Forecaster (Mis-)Behavior*

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Abstract

Professional forecasts are often used to gauge the expectations of households and firms. Recently, the average of such forecasts have been argued to support rational expectation formation with noisy private information. We document that individual forecasts of US GDP and inflation in the Survey of Professional Forecasters overrespond to both private and public information, contradicting, *prima facie*, the assumption of noisy rational expectation formation. We generalize two alternative models of forecaster behavior that focus on strategic diversification and behavioral overconfidence, respectively, to dynamic environments with noisy private information. We find that both models predict overresponses, but only the overconfidence model is simultaneously consistent with a substantial overreaction to public information.

Keywords: Forecaster behavior, rational expectations, bounded rationality

JEL codes: C53, D83, D84

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1 Introduction

Expectations about tomorrow matter for choices today. The design of good macroeconomic policies therefore depends on the way households and firms make predictions about the future. Because individual expectations are typically unobserved, however, it is difficult to discriminate between different models of expectation formation. One exception are professional forecasters, who regularly publish predictions about major macroeconomic and financial variables. Using such forecasts to gauge economic expectations more generally is nevertheless problematic for two reasons: First, professional forecasters are likely to differ from other economic agents in their personal characteristics and information about the state of the economy. And second, their forecasts may be determined by strategic behavior and thus not accurately reflect the mean of their posterior distribution of the variable of interest.

Recently, survey data on professional forecasts have been found to support the main hypotheses of a growing literature on rational expectation formation with imperfect information. Specifically, Coibion and Gorodnichenko (2015) and Dovern et al. (2015) find that on *average* professional forecasts *underreact* to news, in the sense that forecast revisions are too small.[1] This underresponse is in turn consistent with rational behavior by forecasters whose noisy private information optimally dampens individual forecast revisions.

Using the US Survey of Professional forecasters (SPF), we find that *individual* SPF forecasts indeed *overreact* to information, in the sense that their forecast revisions are too large. In particular, we find that positive individual forecast revisions are associated with over-predictions of the variable of interest, and vice versa for negative forecast revisions. Importantly, we also find that a higher average, or 'consensus', forecast is associated with a larger over-prediction in later forecasts.

To explain these facts, we consider two models of expectation formation that have been found to imply overreaction to news. The first is a model of strategic diversification as in Laster et al. (1999), Marinovic et al. (2013), and Ottaviani and Sørensen (2006), where forecasters are paid for correctly predicting outcomes but payoffs are higher when there are only few other correct forecasts. Such a contest motive naturally gives forecasters incentives to distort their forecasts away from the mode of the distribution of forecasts, towards their private information. We compare this 'strategic' model of forecaster behavior to a model of overconfidence: when forecasters are overconfident about the accuracy of their own private information relative to that of others, they naturally overreact to private signals.

We generalize both models to a dynamic environment where agents receive repeated private signals, but also public information in the form of consensus forecasts. Overconfidence naturally predicts overreaction to private signals. Although one might expect this to imply

[1] Coibion and Gorodnichenko (2012) and Fuhrer (2015) provide related evidence.

under-reaction to other sources of information, we show how overconfidence can in fact also rationalize overreaction to public signals, such as consensus forecasts. This is because forecasters who think they have better information than others fail to recognize that the overconfidence of their competitors makes the average forecast more reactive to average news. This makes them infer excessive movements in the fundamental from any given movement in consensus. Overconfidence can thus explain overreactions to consensus, and in fact also over-correction of past deviations from consensus, similar to those observed in the data. Strategic diversification also predicts overreaction and over-correction, but of a magnitude that is substantially smaller than in the data. The reason is simple: strategic forecasters are aware that their competitors over-weigh private signals, and that consensus forecasts thus exaggerate average news. Later forecasts react optimally to the more informative consensus forecasts, contradicting much of the overreaction in the data and implying a lower (if still excessive) weight on private information.

This paper is related to the literature on expectation formation, as well as to studies of the behavior and incentives of professional forecasters. It is most related to Coibion and Gorodnichenko (2015), who show that average forecasts of several key economic variables in both the US SPF and other forecaster surveys exhibit under-responses to news, in the sense that forecast revisions are positively correlated with forecast errors. Their study builds on a number of previous tests of the hypothesis of limited information rational expectations (LIRE), starting with Mankiw et al. (2003) who find support for a sticky-information model in both consumer expectations and SPF forecasts.[2] Our contribution is to study both individual and average SPF forecasts within a consistent framework.[3] We show that forecast revisions are negatively correlated with forecast errors at the individual level, and that there is a significant negative correlation between consensus forecasts and individual forecast errors. We argue that this can be interpreted as overreaction by forecasters to both private and public information.

Although forecaster information is sometimes acknowledged to be an upper bound of that held by the population at large (Andrade and Bihan, 2013), most studies that use forecaster surveys to test the LIRE hypothesis abstract from the particular characteristics that distinguish professional forecasters from the rest of the population. This has attracted criticism (for example by Lamont (2002)) and given rise to a literature that looks at the incentives to distort forecasts away from true conditional expectations of future variables. For example, forecasters might try to mimick their more able collegues (as in Ehrbeck and Waldmann (1996)), or, more generally, change their forecasts as a function of those they anticipate from their colleagues (Lamont, 2002). We focus on one particular incentive distortion that has been found to imply

[2]See Coibion and Gorodnichenko (2015) for a detailed survey of that literature.

[3]Dovern et al. (2015) look at revisions of individual GDP forecasts for a broad set of countries. They argue that these are less supportive of sticky information rational expectation models relative to their noisy information counterpart (Reis, 2006; Woodford, 2001).

overreaction in static environments and is sometimes called the 'contest' hypothesis (Laster et al., 1999; Ottaviani and Sørensen, 2006; Marinovic et al., 2013), whereby the payoff from a correct forecast is higher when only few competitors make the same forecast. In a static context with a common prior, Ottaviani and Sørensen (2006) show how this makes forecasters optimally distort their forecasts away from their posterior mean and towards their idiosyncratic signal. We extend their framework to a dynamic environment with repeated private signals and public information in the form of consensus forecasts. We show that the contest motive implies overreaction to private signals also in that context. Our contribution is to show that the rationality at the core of the model limits the overreaction to public signals, inconsistent with the substantial overresponse that we find in the data.

The literature on forecaster behavior that accounts for their particular strategic incentives typically maintains the assumption of rational expectation formation. Research on alternatives to the LIRE assumption has been very active over recent years, but typically looks at anomalies in financial data, rather than in macroeconomic forecasts. One exception is Bordalo et al. (2018), who document that survey forecasts of credit spreads feature periods of excessive optimism and, more generally, predictable errors. Based on Tversky and Kahneman (1983)'s representative heuristic they explain this by 'diagnostic' expectation formation (Gennaioli and Shleifer, 2010), which exaggerates the probability of future states whose likelihood has increased by recent news. A particular well-studied departure from LIRE is investor overconfidence in the precision of their own knowledge. For example, Ben-David et al. (2013), show how executives are on average overconfident in their ability to predict stock returns, and that this overconfidence correlates with corporate policies (as also shown by Malmendier and Tate (2005)). We compare the LIRE and contest hypotheses to a simple version of overconfidence, which makes forecasters believe that their signals are more precise than those of others. We show how this implies, perhaps surprisingly, overreaction to both private and public signals as well as over-correction of previous deviations from consensus that are not too far from those observed in the data.

Section 2 presents a standard framework of expectation formation with limited noisy information, and shows how this implies under-reaction of average expectations to news. Section 3 uses US SPF data to show how there is indeed under-reaction of average forecasts to average news, as found in Coibion and Gorodnichenko (2015). Individual forecasts of both inflation measures and real gross national output, in contrast, overreact to idiosyncratic news. They also overreact to public news contained in consensus forecasts. Section 4 presents two alternative models and derives their theoretical properties. Section 5 estimates the two models using a simulated method of moment procedure and compares their key predictions to the data. Section 6 concludes. An appendix contains further empirical results and all proofs.

2 A Baseline Model with Private Information

We start with a simple two-period model, to fix ideas about how dispersed information causes under-responses for the average forecast. In the model, a continuum of measure one of private sector forecasters minimize the sum of mean-squared errors of their two sequential forecasts f_{i1} and f_{i2} of the random variable θ,

$$\mathcal{U}_i = \mathbb{E}\left[\theta - f_{i1}\right]^2 + \mathbb{E}\left[\theta - f_{i2}\right]^2, \quad i \in [0,1]. \tag{1}$$

for which they share a common prior $\theta \sim \mathcal{N}\left(\mu, \tau_\theta^{-1}\right)$. At the start of each period, each forecaster observes her own private signal about the fundamental, $x_{it} = \theta + \epsilon_{it}^x$, where $\epsilon_{it}^x \sim \mathcal{N}\left(0, \tau_{xt}^{-1}\right)$ for $t = 1, 2$. In addition, at the start of the second period, all forecasters observe a noisy signal of the consensus (mean) forecast from the first period with a common error $f_c = \int_0^1 f_{it} di + \epsilon_c$, where $\epsilon_c \sim \mathcal{N}\left(0, \tau_c^{-1}\right)$, which captures the fact that consensus is only an imperfect predictor of the fundamental.[4] We can thus summarize an individual forecaster's information sets by $\Omega_{i1} = \{x_{i1}\}$ and $\Omega_{i2} = \{x_{i1}, x_{i2}, f_c\}$ for $t = 1, 2$, respectively.

Individual Forecasts: While simple, the above two-period framework allows us to study several implications of the combined assumption of rational expectations and private information. That is, the assumption that individual forecasts equal their respective conditional expectation, $f_{it} = \mathbb{E}_{it}[\theta] = \mathbb{E}[\theta \mid \Omega_{it}]$. Specifically, it directly follows from the Projection Theorem" that individual forecast errors $u_{it} = \theta - f_{it}$ are orthogonal to any linear combination of elements in the associated information set, Ω_{it}. As a result, the estimate of, for instance, the coefficient β_1 in the regression

$$u_{it} = \alpha_0 + \beta_1 rev_{it} + \eta_{it}, \quad t = \{1.2\} \tag{2}$$

is zero. Here, rev_{it} denotes an individual's forecast revision, $rev_{it} = f_{it} - f_{it-1}$ with $f_{i0} = \mu$ for all $i \in [0,1]$ per assumption, and η_{it} the regression error. An individual's forecast error cannot be correlated with her forecast revision. If it was, the individual could use this correlation to improve her forecast, contradicting the assumption that her forecast was the conditional expectation to start with. Nevertheless, at the level of the average forecast, β_1 does not necessarily equal zero. The Projection Theorem looses its power since no individual possesses the average information set.

Average Forecasts: In fact, as shown by Coibion and Gorodnichenko (2015), the correlation of errors and revisions is positive for average forecasts across a host of setups when individuals have private information. To see this, focus on the first period forecast of individual

[4] This follows the large literature since Hellwig (1980) that uses such non-invertibility shocks to maintain imperfect information in rational expectations models.

$i \in [0,1]$,

$$f_{i1} = \mathbb{E}_{i1}[\theta] = w_x x_{i1} + (1-w_x)\,\mu, \quad w_x = \frac{\tau_{x1}}{\tau_\theta + \tau_{x1}}, \tag{3}$$

This implies that the average forecast is equal to

$$\int_0^1 f_{i1} di = w_x \theta + (1-w_x)\,\mu. \tag{4}$$

since the noise in the private signals cancels on average, $\int_0^1 \epsilon_{i1}^x di = 0$. Because $f_{i0} = \mu$ for all i, the average forecast revision is then simply $rev_1 = w_x(\theta - \mu)$. But this revision is positively correlated with the average forecast error,

$$\nu_{i1} = \theta - \int_0^1 f_{it} di = (1-w_x)(\theta - \mu). \tag{5}$$

In fact, it is straightforward to show that the coefficient b_1 in the regression

$$\nu_t = \alpha_0 + b_1 rev_t + \eta_t, \quad t = 1 \tag{6}$$

where η_t denotes the regression error, is proportional to $b_1 \propto w_x(1-w_x)\tau_\theta^{-1} > 0$. The positive coefficient also carries over to the second-period mean forecast when consensus is not excessively revealing. Individuals, on average, under-respond to new information, as measured by the average forecast revision. Each forecaster down-weighs her own noisy information. But since, on average, the noise terms cancel, this down-weighing of information leads to an apparent under-response to average new information, and hence to a positive correlation between the mean-forecast error, on the one hand, and the mean-forecast revision, on the other hand. We combine these results in Lemma 1.

Lemma 1 *Under the joint assumption of rational expectations and private information, individual forecast revisions do not predict individual forecast errors,* $\beta_1 = 0$. *Mean forecast errors are positively correlated with mean forecast revisions,* $b_1 > 0$ *when* $\tau_\xi < \frac{(\tau_\theta + \tau_{x1})^2}{\tau_\theta} \frac{\tau_{x2}}{\tau_{x1}}$.

3 Empirical Evidence

3.1 The Data

The US Survey of Professional Forecasters (SPF) is an unbalanced panel of individual forecasters that, since 1968, asks its respondents for their forecasts of a number of key macroeconomic and financial variables, and publishes them, in anonymous format but with personal identification numbers, shortly after. We are most interested in forecasts for aggregate output growth and inflation, which most forecasters include in their reports, and which arguably attract most attention. We therefore study SPF forecasts of three variables of interest: First, two measures of price inflation, namely the year-on-year percentage change in the GDP deflator and the consumer price index (CPI, available since 1981). And second the annual growth rate of real gross national output. The questionnaire asks forecasters to report forecasts for Gross National Product until 1991 and for Gross Domestic Product thereafter, and is calculated from nominal forecasts using the GNP deflator until 1981 Q2. For these variables, the survey includes consistent forecasts over the six quarters following the survey quarter. Importantly for our purposes, although the precise schedule over the quarter has changed over time, the administrators of the SPF have consistently published the survey results well before sending out the following questionnaire to their respondents.[5] In other words, the information set of respondents includes the average (or consensus) forecast of the previous quarter.[6]

3.2 Average Forecasts

To set the stage for our analysis of individual forecasts, we first study the properties of average forecasts of output and inflation in the SPF. This confirms the results in Coibion and Gorodnichenko (2015) and Dovern et al. (2015). Specifically, we denote the forecast by individual i made in period t for a variable x in period $t+h$ as $f_{it}x_{t+h}$ and calculate the average as $f_t x_{t+h} = \frac{1}{N_t}\sum_{i=1,..N_t} f_{it}x_{t+h}$.

$$x_{t+h} - f_t x_{t+h} = \alpha + \beta(f_t x_{t+h} - f_{t-1}x_{t+h}) + \nu_t \quad (7)$$

where α is a constant and ν_t an error term.

Table I presents the results. The estimation sample starts in 1970 Q1 for the GDP deflator ("PGDP") and in 1982 Q1 for the CPI. For gross national output we exclude periods where GNP was calculated from nominal output and inflation forecasts to avoid importing the properties of inflation forecasts into those for real output. This yields the starting period 1981 Q3.[7] Revisions of average forecasts for both output growth and inflation are positively associated with forecast errors for all three variables. This effect is strong and highly significant for the GDP deflator, confirming the results in Coibion and Gorodnichenko (2015). It is also significant for gross output, although smaller in magnitude, and even smaller and insignificantly different from zero for the CPI.

[5]See page 8 in https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en.

[6]The number of respondents (including individuals that work for financial institutions and large industrial firms, but also independent forecasting enterprises) fell from over 80 to less than 20 in 1990, when the Federal Reserve Bank of Philadelphia took over the administration, and has fluctuated between 40 and 60 since then. See https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en for more detail on the survey.

[7]Appendix 7.1 shows that our results are robust to the specific sample period.

Table I: Average forecasts

	(1) PGDP	(2) CPI	(3) GDP
Average forecast correction	1.272***	0.346	0.629*
	(0.275)	(0.246)	(0.273)
Constant	-0.0673	-0.189	-0.0762
	(0.0732)	(0.104)	(0.116)
R^2	0.253	0.023	0.059
N	183	137	142

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table presents estimates of equation (7), including forecaster fixed effects. The estimation sample ends in 2016 Q4, and starts in 1970 Q1 for the GDP deflator (PGDP), in 1982 Q1 for the consumer price index (CPI) and in 1981 Q3 for gross national output (GDP). Robust standard errors are used.

3.3 Individual Forecasts

This subsection presents our main empirical results describing the statistical properties of *individual* SPF forecasts. Specifically, we test the implication of rational expectations that forecast errors are uncorrelated with any information, be it public or private to the forecaster, available at the time of the forecast. Figure 1 shows that this implication is in fact not borne out by SPF forecasts for inflation. Specifically, the means of individual forecast errors taken within deciles of the distributions of forecast revisions (top panels) and the consensus forecasts (bottom panels), are negative sloped for the GDP deflator (in the left column), and for the CPI (right colun).

To test the implication of rational expectation more formally, and to derive target moments that can later help identify alternative models, we now first estimate the equivalent of equation (7) using the following benchmark specification

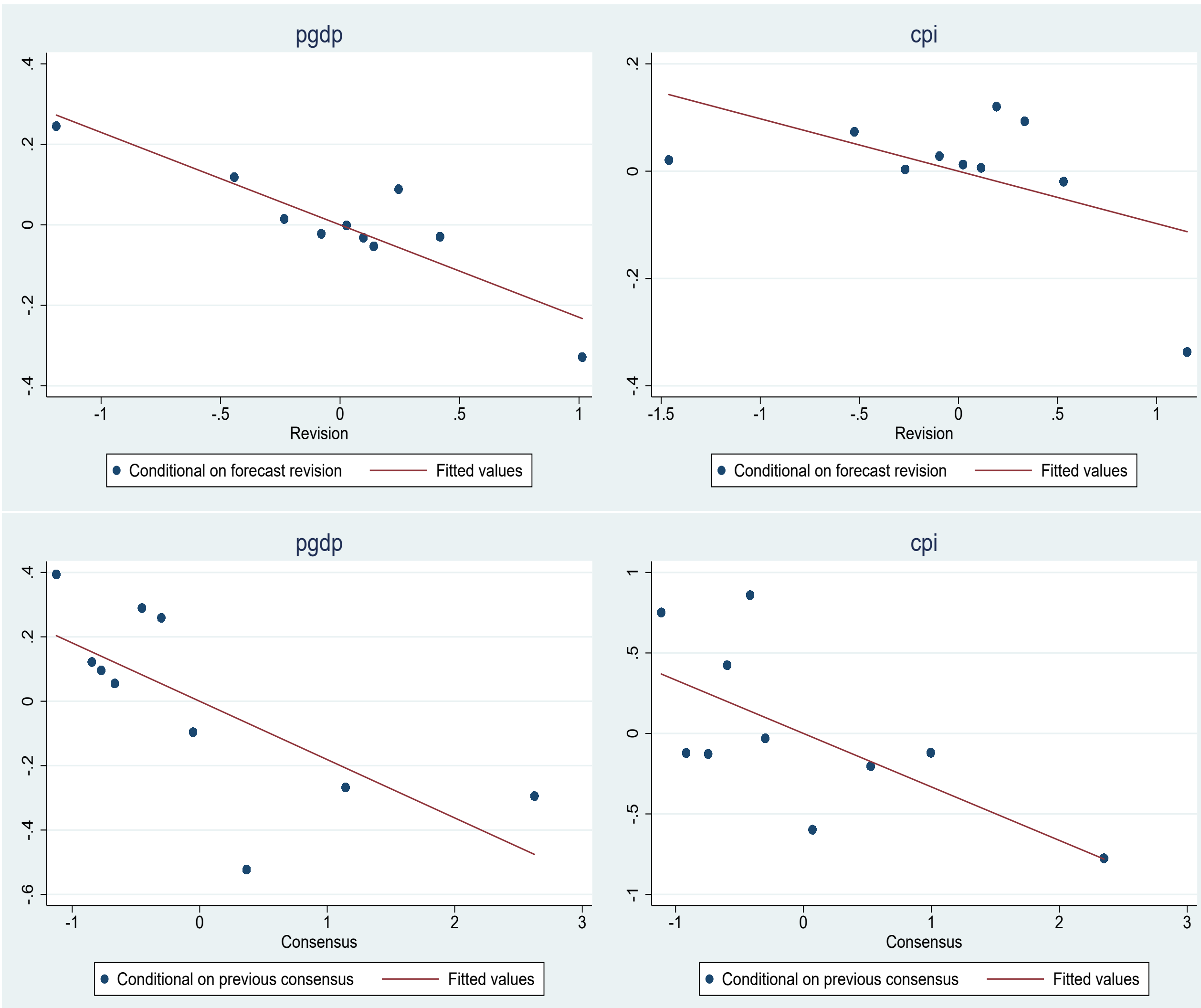
$$x_{t+h} - f_{it}x_{t+h} = \alpha_i + \beta_1(f_{it}x_{t+h} - f_{it-1}x_{t+h}) + \nu_{it} \tag{8}$$

where ν_{it} is an error term and α_i is a forecaster-specific fixed effect, which we include in our benchmark specification but which does not affect the results. Our second estimating equation augments (8) with a specific piece of public information, namely the average, or consensus SPF forecast of the previous survey $f_{t-1}x_{t+h}$

$$x_{t+h} - f_{it}x_{t+h} = \alpha_i + \delta_1(f_{it}x_{t+h} - f_{it-1}x_{t+h}) + \delta_2 f_{t-1}x_{t+h} + \delta_3(f_{it-1}x_{t+h} - f_{t-1}x_{t+h}) + \widehat{\nu}_{it} \tag{9}$$

where we also include the deviation individual forecasts from the consensus $(f_{it-1}x_{t+h} -$

Figure 1: Mean forecast errors



The figure depicts means of forecast errors taken within deciles of the distributions of forecast revisions (top row) and consensus forecasts for the same period in the previous wave of the SPF (bottom row). All variables are demeaned by subtracting the overall mean during the sample. The figure uses the same sample period, starting in 1982 Q1 for the GDP deflator (left column) and the CPI (right column).

$f_{t-1}x_{t+h}$) to capture how strongly forecasters adjust when they deviate from others.

Table II: Individual forecasts - benchmark

	(1) PGDP	(2) PGDP	(3) CPI	(4) CPI	(5) GDP	(6) GDP
Forecast correction	-0.195***	-0.561***	-0.268***	-0.591***	-0.0807	-0.302***
	(0.0428)	(0.0760)	(0.0369)	(0.0517)	(0.0464)	(0.0641)
Previous consensus		-0.200***		-0.591***		-0.238***
		(0.0369)		(0.0377)		(0.0690)
Deviation from previous consensus		-0.790***		-0.732***		-0.629***
		(0.0685)		(0.0509)		(0.0668)
Constant	-0.00520***	0.660***	-0.257***	1.384***	-0.176***	0.446*
	(0.0000535)	(0.123)	(0.00460)	(0.105)	(0.00669)	(0.195)
R^2	0.020	0.224	0.029	0.183	0.002	0.100
N	5059	5059	3594	3594	3806	3806

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table presents estimates of equations (8) and (9), including forecaster fixed effects. The estimation sample ends in 2016 Q4, and starts in 1970 Q1 for the GDP deflator (PGDP), in 1982 Q1 for the consumer price index (CPI) and in 1981 Q3 for gross national output (GDP). Robust standard errors are used.

Table II presents the benchmark results, for the same estimation samples as in the previous section. Columns 1, 3 and 5 show that Coibion and Gorodnichenko (2015)'s test yields opposite results when using individual forecasts: positive revisions in individual forecasts are significantly associated with negative forecast errors. In other words, forecasters on average revise their forecasts by *too much*, and thus *overreact* to the information they receive between SPF survey rounds.

Columns 2, 4 and 6 show that this overreaction holds in response to both individual and aggregate information: individual forecast errors are on average more negative not only when individual revisions are more positive, but also when the previous SPF consensus forecast is higher. Moreover, individuals over-correct their forecasts: when their forecast exceeded the consensus from the previous survey, forecasters on average under-predict the fundamental in the following survey. Importantly, under the assumption of LIRE, all three coefficients should equal zero. That they are significantly negative is thus an indication that forecasters overreact to both public and private information, and over-correct their previous forecast whenever they deviated from consensus.

Note that these results contradict the LIRE hypothesis, but are consistent with the underreaction of average forecasts documented in Section 3.2: individual forecasters react too strongly to individual information, implying a weight on private information in equation (3) that exceeds its rational value. Any weight below one, however, implies a positive estimated coefficient in equation (7). As Table VI in the appendix shows, the results in Table II are robust to a number of changes to the benchmark specification.

4 Overconfidence vs Strategic Incentives

In this Section, we detail two extensions of the model from Section 2. The first introduces a behavioral bias, that of overconfidence in private information; the second a strategic motive for forecaster diversification. Both additions have been proposed previously to explain individual overresponses to private news (see, for instance, Ottaviani and Sørensen (2006)). Yet, as we demonstrate below, their responses to public information differ considerably. This allows us to separate the two explanations in the data.

4.1 Behavioral Overresponses

A substantial literature in psychology has documented a pronounced tendency by individuals to over-emphasize their new information in experiments (see for instance Kahneman and Tversky (1972)). As discussed in Cutler et al. (1990), and more recently for example in Bordalo et al. (2018) and Barberis et al. (2016), such overconfidence could also provide the basis for the apparent overresponses to information observed in financial market forecast data. In this subsection, we show how a simple model of overconfidence can simultaneously rationalize overresponses to both private as well as public information, consistent with what that we observed in the macroeconomic forecast data.

Consider the baseline model from Section 2, but suppose that each forecaster is overconfident in the precision of her own private information. That is, that she believes that the precision of x_{it} is $\tau'_{xt} > \tau_{xt}$, whereas the precision of other forecasters private information is simply τ_{xt}. Each forecaster thus wrongly thinks that her information is better others.

Characterization of Individual Forecasts: An overconfident individual's $t = 1$ forecast is from Bayes' Rule

$$f_{i1} = w'_x x_{i1} + (1 - w'_x)\,\mu, \quad w'_x = \frac{\tau'_{x1}}{\tau_\theta + \tau'_{x1}} > w_x. \tag{10}$$

An overconfident forecasters thus attaches more emphasis to her own private information than the corresponding mean-squared error forecaster from Section 2 ($w'_x > w_x$).

To understand the implication of overconfidence in private information for the $t = 2$ forecast, note that each forecaster attaches more weight to her own private information, but suspects that others, whose signals she thinks are less precise, do not. This has two consequences: First, the average, or consensus forecast becomes more reactive to the fundamental, and its information content increases as all individual forecasters react more to their private information. Second, forecasters fail to acknowledge this increase, and thus infer the precision of the public signal and the value of the fundamental incorrectly. We show how this can make them overreact to the consensus as well as to their private information.

To derive the individual's $t = 2$ forecast, we first need to differentiate between two different consensus (mean) forecasts: (1) the *realized* consensus forecast f_c, and (2) the *suspected* consensus forecast f_{sc}. The former measures the actual, realized consensus forecast value,

$$f_c = \int_0^1 f_{i1} di + \epsilon_c = w_x' \theta + (1 - w_x') \mu. \tag{11}$$

The latter, by contrast, measures the consensus forecast that the overconfident forecaster thinks she observes

$$f_{sc} = \int_0^1 f_{i1} di + \epsilon_c = w_x \theta + (1 - w_x) \mu, \tag{12}$$

where the distinction is due to the failure of the individual forecaster to internalize the overconfidence of others; that is, that all forecasters attach a weight of $w_x' > w_x$ to their own private information at $t = 1$.

The new information derived from the observation of consensus takes the realized consensus value, but treats it as if the forecasters observed the suspected one. As a result, the signal that forecasters infer from the observation of consensus becomes

$$s_c = \frac{1}{w_x} [f_c - (1 - w_x) \mu] = \frac{w_x - w_x'}{w_x} \mu + \frac{w_x'}{w_x} \theta + \frac{1}{w_x} \epsilon_c, \tag{13}$$

The forecasters nevertheless believe that they observe

$$s_{sc} = \theta + \frac{1}{w_x} \epsilon_c. \tag{14}$$

An important distinction between the two is that s_{sc} is less precise than s_c: The mean-squared error of $\mathbb{E}[\theta \mid S_c, \mu]$ is proportional to $(w_x \tau_c)^{-2}$ and $(w_x' \tau_c)^{-2}$, respectively, with $w_x' > w_x$. Forecasters believe that consensus is a less precise predictor of the fundamental than what it actually is. Another central distinction is that overconfident forecasters infer excessive values of the fundamental from consensus. The realized consensus loads onto the fundamental with $w_x'/w_x > 1$, while forecasts suspect that it only loads with one. Consequently, forecasters overrespond to news about the fundamental that is embedded in consensus. The root-cause for this overresponse is once more the failure of forecasters to internalize the overconfidence of others. This, in turn, makes the consensus more responsive to the fundamental than otherwise expected.

We are now ready to state an overconfident individual's $t = 2$ forecast

$$f_{i2} = \left(1 - \gamma'_{x1} - \gamma'_{x2} - \gamma'_c\right)\mu + \gamma'_{x1} x_{i1} + \gamma'_{x2} x_{i2} + \gamma'_c s_c, \tag{15}$$

where $\gamma_{x1} = \tau'_{x1} A^{-1}$, $\gamma_{x2} = \tau'_{x2} A^{-1}$, and $\gamma_c = w_x^2 \tau_c A^{-1}$ with $A = \tau_\theta + \tau'_{x1} + \tau'_{x2} + w_x^2 \tau_c$.

Implications of Overconfidence: Combined, the two forecasts (10) and (15) allow us to show three results. First, that because of the presence of private information, on average forecasters still underrespond to new information, as measured by the average forecast revision. Second, that despite this underresponsiveness at the average level, at the individual level, forecasters instead overrespond. And finally, that due the misinterpretation of consensus, a correlation arises between consensus and individual forecast errors.

The first result is analogous to that in Section 2 and therefore omitted here. To fix ideas about the second, suppose to start that we consider an individual's $t = 1$ forecast. The revision to her forecast relative to that of $f_{i0} = \mu$ is $rev_{i1} = w'_x\left(x_{i1} - \mu\right)$, while the associated forecast error is $\nu_{i1} = \theta - f_{i1} = (1 - w'_x)(\theta + \mu) - w'_x \epsilon^x_{it}$. But, because overconfidence implies an excessive weight on private information $w'_x > w_x$, these two variables are negatively correlated, precisely as in the data. In fact, the correlation is proportional to $\tau'_{x1}\left(1 - \frac{\tau'_{x1}}{\tau_{x1}}\right) < 0$, where $\tau'_{x1} > \tau_{x1}$. Since each forecaster is overconfident in the precision of her own private information, she overresponds to realizations of it. This, in turn, creates a negative correlation between her forecast error, on the one hand, and her forecast revision, on the other hand, that measures the consequences of the new private information.

This overresponse to private information, all else equal, also contributes to an overresponse to *all new* information observed at $t = 2$, as measured by the individual forecast revision between $t = 1$ and $t = 2$. In fact, although individuals underappreciate the informativeness of consensus, caused by the overresponse to private information at $t = 1$, individuals can also overreact to the public information embedded in consensus. Overresponses to the convolution of all new information observed at $t = 2$ can therefore, in part, be driven by an overreaction to public information.

To see this latter effect, consider the forecast error of an individual's $t = 2$ forecast when $\tau_{x1} = \tau_{x2}$ and $\tau'_{x1} = \tau'_{x2}$; that is, in the special case in which both the precision of private information and the extent of overconfidence is constant across the two periods,

$$\begin{aligned} u_{i2} &= \theta - f_{i2} \\ &= \left(1 - 2\gamma'_{x1} - \gamma'_c \frac{w'_{x1}}{w_{x1}}\right)(\theta - \mu) - \gamma'_{x1}(\epsilon^x_{i1} + \epsilon^x_{i2}) - \gamma'_c \frac{1}{w_{x1}} \xi. \end{aligned} \tag{16}$$

After some straightforward but tedious algebra, the correlation between the individual forecast error in (16) and consensus in (13) then becomes

$$\mathbb{C}ov\left(u_{i2}, s_c\right) = \frac{1}{\tau_\theta + 2\tau'_{x1} + w_{x1}^2\tau_\xi}\frac{\tau_{x1} - \tau'_x}{\tau_x(\tau_\theta + \tau'_x)}\left(w_{x1}w'_{x1}\tau_\xi - \tau_\theta\right) \neq 0, \tag{17}$$

which is non-zero and negative for a sufficiently imprecise prior $\tau_\theta \to 0$, for example.

The intuition behind this overresponse to public information rests on overconfident forecasters failure to internalize that the overconfidence of others makes the average forecast more reactive to news about the fundamental. The realized consensus in (13) has a loading on the the fundamental equal to $\frac{w'_{x1}}{w_{x1}} > 1$ while forecasters suspect the loading to be one in (14). This, in turn, makes them infer excessive movements in the fundamental from consensus. As a result, forecasters overrespond to news about the fundamental, and a negative correlation between individual forecast errors and the consensus itself arises.

We summarize these results in Proposition 1.

Proposition 1 *When forecasters are overconfident ($\tau'_{xt} > \tau_{xt}$ for $t = \{1, 2\}$), then there exists a $\tau'_\xi \in \mathbb{R}_+$ and a $\tau'_\theta \in \mathbb{R}_+$ such that for all $\tau_\xi \leq \tau'_\xi$, $\tau_\theta \leq \tau'_\theta$ and for $t = \{1, 2\}$:*

1. *Their average forecast errors are $(+)$ correlated with average forecast revisions.*
2. *Their individual forecast errors are $(-)$ correlated with individual forecast revisions.*
3. *Their individual forecast errors are $(-)$ correlated with consensus realizations.*

Thus, with a modestly informative consensus and prior, the overconfidence model is consistent with the broad qualitative features of the macroeconomic forecaster data.[8]

4.2 Strategic Forecaster Diversification

This subsection considers an alternative model, where forecasters are concerned not just with the long-run accuracy of their forecasts as such, but also with their accuracy relative to that of other forecasters. In fact, whenever clients only purchase a limited number of forecasts, this relative precision, as embodied for example in the quarterly rank-order statistics of news-outlets such as Bloomberg, becomes an important determinant of payoffs. As a result, forecasters become concerned not only about the average long-run accuracy of their individual forecasts but also about their relative accuracy vis-a-vis others.

To capture a particular form of such a concern about relative performance, we extend the standard model of Section 2 to include a contest motive that has been shown to imply overresponse to private news in static environments (Lamont (2002) and Ottaviani and Sørensen

[8] The reason for this dependence on the precision of consensus and the prior is simple: In order to have, for instance, underresponses at the average level, we need forecasters to attach sufficient emphasis onto their own private information, to counteract the overresponse to consensus, for example. This, in turn, requires the consensus and the prior not to be excessively informative.

(2006)). Specifically, we propose a dynamic extension of Ottaviani and Sørensen (2006), where payoffs are divided among all forecasters who correctly predict the fundamental. The model thus closely resembles a simple "winner-takes-all" competition. Note that, although the SPF is in fact an anonymized survey, in the SPF forecasters have no incentive to misrepresent their forecast relative to what they report to their own clients.

Characterization of Individual Forecasts: Consider the model of Section 2. Suppose now instead that an individual forecaster's payoff from her two reported forecasts f_{i1} and f_{i2} is

$$\mathcal{U}_i = U(f_{i1} \mid \Omega_{i1}) + U(f_{i2} \mid \Omega_{i2}), \tag{18}$$

where her per-period payoff is

$$U(f_{it} \mid \Omega_{it}) = \frac{q(f_{it} \mid \Omega_{it})}{\gamma(f_{it} \mid f_{it})}. \tag{19}$$

According to equation (19), forecaster i receives a payoff only if her forecast wins in the sense that it equals the fundamental, $\theta = f_{it}$, which occurs with probability $q(f_{it} \mid \Omega_{it})$. That is, $q(f_{it} \mid \Omega_{it})$ denotes the probability of winning the competition. The component $\gamma(f_{it} \mid \theta)$, by contrast, denotes the density of forecasts released conditional on state θ, and can be interpreted as the "number of winners" among whom the payoff is shared. Since an individual forecaster only wins if $\theta = f_{it}$, $\gamma(f_{it} \mid \theta)$ is evaluated at $\gamma(f_{it} \mid f_{it})$ in (19).

Suppose now that we start from the mean-squared error optimal forecast from Section 2. Is this forecast still optimal? Similar to Ottaviani and Sørensen (2006), we can show that this is indeed not the case. To see this, start from the conditional expectation of the fundamental at time $t = 1$, the posterior mode of the blue line in Figure 2. (The argument for $t = 2$ proceeds analogously, modulo a change in prior and private information.) Now suppose that we increase the weight on private information w_{x1}^c in our forecasts,

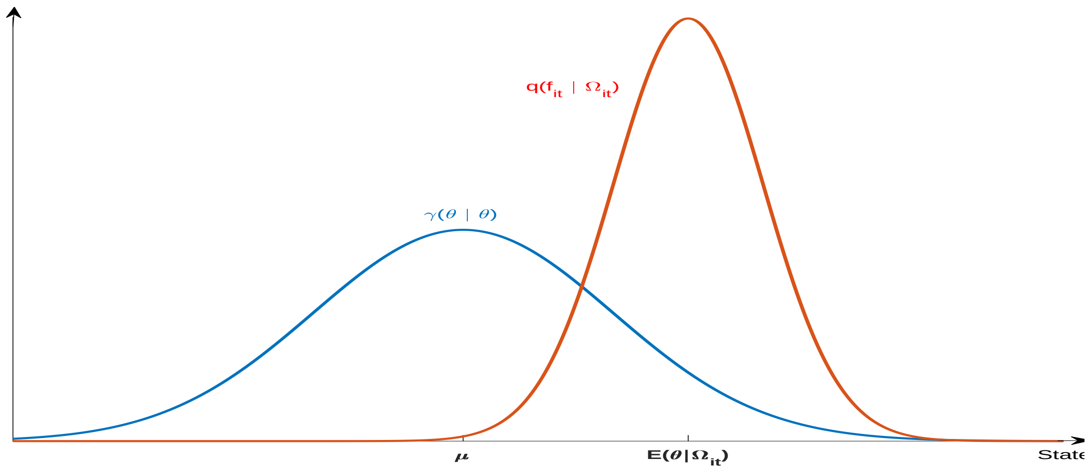
$$f_{i1} = w_x^c x_{i1} + (1 - w_x^c)\,\mu. \tag{20}$$

All else equal, this decreases our probability of winning the contest by twisting the forecast to the right of the mode. But since the posterior distribution is flat at the mode, this loss is of second-order, whereas the associated decrease in the mass of contestants that we believe to win at that point, the red line in Figure 2, is of first order. As a result, forecasters over-emphasize their own private information, $w_x^c > w_x$, to diversify their forecasts.

Lemma 2 *Consider forecaster $i \in [0,1]$ for $t = \{1,2\}$, and suppose $f_{jt} = f_{jt}^{mse}$, $j \neq i$. Then, it is optimal for i to report a f_{it} with a larger emphasis on private information.*

Equilibrium Characterization: We can use Lemma 2 to characterize the symmetric

Figure 2: Strategic Forecaster Diversification



The figure is similar to figure 2 in Ottaviani and Sørensen (2006) and depicts the incentive to deviate from reporting the conditional expectation in the forecasting contest.

Bayesian equilibrium in the model for $t = \{1, 2\}$. This, in turn, shows how the overresponsiveness documented in Lemma 2 carries over to the equilibrium of the model.

Proposition 2 *A unique symmetric linear equilibrium with strategy* $f_{it} = w^c_{xt} z_{it} + (1 - w^c_{xt})\mu_t$ *of the forecasting contest exists.* w^c_{xt} *is here within* $w^c_{xt} \in (0, 1)$ *for* $t = \{1, 2\}$, $\mu_1 = \mu$ *and* $\mu_2 = \mathbb{E}[\theta \mid f_c, \mu]$, *and* $z_{i1} = x_{i1}$ *and* $z_{i2} = \mathbb{E}[\theta \mid x_{i1}, x_{i2}]$. *Forecasters over-emphasize private information relative to what their conditional expectation would dictate.*

Proposition 2 extends the results of Ottaviani and Sørensen (2006) to the dynamic contest case. This, in turn, allows us to detail the implication of strategic diversification for the correlation between individual forecast errors, on the one hand, and their associated forecast revisions and consensus, on the other hand.

Proposition 3 *When forecasters have the contest motive in (19), then there exists a* $\tau^c_\xi \in \mathbb{R}_+$ *and a* $\tau^c_\theta \in \mathbb{R}_+$ *such that for all* $\tau_\xi \leq \tau^c_\xi$, $\tau_\theta \leq \tau^c_\theta$ *and for* $t = \{1, 2\}$

1. *Their average forecast errors are* $(+)$ *correlated with average forecast revisions.*

2. *Their individual forecast errors are* $(-)$ *correlated with individual forecast revision.*

For all $\tau_\xi \in \mathbb{R}_+$, $\tau_\theta \in \mathbb{R}_+$ *and for* $t = 2$, *however:*

1. *Their individual forecast errors are uncorrelated with consensus realizations.*

The intuition behind the first and second result in Proposition 2 once more follows from the presence of private information and the associated overresponses to it, respectively. The new result is the third one. We can illustrate this third result clearly in the case in which $\tau_{x2} \to 0$; that is, in the case in which the second period private signal is completely uninformative.

In that case, an individual forecaster's second-period forecast is

$$f_{i2}^c = w_{x2}^c x_{i1} + (1 - w_{x2}^c)\mathbb{E}[\theta \mid s_c^c], \tag{21}$$

where $w_{x2}^c \in (0, 1)$ and the signal that forecasters infer from the observation of consensus becomes

$$s_c^c = \frac{1}{w_{x1}^c} [f_c - (1 - w_{x1}^c)\, \mu] = \theta + \frac{1}{w_{x1}^c} \epsilon_c. \tag{22}$$

But then we can immediately compute the second-period forecast error,

$$u_{i2}^c = (1 - w_{x2}^c)\,(\theta - \mathbb{E}[\theta \mid s_c^c]) - w_{x2}^c \epsilon_{1t}^x, \tag{23}$$

which is uncorrelated with consensus realization s_c^c from the Projection Theorem since $\theta - \mathbb{E}[\theta \mid s_c^c]$ is uncorrelated with s_c^c. This demonstrates a central difference between the overconfidence model and that of strategic diversification. With strategic diversification, although forecasters over-emphasize their own private information to diversify their forecasts, they still correctly infer information from the first-period consensus. Consequently, their forecast errors in the second-period remain uncorrelated with consensus. Note that our analytical results concern unconditional correlations, unlike the evidence on the joint correlation structure contained in our estimates of (9) in Section 3.3. As we will see, the conditional correlations implied by the strategic diversification model are in fact slightly negative. But the central idea that strategic forecasters correctly back-out information from consensus remains a force that limits the magnitude of any of these correlations.

5 Quantitative Model Comparison

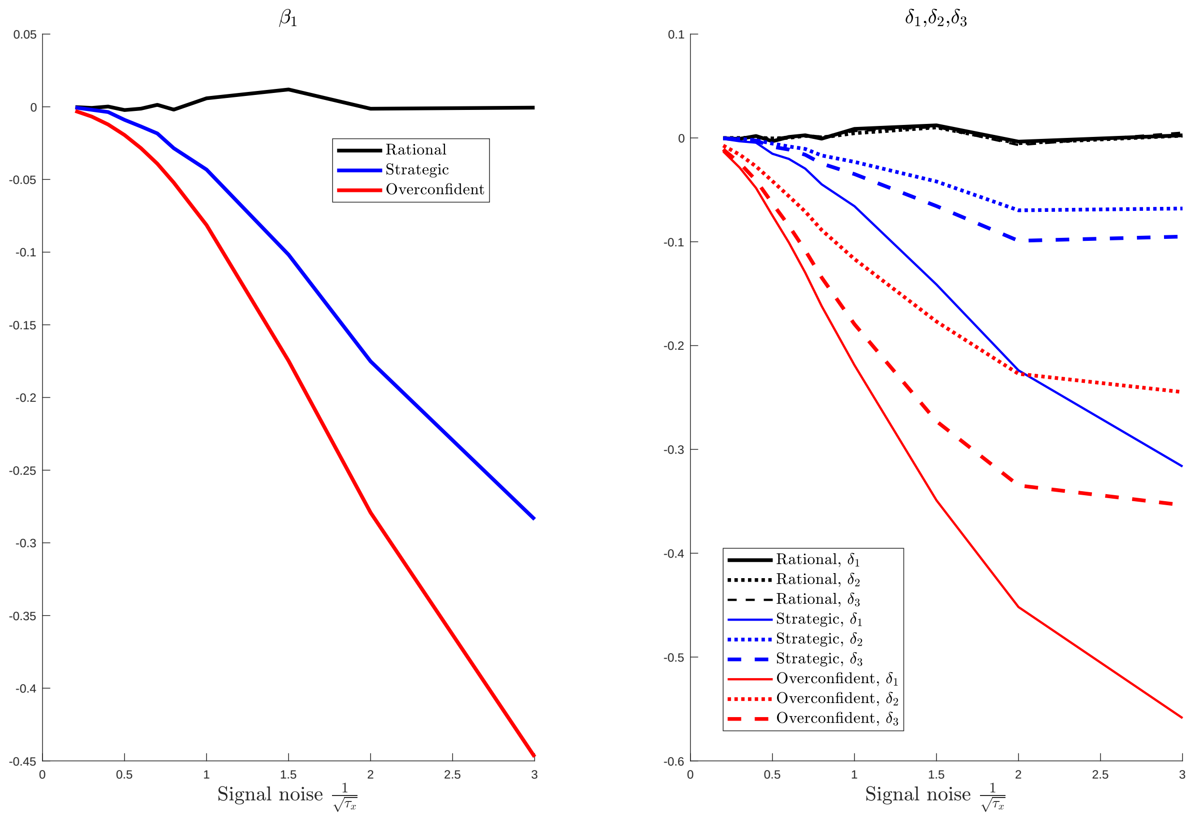
This section compares the implications of three models for forecaster behavior: the LIRE benchmark of Section 2, and the two simple alternative models of strategic diversification and overconfidence presented in Section 4.

5.1 Inspecting the Mechanisms

In order to highlight the model differences we use an illustrative parameterization with a moderately informative consensus forecast and an intermediate value of overconfidence. As a function of the precision of idiosyncratic signal noise $\sigma_x = \frac{1}{\sqrt{\tau_x}}$ with $\tau_{x1} = \tau_{x2} = \tau_x$ (depicted along the bottom axis), Figure 3 depicts the regression coefficients when equations (8) and (9) are estimated using data generated by simulations of the three models.[9] Its left panel shows

[9] Specifically, we normalize τ_θ to 1, set $\sqrt{\tau_\xi} = \frac{1}{2}$ and the overconfidence parameter τ'_{xt} to $4 * \tau_x$.

Figure 3: Coefficient estimates



The figure depicts the regression coefficients β_1 and δ_1 to δ_3 in equation (9) estimated on simulated data from the LIRE benchmark of Section 2 ("Rational"), and the two simple alternative models of strategic diversification ("Strategic") and overconfidence ("Overconfident") presented in Section 4. In the simulations τ_θ is normalized to 1, $\sqrt{\tau_\xi}$ equals $\frac{1}{2}$ and the overconfidence parameter τ'_{xt} is set to $4\tau_x$.

the univariate regression coefficient β_1. Trivially, this coefficient is approximately zero for the LIRE model and whenever private signals perfectly reveal the fundamental ($\sigma_x = 0$). As signal noise rises both strategic diversification and overconfidence predict increasing overreaction to idiosyncratic signals as illustrated by the increasingly negative and, for the chosen value of the overconfidence parameter τ'_{xt}, similar values of the blue and red lines. As the right panel of Figure 3 shows, however, overconfidence seems more consistent with the large negative coefficient estimates for δ_2 and δ_3 in equation (9). In fact, even at high values of signal noise, strategic diversification does not predict values of δ_2 below -0.07. As argued in Section 4.2, the reason is that the rationality at the heart of strategic diversification limits overreaction: strategic forecasters correctly infer the precision of the consensus forecast, which is enhanced by the overreaction in the previous forecast that increases its signal-to-noise ratio, as well as its information about the fundamental. Such optimal inference prevents strong overreaction, and optimally reduces the weight that forecasters give to their private signals from the first and second periods.

While the results in Figure 3 may be illustrative, they are contingent on a particular parameterization of the unknown noise parameters $\tau x1, \tau x2, \tau xi, \tau\theta$ as well as the overconfidence parameters, which we set equal to the same value τ'_x for $t = 1, 2$. The next section therefore takes a simulated method of moment approach that chooses the parameter combinations that best explain the regression coefficients in Figure 3.

5.2 Model Estimation

This section compares the ability of the three models to explain the overreaction, and overcorrection documented in Section 3.3 when parameters are chosen to minimize the difference between the regression coefficients of equations (8) and (9) implied by the model and those found in the data. Rather than in the particular estimated parameter values, we are interested in whether the admittedly simple models can rationalize the data moments for any parameter combination. For the estimation, the criterion we choose to minimize is

$$\Lambda(\tau) \quad = \quad [\hat{m} - m(\tau)]' W^{-1} [\hat{m} - m(\tau)]$$

where $\hat{m}$ is a vector of target moments of SPF data and $m(\tau)$ is the vector of simulated moments as a function of the parameter vector $\tau = \{\tau_{x1}, \tau_{x2}, \tau_\xi, \tau_\theta\}$ and, when applicable, the overconfidence parameters, which we set equal to the same value τ'_x for $t = 1, 2$. For our estimation we use a diagonally weighted minimum distance procedure, corresponding to a weighting matrix W that has the variances of the moments on the diagonal and is zero everywhere else. Apart from the four regression coefficients β_1 and δ_1 to δ_3 we also include the standard deviation of forecast errors $\sigma_{fc_{err}}$ among our target moments, as a measure of the mean forecast error, presumably one of the most important statistics of economic forecasts.[10]

Tables III to V present the results. Not surprisingly the LIRE model cannot generate coefficients that are economically different from zero. Strategic diversification according to the contest hypothesis can generate substantial overreaction to individual signals, and substantial over-correction of previous deviations from consensus as shown by the negative values of coefficients β_1, δ_1 and δ_3. In line with Figure 3, the model generates these values at extreme levels of noise in idiosyncratic signals (on the upper bound of our parameter grid). Noise in the consensus forecast, in contrast, is estimated to be small. Consistent with the results in Section 4.2, there is no economically significant overreaction to consensus forecasts, however. Moreover, the forecast error dispersion is substantially smaller than in the data for CPI and gross national output forecasts. This holds for forecasts of both inflation measures and growth in gross national output (apart from the error dispersion of GDP deflator forecasts, which is well matched by the model).

The overconfidence model predicts substantial overreaction to both idiosyncratic signals and the consensus forecast, as well as substantial over-correction of previous deviations from consensus. The model thus qualitatively matches the evidence well, although it does not match all coefficients perfectly, which is perhaps not surprising given its simplicity. For example, the reaction to consensus, as measured by the coefficient δ_2, is somewhat smaller than in the data, and more substantially so for the GDP deflator in Table III. The over-correction of deviations from consensus, indicated by estimates of δ_3 is stronger than that observed in the data in the case of forecasts of gross national output and the GDP deflator. The estimated values of the overconfidence parameter τ'_x are substantial, ranging from 0.2 to 0.4. The idiosyncratic signal noise is on average half as large as that in the contest model.

Table III: SMM estimation: PGDP data

	β_{11}	δ_1	δ_2	δ_3	σ_{fcerr}	σ_{x1}	σ_{x2}	σ_{xi}	σ_θ	τ'_x
Data	-0.19	-0.56	-0.79	-0.20	1.49					
Rational	-0.01	-0.00	-0.02	-0.02	0.96	5.00	5.00	5.00	1.00	
Strategic	-0.24	-0.45	0.01	-0.40	0.96	5.00	5.00	0.30	1.00	
Overconfident	-0.11	-0.75	-0.23	-0.74	1.50	3.00	2.00	1.00	1.00	0.20

6 Conclusion

Previous research has shown that *average* professional forecasts are in line with the limited information rational expectations hypothesis. By contrast, we show that *individual* forecasts in the US survey of professional forecasters contradict simple versions of the noisy rational

[10]We calculate the variance of $\sigma_{fc_{err}}$ using a bootstrap procedure.

Table IV: SMM estimation: CPI data

	β_{11}	δ_1	δ_2	δ_3	σ_{fcerr}	σ_{x1}	σ_{x2}	σ_{xi}	σ_θ	τ'_x
Data	-0.27	-0.59	-0.73	-0.59	1.27					
Rational	-0.01	-0.00	-0.02	-0.02	0.96	5.00	5.00	5.00	1.00	
Strategic	-0.31	-0.49	-0.03	-0.34	0.96	5.00	5.00	0.40	1.00	
Overconfident	-0.30	-0.54	-0.40	-0.43	1.25	2.00	5.00	5.00	1.00	0.40

Table V: SMM estimation: GDP data

	β_{11}	δ_1	δ_2	δ_3	σ_{fcerr}	σ_{x1}	σ_{x2}	σ_{xi}	σ_θ	τ'_x
Data	-0.08	-0.30	-0.63	-0.24	1.52					
Rational	0.02	0.02	-0.00	-0.00	0.97	5.00	5.00	3.00	1.00	
Strategic	-0.24	-0.45	0.01	-0.40	0.96	5.00	5.00	0.30	1.00	
Overconfident	-0.01	-0.05	0.01	-0.05	0.20	7.00	0.20	0.20	1.00	0.30

Tables III to V report estimates of the LIRE benchmark of Section 2 ("Rational"), and the two simple alternative models of strategic diversification ("Strategic") and overconfidence ("Overconfident") presented in Section 4. All models are estimated using a simulated method of moment procedure. Apart from the four regression coefficients β_1 and δ_1 to δ_3 we also include the standard deviation of forecast errors $\sigma_{fc_{err}}$ among the target moments. Columns 1 to 5 report their values in the data and the models, while columns 6 to 10 report the parameter estimates. Values of σ_{x1}, σ_{x2} and σ_ξ equal to 5 are on the upper bound of our parameter grid, where $\sigma_{x1} = \frac{1}{\sqrt{\tau_{x1}}}$, for example.

expectations hypothesis. Specifically, forecast errors violate the Projection Theorem as they are significantly correlated with past idiosyncratic variables, such as forecast revisions and deviations of individual forecasts from the previous consensus, but also with public variables, namely the previous consensus forecast itself. In fact, we find that forecasters seem to overreact to both private and public information, and to over-correct their previous deviations from consensus.

We investigate dynamic versions of two alternative models that have been proposed to explain overresponses to news, based on, respectively, strategic diversification within the forecast distribution, also called the 'contest' motive, and behavioral overconfidence. We find that both models can explain overreaction to private signals. Although one might expect such overresponses to imply under-reaction to other sources of information, we show how overconfidence can in fact also rationalize overreaction to public signals, which in our case are the consensus forecasts. This is because forecasters fail to recognize that the overconfidence of their competitors makes the average forecast more reactive to average news. This makes them infer excessive movements in the fundamental from any given movement in consensus. As a result, overconfidence can explain overreactions to consensus similar to those observed in the data. Strategic diversification, in contrast, does not give rise to substantial overresponses to endogenous public signals in the form of consensus forecasts, because the rationality at the heart of strategic diversification limits overreaction.

We think these results are interesting for two reasons. First, they illustrate how overconfidence in private signals naturally leads to overreaction to endogenous public signals. This effect is, in fact, independent of the particular environment we consider: whenever individuals fail to recognize that the overconfidence of others makes public signals – be they equilibrium objects such as prices or quantities, or just averages of individual actions as in our context – more reactive to fundamentals, they are likely to overreact. Second, we think that the evidence in favor of a behavioral explanation of forecaster behavior should motivate researchers to incorporate specific behavioral restrictions into the standard limited information rational expectations paradigm and test the resulting predictions against the data (see, for instance, Kohlhas and Walther (2018)).

Our results naturally imply some specific opportunities for further research. First, it would be interesting to see if our empirical findings also translate to other surveys of professional forecasters. In fact, in ongoing work, we also find overreaction to new information in the European survey of professional forecasters. Second, it would be interesting to investigate other models of strategic forecasting in the light of our empirical findings. While we focused on the 'contest' motive to strategically diversify forecasts, which has been found to imply overreaction to private signals in simpler environments, other studies have emphasized incentives to herd (see for example, the studies cited in Lamont (2002)). While herding typically leads to under-reaction to private signals, more in-depth analyses of alternative models would be welcome. Finally, it seems crucial to investigate whether the overconfidence for which we find evidence is specific to forecasters, or a feature of expectation formation more generally.

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7 Appendix

7.1 Robustness of empirical results

Table VI: Individual forecasts - robustness

	(1) PGDP	(2) PGDP	(3) CPI	(4) CPI	(5) GDP	(6) GDP
Forecast correction	-0.798***	-0.783***	-0.576***	-0.505***	-0.296***	-0.222**
	(0.0326)	(0.0444)	(0.0511)	(0.0525)	(0.0623)	(0.0724)
Previous consensus	-0.220***	-0.506***	-0.438***	-0.678***	-0.246***	-0.620***
	(0.0162)	(0.0411)	(0.0225)	(0.0503)	(0.0681)	(0.0546)
Deviation from previous consensus	-0.916***	-0.879***	-0.765***	-0.742***	-0.628***	-0.671***
	(0.0359)	(0.0423)	(0.0422)	(0.0617)	(0.0651)	(0.0731)
Constant	0.136**	0.740***	0.938***	1.485***	0.243	1.544***
	(0.0506)	(0.0865)	(0.0736)	(0.121)	(0.209)	(0.153)
R^2		0.435		0.136		0.160
N	3747	2896	3594	2911	3806	3009

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table presents estimates of equation (9), including forecaster fixed effects, for alternative estimation samples starting in 1981 Q3 (odd columns) and 1993 Q1 (even columns). All estimation samples end in 2016 Q4. Robust standard errors are used.

7.2 Proofs of Lemmas and Propositions

Lemma 1: The first part of the statement follows directly from the Projection Theorem, and we therefore do not discuss it further. The second part is shown in the main text for $t = 1$. The proof of the second part $t = 2$ proceeds as follows: Consider forecaster $i \in [0, 1]$'s second-period forecast

$$f_{i2} = \mathbb{E}_{i2}[\theta] = \gamma_{x1}x_{i1} + \gamma_{x2}x_{i1} + \gamma_c s_c + (1 - \gamma_{x1} - \gamma_{x2} - \gamma_c)\mu, \quad \gamma_x^1 = \frac{\tau_{x1}}{\tau_{x1} + \tau_{x2} + w_x^2\tau_\xi + \tau_\theta}.$$

This implies that the average forecast is equal to

$$f_2 = (\gamma_{x1} + \gamma_{x2} + \gamma_c)\theta + \gamma_c \frac{1}{w_x}\xi + (1 - \gamma_{x1} - \gamma_{x2} - \gamma_c)\mu$$

such that the average forecast revision becomes

$$rev_2 = (\gamma_{x1} + \gamma_{x2} + \gamma_c - w_x)\theta + \gamma_c \frac{1}{w_x}\xi + (w_x - \gamma_{x1} - \gamma_{x2} - \gamma_c)\mu$$

Table VII: Individual forecasts - robustness

	(1) PGDP	(2) PGDP	(3) CPI	(4) CPI	(5) GDP	(6) GDP
Previous consensus	-0.175***		-0.437***		-0.202***	
	(0.0342)		(0.0430)		(0.0599)	
Deviation from previous consensus		-0.460***		-0.397***		-0.476***
		(0.0600)		(0.0674)		(0.0602)
Constant	0.635***	0.00873	1.031***	-0.334***	0.376*	-0.385***
	(0.120)	(0.0509)	(0.126)	(0.0395)	(0.167)	(0.0848)
R^2	0.018		0.052		0.007	
N	6780	5059	4677	3594	4967	3806

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table presents the estimated coefficients in a regression of individual forecast errors on the previous consensus, and the deviation of individual forecasts from previous consensus, respectively, including forecaster fixed effects. The estimation sample ends in 2016 Q4, and starts in 1970 Q1 for the GDP deflator (PGDP), in 1982 Q1 for the consumer price index (CPI) and in 1981 Q3 for gross national output (GDP). Robust standard errors are used.

with associated average forecast error

$$u_2 = (1 - \gamma_{x1} - \gamma_{x2} - \gamma_c)(\theta - \mu) - \gamma_c \frac{1}{w_x}\xi.$$

The covariance between the two is $\mathbb{C}ov\,(rev_2, u_2) \propto \left(\frac{\tau_{x1}+\tau_{x2}}{\tau_{x1}+\tau_{x2}+\tau_\theta+w_x^2\tau_\xi} - \frac{\tau_{x1}}{\tau_{x1}+\tau_\theta}\right) \neq 0.$ $\square$

Proposition 1: Let us start from the bottom up. The second-period forecast error of an individual equals

$$u_{i2} = \left(1 - \gamma'_{x1} - \gamma'_{x2} - \gamma'_c \frac{w'_{x1}}{w_{x1}}\right)(\theta - \mu) - \gamma'_{x1}\epsilon^x_{i1} - \gamma'_{x2}\epsilon^x_{i2} - \gamma'_c \frac{1}{w_{x1}}\xi,$$

while the observed consensus is equal to

$$s_c = \frac{w_x - w'_x}{w_x}\mu + \frac{w'_x}{w_x}\theta + \frac{1}{w_x}\epsilon_c.$$

The covariance between the two is thus

$$\mathbb{C}ov\,(s_c, u_{i2}) = \left(1 - \gamma'_{x1} - \gamma'_{x2} - \gamma'_c \frac{w'_{x1}}{w_{x1}}\right)\frac{w'_x}{w_x}\frac{1}{\tau_\theta} - \gamma'_c \frac{1}{w_{x1}^2\tau_\xi} \neq 0,$$

where $\mathbb{C}ov\,(s_c, u_{i2}) \neq 0$ generically for $w'_{x1} > w_{x1}$.

Consider now instead an individuals first-period forecast error, which is

$$u_{i1} = (1 - w'_x)(\theta - \mu) - w'_x \epsilon^x_{i1},$$

while the associated forecast revision is $rev_{i1} = w'_x\left(\theta + \epsilon^x_{i1} - \mu\right)$. The covariance with u_{i1} is

$$\mathbb{C}ov\left(rev_{i1}, u_{i1}\right) = w'_x\left[(1 - w'_x)\frac{1}{\tau_\theta} - w'_x\frac{1}{\tau^1_x}\right] = \left(w'_x\right)\left(\frac{1}{\tau'_{x1}} - \frac{1}{\tau_{x1}}\right) < 0.$$

By contrast, the forecast revision for $t = 2$ is

$$rev_{i2} = \left(w'_x - \gamma'_{x1} - \gamma'_{x2} - \gamma'_c\frac{w'_{x1}}{w_{x1}}\right)(\mu - \theta) + \left(\gamma'_{x1} - w'_x\right)\epsilon^x_{i1} + \gamma'_{x2}\epsilon^x_{i2} + \gamma'_c\frac{1}{w_{x1}}\xi.$$

Consider now the case in which $\tau_{x1} = \tau_{x2} = \tau_x$ and $\tau'_{x1} = \tau'_{x2} = \tau'_x$. In that case,

$$\mathbb{C}ov\left(rev_{i2}, u_{i2}\right) = \left(\tau_x - \tau'_x\right)\frac{A}{\tau_x\left(\tau_\theta + \tau'_x\right)^2\left[\tau^2_x\left(\tau_\theta + \tau_\xi\right) + 2\tau^2_\theta\left(\tau_x + \tau'_x\right) + 2\tau^2_x\tau'_x + \tau^3_\theta + 4\tau_\theta\tau_x\tau'_x\right]^2} \neq 0,$$

where A is some a large expression with limits, $\lim A_{\tau_\theta\to 0} = -\tau_\xi\tau^4_x\left(\tau'_x\right)^2 < 0$ and $\lim A_{\tau_\xi\to 0} > 0$.

Last, the average forecast error for $t = 1$ is

$$u_1 = (1 - w'_x)(\theta - \mu)$$

with associated forecast revision $rev_1 = w'_x\left(\theta - \mu\right)$. The covariance of which with u_1 is

$$\mathbb{C}ov\left(rev_1, u_1\right) = w'_x(1 - w'_x) > 0$$

since $w'_x \in (0, 1)$. The average forecast error for $t = 2$ is in contrast

$$u_2 = \left(1 - \gamma'_{x1} - \gamma'_{x2} - \gamma'_c\frac{w'_{x1}}{w_{x1}}\right)(\theta - \mu) - \gamma'_c\frac{1}{w_{x1}}\xi$$

with forecast revision

$$rev_2 = -\left(w'_x - \gamma'_{x1} - \gamma'_{x2} - \gamma'_c\frac{w'_{x1}}{w_{x1}}\right)(\theta - \mu) + \gamma'_c\frac{1}{w_{x1}}\xi = -u_2 + u_1$$

the covariance of which is

$$\mathbb{C}ov\left(rev_2, u_2\right) = -\mathbb{V}\left[u_2\right] + \mathbb{C}ov\left[u_1, u_2\right]$$

such that $\mathbb{C}ov\left(u_1, u_2\right) \gtreqless 0$ since $\mid \mathbb{C}ov\left(u_1, u_2\right) \mid \leq \max\left\{\mathbb{V}\left[u_2\right], \mathbb{V}\left[u_1\right]\right\}$. □

Lemma 2: The proof is identical modulo a change of prior and private information to that of the static case in Ottaviani and Sorensen (2006). The prior μ_t for $t = 1$ and $t = 2$ is equal to $\mu_1 = \mu$ and $\mu_2 = \mathbb{E}[\theta \mid f_c, \mu]$, respectively. The private information is $z_{i1} = x_{i1}$ and $z_{i2} = \mathbb{E}[\theta \mid x_{i1}, x_{i2}]$. □

Proposition 2: The proof follows that of Ottaviani and Sorensen (2006) modulo a change of prior and private information to μ_t and z_{it}, respectively. □

Proposition 3: We once more start from the bottom. With the exception of a change in private information from x_{i1} to z_{i1}, the proof of the third result in Proposition 3 proceeds precisely the derivations shown in Section 4.2. We now show the second and third result for the $t = 1$ case. The results for $t = 2$ follow similar steps to those for $t = 1$ (see also the proof of Proposition 1).

An individual forecaster's $t = 1$ forecast equals

$$f_{i1} = Ax_{i1} + (1 - A)\,\mu, \quad A \in (0, 1)$$

with associated forecast error and revisions

$$\begin{aligned} u_{i1} &= (1 - A)(\theta - \mu) - A\epsilon^x_{i1} \\ rev_{i1} &= A(\theta + \epsilon^x_{i1} - \mu). \end{aligned}$$

Thus,

$$\mathbb{C}ov\,(rev_{i1}, u_{i1}) = A\left[(1 - A)\frac{1}{\tau_\theta} - A\frac{1}{\tau_{x1}}\right] < 0$$

since $A > w_x$. Last, the associated average forecast for $t = 1$ is

$$f_1 = A\theta + (1 - A)\,\mu, \quad A \in (0, 1)$$

with forecast error and revision

$$\begin{aligned} u_1 &= (1 - A)(\theta - \mu) \\ rev_1 &= A(\theta - \mu). \end{aligned}$$

Hence,

$$\mathbb{C}ov\,(rev_1, u_1) = A(1 - A)\frac{1}{\tau_\theta} > 0$$

since $A \in (0, 1)$. This completes the proof. □