

*Online Appendix to:*  
**“An Informational Rationale for Action over Disclosure”**

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## Contents of Online Appendix:

This *Online Appendix* details additional extensions of the models analyzed in the paper.

*Appendix C* considers four variations of the baseline model; two pertain to changes in the payoff structure, one to the information structure, and one to the shock structure. These are: *(i)* the introduction of direct strategic complementarity; *(ii)* the existence of inefficient disturbances; *(iii)* the presence of idiosyncratic noise in the policy maker’s instrument rule combined with a noisy signal of the policy instrument; and *(iv)* binary signals combined with a beta distributed fundamental. The analysis confirms that the superiority of instrument policy does not depend unduly on either of these extensions. Combined, they provide the necessary bridge between the results from our baseline model and those from the business cycle applications considered in the paper in addition to in this *Online Appendix*.

*Appendix D* considers the micro-founded business cycle model analyzed in Section 5 but supposes instead that *(a)* both firms and workers make their employment decisions under incomplete dispersed information; that *(b)* the policy maker corresponds to the economy-wide tax authority; and that *(c)* firms and workers directly learn from current taxes about expected future tax rates. Unlike in the main text, equilibrium prices here also clear commodity markets. The model provides an important example of an economy in which both firm and worker choices are *conditionally efficient*, despite the presence of direct strategic interactions and incomplete dispersed information. The learning externality is the only inefficiency in the use of information. All results therefore mirror those from the baseline model. I also in this *Appendix* consider an additional extension where firms always know their own productivity level before setting prices in the business cycle model analyzed in Section 5. I show how all results continue to hold.

## Appendix C: Extensions and Variations

This Appendix discusses various extensions and variations of the baseline model.

### C. 1 Direct Strategic Complementarity (and the Crudeness of Policy)

An important extension of the baseline model is the introduction of direct strategic complementarity. As [Morris and Shin \(2002\)](#) first made clear, the presence of direct strategic complementarity has important consequences for the desirability of public information disclosure. [Angeletos and Pavan \(2007\)](#) subsequent contribution demonstrates that whether additional public information is detrimental or beneficial for welfare depends critically on whether the “equilibrium degree of coordination” between individual actions is above or below the “socially optimal degree”.

Suppose, for instance, that there is too little coordination between people’s actions compared to what is socially optimal. Releasing additional public information, in this case, helps people better coordinate their actions as it informs them about the activities of others. An extension worthwhile exploring is thus whether our preference for instrument over communication policy extends to situations like these with *insufficient payoff coordination*. This extension will also demonstrate how our main results extend to cases where both communication and instrument policy are “crude”, in the sense that they attempt to use the policy maker’s information to attain multiple, rival objectives. The following utility function delivers such an instance,<sup>1</sup>

$$U_i = -\mathbb{E}_i [a_i - (1 - r)(\theta - m) - r\bar{a}]^2, \quad \bar{a} = \int_0^1 adi, \quad (1)$$

$$\propto -\mathbb{E}_i [\bar{a} - (\theta - m)]^2 - \left(\frac{1}{1 - r}\right)^2 \mathbb{E}_i [a_i - \bar{a}]^2, \quad (2)$$

when  $r \in (0, 1)$  and the social loss function equals  $\mathcal{W} = -\mathbb{E} \int_0^1 U_i di$ . The difference between the equilibrium degree of payoff coordination and the socially optimal is  $(r - 1)r < 0$ .<sup>2</sup>

In the absence of instrument policy and keeping the informativeness of the economy-wide outcome constant, (1) makes public information disclosure invariably beneficial. Nevertheless, despite this counteracting motive, the combination of complete opacity and active instrument policy is here still optimal. The presence of insufficient payoff coordination does not overturn the superiority of instrument policy (*cf.* [James and Lawler, 2011](#)).

**Proposition.** *The optimal policy is complete opacity,  $\tau_\omega^* \rightarrow 0$ , combined with active instrument policy,  $\phi_r^* \in \mathbb{R}_+$ . The informativeness of the endogenous public signal,  $a$ , under the optimal policy can be either above or below that under full disclosure,  $k_{0,r}^* \gtrless k_{0,r}^d$ .*

In fact, instrument policy has two separate advantages over communication policy in (1) – the same two advantages as detailed in Section 5. First, it is able to control the emphasis on private and policy maker information without adding noise to the information structure, the

<sup>1</sup>This utility function is also used in, for instance, [Hellwig and Veldkamp \(2009\)](#) and [Hellwig et al. \(2012\)](#).

<sup>2</sup>The equilibrium degree of payoff coordination equals  $r$ ; the socially optimal is  $(2 - r)r$ .

advantage from the baseline model. And second, it can increase the equilibrium emphasis on the policy maker's information, even beyond the full disclosure case. (Note that increases to  $\phi$  can arbitrarily increase the loading on  $z$  in (4.3).) This allows the policy maker to create even more coordination than that achieved with full disclosure.

While the first advantage is present irrespective of parameter values, the second depends on whether the learning externality dominates the lack of payoff coordination or *vice versa*. The learning externality, all else equal, causes the weight on policy maker information under full disclosure to be *above* the socially optimal. This induces the policy maker to set  $\phi_r^* < \hat{\phi}_r$  and  $\tau_\omega \rightarrow 0$ , where  $\phi = \hat{\phi}_r$  replicates the full disclosure outcome, such as to increase the relative emphasis on private information  $k_{0,r}^* > k_{0,r}^d$  but without the introduction of noise to the information structure. The lack of payoff coordination, by contrast, has almost the exact opposite implication. It causes, all else equal, the weight on policy maker information under full disclosure to be *below* the socially optimal. That is, to alleviate the lack of payoff coordination, we need less use of private information than under full disclosure. The best means for the policy maker to achieve an outcome where  $k_{0,r}^* < k_{0,r}^d$  is, however, still only with instrument policy. Only with  $\phi_r^* > \hat{\phi}_r$  and  $\tau_\omega \rightarrow 0$  can the policy maker increase coordination beyond the full disclosure case and also avoid the added noise from communication policy.<sup>3</sup>

Thus, while our basic rationale for instrument policy is unaffected by the presence of direct strategic complementarity, its existence does provide instrument policy with yet another advantage. When the learning externality is relatively strong ( $k_{0,r}^* > k_{0,r}^d$ ), the exclusive use of instrument policy is optimal *only* because it avoids the introduction of noise to the information structure. When the lack of payoff coordination, by contrast, is relatively severe ( $k_{0,r}^* < k_{0,r}^d$ ), the sole use of instrument policy is optimal *partially also* because it can increase coordination beyond the full disclosure case. By comparing  $k_{0,r}^*$  with  $k_{0,r}^d$ , one can determine the existence of the second advantage of instrument over communication policy.<sup>4</sup>

**Proof of Proposition:** Using (1), an individual's decision rule becomes,

$$a_i = \mathbb{E}_i [(1 - r)(\theta - m) + r\bar{a}]. \quad (3)$$

Applying a similar *guess and verify procedure* to that in Subsection 2.2 now shows that the

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<sup>3</sup>Surprisingly, full disclosure combined with active instrument policy is here not optimal. Under full disclosure, the policy maker cannot increase the equilibrium loading on  $z$  above the full disclosure case. The policy instrument is fully known under full disclosure. So the policy maker cannot use expected instrument policy to further increase the amount of coordination. Only under partial- or complete opacity can the policy maker increase equilibrium coordination beyond the full disclosure case. Furthermore, because the value of added payoff coordination depends on the coefficient  $r$ , there can exist a value of  $r \in (0, 1)$  such that  $k_{0,r}^* = k_{0,r}^d$ . With the exception of this knife-edge case, however, instrument policy is *uniquely optimal*.

<sup>4</sup>The alternate case of  $r \in (-1, 0)$  creates *direct strategic substitutability*. It details the inherently less interesting case where the equilibrium degree of coordination is always above the socially optimal one – both because of the learning externality and the excess payoff coordination. The learning externality, in this instance, therefore merely reinforces the conclusions from the setup with only direct strategic substitutability and  $k_0^* > k_0^d$ .

unique linear Bayesian equilibrium action for person  $i$  in this economy equals,<sup>5</sup>

$$a_i = k_0 x_i + k_1 y + k_2 \omega, \quad y = \theta + \frac{1}{k_0} \epsilon_a,$$

where  $k_0 \in (0, 1)$  and the coefficients solve the equations,

$$k_0 = (1-r)\tau_x \frac{\tau_\omega + (1-\phi)\tau_z}{(\tau_z + \tau_\omega) [\tau_x(1-r) + \tau_a k_0^2] + \tau_z \tau_\omega}, \quad k_1 = \frac{\tau_a k_0^2}{\tau_x} \frac{1}{1-r} k_0 \quad (4)$$

$$k_2 = \tau_\omega \frac{\tau_z - \phi [\tau_x(1-r) + \tau_a k_0^2 + \tau_z]}{(\tau_z + \tau_\omega) [\tau_x(1-r) + \tau_a k_0^2] + \tau_z \tau_\omega}. \quad (5)$$

To demonstrate the optimality of complete opacity combined with active instrument policy, I follow the same three step procedure as in the Proof of Theorem 1.

*Step 1:* Repeated use of the equilibrium coefficients shows that the term which matters for social welfare equals,

$$\begin{aligned} \Delta_i^r &= a_i - (1-r)(\theta - m) - r \int_0^1 a_i di, \\ &= k_0 \epsilon_x^i + \frac{k_1}{k_0} (1-r) \epsilon_a + (\phi + k_2) (1-r) \epsilon_z + k_2 (1-r) \epsilon_\omega. \end{aligned}$$

Thus, after a few, simple derivations, also using that  $\sum_{j=1}^3 k_j = 1 - \phi$ ,

$$\mathcal{W}_r = -\mathbb{E}[U_i] = \frac{\tau_x + \tau_a k_0^2}{\tau_x^2} k_0^2 + \left[ \frac{\tau_x(1-r) + \tau_a k_0^2}{\tau_x} k_0 - (1-r) \right]^2 \frac{1}{\tau_z} + k_2^2 (1-r)^2 \frac{1}{\tau_\omega}. \quad (6)$$

*Step 2 and 3:* Consider now the only term that depends on  $\tau_\omega$ ,  $k_2^2 (1-r)^2 \frac{1}{\tau_\omega}$ . Because this term is similar to that from Theorem 1, it once more follows that complete opacity combined with active instrument policy is uniquely optimal *iff.*  $k_{0,r}^* \equiv \arg \min_{k_0} \mathcal{W}_r(k_0, \tau_\omega \rightarrow 0) \neq k_{0,r}^d$ , where  $k_{0,r}^d$  uniquely solves  $k_{0,r}^d = \frac{\tau_x(1-r)}{\tau_x(1-r) + \tau_a (k_{0,r}^d)^2 + \tau_z}$ . All that remains is to characterize  $k_{0,r}^*$ . Minimizing  $\mathcal{W}_r$  when  $\tau_\omega \rightarrow 0$  shows that  $k_{0,r}^* > 0$  equals the unique solution to,<sup>6</sup>

$$k_{0,r}^* = D(k_{0,r}^*) \equiv \frac{\tau_x(1-r)}{\tau_x(1-r) + \tau_a k_{0,r}^{*2} + \tau_z - \tau_z \left[ \frac{\tau_a k_{0,r}^{*2} - r\tau_x}{\tau_x(1-r) + 3\tau_a k_{0,r}^{*2}} \right]} \geq k_{0,r}^d, \quad (7)$$

which is implemented by  $\phi_r^* = 1 - \frac{\tau_x(1-r) + \tau_a k_{0,r}^{*2}}{\tau_x(1-r)} k_{0,r}^* \leq \frac{\tau_z}{\tau_x(1-r) + \tau_a k_0^2 + \tau_z} \equiv \hat{\phi}_r$ . When  $r$  is small, (7) shows that the baseline result continues to hold and  $k_{0,r}^* > k_{0,r}^d$ . However, comparing the fixed-point equations for  $k_{0,r}^*$  and  $k_{0,r}^d$  shows that the right-hand side of the equation for  $k_{0,r}^d$  is

<sup>5</sup>I derive  $a_i$  in terms of  $y$  instead of  $a$ , to simplify the subsequent derivations.

<sup>6</sup>The uniqueness of  $k_{0,r}^*$  follows from the strict pseudo-convexity of  $\mathcal{W}_r$  when  $\tau_\omega \rightarrow 0$ . A solution to  $\frac{d\mathcal{W}_r}{dk_0} = 0$  [or (7)] is therefore the unique global minimum. But such a solution always exists since  $D(0) > 0$  and  $\lim_{k_0 \rightarrow \infty} D(k_0) = 0$ , which combined with the continuity of  $D$  ensures a crossing with the 45°-line at  $k_{0,r}^* > 0$ .

greater than  $D(k_{0,r}^d)$  whenever  $r\tau_x > \tau_a (k_{0,r}^d)^2$  – or equivalently when  $r > \tau_a \frac{\tau_x(1-r)^2}{(\tau_x+\tau_z)^2}$ .<sup>7</sup> Thus, if there exists values of  $r \in \left[ \tau_a \frac{\tau_x(1-r)^2}{(\tau_x+\tau_z)^2}, 1 \right)$ , we have that  $k_{0,r}^* \leq k_{0,r}^d$  since the crossing with the 45°-line is in both cases from above (see (7) and the definition of  $k_{0,r}^d$ ). While complete opacity combined with active instrument policy is always optimal, there can exist a value of  $r \in (0, 1)$  such that full disclosure also attains the welfare optimum. □

## C.2 Inefficient Disturbances

Perhaps a more immediate concern is how our results extend to situations where responses to the fundamental are inefficient even under full information, as with mark-up shocks in business cycle models. Hellwig (2005) and Angeletos and Pavan (2007) demonstrate how the optimality of public information disclosure depends critically on the efficiency of the underlying disturbance. The following utility function delivers an example of an *inefficient disturbance*,<sup>8</sup>

$$U_i = -\frac{1}{2}\mathbb{E}_i [a_i - (\theta - m)]^2 - \frac{1}{2}\mathbb{E}_i [\bar{a} - (\lambda\theta - m)]^2, \quad \lambda \in (0, 1). \quad (8)$$

where a person’s optimal action still equals  $a_i = \mathbb{E}_i [\theta - m]$  and  $\mathcal{W} = -\mathbb{E} \int_0^1 U_i di$ . Here, even if people perfectly observe the fundamental, responses to it are still suboptimal: If  $\theta \in \Omega_i$ , the equilibrium response is  $a_i^{eq} = \theta$ , while the socially optimal is  $a_i^{fb} = \frac{1+\lambda}{2}\theta$ . People overreact to the fundamental compared to what is socially optimal. This excess response, in turn, does not follow from any inefficiency in the amount of payoff coordination. The model, like the baseline version, has *efficient payoff coordination*: the equilibrium degree of payoff coordination equals the socially optimal one.<sup>9</sup> Indeed, the source of inefficiency is entirely due to  $\lambda < 1$ .

The inefficiency of the fundamental disturbance provides yet another reason for complete opacity. Even when the instrument is mute and keeping the informativeness of the economy-wide outcome constant, the optimal policy is  $\tau_\omega \rightarrow 0$  when  $\lambda < \frac{\tau_x}{\tau_\theta + \tau_x + \tau_a k_0^d}$ . Providing additional information, in this case, merely exacerbates the excess response to the fundamental, making any disclosure suboptimal (see also Angeletos and Pavan, 2007).

Once we allow for the use of the instrument and internalize the informativeness of the economy-wide outcome, it is thus hardly surprising that the fully optimal policy once more features complete opacity combined with active instrument policy.<sup>10</sup>

<sup>7</sup>The latter follows from the fixed-point equation for  $k_{0,r}^d$ :  $r\tau_x > \tau_a (k_{0,r}^d)^2$  iff.  $k_{0,r}^d > \frac{\tau_x(1-r)}{\tau_x+\tau_z}$ . But because the unique crossing between the right-hand side of the equation for  $k_{0,r}^d$  and the 45°-line is from above,  $k_{0,r}^d > \frac{\tau_x(1-r)}{\tau_x+\tau_z}$  iff.  $r > \tau_a \frac{\tau_x(1-r)^2}{(\tau_x+\tau_z)^2}$ , where I have used the right-hand side of the equation for  $k_{0,r}^d$  evaluated at  $\frac{\tau_x(1-r)}{\tau_x+\tau_z}$ .

<sup>8</sup>I assume in (8) that  $\theta \sim N(0, 1/\tau_\theta)$ . This ensures that utility is bounded.

<sup>9</sup>In both cases, equal to zero.

<sup>10</sup>The alternate case where  $\lambda > 1$  details a situation where people respond too little to the fundamental. In this case, the optimal policy is once more complete opacity combined with active instrument policy. But like in the “*Direct Strategic Complementarity*” case, there could now be two root causes of the superiority of instrument policy. Depending on parameter values, it could in part be driven by instrument policy’s ability to create a greater reaction to  $z$  than that achieved with full disclosure. This excess reaction then, in turn, partially alleviates people’s insufficient response to the fundamental.

**Proposition.** *The unique optimal policy is complete opacity,  $\tau_\omega^* \rightarrow 0$ , combined with active instrument policy,  $\phi_\lambda^* < \hat{\phi}$ . The informativeness of the endogenous public signal,  $a$ , under the optimal policy is always greater than that under full disclosure,  $k_{0,\lambda}^* > k_0^d$ .*

The added rationale combines with that identified in the baseline model to make complete opacity uniquely optimal. Furthermore, because of the excess response to the fundamental, the optimal level of instrument policy  $\phi_{\lambda \in (0,1)}^* < \phi_{\lambda=1}^*$ . A smaller  $\lambda$ , all else equal, increases the excess response to the fundamental. But this optimally necessitates less use of the policy maker's information, to counteract the excess response, and thus a smaller value of  $\phi_\lambda^*$ . Under the optimal policy, it therefore still holds that  $\phi_{\lambda \in (0,1)}^* < \hat{\phi}$  (and thus that  $k_{0,\lambda}^* > k_0^d$ ) since  $\phi_{\lambda=1}^* < \hat{\phi}$  because of the learning externality.

**Proof of Proposition:** Since an individual's decision rule still equals  $a_i = \mathbb{E}_i[\theta - m]$ , it follows from Proposition 1 that,

$$a_i = k_0 x_i + k_1 y + k_2 \omega, \quad y = \theta + \frac{1}{k_0} \epsilon_a,$$

where the coefficients equal the unique solutions to,

$$k_0 = \tau_x \frac{\tau_\omega + (1 - \phi)\tau_z}{(\tau_z + \tau_\omega) [\tau_\theta + \tau_x + \tau_a k_0^2] + \tau_z \tau_\omega}, \quad k_1 = \frac{\tau_a k_0^2}{\tau_x} k_0 \quad (9)$$

$$k_2 = \tau_\omega \frac{\tau_z - \phi [\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z]}{(\tau_z + \tau_\omega) [\tau_\theta + \tau_x + \tau_a k_0^2] + \tau_z \tau_\omega}. \quad (10)$$

I once more show that complete opacity combined with active instrument policy is optimal by using the same three step procedure as in the Proof of Theorem 1.

*Step 1:* Repeatedly using the equilibrium coefficients shows that,

$$\begin{aligned} \mathcal{W}_\lambda &= -\mathbb{E}[U_i] \quad (11) \\ &= \left( \lambda + \frac{\tau_\theta}{\tau_x} k_0 - 1 \right)^2 \frac{1}{\tau_\theta} + \left( \frac{\tau_\theta + \tau_x + 2\tau_a k_0^2}{\tau_x^2} \right) k_0^2 + 2 \left( \frac{\tau_\theta + \tau_x + \tau_a k_0^2}{\tau_x} k_0 - 1 \right)^2 \frac{1}{\tau_z} + 2k_2^2 \frac{1}{\tau_\omega}. \end{aligned}$$

*Step 2 and 3:* Since the only term that depends on  $\tau_\omega$ ,  $k_2^2 \frac{1}{\tau_\omega}$ , is almost identical to that from Theorem 1, it once more follows that complete opacity combined with active instrument policy is uniquely optimal *iff.*  $k_{0,\lambda}^* \equiv \arg \min_{k_0} \mathcal{W}_\lambda(k_0, \tau_\omega \rightarrow 0) \neq k_0^d$ , where  $k_0^d$  uniquely solves  $k_0^d = \frac{\tau_x}{\tau_\theta + \tau_x + \tau_a (k_0^d)^2 + \tau_z} > 0$ . All that remains is to show that  $k_{0,\lambda}^*$  differs from  $k_0^d$ . Minimizing (11) when  $\tau_\omega \rightarrow 0$  illustrates that  $k_{0,\lambda}^* > 0$  equals the unique solution to,<sup>11</sup>

$$k_{0,\lambda}^* = D(k_{0,\lambda}^*) \equiv \frac{\tau_x + \frac{(1-\lambda)\tau_x}{2\tau_\theta + 2\tau_x + 6\tau_a k_{0,\lambda}^{*2}}}{\tau_\theta + \tau_x + \tau_a k_{0,\lambda}^{*2} + \tau_z \left( \frac{2\tau_\theta + \tau_x + 4\tau_a k_{0,\lambda}^{*2}}{2\tau_\theta + 2\tau_x + 6\tau_a k_{0,\lambda}^{*2}} \right)} > k_0^d. \quad (12)$$

<sup>11</sup>The uniqueness of  $k_{0,\lambda}^*$  once more follows from the strict pseudo-convexity of  $\mathcal{W}_\lambda$  when  $\tau_\omega \rightarrow 0$ . A solution to  $\frac{d\mathcal{W}_\lambda}{dk_0} = 0$  [or (12)] is therefore the unique global minimum. However, such a solution  $k_{0,\lambda}^* > 0$  always exists since  $D(0) > 0$  and  $\lim_{k_0 \rightarrow \infty} D(k_0) = 0$ , which combined with the continuity of  $D$  ensures a crossing with the 45°-line at a positive point.

Since  $\frac{dk_0}{d\phi} < 0$ , it follows that  $\phi_\lambda^* < \hat{\phi} \equiv \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z}$ , where  $\hat{\phi}$  replicates the full disclosure outcome,  $k_0^d$ . Last,  $\frac{dk_0}{d\lambda} < 0$  from (12). □

### C.3 Noisy, Observable Instruments

Last, an equally pressing extension is to abandon a specific assumption that I made for tractability purposes. Specifically, that because of our one-shot model people cannot learn from the history of past and current instruments about the policy maker's beliefs. I here illustrate why this assumption does not critically affect our main results; I also discuss in more detail why the noise disturbance  $\epsilon_m$  does not affect our main rationale for instrument policy. *Online Appendix D.1* below expands on this robustness towards the *signaling role of policy instruments* by studying a micro-founded business cycle model in which agents directly observe a policy instrument.

A way to explore the effect of adding past and current instruments to agents' information sets still within the confines of our framework is to extend the baseline model with an additional public signal  $s$ ,

$$s = \phi z + \epsilon_s, \quad m = \phi z + \epsilon_m, \quad (13)$$

where  $\epsilon_s \sim \mathcal{N}(0, 1/\tau_s)$  is independent of all other stochastic disturbances. The signal  $s$  provides a crude summary measure that substitutes for the information that in a dynamic setting could be contained in past and current instruments (see *Online Appendix D.1* for an example). When  $\phi$  is relatively large,  $s$  is very informative about the policy maker's beliefs. This corresponds to the situation where past and current instruments provide a lot of information about the policy maker because changes to his instrument to, a large extent, reveal his beliefs. Because of  $\epsilon_s$ , however, the summary measure does not perfectly reveal  $z$ . This is important; it allows communication policy to still provide additional information about the policy maker's beliefs.

While this extension provides additional information about the policy maker's beliefs it, however, does not meaningfully alter our results.

**Proposition.** *The unique optimal policy is still complete opacity,  $\tau_\omega^* \rightarrow 0$ , combined with active instrument policy,  $\phi^* < \hat{\phi}$ . The informativeness of the endogenous public signal,  $a$ , under the optimal policy is always greater than that under full disclosure,  $k_0^* > k_0^d$ .*

While  $s$  provides a lower-bound to the knowledge about the policy maker's beliefs, it does not alter that decreases in  $\phi$  from  $\hat{\phi}$  increase the informativeness of the economy-wide outcome, alleviating the consequences of the learning externality. Increases in  $\tau_s$  do, of course, diminish the benefit of decreasing  $\phi$  below  $\hat{\phi}$ . Decreases in  $\phi$  come at a more substantial cost in terms of decreasing the knowledge about the policy makers beliefs. But for a finite  $\tau_s \in \mathbb{R}_+$ , it is still optimal to rely solely on instrument policy since it avoids the introduction of additional noise.

Let me now briefly turn to why the noise disturbance  $\epsilon_m$  in (13) also does not affect our basic rationale for instrument policy. The reason is that the welfare costs from  $\epsilon_m$  do not influence the welfare benefits of changes to  $\phi$  – precisely as in standard monetary policy rules. Decreases

in  $\phi$  still directly affect the stability of the effective state of the economy, and hence people's incentives to rely on their own private information. And they still do so without the *introduction of additional noise* to the information structure, unlike communication policy. The presence of the added disturbance  $\epsilon_m$  therefore does not influence the characteristics of the optimal policy.

**Proof of Proposition:** The unique linear Bayesian equilibrium action for person  $i$  now equals,

$$a_i = k_0 x_i + k_1 y + k_2 \tilde{s} + k_3 \omega, \quad y = \theta + \frac{1}{k_0} \epsilon_a,$$

where  $\tilde{s} \equiv z + \frac{1}{\phi} \epsilon_s$  and the coefficients solve the equations,

$$k_0 = \tau_x \frac{\tau_\omega + (1 - \phi) \tau_z + \tau_s \phi^2}{(\tau_\omega + \tau_z) (\tau_x + \tau_a k_0^2) + \tau_\omega \tau_z + \tau_s \phi^2 (\tau_x + \tau_a k_0^2 + \tau_z)}, \quad k_1 = \frac{\tau_a k_0^2}{\tau_x} k_0 \quad (14)$$

$$k_2 = \tau_s \phi^2 \frac{\tau_z - \phi (\tau_x + \tau_a k_0^2 + \tau_z)}{(\tau_\omega + \tau_z) (\tau_x + \tau_a k_0^2) + \tau_\omega \tau_z + \tau_s \phi^2 (\tau_x + \tau_a k_0^2 + \tau_z)}, \quad k_3 = \frac{\tau_\omega}{\tau_s \phi^2} k_2. \quad (15)$$

*Approach:* I proceed in four steps: First, I derive an expression for the equilibrium welfare loss. Second, I use that expression to show how complete opacity combined with active instrument policy can achieve a strictly lower welfare loss than that any *partial disclosure* policy where  $\phi \neq \hat{\phi} = \frac{\tau_z}{\tau_x + \tau_a k_0^2 + \tau_z}$  and  $\tau_\omega$  is finite. Third, I show that those policies where  $\phi = \hat{\phi}$  achieve the same level of welfare as full disclosure. Last, I show how the best complete opacity policy dominates the full disclosure case and characterize the optimal policy.

*Step 1: Equilibrium Welfare:* Using the equilibrium coefficients repeatedly demonstrates that,

$$\mathcal{W}_s = \frac{1}{2} \left[ \frac{\tau_x + \tau_a k_0^2}{\tau_x^2} k_0^2 + \left( \frac{\tau_x + \tau_a k_0^2}{\tau_x} k_0 - 1 \right)^2 \frac{1}{\tau_z} + \left( \frac{k_2}{\phi} \right)^2 \frac{1}{\tau_s} + k_3^2 \frac{1}{\tau_\omega} + \frac{1}{\tau_m} \right]. \quad (16)$$

*Step 2 and 3: The Weak Optimality of Opacity:* Once more applying the equilibrium coefficients in (14) and (15) repeatedly shows that,<sup>12</sup>

- All *partial disclosure* policies where  $\phi^p \neq \hat{\phi}$  and  $\tau_\omega^p$  finite are strictly dominated in welfare terms by the *complete opacity* policy  $\phi^o = \beta_\theta + \phi^p (1 - \beta_z)$ , where  $\beta_\theta$  and  $\beta_z$  denote the

<sup>12</sup>Alternatively, consider  $\Delta_i$  under *partial disclosure*. Adjusted for the change in the information structure and the instrument rule, this expression equals when  $\phi^p \neq \hat{\phi}$  and  $\tau_\omega^p$  is finite,

$$\Delta_i^p = \mathbb{E}_i^o [\theta] - \theta + [\beta_\theta + \phi^p (1 - \beta_z)] (z - \mathbb{E}_i^o [\theta]) + (\beta_\theta - \phi^p \beta_z) \epsilon_\omega + \epsilon_m, \quad (17)$$

where  $\mathbb{E}_i^o [\theta]$  denotes the expectation of  $\theta$  based on the complete opacity information set,  $\Omega_i^o = \{x_i, y, \tilde{s}\}$ , and I have once more used the decomposition  $\mathbb{E}_i^p [\cdot] = \mathbb{E}_i^o [\cdot] + \beta (\omega - \mathbb{E}_i^o [\cdot])$  for both  $\mathbb{E}_i^p [\theta]$  and  $\mathbb{E}_i^p [z]$ . Now consider the expression for  $\Delta_i$  under *complete opacity*,

$$\Delta_i^o = \mathbb{E}_i^o [\theta] - \theta + \phi^o (z - \mathbb{E}_i^o [\theta]) + \epsilon_m. \quad (18)$$

Comparing (17) and (18) shows that when  $\phi^o = \beta_\theta + \phi^p (1 - \beta_z)$ ,

$$\mathcal{W}(\phi^p, \tau_\omega^p) - \mathcal{W}(\phi^o, \tau_\omega \rightarrow 0) = (\beta_\theta - \phi^p \beta_z)^2 \frac{1}{2\tau_\omega^p} > 0 \quad \forall \phi^p \neq \frac{\beta_\theta}{\beta_z} = \hat{\phi}.$$

But when  $\phi^p = \hat{\phi}$ ,  $\mathcal{W}(\phi^p, \tau_\omega^p) = \mathcal{W}(\hat{\phi}, \tau_\omega \rightarrow 0) = \mathcal{W}(\phi \in (0, 1), \tau_\omega \rightarrow \infty)$ .



projection coefficients of  $\theta$  and  $z$  onto  $\mathcal{F}_\omega = \{\omega - \mathcal{P}_{\Omega_i^c}\omega\}$ , respectively.

- Those *partial disclosure* policies where  $\phi^p = \hat{\phi}$ , however, all attain the same welfare outcome as *full disclosure*, which outcome can also be replicated under complete opacity with  $\phi^o = \hat{\phi}$ .

*Step 4: The Strict Optimality of Opacity:* Consider the policy maker's problem under complete opacity,

$$\begin{aligned} \min_{\phi^o} \mathcal{W}_s &= \frac{1}{2} \left[ \frac{\tau_x + \tau_a k_0^2}{\tau_x^2} k_0^2 + \left( \frac{\tau_x + \tau_a k_0^2}{\tau_x} k_0 - 1 \right)^2 \frac{1}{\tau_z} + \left( \frac{k_2}{\phi^o} \right)^2 \frac{1}{\tau_s} + \frac{1}{\tau_m} \right]. \quad (19) \\ \text{s.t.} & \quad (14) \quad \text{and} \quad (15). \end{aligned}$$

Next, let us decompose,

$$2\mathcal{W}_s = f(k_0) + \left( \frac{k_2}{\phi^o} \right)^2 \frac{1}{\tau_s} + \frac{1}{\tau_m},$$

where  $f$  is the strictly pseudo-convex welfare function from the Proof of Theorem 1,

$$f(k_0) \equiv \frac{\tau_x + \tau_a k_0^2}{\tau_x^2} k_0^2 + \left( \frac{\tau_a k_0^2}{\tau_x} k_0 + k_0 - 1 \right)^2 \frac{1}{\tau_z}.$$

It follows that  $k_{0,f}^* > k_0^d$ , where  $k_{0,f}^*$  denotes the unique value of  $k_0$  that minimizes  $f$ .

Suppose now that we start at  $\phi^o = \hat{\phi}$ , and hence with  $k_0 = k_0^d$ . In that case,

$$\mathcal{W}_s = f(k_0^d) + 0 + \frac{1}{\tau_m},$$

since  $k_2 = 0$  when  $\phi^o = \hat{\phi}$ . Consider now an infinitesimal increase in  $k_0$  from  $k_0^d$  to  $k_0'$  consistent with (14). Since  $k_{0,f}^* > k_0^d$  and  $f$  is strictly pseudo-convex, it follows that  $f(k_0') < f(k_0^d)$ . However, it also implies that  $k_2^2(k_0') \frac{1}{\phi^o(k_0')^2 \tau_s} > 0$ . But since  $\phi^o = \hat{\phi}$  attains the global minimum for  $k_2^2 \frac{1}{\phi^o^2 \tau_s}$ , this latter effect is only of second order. By contrast, the former effect is of first order. Thus, we can conclude that  $k_0^* > k_0^d$  and  $\phi^* < \hat{\phi}$ ; the latter follows from that  $\frac{dk_0}{d\phi} |_{k_0=k_0^d} < 0$ .<sup>13</sup>

Finally, it follows from (15) and (19) that when  $\tau_s \rightarrow 0$  the optimal policy tends towards that from Section 4; for  $\tau_s \rightarrow \infty$  the optimal policy, in contrast, tends towards full disclosure. □

## C.4 Alternative Shock Structures

Several of the convenient properties of normal distributions that I used in Sections 2 and 3 are enjoyed by many other pairs of prior and likelihood. In fact, Ericson (1969), DeGroot (1970) and Vives (2010) show how the pair normal-normal is merely one example of the class for which conditional expectations are linear, and hence the main decomposition used to arrive at Theorem 1 holds exactly. The decomposition, for instance, holds for many of the most commonly used

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<sup>13</sup>The uniqueness of  $k_0^*$  follows from the strict pseudo-convexity of  $\mathcal{W}_s$  when using that  $\left( \frac{k_2}{\phi} \right)^2 = \left( \frac{1}{\phi} \right)^2 \left( \phi + \frac{p_a k_0^2}{p_x} k_0 + k_0 - 1 \right)^2$  and from the quasi-convexity of the constraint (14).

distributions when combined with natural priors, in addition to the important case where we restrict agents to only construct linear-best predictors (see [Brockwell and Davis, 2009](#)).<sup>14</sup> Below, I provide a simple example with binary signals.

Suppose that  $\theta$  is drawn from a beta distribution between zero and one, where  $\alpha > 0$  and  $\beta > 0$ . The private information of individual  $i$  is summarized by an independent bernoulli trial  $x_i$  with parameter  $\theta$ . Consistent with Section 2, the policy maker's own private information is also assumed to be an independent bernoulli trial  $z$  with  $\theta$ . I here discount the presence of an endogenous public signal  $a$  for simplicity. The key step is to demonstrate how complete opacity combined with active instrument policy can replicate the full disclosure outcome. When full disclosure does not attain the constraint efficient outcome, complete opacity then dominates because it avoids the added obfuscation associated with partial disclosure. The rest of the model follows that in Section 2.

**Complete Opacity:** To start, let me define  $\tau \equiv \mathbb{E}[\mathbb{V}[x_i | \theta]]^{-1} = \frac{1}{\alpha + \beta} \tau_\theta$ , where  $\tau_\theta$  denotes the precision of  $\theta$ . A simple application of bayes' rule shows that the posterior distribution of  $\theta$  conditional on  $x_i$  is  $beta(\alpha + x_i, \beta + 1 - x_i)$ . Thus,

$$\mathbb{E}[\theta | x_i] = \frac{\alpha + x_i}{1 + \alpha + \beta} = \frac{\tau}{\tau + \tau_\theta} x_i + \frac{\tau_\theta}{\tau + \tau_\theta} \mathbb{E}[\theta].$$

Consider now the deviation from the effective state of the economy,  $\Delta_i$ , under complete opacity,

$$\Delta_i^o = \mathbb{E}[\theta | x_i] - \theta + m - \mathbb{E}[m | x_i] = \mathbb{E}[\theta | x_i] - \theta + \phi(z - \mathbb{E}[\theta | x_i]) + \epsilon_m. \quad (20)$$

**Full Disclosure:** Suppose that the policy maker instead fully discloses his own information, and notice that  $\mathbb{E}[\mathbb{V}[z | \theta]]^{-1} = \tau$ . We can then apply bayes' rule one more time to show that the posterior distribution of  $\theta$  conditional on  $x_i$  and  $z$  also is  $beta(\alpha + x_i + z, \beta + 2 - x_i - z)$  with,

$$\begin{aligned} \mathbb{E}[\theta | x_i, z] &= \frac{\alpha + x_i + z}{2 + \alpha + \beta} = \frac{\tau}{2\tau + \tau_\theta} x_i + \frac{\tau}{2\tau + \tau_\theta} z + \frac{\tau_\theta}{2\tau + \tau_\theta} \mathbb{E}[\theta] \\ &= \mathbb{E}[\theta | x_i] + \frac{\tau}{2\tau + \tau_\theta} (z - \mathbb{E}[\theta | x_i]). \end{aligned}$$

We can now compute the deviation from the effective state of the economy under full disclosure,

$$\Delta_i^d = \mathbb{E}[\theta | x_i, z] - \theta + \epsilon_m = \mathbb{E}[\theta | x_i] - \theta + \frac{\tau}{2\tau + \tau_\theta} (z - \mathbb{E}[\theta | x_i]) + \epsilon_m. \quad (21)$$

All that is left is to compare (20) and (21) from which the below Proposition follows.

**Proposition.** *Complete opacity with active instrument policy, where  $\phi = \hat{\phi} \equiv \frac{\tau}{2\tau + \tau_\theta}$ , replicates the full disclosure outcome under the beta-binomial, affine information structure.*

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<sup>14</sup>The decomposition, for example, holds for affine information structures with beta-binomial or gamma-poisson combinations of prior and likelihood. Other cases are when the observations are negative binomial, gamma or exponential when assigned natural conjugate priors.

## Appendix D: Alternative Business Cycle Applications

This Appendix discusses the alternative business cycle applications mentioned in the paper.

### D.1. An RBC Model of Tax Policy

The model is identical to that from Section 5 with the exception of (a) the timing and information structure; (b) the set of policy instruments available to the policy maker – in this case, the economy-wide tax authority; and (c) that workers and firms here directly observe and learn from a policy instrument. The model resembles that of [Angeletos \*et al.\* \(2016\)](#).<sup>15</sup>

**Timing Structure:** I alter the timing structure as follows: I assume that workers are now sent to each island at the first stage of each period. At that moment, local labor markets open, workers decide how much labor to supply and firms how much to demand. The local wage adjusts to clear the market and labor taxes are paid. At this stage, firms and workers have perfect information about local productivity but imperfect information about the productivity of, and hence demand from, other islands. After employment and production decisions are made, the economy moves to the second stage, where all information that was previously dispersed becomes publicly known. Commodity markets now open and commodity prices adjust to clear them. Firm sales taxes are here also paid.

**Households and Firms:** The economy-wide tax authority sets labor income taxes and firm sales taxes. The household's budget constraint is therefore now given by,

$$\int_0^1 P_{it} C_{it} di + M_t^d \leq \int_0^1 \Pi_{it} di + (1 - T_{wt}) \int_0^1 W_{it} L_{it} di + M_{t-1}^d + T_t, \quad (22)$$

where  $T_{wt}$  denotes the *economy-wide labor income tax*. Firm profits by contrast now equal,

$$\Pi_{it} = (1 - T_{st}) P_{it} Y_{it} - W_{it} L_{it}, \quad (23)$$

where  $T_{st}$  denotes the *economy-wide sales tax*.

**Active Tax Policy (and Constant Monetary Policy):** The economy-wide tax authority has access to three separate tools: (i) labor taxes  $T_{wt}$ , (ii) sales taxes  $T_{st}$ , and last (iii) the precision of its disclosure  $\omega_t$  about its own beliefs  $z_t$  about aggregate productivity. In line with the baseline model and its extensions, I assume that sales and labor income taxes are set in accordance with,

$$\log(1 - T_{wt}) = \phi_w z_t + \epsilon_{wt}, \quad \log(1 - T_{st}) = \phi_0 + \phi_s z_t, \quad (24)$$

where  $\{\phi_0, \phi_w, \phi_s\} \in \mathbb{R}^3$  and  $z_t = \theta_t + \epsilon_{zt}$ .<sup>16</sup> The error terms  $\epsilon_{wt}$  and  $\epsilon_{zt}$  are assumed white noise normal with precision  $\tau_w$  and  $\tau_z$  and independent of all other random disturbances. Com-

<sup>15</sup>I also here consider only one level of CES aggregator with elasticity  $\sigma$ . This is merely to simplify the exposition as I do not consider inefficient disturbances within this model.

<sup>16</sup>Here,  $\phi_0$  denotes the standard optimal production subsidy.

munication policy is set as in the baseline model. To make the exposition as simple as possible, I assume that  $\phi_w = -\phi_s = \phi$  in what follows.<sup>17</sup>

Because commodity prices can freely adjust to clear commodity markets, the central bank, unlike the tax authority, cannot here influence equilibrium outcomes through changes in its instrument, the money supply  $M_t^s$ . I therefore, for simplicity, assume that it keeps the money supply constant and does not disclose any information.

**Information Structure:** To close the model, I need to specify the information structure. Similar to the baseline model and its extensions, firms' and workers' information sets equal,

$$\Omega_{it} = \{x_{i\tau}, \omega_\tau, y_\tau, \log(1 - T_{wt})\}_{\tau=-\infty}^{\tau=t}, \quad (25)$$

where  $y_t = \log(Y_t) + \epsilon_{at}$  and  $\epsilon_{at} \sim \mathcal{WN}(0, 1/\tau_a)$  is independent of all other shocks. I hence assume that firms and workers observe a noisy signal of the economy-wide level of output and use this signal to estimate demand from other islands.<sup>18</sup> Importantly in (25), neither firms nor workers observe the sales tax at the time of decision making. Labor supply and demand decisions are therefore partially pre-determined and expected tax policy thus matters for equilibrium outcomes, even within periods. Unlike the baseline model, however, firms and workers here also observe and learn about the tax authority's beliefs from *the current setting of a policy instrument*, the income tax. This allows current tax choices to signal the tax authority's beliefs and hence future second-stage tax rates. Specifically, the more active the income tax is, that is the larger  $|\phi|$  is, the more informative the income tax is of the tax authority's private information, and thus about the future sales tax. This is similar to the outcome of the quantitative framework used by Melosi (2016) to study the signaling effects of monetary policy rates.

**Equilibrium Characterization:** Solving for the labor market equilibrium on island  $i$  allows us to show that the level of output in the economy is pinned down by:

**Lemma 1 (D).** *The equilibrium level of output on island  $i$  is determined by,*

$$y_{it} = \varsigma x_{it} + (1 - \varsigma) \mathbb{E}_{it} [\bar{y}_t] + \frac{\phi \varsigma}{1 + \eta} (z_t - \mathbb{E}_{it} [z_t]) + \frac{\varsigma}{1 + \eta} \epsilon_{wt}, \quad (26)$$

where  $\varsigma \equiv \frac{(1+\eta)\sigma}{1+\eta\sigma} > 1$  and  $\bar{y}_t = \log(Y_t)$ .<sup>19</sup>

Equation (26) closely resembles the decision rule analyzed in Section 5. First, since  $\varsigma > 1$ , there are direct strategic interactions between individual island outputs. And second, because of the noisy income tax, individual island output is noisy; it depends directly on the disturbance  $\epsilon_{wt}$ . Both features resemble those we saw firm prices exhibit in Section 5.

<sup>17</sup>This does not crucially affect our main results. Relaxing the constraint allows the tax authority to better internalize the learning externality. This, in turn, allows the policy maker to optimally make endogenous public statistics more informative than that achieved under the optimal policy with the constraint in place. However, all comparisons to the full disclosure outcome remain the same. Despite the added degree of freedom, the optimal policy when  $\phi_w \neq \phi_s$  still does not attain the constrained efficient outcome.

<sup>18</sup>The noise in the observation of  $y_t$  can once more be attributed to the statistical error that occurs when island inhabitants observe only a random sample of other islands outputs (cf. Lorenzoni, 2010).

<sup>19</sup>Equation (26), for simplicity, ignores unimportant constant terms (see Proof below).

**Optimal Tax Policy:** The tax authority seeks to maximize the ex-ante expected utility of the representative household by adjusting its three tools: (i) labor taxes, (ii) sales taxes, and last (iii) the precision of the signal that it sends about its own beliefs about aggregate productivity. Conditional on an optimal production subsidy, welfare in the economy equals:

**Lemma 2 (D).** *There exists a function  $f : \mathbb{R}_+ \rightarrow \mathbb{R}$  such that the ex-ante level of welfare equals,*

$$\mathcal{W} = f(\Lambda), \quad \Lambda = \mathbb{E}[\bar{y}_t - y_t^*]^2 + \frac{1}{\varsigma} \mathbb{E}[(y_{it} - y_{it}^*) - (\bar{y}_t - y_t^*)]^2 \quad (27)$$

where  $y_{it}^*$  and  $y_t^* = \theta_t$  denote the first-best levels of local and economy-wide output, respectively. Moreover,  $\mathcal{W}$  attains its maximum (the first best level) at  $\Lambda = 0$  and is strictly decreasing in  $\Lambda$ .

The social welfare loss function  $\Lambda$  is identical to that studied in Section 5. Equation (5.2) and (5.9) can directly be used to re-write (5.11) into (27). Using an identical approach to that applied in Section 5 to solve for optimal policy, then once more shows that the unique best policy features complete opacity combined with active instrument policy:

**Proposition.** *The unique optimal policy is complete opacity,  $\tau_\omega \rightarrow 0$ , combined with active tax policy,  $\phi = \phi_{tp}^* \in (0, 1 + \eta)$ . The informativeness of economy-wide output,  $y_t$ , under the optimal policy is always above that achieved with full disclosure,  $k_0^* > k_0^d$ .*

Unlike in Section 5, however, the informativeness of the endogenous public signal  $y_t$  is here always above that achieved with full disclosure. It holds for all parameter values that  $k_0^* > k_0^d$ , exactly like in the baseline model. Since  $k_0^* > k_0^d$ , we moreover conclude that it is always the absence of added noise associated with communication policy that causes instrument policy to dominate. In contrast, instrument policy's ability to also produce more coordination than that achieved with full disclosure does not here contribute to its superiority.

The sharpness of the above Proposition is a result of the direct strategic interactions between islands being *conditionally efficient*: conditional on the informativeness of output, the equilibrium degree of coordination between local islands always equals the socially optimal one (see also Angeletos *et al.*, 2016).<sup>20</sup> Any wedge between the overall equilibrium and the socially optimal degree of coordination is therefore entirely due to the learning externality, and hence optimally  $k_0^* > k_0^d$ . This business cycle application thus closely resembles the baseline model. In both cases, the key inefficiency is the learning externality. This, in turn, pushes the policy maker towards making the endogenous public statistic more informative than what would be achieved with full disclosure, and to use instrument policy to do so because it avoids the introduction of added noise to the information structure.

Last, notice how the close similarity to the results from the baseline model occurs despite that workers and firms here learn from *current income taxes* about *subsequent sales taxes*. Although, the current income tax signals the tax authority's information, it does not alter that decreases in  $\phi$  increase the informativeness of output, alleviating the consequences of the learning externality.

<sup>20</sup>Both are, in this case, equal to  $1 - \varsigma < 0$ .

Clearly, the better a signal current income taxes provide of the tax authority's information (the lower  $\tau_\omega$  is, for instance) the less benefit there is to decrease  $\phi$  below the level which replicates  $k_0^d$  under complete opacity. But so long as the income tax does not *perfectly* reveal the tax authority's information, it is still optimal to rely exclusively on instrument policy since it avoids the introduction of additional noise.

**Proof of Lemma 1(D):** The steps used to prove Lemma 1(D) follow those in [Angeletos et al \(2016\)](#). But I here extend the proof to deal with the explicitly endogenous information structure as well as the separate labor and sales taxes.<sup>21</sup>

The representative firm on island  $i$ 's problem is to,

$$\begin{aligned} \max_{Y_i} \mathbb{E}_i [\Pi_i] &= \mathbb{E}_i \left[ \frac{U'(Y)}{P} \left( DY_i^{1-\frac{1}{\sigma}} - W_i N_i \right) \right] \\ \text{s.t. } Y_i &= X_i L_i, D \equiv (1 - T_s) P Y_i^{\frac{1}{\sigma}}, \end{aligned}$$

where for convenience I have dropped time subscripts and  $Y = C$ . The sufficient first order condition to this problem is,

$$\mathbb{E}_i \left[ \frac{U'(Y)}{P} \right] W_i = \left( 1 - \frac{1}{\sigma} \right) X_i \mathbb{E}_i \left[ (1 - T_s) Y_i^{-\frac{1}{\sigma}} Y_i^{\frac{1}{\sigma}-1} \right]. \quad (28)$$

The worker sent to island  $i$  maximizes its contribution to the representative household's utility. She therefore supplies labor until,

$$(1 - T_w) \mathbb{E}_i \left[ \frac{U'(Y)}{P} \right] W_i = \left( \frac{Y_i}{X_i} \right)^\eta. \quad (29)$$

Equating (28) and (29) provides us with,

$$\left( \frac{Y_i}{X_i} \right)^\eta = \left( 1 - \frac{1}{\sigma} \right) (1 - T_w) X_i \mathbb{E}_i \left[ (1 - T_s) Y_i^{-\frac{1}{\sigma}} Y_i^{\frac{1}{\sigma}-1} \right]. \quad (30)$$

To show Lemma 1, and hence characterize island  $i$  output, we thus need to derive expressions for the different terms in (30). To do so, I first conjecture and later verify that,

$$y_i = \underline{k} + k_0 x_i + k_1 y^s + k_2 \tilde{T}_w + k_3 \omega + k_4 \log(1 - T_w), \quad (31)$$

where  $k_j$ ,  $j = \{ \cdot, 0, 1, 2, 3, 4 \}$ ,  $y^s = \theta + \frac{1}{k_0} \epsilon_a$  denotes the orthogonalized version of  $y = \log(Y) + \epsilon_a$  and  $\tilde{T}_w = z + \frac{1}{\phi} \epsilon_s$ .<sup>22</sup> It follows from (31) and (5.2) that  $Y$  is log-normal. Using this conjecture combined with  $U'(Y) = Y^{-1}$  and our tax rules from the body of this paper allows us to re-state

<sup>21</sup>I once more use that if  $Q \sim LN(\mu, \sigma^2)$  then  $\mathbb{E}[Q^\delta] = \mathbb{E}[Q]^\delta \exp\left[\frac{\delta}{2}(\delta-1)\sigma^2\right]$ .

<sup>22</sup>Indeed, to also account for the possibility of any additional information about the tax authority's beliefs besides the observation of the income tax and the tax authority's own disclosure, I assume that firms and workers learn from  $\tilde{T}_w$  instead of  $\log(1 - T_w)$ , where  $\epsilon_s \sim \mathcal{N}(0, \tau_s^{-1})$  with  $\tau_s \geq \tau_w$ . As the Proof shows, neither of my main results are affected by the presence of additional information.

(30) as,

$$\left(\eta + \frac{1}{\sigma}\right) y_i = H + (1 + \eta) x_i + \left(\frac{1}{\sigma} - 1\right) \mathbb{E}_i[\bar{y}] + \phi(z - \mathbb{E}_i[z]) + \epsilon_w, \quad (32)$$

where  $\bar{y} \equiv \log(Y)$ ,  $H \equiv \phi_0 + \log\left(1 - \frac{1}{\sigma}\right) + \frac{\sigma_D^2}{2}$  and  $\sigma_D^2 \equiv \mathbb{V}\left[(1 - T_s) Y_i^{-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}-1}\right]$ .

What remains to show is to verify (31) and characterize the coefficients  $k_j$ ,  $j = \{1, 0, 1, 2, 3, 4\}$ .

Solving island inhabitants' signal extraction problem yields,

$$\begin{aligned} \mathbb{E}_i[\theta] &= w_x x_i + w_y y^s + w_T \tilde{T}_w + w_\omega \omega, & w_x &= \frac{\tau_x (\tau_\omega + \tau_z + \phi^2 \tau_s)}{(\tau_\omega + \phi^2 \tau_s) (\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z) + \tau_z (\tau_\theta + \tau_x + \tau_a k_0^2)} \\ \mathbb{E}_i[z] &= v_x x_i + v_y y^s + v_T \tilde{T}_w + v_\omega \omega, & v_x &= \frac{\tau_x \tau_z}{(\tau_\omega + \phi^2 \tau_s) (\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z) + \tau_z (\tau_\theta + \tau_x + \tau_a k_0^2)}. \end{aligned}$$

Combined with (5.2), (32) and the conjecture in (31) this in turn shows that,

$$\begin{aligned} \left(\eta + \frac{1}{\sigma}\right) y_i &= H - \frac{1}{2} \left(\frac{1 - \sigma}{\sigma}\right)^2 \mathbb{V}[\bar{y}] + \left(\frac{1}{\sigma} - 1\right) \underline{k} + (1 + \eta) x_i + \phi \left[ z - (v_x x_i + v_y y^s + v_T \tilde{T}_w + v_\omega \omega) \right] \\ &+ \left(\frac{1}{\sigma} - 1\right) \left[ k_0 (w_x x_i + w_y y^s + w_T \tilde{T}_w + w_\omega \omega) + k_1 y^s + k_2 \tilde{T}_w + k_3 \omega + k_4 \log(1 - T_w) \right] + \epsilon_w, \end{aligned}$$

and hence that,

$$k_0 = \varsigma + (1 - b) k_0 w_x - \phi \frac{\varsigma}{1 + \eta} v_x \quad (33)$$

$$k_1 = (1 - \varsigma) (k_1 + k_0 w_y) - \phi \frac{\varsigma}{1 + \eta} v_y \quad (34)$$

$$k_2 = (1 - \varsigma) (k_2 + k_0 w_T) - \phi \frac{\varsigma}{1 + \eta} v_T \quad (35)$$

$$k_3 = (1 - \varsigma) (k_3 + k_0 w_\omega) - \phi \frac{\varsigma}{1 + \eta} v_\omega \quad (36)$$

$$k_4 = (1 - \varsigma) k_4 + \frac{\varsigma}{1 + \eta} \quad (37)$$

$$\underline{k} = (1 - \varsigma) \underline{k} + \frac{\varsigma}{1 + \eta} \left[ H - \frac{1}{2} \left(\frac{1 - \sigma}{\sigma}\right)^2 \mathbb{V}[\bar{y}] \right], \quad (38)$$

where  $\varsigma \equiv \frac{\sigma(1+\eta)}{1+\eta\sigma}$ . This verifies our conjecture in (31), and thus that  $y^s = \theta + \frac{1}{k_0} \epsilon_a$ .<sup>23</sup>

The equilibrium described by (33) to (38) is, moreover, unique. Equation (33) implies that  $k_0$  solves  $L(k_0) = R(k_0)$ , where,

$$L(k_0) \equiv (1 + \eta) \varsigma \left[ (\tau_\omega + \tau_z + \phi^2 \tau_s) \tau_a k_0^2 + (\tau_\omega + \phi^2 \tau_s) (\tau_\theta + \tau_x + \tau_z) + \tau_z (\tau_\theta + \tau_x) \right]$$

$$\begin{aligned} R(k_0) &\equiv \phi b \tau_z \tau_x \\ &+ (1 + \eta) k_0 \left[ (\tau_\omega + \tau_z + \phi^2 \tau_s) \tau_a k_0^2 + (\tau_\omega + \phi^2 \tau_s) (\tau_\theta + \tau_x + \tau_z) + \tau_z (\tau_\theta + \tau_x) \right]. \end{aligned}$$

But since  $L(0) > R(0) > 0$ ,  $R(b) > L(b) > 0$  and both  $L$  and  $R$  are strictly convex, it follows that there exists a unique solution  $k_0 \in (0, \varsigma)$ . Furthermore, since  $\frac{\partial R}{\partial k_0} > 0$  and  $L$  achieves its

<sup>23</sup>I assume that  $\phi \in [0, 1 + \eta]$ . As I show below, the optimal  $\phi$  will always be within this range.

minimum at  $k_0 = 0$  where  $L(0) > R(0)$ , we can rule out any negative solutions. Finally, since  $\frac{\partial^2 R}{\partial k_0^2} > \frac{\partial^2 L}{\partial k_0^2} > 0$  whenever  $k_0 > \varsigma$  shows that there are also no further solutions above  $\varsigma$ . Since (33) thus has a unique solution, so too does (34) to (38).  $\square$

**Proof of Lemma 2(D):** See Angeletos et al (2016) and the Appendix to this paper.

The first best full information level of local-island and economy-wide output equal, respectively,<sup>24</sup>

$$y_i^* = \varsigma x_i + (1 - \varsigma) y^*, \quad y^* = \theta. \quad \square$$

**Proof of Proposition:** I use a similar approach to that applied to the “Noisy, Observable Instrument” extension in Appendix C.

*Step 1: Equilibrium Welfare:* Using the expression for the social welfare loss function combined with (31) and (33) to (37) shows after some straightforward but tedious algebra that,

$$\begin{aligned} \Lambda &= \frac{\tau_\theta + b\tau_x + \tau_a k_0^2}{\tau_x^2} \left( \frac{k_0 - b}{b} \right)^2 \\ &+ \left( \frac{\tau_\theta + \tau_a k_0^2}{\tau_x} \frac{k_0 - b}{b} + k_0 - 1 \right)^2 \frac{1}{\tau_z} + k_2^2 \frac{1}{\phi^2 \tau_s} + k_3^2 \frac{1}{\tau_w} + k_4^2 \frac{1}{\tau_w}, \end{aligned} \quad (39)$$

where  $k_4^2 \frac{1}{\tau_w}$  is independent of policy.

*Step 2 and 3: The Weak Optimality of Opacity:* Repeatedly using the equilibrium conditions (33), (35) and (36) combined with the social welfare loss function in (39) then shows that,

- All *partial disclosure* policies where  $\phi^p \neq k_0 \left( \frac{1-\sigma}{\sigma} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z}$  are strictly dominated in welfare terms by the *complete opacity* policy  $\phi^o = k_0 \left( \frac{1-\sigma}{\sigma} \right) \beta_\theta + \phi^p (1 - \beta_z)$ , where  $\beta_\theta$  and  $\beta_z$  denote the projection coefficients of  $\theta$  and  $z$  onto  $\mathcal{F}_\omega = \{\omega - \mathcal{P}_{\Omega_i^o} \omega\}$ , respectively.
- Those *partial disclosure* policies where  $\phi^p = k_0 \left( \frac{1-\sigma}{\sigma} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z}$ , in contrast, achieve the same level of welfare loss as any *full disclosure* policy, which outcome can also be replicated under complete opacity with  $\phi^o = k_0 \left( \frac{1-\sigma}{\sigma} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z}$ .

*Step 4: The Strict Optimality of Opacity:* All that is left to show is that  $\phi_{tp}^{o,*} \neq k_0 \left( \frac{1-\sigma}{\sigma} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z}$ , or equivalently that the optimal  $k_0$  coefficient differs from  $k_0^d = b \frac{\tau_\theta + \tau_x + \tau_a (k_0^d)^2 + \tau_z}{\tau_\theta + b\tau_x + \tau_a (k_0^d)^2 + \tau_z}$ . To do so, consider the social loss function under complete opacity,

$$\Lambda = f(k_0) + k_2^2 \frac{1}{\phi^2 \tau_s} + k_4^2 \frac{1}{\tau_w}, \quad (40)$$

where I have used that  $k_3^2 \frac{1}{\tau_w} \rightarrow 0$  when  $\tau_w \rightarrow 0$  and  $f$  is the strictly pseudo-convex function,

$$f(k_0) \equiv \frac{\tau_\theta + b\tau_x + \tau_a k_0^2}{\tau_x^2} \left( \frac{k_0 - b}{b} \right)^2 + \left( \frac{\tau_\theta + \tau_a k_0^2}{\tau_x} \frac{k_0 - b}{b} + k_0 - 1 \right)^2 \frac{1}{\tau_z}.$$

<sup>24</sup>I here ignore some unimportant constant terms for brevity.



Similar steps to those from the Proof of Theorem 1 then demonstrate that  $k_{0,f}^* > k_0^d$ , where  $k_{0,f}^*$  denotes the unique value of  $k_0$  that minimizes  $f$ .

Suppose now that we start at  $k_0 = k_0^d$ , and hence with  $\phi^o = k_0 \left( \frac{1-\sigma}{\sigma} \right) \frac{\tau_z}{\tau_x + \tau_a k_0^2 + \tau_z}$ . In that case,

$$\Lambda = f \left( k_0^d \right) + 0 + k_4^2 \frac{1}{\tau_w},$$

since  $k_2 = 0$  when  $k_0 = k_0^d$ . Consider now an infinitesimal increase in  $k_0$  from  $k_0^d$  to  $k_0'$ . Since  $k_{0,f}^* > k_0^d$  and  $f$  is strictly-pseudo convex, it follows that  $f(k_0') < f(k_0^d)$ . However, it also implies that  $k_2^2(k_0') \frac{1}{\phi^2 \tau_s} > 0$ . But since  $k_0 = k_0^d$  achieves the *global minimum* for  $k_2^2 \frac{1}{\phi^2 \tau_s}$ , this latter effect is only of second order. By contrast, the former effect is of first order. Hence, we conclude that  $k_0^* > k_0^d$  and  $\phi_{tp}^{o,*} < k_0 \left( \frac{1-\sigma}{\sigma} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a k_0^2 + \tau_z}$ , where the latter follows from that  $\frac{dk_0}{d\phi} |_{k_0=k_0^d} < 0$ .<sup>25</sup>  $\square$

## D.2. Known Productivity Levels

The model is identical to that from Section 5 with the exception that firms here always observe their own island-specific productivity level before setting prices. That is,  $\chi_{it} = x_{it} \in \Omega_{it}$  (see also Section 5.3 and *Online Appendix D.1* for related examples where  $x_{it} \in \Omega_{it}$ ).

**Equilibrium Characterization:** Following the same two-step procedure detailed in Section 5 shows after some straightforward-but-tedious derivations that:

**Lemma 3 (D).** *The unique linear equilibrium price that firms on island  $i \in [0, 1]$  set equals,*

$$p_{it} = \mathbb{E}_{it} \left[ \xi \bar{p}_t + (1 - \xi) m_t \right] - (1 - \xi) x_{it} = \kappa_0 x_{it} + \kappa_1 p_t + \kappa_2 \omega_t + \kappa_3 \hat{m}_t, \quad (41)$$

where  $\bar{p}_t = \log(P_t)$  and  $\xi \equiv \frac{\eta(\rho-1)}{1+\rho\eta} \in (0, 1)$  determines the extent of strategic complementarity. The coefficients  $\kappa_1$ ,  $\kappa_2$  and  $\kappa_3$  are functions of  $k_0 \in \mathbb{R}$  and the parameters of the model.

The only difference between Lemma 3(D) and Lemma 2 is that firms now always set prices *after observing* their own productivity level;  $x_{it}$  is known to firms on island  $i$ .

**Optimal Policy:** The central bank once more seeks to maximize the *ex-ante* expected utility of the representative household (5.11) by adjusting its two levers: (i) the money supply and (ii) the precision of the signal that it sends about its own beliefs about aggregate productivity. Using once more the primal approach shows that:

**Proposition.** *The optimal policy when  $x_{it} \in \Omega_{it}$  is complete opacity,  $\tau_\omega \rightarrow 0$ , combined with active monetary policy,  $\phi = \phi_{mp}^* > 0$ . The informativeness of the price level,  $p_t$ , under the optimal policy is either greater or smaller than that achieved with full disclosure,  $|\kappa_0^*| \gtrless |\kappa_0^*|$ .*

The combination of complete opacity and active monetary policy is thus once more optimal. It once more allows the central bank to better trade-off, on the one hand, the use of its own

<sup>25</sup>The uniqueness of  $k_0^*$  follows from an identical argument to that applied to “*Noisy, Observable Instrument*” extension. It remains to check that  $\phi_{tp}^{o,*} \leq 1 + \eta$ . This, however, follows immediately from  $k_0^* > k_0^d$ .

information versus, on the other hand, the use of firms' own private information. That firm private information is here comprised of physical productivity does not matter for our rationale; it is only of consequence for the precise parameters for which  $|\kappa_0^*| \gtrless |\kappa_0^*|$  (see Proof below).

**Proof of Lemma 3(D):** The steps used to prove Lemma 3(D) follow those used to show Lemma 2. The price that island  $i \in [0, 1]$  firms set is equal to,

$$\begin{aligned} p_i &= \left( \frac{\bar{\kappa} + \gamma}{1 + \sigma\eta} \right) + \mathbb{E}_i \left[ \frac{\eta(\sigma - 1)}{1 + \sigma\eta} \bar{p} + \frac{1 + \eta}{1 + \sigma\eta} (m - x_i) \right], \\ &\equiv \bar{\kappa} + \mathbb{E}_i [\xi \bar{p} + (1 - \xi)(m - x_i)], \end{aligned} \quad (42)$$

I first conjecture and later verify that,

$$p_i = \tilde{\kappa} + \kappa_0 x_i + \kappa_1 y^s + \kappa_2 \tilde{m} + \kappa_3 \omega, \quad (43)$$

where  $\kappa_j \in \mathbb{R}$ ,  $j = \{\tilde{\cdot}, 0, 1, 2, 3\}$ ,  $y^s = \theta + \frac{1}{\kappa_0} \epsilon_a$  denotes the orthogonalized version of  $p = \log(P_t) + \epsilon_a$  and  $\tilde{m} = z + \frac{1}{\phi} \epsilon_s$ .

Solving firms' signal extraction problem now provides us with,

$$\begin{aligned} \mathbb{E}_i[\theta] &= w_x x_i + w_y y^s + w_m \tilde{m} + w_\omega \omega, \quad w_x = \frac{\tau_x (\tau_\omega + \tau_z + \phi^2 \tau_s)}{(\tau_\omega + \phi^2 \tau_s) (\tau_\theta + \tau_x + \tau_a \kappa_0^2 + \tau_z) + \tau_z (\tau_\theta + \tau_x + \tau_a \kappa_0^2)} \\ \mathbb{E}_i[z] &= v_x x_i + v_y y^s + v_m \tilde{m} + v_\omega \omega, \quad v_x = \frac{\tau_x \tau_z}{(\tau_\omega + \phi^2 \tau_s) (\tau_\theta + \tau_x + \tau_a \kappa_0^2 + \tau_z) + \tau_z (\tau_\theta + \tau_x + \tau_a \kappa_0^2)}. \end{aligned}$$

Using these expressions in conjunction with (43), (42) and (6.2) then shows that,

$$\begin{aligned} p_i &= \bar{\kappa} + \frac{1 - \sigma}{2} \xi \mathbb{V}[\bar{p}] - (1 + \xi) x_i \\ &+ \mathbb{E}_i [\xi (\tilde{\kappa} + \kappa_0 \theta + \kappa_1 y^s + \kappa_2 \tilde{m} + \kappa_3 \omega) + (1 - \xi)(m_{-1} + \phi_0 + \phi z)], \end{aligned}$$

and hence that,

$$\begin{aligned} p_i &= \bar{\kappa} + \frac{1 - \sigma}{2} \xi \mathbb{V}[\bar{p}] + (1 - \xi)(m_{-1} + \phi_0) - (1 + \xi) x_i + \xi (\tilde{\kappa} + \kappa_1 y^s + \kappa_2 \tilde{m} + \kappa_3 \omega) \\ &+ \xi \kappa_0 (w_x x_i + w_y y^s + w_m \tilde{m} + w_\omega \omega) + (1 - \xi) \phi (v_x x_i + v_y y^s + v_m \tilde{m} + v_\omega \omega). \end{aligned}$$

Thus,

$$\kappa_0 = \xi - 1 + \xi \kappa_0 w_x + (1 - \xi) \phi v_x \quad (44)$$

$$\kappa_1 = \xi \kappa_1 + \xi \kappa_0 w_y + (1 - \xi) \phi v_y \quad (45)$$

$$\kappa_2 = \xi \kappa_2 + \xi \kappa_0 w_m + (1 - \xi) \phi v_m \quad (46)$$

$$\kappa_3 = \xi \kappa_3 + \xi \kappa_0 w_\omega + (1 - \xi) \phi v_\omega \quad (47)$$

$$\tilde{\kappa} = \xi \tilde{\kappa} + \bar{\kappa} + \xi \frac{1 - \sigma}{2} \mathbb{V}[\bar{p}] + (1 - \xi)(m_{-1} + \phi_0), \quad (48)$$

where the uniqueness of  $\kappa_0$ , and thus of the solution to (44) to (48), follows from a similar argument to that used to prove Lemma 2. □

**Proof of Proposition:** The approach used is identical to that employed in Section 5. I therefore only provide a sketch of the Proof.

*Step 1: Equilibrium Welfare:* Using the expression for  $\Lambda$  combined with  $Y_i = (P_i/P)^{-\sigma}Y$ , (43) and (44) to (48) shows after some routine algebra that,

$$\begin{aligned} \Lambda &= \left( \frac{\tau_\theta + \tau_a \kappa_0^2 + \sigma \tau_x (1 - \xi)}{\tau_x^2 (1 - \xi)^2} \right) (\kappa_0 + 1 - \xi)^2 \\ &+ \left[ 1 + \kappa_0 + \frac{\tau_\theta + \tau_a \kappa_0^2}{(1 - \xi) \tau_x} (\kappa_0 + 1 - \xi) \right]^2 \frac{1}{\tau_z} + \left( \frac{\kappa_2}{\phi} \right)^2 \frac{1}{\tau_s} + \kappa_3^2 \frac{1}{\tau_\omega}. \end{aligned} \quad (49)$$

*Step 2 and 3: The Weak Optimality of Opacity:* Repeated use of the equilibrium conditions (44), (46) and (47) combined with the loss function in (49) then shows that:

- All *partial disclosure* policies where  $\phi^p \neq \kappa_0 \left( \frac{\xi}{\xi-1} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a \kappa_0^2 + \tau_z}$  are strictly dominated by the *complete opacity* policy  $\phi^o = \kappa_0 \left( \frac{\xi}{\xi-1} \right) \beta_\theta + \phi^p (1 - \beta_z)$ , where  $\beta_\theta$  and  $\beta_z$  denote the projection coefficients of  $\theta$  and  $z$  onto  $\mathcal{F}_\omega = \{\omega - \mathcal{P}_{\Omega_i^o} \omega\}$ , respectively, where  $\Omega_i^o = \{x_{i\tau}, p_\tau, \Delta \hat{m}_\tau, m_{\tau-1}\}_{\tau=-\infty}^{\tau=t}$ .
- Those *partial disclosure* policies where  $\phi^p = \kappa_0 \left( \frac{\xi}{\xi-1} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a \kappa_0^2 + \tau_z}$ , by contrast, achieve the same level of welfare loss as any *full disclosure* policy, which outcome can also be replicated under complete opacity with  $\phi^o = \kappa_0 \left( \frac{\xi}{\xi-1} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a \kappa_0^2 + \tau_z}$ .

*Step 4: The Strict Optimality of Opacity:* It remains to check when  $\phi_x^{o,mp} \neq \kappa_0 \left( \frac{\xi}{\xi-1} \right) \frac{\tau_z}{\tau_\theta + \tau_x + \tau_a \kappa_0^2 + \tau_z}$ , or equivalently when  $\kappa_{*,0}^{mp} \neq \kappa_0^d = \xi - 1 + \xi \kappa_0^d \frac{\tau_x}{\tau_\theta + \tau_x + \tau_a (\kappa_0^d)^2 + \tau_z}$ . To do so, consider  $\Lambda$  when  $\tau_\omega \rightarrow 0$ ,

$$\Lambda = f(\kappa_0) + \left( \frac{\kappa_2}{\phi} \right)^2 \frac{1}{\tau_s},$$

where I have used that  $\kappa_3^2 \frac{1}{\tau_\omega} \rightarrow 0$  when  $\tau_\omega \rightarrow 0$  and  $f$  is the strictly pseudo-convex function,

$$f(\kappa_0) \equiv \left( \frac{\tau_\theta + \tau_a \kappa_0^2 + \sigma \tau_x (1 - \xi)}{\tau_x^2 (1 - \xi)^2} \right) (\kappa_0 + 1 - \xi)^2 + \left[ 1 + \kappa_0 + \frac{\tau_\theta + \tau_a \kappa_0^2}{(1 - \xi) \tau_x} (\kappa_0 + 1 - \xi) \right]^2 \frac{1}{\tau_z}.$$

Now, consider the unique value of  $\kappa_0$  that minimizes  $f$ ,

$$\kappa_0^{*,f} = \xi - 1 + \xi \kappa_0^{*,f} \frac{\tau_x}{\tau_\theta + \tau_x + \tau_a \left( \kappa_0^{*,f} \right)^2 + \frac{\tau_\theta + \rho(1-\xi)\tau_x + 2\left(\kappa_0^{*,f}\right)^2 + \tau_a \kappa_0^{*,f}(1-\xi)}{\tau_\theta + (1-\xi)\tau_x + 3\left(\kappa_0^{*,f}\right)^2 + 2\tau_a \kappa_0^{*,f}(1-\xi)} \tau_z} \begin{matrix} \leq \\ \geq \end{matrix} \kappa_0^d \quad (50)$$

An identical line of argument to that used in the “*Noisy, Observable Instrument*” extension in Appendix C, also used in Section 5, then establishes the rest of the Proposition. □

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