{-1}\mathcal{U}_{\bar{a}m}$
- $\mathcal{U}_m(0,0,0,0,0) = -\delta(\mathcal{U}_a(0,0,0,0,0) + \mathcal{U}_{\bar{a}}(0,0,0,0,0))$
- $\mathcal{U}_{m\theta} = -\delta(\mathcal{U}_{a\theta} + \mathcal{U}_{\bar{a}\theta})$.

These conditions can alternatively be stated more succinctly as:

$$\begin{aligned} & \mathcal{U}_m(0,0,0,0,0) = -\delta(\mathcal{U}_a(0,0,0,0,0) + \mathcal{U}_{\bar{a}}(0,0,0,0,0)) \\ & \eta_2 = \eta_2^\star = -\frac{\mathcal{W}_{mm}}{\mathcal{W}_{\bar{a}m}} = \delta, \quad \eta_1^\star = \delta\frac{\mathcal{W}_{m\theta}}{\mathcal{W}_{mm}}. \end{aligned}$$

Setting $U = V + \frac{1}{2}\mathcal{U}_{\sigma\sigma}\sigma_a^2$ then completes the proof. □

Proof of Lemma 3. The proof follows from a comparison of the difference between agents' actions and the policy instrument, the term that matters for social welfare under the conditions for Lemma 2.

Consider any *complete opacity policy* (o) with active use of the policy instrument ($\tau_\omega^o \to 0$, $\phi^o \in \mathbb{R}$). As in (4.5) we have that:

$$\begin{aligned} \Delta^o & \equiv a_i - \delta m^o \\ & = \mathbb{E}\left[(1-\alpha)(\eta_0 + \eta_1\theta) + \alpha\bar{a}(\theta, y, \omega) \mid \chi_i, \gamma\right] - \delta\phi^o\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right). \end{aligned} \tag{B.41}$$

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40 That is, I replace the term $(m - \theta)^2$ in both cases with m^2.

41 These examples may lead the reader to conjecture that partial disclosure $\tau_\omega \in \mathbb{R}_+$ can never be optimal. This conjecture is, however, incorrect. For example, consider the simple case in which $\mathcal{U} = -\frac{1}{2}(a_i - m - \theta)^2$ when $\tau_y = \tau_{x\bar{a}} \to 0$. Then, all values of $\tau_\omega \in \mathbb{R}_+$ are optimal if combined with $\phi = -\frac{\tau_z}{\tau_\theta + \tau_{x\theta} + \tau_z}$. In fact, all of these values achieve the same welfare as the full disclosure allocation with $\phi = 0$.

Consider now instead any *partial disclosure policy* (p) with active use of the policy instrument $(\tau_\omega^p \in \mathbb{R}_+, \phi^p \in \mathbb{R})$. The difference between an agent's action and the policy instrument is, in this case,

$$\begin{aligned}\Delta^p &\equiv a_i - \delta m^p \\ &= \mathbb{E}\left[(1-\alpha)(\eta_0 + \eta_1\theta) + \alpha\bar{a}(\theta, y, \omega) \mid \chi_i, \gamma, \omega\right] - \delta\phi^p\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma, \omega\right]\right) \\ &= \mathbb{E}\left[(1-\alpha)(\eta_0 + \eta_1\theta) + \alpha\bar{a}(\theta, y, \omega) \mid \chi_i, \gamma\right] - \delta\phi^p\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right) \\ &+ \beta_{a\omega}\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right) + \beta_{a\omega}\epsilon_\omega + \delta\phi^p\beta_{z\omega}\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right) + \delta\phi^p\beta_{z\omega}\epsilon_\omega,\end{aligned}$$

where I have also used that

$$\mathbb{E}\left[e \mid \chi_i, \gamma, \omega\right] = \mathbb{E}\left[e \mid \chi_i, \gamma\right] + \beta_{e\omega}\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right) + \beta_{e\omega}\epsilon_\omega$$

for e equal to $e = (1-\alpha)(\eta_0 + \eta_1\theta) + \alpha\bar{a}(\theta, y, \omega)$ and $e = z$, respectively. Thus,

$$\begin{aligned}\Delta^p \equiv a_i - \delta m^p = \mathbb{E}\left[(1-\alpha)(\eta_0 + \eta_1\theta) + \alpha\bar{a}(\theta, y, \omega) \mid \chi_i, \gamma\right] \\ + \left(\beta_{a\omega} - \delta\phi^p(1-\beta_{z\omega})\right)\left(z - \mathbb{E}\left[z \mid \chi_i, \gamma\right]\right) + \left(\beta_{a\omega} + \delta\phi^p\beta_{z\omega}\right)\epsilon_\omega.\end{aligned} \tag{B.42}$$

Comparing (B.41) with (B.42) then shows that if $-\delta\phi^o = \beta_{a\omega} - \delta\phi^p(1-\beta_{z\omega})$, then the only difference between (B.41) and (B.42) is the noise term $(\beta_{a\omega} + \delta\phi^p\beta_{z\omega})\epsilon_\omega$, due to the policy-maker's partial disclosure.[42] Squaring this term and taking ex-ante expectations illustrates the additional welfare cost of noisy disclosure – added noise, which is proportional in welfare terms to $(\beta_{a\omega} + \delta\phi^p\beta_{z\omega})^2 \frac{1}{\tau_\omega^p}$.[43] □

Proof of Proposition 6. The proof proceeds in two steps. The first step shows that complete opacity combined with active instrument policy *strictly* dominates any partial or full disclosure policy. The second step then derives an expression for the optimal instrument rule under complete opacity.

Step 1: The proof of Lemma 3 shows that there exists a complete opacity policy combined with active instrument policy $\left(\tau_\omega^o \to 0, \phi^o \in \mathbb{R}\right)$ that strictly dominates any other policy, in which $\tau_\omega \nrightarrow \infty$ or $-\delta\phi \neq \beta_{a\omega}/\beta_{z\omega} = \tilde{\phi}$. But since any policy in which $-\delta\phi = \tilde{\phi}$ or $\tau_\omega \to \infty$ results in a weight on private (and hence public) information equal to the full disclosure weight, $\kappa = \eta_1(1-\alpha)\frac{\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2}{\tau_\theta + \tau_y\kappa^2 + \tau_{x\theta} + \tau_{x\bar{a}}\kappa^2 + \tau_z}$ (cf. Example 1), all that needs to be established is that the full disclosure weight is sub-optimal. This, however, follows immediately from the inefficient use of information ($\alpha \neq \alpha^\star$ or $\eta \neq \eta^\star$) assumed in the proposition. We also conclude that $-\delta\phi^\star \neq \tilde{\phi}$.

Step 2: Consider the expression for social welfare (Lemma 1)

$$\mathbb{E}\mathcal{W} = \mathbb{E}\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right) + \frac{1}{2}\mathcal{W}_{\bar{a}\bar{a}}\mathbb{E}\left(\bar{a} - \hat{a}^\star\right)^2 + \frac{1}{2}\mathcal{W}_{\sigma\sigma}\mathbb{E}\left(a_i - \bar{a}\right)^2.$$

Using the equilibrium coefficients in the proof of Proposition 1 then shows that[44]:

[42] Analogous to the main text, it follows that if $-\delta\phi^o = \beta_{a\omega} - \delta\phi^p(1-\beta_{z\omega})$, then $\kappa^o = \kappa^p$, so that the informativeness of the signals of the average actions are also the same across the two policy cases.

[43] Finally, notice also that $\beta_{a\omega} + \delta\phi^p\beta_{z\omega} = 0$ if and only if $\delta\phi^p = \beta_{a\omega}/\beta_{z\omega} = \tilde{\phi}$.

[44] I throughout this proof ignore irrelevant constant terms in agents' actions and payoffs.

- $a_i - \bar{a} = k_0 \epsilon^i_{x\theta} + k_1 \frac{1}{\kappa} \epsilon^i_{x\bar{a}}$
- $\bar{a} - \hat{a}^\star = \left(k_0 + k_1 + k_2 + k_3 - \eta_1^\star - \eta_2^\star \phi\right) \theta + k_2/\kappa \epsilon_y + (k_3 - \phi \eta_2^\star)\epsilon_z + k_3 \epsilon_\omega,$

where

- $k_0 + k_1 + k_2 + k_3 - \eta_1^\star - \eta_2^\star \phi = \eta_1 - \eta_1^\star + \left(\eta_2 - \eta_2^\star\right) \phi - \frac{\tau_\theta}{(1-\alpha)(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2)} \kappa$
- $k_2 = \frac{\tau_y \kappa^2}{(1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)} \kappa$
- $k_3 = \eta_1 + \eta_2 \phi - \eta_1 \frac{\kappa}{\kappa(\phi=0, \tau_\omega \to 0)}$
- $k_3 - \eta_2^\star \phi = \eta_1 + (\eta_2 - \eta_2^\star)\phi - \eta_1 \frac{\kappa}{\kappa(\phi=0, \tau_\omega \to 0)}.$

Thus,

$$\begin{aligned} \mathbb{E}\mathcal{W} = {} & \mathbb{E}\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right) + \frac{1}{2} \mathcal{W}_{\sigma\sigma} \frac{1}{\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2} \kappa^2 \\ & + \frac{1}{2} \mathcal{W}_{\bar{a}\bar{a}} \left\{ \left(\eta_1 - \eta_1^\star + \left(\eta_2 - \eta_2^\star\right) \phi - \frac{\tau_\theta}{(1-\alpha)(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2)} \kappa \right)^2 \frac{1}{\tau_\theta} \right. \\ & + \frac{\tau_y \kappa^2}{(1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)} \kappa^2 + \left(\eta_1 + (\eta_2 - \eta_2^\star)\phi - \eta_1 \frac{\kappa}{\kappa(\phi = 0, \tau_\omega \to 0)} \right)^2 \frac{1}{\tau_z} \\ & \left. + \left(\eta_1 + \eta_2 \phi - \eta_1 \frac{\kappa}{\kappa(\phi = 0, \tau_\omega \to 0)} \right)^2 \frac{1}{\tau_\omega} \right\}, \end{aligned} \tag{B.43}$$

where I have also used that $k_0^2 \frac{1}{\tau_{x\theta}} + k_1^2 \frac{1}{\tau_{x\bar{a}}\kappa^2} = \frac{1}{\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2} \kappa^2$. It remains to characterize $\mathbb{E}\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right)$.

But now notice that

$$\begin{aligned} \mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right) = {} & \mathcal{U}\left(\hat{a}^\star, \hat{a}^\star, 0, \theta, m\right) \\ = {} & \mathcal{W}_{\bar{a}\theta} \hat{a}^\star \theta + \mathcal{W}_{m\theta} m \theta + \frac{1}{2} \mathcal{W}_{\bar{a}\bar{a}} \hat{a}^{\star 2} + \frac{1}{2} \mathcal{W}_{mm} m^2 + \frac{1}{2} \mathcal{W}_{\bar{a}m} \hat{a}^\star m \\ & + \text{zero-mean terms}, \end{aligned}$$

such that

$$\begin{aligned} \mathbb{E}\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right) = {} & \frac{1}{2} \frac{1}{\tau_\theta \tau_z} \mathcal{W}_{\bar{a}\bar{a}} \left\{ \phi^2 \left(\tau_\theta + \tau_z\right) \left(\mathcal{W}_{\bar{a}m}^2 - \mathcal{W}_{\bar{a}\bar{a}} \mathcal{W}_{mm} \right) \right. \\ & \left. + 2\phi \tau_z \left(\mathcal{W}_{\bar{a}m} \mathcal{W}_{\bar{a}\theta} - \mathcal{W}_{\bar{a}\bar{a}} \mathcal{W}_{m\theta} \right) \right\}, \end{aligned} \tag{B.44}$$

where I have also used the expression for the efficient action in Proposition 3.

The above derivations of social welfare have so far been done for any economy. But now, notice that, for the class of economies covered by Lemma 2, (i) $\eta_2 = \eta_2^\star = \delta$, while (ii) the efficient, full information welfare expression $\mathbb{E}\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right) = 0$. Combining these result with the fact that $\mathbb{E}\mathcal{W}$ in (B.43) is only finite when $\tau_\omega \to 0$ iff. $\eta_2 \phi = \eta_1 - \eta_1 \frac{\kappa}{\kappa(\phi=0, \tau_\omega \to 0)}$, then provides the expression for the optimal instrument rule, when I also condition on the optimal weight on private information ($\kappa = \kappa^\star$). □

Proof of Proposition 7. The proof uses the welfare expressions in (B.44) and (B.43).

The first part of the proposition follows from that if $\eta_2 = \eta_2^\star = \mathcal{U}_{m\theta} = 0$, then

$$\begin{aligned}\mathbb{E}\mathcal{W} = &-\frac{1}{2}\frac{\tau_\theta+\tau_z}{\tau_\theta\tau_z}\mathcal{W}_{\bar{a}\bar{a}}^2\mathcal{U}_{mm}\phi^2 + \frac{1}{2}\mathcal{W}_{\sigma\sigma}\frac{1}{\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2}\kappa^2 \\ &+\frac{1}{2}\mathcal{W}_{\bar{a}\bar{a}}\Bigg\{\left(\eta_1-\eta_1^\star-\frac{\tau_\theta}{(1-\alpha)(\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2)}\kappa\right)^2\frac{1}{\tau_\theta}+\frac{\tau_y\kappa^2}{(1-\alpha)\left(\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2\right)}\kappa^2 \\ &+\left(\eta_1-\eta_1\frac{\kappa}{\kappa(\phi=0,\tau_\omega\to 0)}\right)^2\frac{1}{\tau_z}+\left(\eta_1-\eta_1\frac{\kappa}{\kappa(\phi=0,\tau_\omega\to 0)}\right)^2\frac{1}{\tau_\omega}\Bigg\}.\end{aligned} \tag{B.45}$$

This shows that $\phi^\star = 0$, since changes in ϕ also do not affect κ when $\eta_2 = 0$.

The second part of the proposition then follows straight-forwardly from $\phi^\star = 0$ and the proof of Proposition 7 and Corollary 4 in Angeletos and Pavan (2007).[45] □

Proof of Corollary 5. Consider social welfare in (B.44) and (B.43) if $\eta_2 = \eta_2^\star = 0$,

$$\begin{aligned}\mathbb{E}\mathcal{W} = &-\frac{1}{2}\frac{1}{\tau_\theta\tau_z}\mathcal{W}_{\bar{a}\bar{a}}^2\left\{(\tau_\theta+\tau_z)\mathcal{U}_{mm}\phi^2+2\tau_z\mathcal{U}_{m\theta}\phi\right\} + \frac{1}{2}\mathcal{W}_{\sigma\sigma}\frac{1}{\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2}\kappa^2 \\ &+\frac{1}{2}\mathcal{W}_{\bar{a}\bar{a}}\Bigg\{\left(\eta_1-\eta_1^\star-\frac{\tau_\theta}{(1-\alpha)(\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2)}\kappa\right)^2\frac{1}{\tau_\theta}+\frac{\tau_y\kappa^2}{(1-\alpha)\left(\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2\right)}\kappa^2 \\ &+\left(\eta_1-\eta_1\frac{\kappa}{\kappa(\phi=0,\tau_\omega\to 0)}\right)^2\frac{1}{\tau_z}+\left(\eta_1-\eta_1\frac{\kappa}{\kappa(\phi=0,\tau_\omega\to 0)}\right)^2\frac{1}{\tau_\omega}\Bigg\}.\end{aligned} \tag{B.46}$$

But since κ is unaffected by changes in ϕ when $\eta_2 = 0$, it immediately follows that

$$\phi^\star = \hat{\phi}^\star = -\frac{\tau_z}{\tau_\theta+\tau_z}\frac{\mathcal{U}_{\theta m}}{\mathcal{U}_{mm}} \neq 0.$$

Appendix C. Extended instrument rules

Proof of Lemma 4. The proof relies on the steps used in the proof of Proposition 1 and 3.

Applying the same steps as in *Part (i)* of the proof of Proposition 1 but with (5.1) shows that

$$a_i = \mathbb{E}\left[(1-\alpha)\,\hat{a}+\alpha\bar{a} \mid \chi_i,\gamma,\omega\right],$$

where $\hat{a} = \eta_0 + \eta_1(\psi_\theta,\psi_{\bar{a}})\,\theta$, $\alpha(\psi_{\bar{a}}) = -\frac{\mathcal{U}_{a\bar{a}}+\mathcal{U}_{am}\psi_{\bar{a}}}{\mathcal{U}_{aa}}$, and $\eta_1 = -\frac{\mathcal{U}_{a\theta}+\mathcal{U}_{am}\psi_\theta}{\mathcal{U}_{aa}+\mathcal{U}_{a\bar{a}}+\mathcal{U}_{am}\psi_{\bar{a}}}$.

By contrast, the steps in *Part (i)* and *(ii)* of the proof of Proposition 3 demonstrate that

$$a_i^\star = \mathbb{E}\left[\left(1-\alpha^\star\right)\hat{a}^\star+\alpha^\star\bar{a} \mid \chi_i,\gamma,\omega\right],$$

where $\hat{a}^\star = \eta_0^\star + \eta_1^\star(\psi_\theta,\psi_{\bar{a}})\,\theta$, $\eta_1^\star = -\frac{\mathcal{W}_{\bar{a}\theta}+\mathcal{W}_{\bar{a}m}\psi_\theta}{\mathcal{W}_{\bar{a}\bar{a}}+\mathcal{W}_{\bar{a}m}\psi_{\bar{a}}}$, and $\alpha^\star \in \mathbb{R}$ is once more defined by equating the equilibrium and socially optimal weight on private information.

Thus, it follows that we can always set (i) $\psi_{\bar{a}}$ such that $\alpha(\psi_{\bar{a}}) = \alpha^\star$, and (ii) ψ_θ such that $\eta_1 = \eta_1^\star$.[46] □

[45] Intuitively, full disclosure is optimal if $\alpha^\star = \alpha$. But suppose now that $\alpha^\star > \alpha$. Then, the efficient weight on private information c is below the full disclosure outcome (κ when $\tau_\omega \to \infty$). This, in turn, shows that full disclosure remains optimal in (B.45).

[46] Any discrepancy between η_0 and $\eta_0^\star$ can be corrected by an appropriately set constant term in the policy rule.

Proof of Proposition 8. Lemma 1 shows that the social welfare function equals

$$\mathbb{E}\left[\mathcal{W}\right]=\mathbb{E}\left[\mathcal{W}\left(\hat{a}^{\star},0,\theta,m\right)\right]+\frac{1}{2}\mathcal{W}_{\bar{a}\bar{a}}\mathbb{E}\left[\left(\bar{a}-\hat{a}^{\star}\right)^{2}\right]+\frac{1}{2}\mathcal{W}_{\sigma\sigma}\mathbb{E}\left[\left(a_i-\bar{a}\right)^{2}\right].$$

Now, notice that for each $\tau_\omega \in \mathbb{R}$

- If we set ψ_θ and $\psi_{\bar{a}}$ set such that $a_i = a_i^\star$ for all realizations of (χ_i, γ, ω), then by definition we maximize the latter two terms, which describe welfare effects from dispersed information.
- By contrast, the values of ψ_θ and $\psi_{\bar{a}}$ that maximizes the first term, which describes welfare in the efficient full information case, are given by the solution to:

$$\mathbb{E}\left[\mathcal{W}_{\bar{a}_{|\bar{a}=\hat{a}^\star}}\frac{\partial \bar{a}}{\partial \psi_j}+\mathcal{W}_{\sigma_{|\sigma=0}}\frac{\partial \sigma_a}{\partial \psi_j}+\mathcal{W}_{m_{|\bar{a}=\hat{a}^\star}}\frac{\partial m}{\partial \psi_j}\right]=0$$

  for $j=\{\theta,\bar{a}\}$. Yet the solution to these equations generically differs from the values of ψ_θ and $\psi_{\bar{a}}$ that equate $a_i = a_i^\star$ for all realizations of (χ_i, γ, ω) (cf. the proof of Lemma 4).

Thus, we conclude that an economy (generically) retains the inefficiency of its use of information under the optimal value of ψ_θ and $\psi_{\bar{a}}$, because of the full information role of the policy instrument. □

Proof of Proposition 9. The proof follows the exact same steps as those in the proof of Proposition 8. The only distinction is that I also condition on complete opacity $\tau_\omega \to 0$.[47] □

Appendix D. Optimal policy in the general case

The proof of Proposition 6 combined with the proof of Corollary 1 show that we can write the policymaker's optimal policy problem in general as follows:

$$\begin{aligned}\max_{\phi\in\mathbb{R},\tau_\omega\in\mathbb{R}_+}\ \mathbb{E}\mathcal{W}=&\ \mathbb{E}\mathcal{W}\left(\hat{a}^{\star},0,\theta,m\right)+\frac{1}{2}\mathcal{W}_{\sigma\sigma}\frac{1}{\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2}\kappa^2 \\ &+\frac{1}{2}\mathcal{W}_{\bar{a}\bar{a}}\Bigg\{\left(\eta_1-\eta_1^\star+\left(\eta_2-\eta_2^\star\right)\phi-\frac{\tau_\theta}{(1-\alpha)(\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2)}\kappa\right)^2\frac{1}{\tau_\theta} \\ &+\frac{\tau_y\kappa^2}{(1-\alpha)\left(\tau_{x\theta}+\tau_{x\bar{a}}\kappa^2\right)}\kappa^2+\Bigg(\eta_1+(\eta_2-\eta_2^\star)\phi \\ &-\eta_1\frac{\kappa}{\kappa(\phi=0,\tau_\omega\to 0)}\Bigg)^2\frac{1}{\tau_z} \\ &+\left(\eta_1+\eta_2\phi-\eta_1\frac{\kappa}{\kappa(\phi=0,\tau_\omega\to 0)}\right)^2\frac{1}{\tau_\omega}\Bigg\} \\ &s..t.\end{aligned} \tag{D.47}$$

[47] By contrast, under full disclosure, changes in ϕ would not alter the equilibrium use of information, because the portion of the fluctuations of the policy instrument caused by z would be completely anticipated.

$$\kappa = \eta_1 (1-\alpha) \left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right) \frac{1 + \eta_2 \phi \frac{\tau_z}{\tau_\omega + \tau_z}}{\tau_\theta + \tau_y \kappa^2 + (1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right) + \frac{\tau_\omega \tau_z}{\tau_\omega + \tau_z}} \tag{D.48}$$

$$\begin{aligned}\mathbb{E}\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right) = \frac{1}{2}\frac{1}{\tau_\theta \tau_z}\mathcal{W}_{\bar{a}\bar{a}}\Big\{&\phi^2 (\tau_\theta + \tau_z)\left(\mathcal{W}_{\bar{a}m}^2 - \mathcal{W}_{\bar{a}\bar{a}}\mathcal{W}_{mm}\right) \\ &+ 2\phi\tau_z \left(\mathcal{W}_{\bar{a}m}\mathcal{W}_{\bar{a}\theta} - \mathcal{W}_{\bar{a}\bar{a}}\mathcal{W}_{m\theta}\right)\Big\}.\end{aligned} \tag{D.49}$$

The Lagrangian for this problem is

$$\begin{aligned}\mathcal{L} = \mathbb{E}\mathcal{W} + \lambda \left(\kappa - \eta_1 (1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)\frac{1 + \eta_2\phi\frac{\tau_z}{\tau_\omega + \tau_z}}{\tau_\theta + \tau_y\kappa^2 + (1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right) + \frac{\tau_\omega\tau_z}{\tau_\omega + \tau_z}}\right) \\ + \mu\Bigg(\mathbb{E}\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right) - \frac{1}{2}\frac{1}{\tau_\theta\tau_z}\mathcal{W}_{\bar{a}\bar{a}}\Big\{\phi^2(\tau_\theta + \tau_z)\left(\mathcal{W}_{\bar{a}m}^2 - \mathcal{W}_{\bar{a}\bar{a}}\mathcal{W}_{mm}\right) \\ + 2\phi\tau_z\left(\mathcal{W}_{\bar{a}m}\mathcal{W}_{\bar{a}\theta} - \mathcal{W}_{\bar{a}\bar{a}}\mathcal{W}_{m\theta}\right)\Big\}\Bigg),\end{aligned}$$

where λ and μ denote Lagrange multipliers. Thus, the first-order conditions for this problem are[48]:

$$\partial\mathcal{L}/\partial\tau_\omega : \mathcal{W}_{\bar{a}\bar{a}}\left\{-\left(\eta_1 + \eta_2\phi - \eta_1\frac{\kappa}{\underline{\kappa}}\right)^2\frac{1}{\tau_\omega^2} - \lambda\frac{\partial\kappa^{RHS}}{\partial\tau_\omega}\right\} = 0$$

$$\begin{aligned}\partial\mathcal{L}/\partial\phi : &\frac{\partial\mathbb{E}\mathcal{W}\left(\hat{a}^\star, 0, \theta, m\right)}{\partial\phi} - \lambda\frac{\partial\kappa^{RHS}}{\partial\phi} \\ &+ \mathcal{W}_{\bar{a}\bar{a}}\Bigg\{\left(\eta_1 - \eta_1^\star + \left(\eta_2 - \eta_2^\star\right)\phi - \frac{\tau_\theta}{(1-\alpha)(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2)}\kappa\right)\frac{\eta_2 - \eta_2^\star}{\tau_\theta} \\ &+ \left(\eta_1 + (\eta_2 - \eta_2^\star)\phi - \eta_1\frac{\kappa}{\underline{\kappa}}\right)\frac{\eta_2 - \eta_2^\star}{\tau_z} + \left(\eta_1 + \eta_2\phi - \eta_1\frac{\kappa}{\underline{\kappa}}\right)\frac{\eta_2}{\tau_\omega}\Bigg\} = 0\end{aligned}$$

$$\begin{aligned}\partial\mathcal{L}/\partial\kappa : &\lambda + \mathcal{W}_{\sigma\sigma}\frac{\tau_{x\theta}\kappa}{\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)^2} + \mathcal{W}_{\bar{a}\bar{a}}\Bigg\{-\Bigg(\eta_1 - \eta_1^\star + \left(\eta_2 - \eta_2^\star\right)\phi \\ &- \frac{\tau_\theta}{(1-\alpha)(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2)}\kappa\Bigg)\frac{\tau_{x\theta} - \tau_{x\bar{a}}\kappa^2}{(1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)^2} \\ &+ \frac{2\tau_y\tau_{x\theta}\kappa^3 + \tau_y\tau_{x\bar{a}}\kappa^5}{(1-\alpha)\left(\tau_{x\theta} + \tau_{x\bar{a}}\kappa^2\right)^2} - \left(\eta_1 + (\eta_2 - \eta_2^\star)\phi - \eta_1\frac{\kappa}{\underline{\kappa}}\right)\frac{\eta_1}{\tau_z}\frac{\partial\left(\frac{\kappa}{\underline{\kappa}}\right)}{\partial\kappa} \\ &- \left(\eta_1 + \eta_2\phi - \eta_1\frac{\kappa}{\underline{\kappa}}\right)\frac{\eta_1}{\tau_\omega}\frac{\partial\left(\frac{\kappa}{\underline{\kappa}}\right)}{\partial\kappa}\Bigg\} = 0,\end{aligned}$$

where I have defined $\underline{\kappa} \equiv \kappa(\phi = 0, \tau_\omega \to 0)$ and κ^{RHS} denotes the right-hand side of (D.48).

Remark 1. The solution to the policymaker's problem $\phi^\star$ and $\tau_\omega^\star$ solves the three first-order conditions.

[48] The sufficiency of the first-order conditions follows from the strict pseudo-concavity of the welfare function $\mathbb{E}\mathcal{W}$. Thus, if a solution to the first-order conditions exists, it describes the unique optimal policy.

Remark 2. (*Exclusive Use of Instrument Policy*) Let $\phi = \phi^\star$, then it follows that the *necessary and sufficient* condition for $\tau_\omega^\star \to 0$ is $\frac{\partial \mathcal{L}}{\partial \tau_\omega} |_{\phi=\phi^\star, \tau_\omega \to 0} < 0$. Furthermore, it follows from (D.47) that, in this case, the optimal value of the policy instrument solves $\phi^\star = \frac{\eta_1}{\eta_2} \left(1 - \frac{\kappa(\phi=\phi^\star, \tau_\omega^\star \to 0)}{\kappa(\phi=0, \tau_\omega \to 0)}\right)$.

Remark 3. (*Exclusive Use of Communication Policy*) By contrast, let $\phi = 0$ and $\tau_\omega = \tau_\omega^\star$. Then, it follows that the *necessary and sufficient* condition for $\phi^\star = 0$ is $\frac{\partial \mathcal{L}}{\partial \phi} |_{\phi=0, \tau_\omega = \tau_\omega^\star} = 0$.

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