The Impossibility of Krusell-Smith Equilibria*

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Abstract

Modern solutions of macroeconomic models with wealth inequality and aggregate shocks often rely on the assumption of *limited* but *common* information among households. We show that this assumption is inconsistent with rational information choice. To do so, we embed information choice into a workhorse heterogeneous-agent economy. First, we demonstrate that the benefits of acquiring more precise information about the current state of the economy depend crucially on household wealth. Second, because of such heterogeneous benefits to information acquisition and the strategic substitutability of savings choices, equilibria in which households acquire the *same* information do not exist for plausible costs of acquiring information. Our results further imply that a representative-agent equilibrium may not exist even in the absence of exogenous sources of wealth heterogeneity.

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1 Introduction

In the aftermath of the Great Recession, macroeconomics has had a renewed focus on the heterogeneous effects of economic shocks on households. The workhorse model of household heterogeneity in macroeconomics—combining aggregate risk and incomplete markets—is analytically and numerically intractable. This intractability stems from the entire wealth distribution, an infinite-dimensional object, being a state-variable (Den Haan *et al.*, 2010). To make progress, macroeconomists commonly combine the standard model with a notion of *boundedly-rational equilibria*, following the pioneering work of Krusell and Smith (1998): Instead of possessing full information about the evolution of infinite-dimensional state variables, agents only use a subset of state variables to forecast future prices. Crucially, this limited information is assumed to be the *same across all agents* in the economy.[1]

In this paper, we study households' information choices in a standard infinite-horizon incomplete markets model with aggregate risk. In contrast to the previous literature, we allow households' information choices to be optimal, and to depend on individual wealth, income, and other state variables. As a result, the extent to which households have limited information in our framework is a result of households' optimal choices, and not a restriction on the information sets imposed by the researcher. Through this lens, we propose a microfoundation for equilibria in economies with distributions of heterogenous agents and aggregate risk. We show that small deviations in assumptions can lead to drastically different outcomes.

We start by showing that heterogeneity in wealth and employment status naturally implies heterogeneity in the incentives to acquire information. In our model, low-wealth households, who save little regardless of aggregate conditions, decide not to pay even small costs of information acquisition. Similarly, depending on risk aversion, a constant savings rule may have low utility costs for households with substantial levels of financial wealth. By contrast, households that are neither poor nor rich but whose lifetime income is dominated by future wages, value information about current state variables highly. This is because it allows such households to better predict future incomes, and thus to make better savings choices.

Importantly, such heterogeneous benefits of information imply that Krusell-Smith type equilibria, in which agents with different wealth make the same information choice *in each period,* are not robust to the introduction of information costs. Some households would not find it optimal to pay even small such costs of acquiring information at some point.

We then explore how the heterogenous incentives to acquire information at at the micro-level interact with aggregate dynamics. We find that household information choices are strategic substitutes: individual benefits of information fall as the average degree of information in

[1] In fact, Krusell and Smith (1998) describe their method as based on a *behavioral* assumption of limited "perception" (p. 874).

the economy rises. All else equal, a large number of informed households reduces the volatility of savings, and hence that of the aggregate capital stock. As result, higher levels of household informativeness strongly dampen economic fluctuations. This, in turn, lowers the individual benefits of acquiring information about the current state of the economy by decreasing the volatility of future prices and wages. We show how such strategic substitutability in information choice naturally implies that homogeneous-information equilibria, where households make a once-and-for-all information choice, may not exist. Moreover, even without (exogenous) income and wealth heterogeneity, we show that a representative-agent equilibrium with information choice may not be present.

To illustrate our main results, we first focus on a simplified, two-period version of the standard neoclassical economy. With log-preferences, households naturally split into three groups, according to their first-period resources: The first group are poor households, for whom costs of information acquisition outweigh the limited benefits they can obtain from information. The second group consists of those households who are rich enough for consumption to be approximately unaffected by future wages. They will not pay any utility cost of information even if it perfectly reveals future wages and returns, and therefore do not acquire information either.[2] The final group consists of households with an intermediate level of current resources. They, in contrast, strictly benefit from information as it improves their savings choices.

The three groups further interact in general equilibrium. Because the savings of informed households are high when they expect low capital in the second period (and vice versa), informed savings reduce the dispersion of future wages and interest rates, and thus lower the cost of uninformed savings decisions today. The more middle-income earners there are, the more important this strategic substitutability becomes. Consequently, the larger is the range of information cost parameters for which there is no homogenous equilibrium in pure strategies. By implication, there may not exist a representative-agent equilibrium even in an economy that consists of ex-ante identical agents with identical wealth.

Finally, to quantify our results, and to study the relationship between information choice and *endogenous* inequality in wealth and consumption, we study once-and-for-all information choice in the full, infinite-horizon economy model with aggregate and idiosyncratic risk and incomplete markets. We show that the benefits of acquiring information are strongly heterogeneous along the equilibrium distribution of wealth and employment status, for the reasons discussed above. We then quantify the relationship between the average degree of household informativeness and individual costs of uninformed decisions. In the standard Krusell-and-Smith economy, where all agents have (limited) information about the aggregate state of

[2]This result depends, however, on the assumption of log-preferences: with higher risk-aversion, there is a wealth threshold beyond which households always acquire information.

the economy, uninformed savings decisions are not very costly; the capital stock is tightly distributed around its steady-state level. Indeed, the average utility implied by uninformed decisions is not much different from that implied by informed choices: utility losses average to only 0.18 percent in consumption equivalent terms, and even the maximum loss are only equal to 0.7 percent. When all individuals make uninformed savings choices, in contrast, the variance of the capital stock is 35 percent higher. This increases the average (maximum) utility loss from uninformed savings to 0.98 percent (3.95 percent), a more than fivefold increase. We use these results to quantify the range of information costs where no homogeneous-information equilibrium exists. On balance, we find that for plausible costs of acquiring information, no Krusell-Smith type equilibria exists.

Organization Section 2 presents the economic environment, while Section 3 derives analytical results that characterize households' benefits from additional information. Section 4 quantitatively assesses households incentive to acquire information. We conclude in Section 5. An appendix contains all proofs, in addition to further quantitative results.

2 Economic Environment

We consider a standard incomplete markets economy with aggregate risk. The environment closely follows that of Krusell and Smith (1998) but with a modified information structure.

2.1 Technology and Preferences

Firms: The production sector consists of a representative competitive firm, which maximizes profits. Output Y_t is produced in accordance with a Cobb-Douglas production function that aggregates economy-wide labor services and capital:

$$Y_t = z_t K_t^{\alpha} L_t^{1-\alpha}, \quad \alpha \in [0,1], \tag{2.1}$$

where K_t and L_t denote economy-wide capital and labor in period t, respectively. Total factor productivity z_t is stochastic and follows a first-order Markov process that takes on two values $z_t \in \{z_l, z_h\}$ with $z_h > z_l$. We assume that markets for labor and capital are competitive, so that factor prices for labor w_t and capital r_t are given by their marginal product:

$$w_t = z_t(1-\alpha)K_t^{\alpha} L_t^{-\alpha}, \quad r_t = z_t \alpha K_t^{\alpha-1} L_t^{1-\alpha}. \tag{2.2}$$

Households: The household sector comprises of a continuum of ex-ante identical households of unit mass, who have logarithmic preferences over non-durable consumption:

$$\mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log c_t, \tag{2.3}$$

where $\beta \in (0,1)$ denotes the time discount factor and c_t the household's consumption at time t. Each household is endowed with $\bar{l}$ units of time, which it supplies inelastically to the labor market. Household labor productivity ϵ_t is stochastic and can take on two values $\epsilon_t \in \{0,1\}$, which we interpret as unemployment and employment, respectively. We assume that ϵ_t follows a two-state, first-order Markov process $\Pi_{z',\epsilon'|z,\epsilon}$, which depends on both ϵ_t and z_t. A household earns wage w_t when employed and receives unemployment benefits μw_t when unemployed, where $\mu \in (0,1)$. We assume that households cannot borrow but can only save in physical capital k_t, whose net return is $r_t - \delta$, where $\delta \in (0,1)$ denotes the rate of depreciation on capital. Household consumption choices are restricted by the per-period budget constraint:

$$c_t + k_{t+1} = (1-\tau)\epsilon_t w_t \bar{l} + \mu(1-\epsilon_t) w_t \bar{l} + (1 + r_t - \delta)\, k_t, \tag{2.4}$$

where τ denotes the tax rate on labor income. We denote the right-hand side of (2.4) by $m_t \equiv (1-\tau)\epsilon_t w_t + \mu(1-\epsilon_t)w_t + (1 + r_t - \delta)\, k_t$, and refer to m_t as household *cash-at-hand.* A household seeks to maximize its utility in (2.3) subject to the budget constraint in (2.4).

Finally, we assume that the share of households in a given idiosyncratic employment state only depends on current total factor productivity z_t. Hence, the unemployment rate is a function only of z_t, and thus only takes on two values u_h and u_l with $u_h > u_l$.

2.2 Government

The government runs a balanced-budget unemployment insurance scheme, such that $\tau_t = \frac{\mu u_t}{L_t}$, where L_t and $u_t = l - L_t$ are the employment and unemployment rates, respectively.

2.3 Timeline and Information Structure

The economy proceeds through two stages. In the first stage, at the start of period $t = 0$, households choose once-and-for-all which signals $\mathcal{I}_t$ they want to receive about the current state of the economy (described below) at fixed utility cost $\kappa(\mathcal{I}_t)$. We restrict households to making the same information choice in each period, and we impose a maximum signal set $\mathcal{I}_t^{\max}$ that contains the signals that the household can choose between. We assume that $\mathcal{I}_t^{\max}$ can include current market signals such as aggregate capital. The households information set Ω_t accumulates in future periods $t > 0$ according to $\Omega_t = (\mathcal{I}_s)_{s=0}^{t}$. Once households have committed to their information choices, the economy transitions to the second stage. Nature determines the realization of the innovations ϵ_t and z_t for each $t > 0$. Conditional on their information choices, households make consumption and savings decisions, firms produce, and goods and input market prices adjust to clear markets.

2.4 Discussion

The state space of our economy is highly multi-dimensional. Even if households have full information about all shocks, because their saving choices are non-linear functions of capital holdings, there is no law of motion for aggregate capital that households can use to accurately predict future wages and returns. The state of the economy comprises the entire joint distribution of capital and employment status (e.g., Krueger *et al.*, 2016).

Motivated by this fact, standard solutions of incomplete market economies with aggregate risk (e.g., Krusell and Smith, 1998) assume that households employ a law of motion that uses information only about a limited set of moments of the cross-sectional distribution. Indeed, predictions of future prices are often extremely accurate in standard models when households' information comprises *only* the current level of productivity z_t and the current mean of the capital distribution $\overline{k}_{t+1}$ (equal to the aggregate capital stock K_t) (Den Haan *et al.*, 2010). Consistent with this convention we take as a benchmark $\mathcal{I}_t^{\max} = \{z_t, K_t\}$. Consistent with rational expectations, we assume that agents use the equilibrium law of motion H to update their posterior distribution about future variables, conditional on their information.

Trivially, whenever $\mathcal{I}_t$ is constant over time and identical for all households, and includes z_t and K_t, our approach is identical to that in Krusell and Smith (1998). However, in contrast to Krusell and Smith (1998), we also consider information sets that do not include aggregate productivity z_t, or the mean of the capital distribution K_t, and we crucially allow information sets to differ across households.

Furthermore, while we retain the requirement that, in equilibrium, households' perceived law of motion H captures the conditional distribution of elements in $\mathcal{I}_t$, we do not necessarily require H to describe their dynamics accurately in a statistical sense. In other words, we allow households to make non-negligible expectational errors in equilibrium.

Finally, we note that the presence of heterogeneity in information further complicates the state space meaningfully. Since households can learn about the state of the economy from market outcomes, such as the cross-sectional average of capital holdings, households need to not only form beliefs about the cross-sectional distribution of capital and employment, but also about each household's beliefs about it, and so on ad infinitum (Townsend, 1983). Section 4 describes how we simplify this double-infinity state-space.

2.5 Household Problem and Equilibrium

Given our timeline, the description of the equilibrium is as follows:

Let $S = (\Gamma, z)$, where Γ denotes the cross-sectional distribution of capital and employment at $t \geq 0$. We denote an individual household's first-order belief about S by $\mathcal{P}_i(S)$, where subscript $i \in [0,])$ identifies the i's household. Household i's second-order belief about household j's belief is denoted as $\mathcal{P}_{ij}(S)$, and so on ad infinitum. Individual household beliefs are summarized by: $p_i = \left\{\mathcal{P}_i, (\mathcal{P}_{ij})_{j\in[0,1]}, ..., (\mathcal{P}_{ij...k})_{j,...,k\in[0,1]^{n-1}}, ...\right\}$. Let $\mathcal{P}$ denote the set of all such beliefs $\mathcal{P} = \left\{(\mathcal{P}_i)_{i\in[0,1]}, (\mathcal{P}_{ij})_{i,j\in[0,1]^2}, ..., (\mathcal{P}_{ij...k})_{i,j,...,k\in[0,1]^n}, ...\right\}$. The aggregate state of the economy can then be described by $\Sigma = (S, \mathcal{P})$, while the individual state variables are $\sigma_i = (m_i, \epsilon_i, \mathcal{I}_i, \Sigma, p_i)$. Households solve their dynamic problem in two stages.

At the start of $t = 0$, households choose what information to acquire each period $\mathcal{I} \in \mathcal{I}^{max}$:[3]

$$V(m, \epsilon, p_{-1}, \Sigma_{-1}) = \max_{\mathcal{I}} \mathbb{E}\left[W(m, \epsilon, p, \Sigma) - \kappa(\mathcal{I}) \mid \Omega_{-1}\right], \tag{2.5}$$

where V and W denote a household's value functions before and after information choice, respectively. Information acquisition entails a utility cost κ per signal acquired in $\mathcal{I}$. We note that households' expectations in the first stage are computed using their ex-ante prior p_{-1}. We assume that every period households rationally use their information, together with the equilibrium law of motion for the aggregate state, which we denote by H, i.e., $\Sigma = H(\Sigma_{-1}, z, (\mathcal{I})_i)$, and the exogenous transition matrix Π^z, to form a prior about today's state variables from yesterday's posterior.

After deciding on their information set $\mathcal{I}$, every period $t \geq 0$, households choose consumption c and savings k' given their updated information set $\Omega = \{\Omega_{-1} \cup \mathcal{I}\}$:

$$W(m, \epsilon, p, \Sigma) = \max_{c,k'\geq 0} \log c + \beta\mathbb{E}\left[W(m', \epsilon', p, \Sigma) \mid \Omega\right] \tag{2.6}$$

$$\text{subj. to}$$

$$c + k' = y$$

$$m' = r(\Sigma')k' + w(\Sigma')L'\epsilon + (1-\delta)k' \tag{2.7}$$

We let g denote the function that characterizes a household's savings choice $k' = g(\sigma)$, and ι the function that characterizes its information choice $\mathcal{I}_t = \iota(\sigma_{-1})$. Finally, today's posterior beliefs p are linked to yesterday's p_{-1} through Bayes' Rule and the information choice $\mathcal{I}$.

Given this formulation of a household's problem, the definition of a *Recursive Competitive Imperfect Information Equilibrium* (RIICE) straightforwardly extends the standard definition

[3]We once again abstract from the subscript i, to ease notation.

of a RCE: A RIICE is a law of motion $H : \Sigma' = H(\Sigma, z, (\mathcal{I})_i)$, a pair of individual functions W and V, which describe the value functions at the two separate stages of a household's problem, a pair of policy functions (g, i), and pricing functions (r, w) such that (i) (W, V, g, ι) solves a household's problem, (ii) r and w are competitive, (iii) H is generated by $\mathcal{I}$ and g and Bayes' Rule, using the information contained in $\mathcal{I}$ and current beliefs summarized in $\mathcal{P}$.

3 Analytical Results in a Simplified Model

This section analyzes a simplified version of our baseline economy, whose analytical tractability allows us to highlight the forces that determine households' information choices. We collapse all future periods into a second period $t = 1$, and consider information choices in a first period $t = 0$. We proceed in two steps. First, because the relevant variables for consumption and savings choices are future prices, we initially let households acquire information directly about second-period prices. This allows us to highlight how households' information choices differ across the wealth distribution. Second, we proceed to characterize equilibrium information choices, where period-two prices are determined by exogenous productivity shocks and the capital stock that results from period-one savings decisions. We show that a pure strategy equilibrium may not exist for households' information choices.

3.1 Heterogenous Benefits of Information

Consider a household with cash-at-hand m_0, who has the option to purchase *perfect information* about period-two wages w_1 and returns r_1, before choosing period-one consumption c_0 and savings k_1. The household enters the initial period with a non-degenerate prior distribution $\Phi(w_1, r_1)$ over w_1 and r_1 on the bounded support $\Psi = [\underline{w}, \overline{w}] \times [\underline{r}, \overline{r}]$ for $\underline{w} < \overline{w}$ and $\underline{r} < \overline{r}$. The household's Euler equations for k_1, with and without information choice are, respectively:

$$\frac{1}{m_0 - k_1} \geq \beta \frac{1}{k_1 + \frac{w_1}{r_1}}, \quad \frac{1}{m_0 - k_1} \geq \beta \mathbb{E}_\Phi \left[\frac{1}{k_1 + \frac{w_1}{r_1}} \right], \tag{3.1}$$

where $\mathbb{E}_\Phi [\cdot]$ denotes the expectation operator with respect to Φ. Equation (3.1) holds with equality whenever $k_1 > 0$. Proposition 1 characterizes a household's expected benefits from acquiring information, using the household's utility in (2.3) and log-linear approximation of (3.1) around the perfect information choice.

Proposition 1. *Consider households in the simplified two-period version of the model with an exogenous prior about second-period prices w_1 and r_1:*

(i) There exists a threshold $\underline{m} \equiv \underline{w} (\beta \overline{r})^{-1} > 0$ such that households, whose first-period cash-at-hand m_0 is less than $\underline{m}$, have zero benefit of acquiring information.

(ii) There exists another threshold $\bar{m} > \underline{m}$ such that households whose first-period cash-at-hand m_0 exceeds $\bar{m}$, have a strictly positive benefit of acquiring information, which decreases towards zero in cash-at-hand m_0.

(iii) Finally, there exists cash-at-hand values $m_0 \in (\underline{m}, \bar{m})$ for which the benefit of acquiring information is strictly positive and increases in cash-at-hand m_0.

Proposition 1 shows that expected benefits of information follow an inverted u-shape in cash-at-hand. The reason for this shape arises from the effects of household wealth on savings choices k_1, the sole inter-temporal decision that households make.

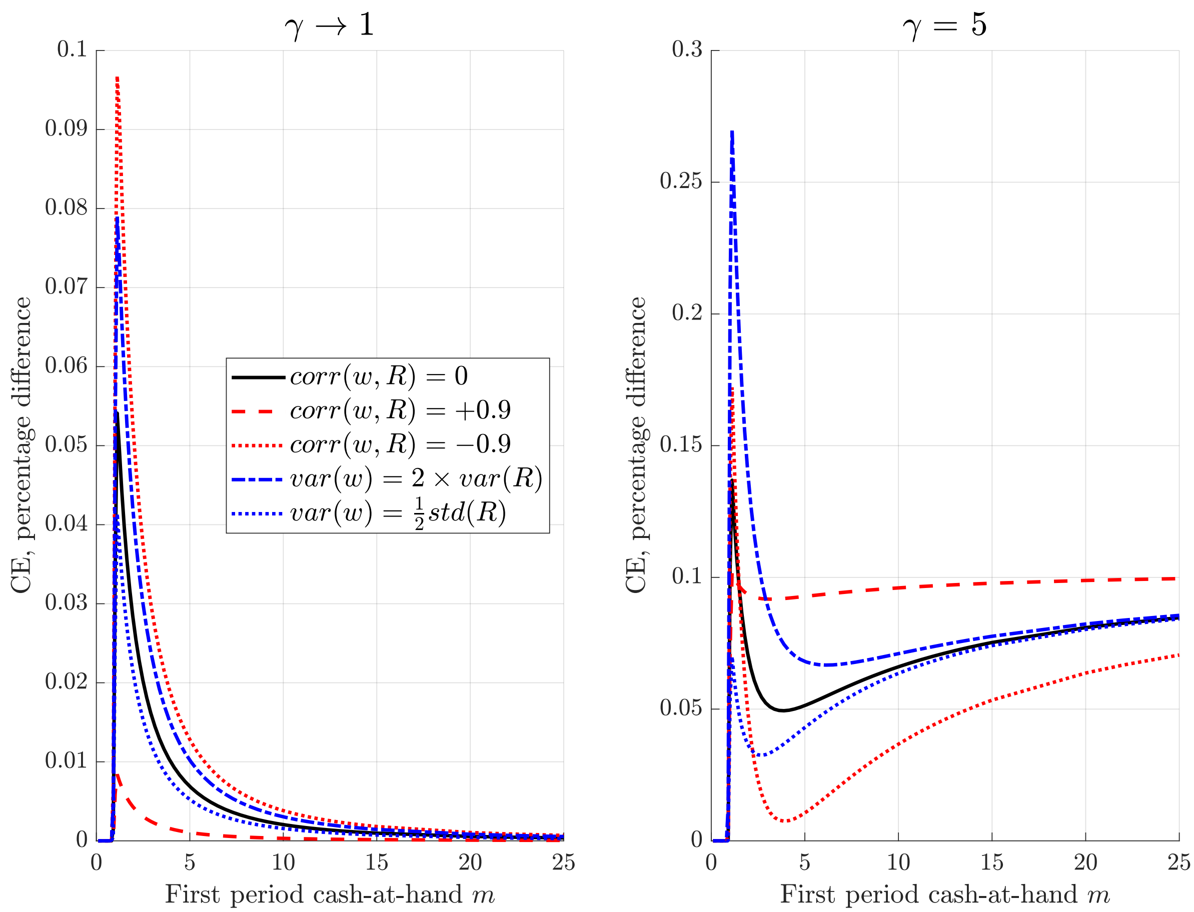
The first part of the proposition shows that households whose income is low enough to save zero $k_1 = 0$, irrespective of the present discounted value of future wages $\frac{w_1}{r_1}$, will never pay for information. Households for which $m < \underline{m}$ are *not* on their Euler equation (3.1 holds with strict inequality). As a result, these households choose to save zero irrespective of future wages or returns, and hence do not value information that helps better predict these prices.

By contrast, the second part of the proposition shows that households with interior savings choices, those for which $m_0 > \bar{m}$, *do value* acquiring information. This is because additional information helps these household make better savings choices; savings choices that are more in line with future wages and returns. However, a key feature of this incentive to acquire information is that it decreases with household wealth. Increases in m_0 make a household (with log-preferences) savings choices less responsive to future wages and returns, and hence decreases the expected benefit of information acquisition. Indeed, for households that are sufficiently rich $\left(\frac{\overline{w}}{m_0} \to 0\right)$, the expected benefit of information about future prices convergences back to zero. We discuss below how these conclusions are modified in the case where households have a degree of relative risk aversion above one.

Finally, the last part of the proposition follows from the continuity of a household's utility function. It shows that there exists cash-at-hand values $m_0 \in (\underline{m}, \bar{m})$ for which the benefit of information is strictly positive. However, unlike households for which $m_0 > \bar{m}$, the expected benefit here increases with cash-at-hand. This is because the increase in household savings k_1, due to an increase in m_0, dominates the decreased sensitivity of household savings.

Figure 1 illustrates Proposition 1 numerically, by depicting the expected utility loss from uninformed savings choices for different prior distributions. We transform the utility losses into percentage differences in permanent consumption as a function of first period cash-at-hand m_0. With log-preferences (left-hand panel), losses follow an inverse u-shape pattern, and approach zero as first-period cash-at-hand m_0 rises. Expected losses are furthermore lower when wages and interest rates are perceived to be less volatile, or co-move more strongly. The latter arises because the present discounted value of future wage payments $\frac{w_1}{r_1}$ that determine informed savings in (3.1) is less variable when wages co-move more with interest rates. This will be important later.

Figure 1: Utility losses in the Two-Period Model



The figure depicts the expected certainty-equivalent (CE) utility loss from not acquiring information in the simple two-period model with $\beta = 0.99$ and a joint normal distribution for w_1 and r_1 with means of 1 and standard deviations of 5 percent. A household's felicity function is given by $u(c) \equiv \frac{c^{1-\gamma}-1}{1-\gamma}$. The left-hand panel considers the case of $\gamma \to 1$ (logarithmic utility); the right-hand panel that of $\gamma = 5$ (high risk-aversion).

Risk-Aversion and the Benefits of Information. The benefits of information at high-levels of cash-at-hand depend crucially on households' relative risk-aversion. With a higher level of risk-aversion than in (2.3), expected losses are higher and no longer inverse u-shaped. Instead, if we assume a higher-level of risk-aversion, losses have a local minimum at intermediate values for m_0, where future income is most "diversified" across wages and returns on savings. This is, moreover, a more powerful force when wages and interest rates correlate negatively. The right-hand side panel of Figure 1 demonstrates these results for the case in which households' per-period utility function is equal to $u(c) \equiv \frac{c^{1-\gamma}-1}{1-\gamma}, \gamma > 1$. As cash-at-hand m_0 rises further, expected losses then converge to a strictly positive limit. The difference between the right- and left-hand side panel of Figure 1, where $\gamma \to 1$ as in our baseline analysis, arises because increased risk-aversion means that households dislike variations in future consumption more. Households have a greater preference for aligning their savings choices with future interest rates, and hence value information more.

3.2 Equilibrium Information Choice

The utility benefits of information depend on the joint distribution of wages and rates of return on capital (e.g., Figure 1). In equilibrium, these in turn depend on aggregate productivity, as well as (other) households' savings and information choices. In this subsection, we illustrate the consequences of this circular relationship for the existence of an equilibrium information choice, and show how one force pushes households towards never acquiring information.

Information about Productivity z_t: We start by analyzing a household's incentive to acquire information productivity z_t. Consider equation (2.2) at time $t = 1$. This equation shows that information about second-period factor prices is embedded in information about future productivity z_1 and current aggregate savings K_1. Knowledge of z_1 and K_1 suffice to predict the future value of wages w_1 and the rate of return on capital r_1 since in equilibrium aggregate labor supply $L_t = 1$. Consider now the Euler equation (3.1). This shows that (i) savings choices k_1 either depend only on the present discounted value of future wage payments $\frac{w_1}{r_1}$ (if the household is on its Euler equation); or (ii) are independent of future wages and returns altogether ($k_1 = 0$ if the household is off its Euler equation). As a result, aggregate savings K_1 are unaffected by movements in wages and returns that are caused by productivity z_t. Indeed, directly substituting (2.2) into (3.1) shows that

$$\frac{1}{m_0 - k_1} \geq \beta \frac{1}{k_1 + \frac{\alpha}{1-\alpha} K_1}, \quad \frac{1}{m_0 - k_1} \geq \beta \mathbb{E}_\Phi \left[\frac{1}{k_1 + \frac{\alpha}{1-\alpha} K_1} \right], \tag{3.2}$$

which is independent of productivity z_1. We conclude that households that know the current distribution of cash-at-hand $(m_{0i})_{i\in[0,1]}$, and therefore correctly anticipate aggregate savings K_1, do not wish to acquire additional information about productivity z_1.

Proposition 2. *When households know the distribution of cash-at-hand $(m_{0i})_{i\in[0,1]}$, a unique equilibrium exists; no household chooses to acquire information about current productivity z_1.*

Information about Capital K_t: By contrast, households do have an incentive to acquire information about the capital stock K_1, and hence about the distribution of cash-at-hand

$(m_{0i})_i$. To illustrate the equilibrium consequences of households' incentive to acquire information about capital, we proceed in two steps. First, we consider the special case in which all households have the same level of cash-at-hand $\tilde{m}_0$ but are unaware of this equality. This will allow us to show that a representative household equilibrium does not necessarily exist. We then proceed with the case in which households are heterogenous in their cash-at-hand.

Representative Agent Case: Suppose that each household has the prior about average cash-at-hand $\tilde{m}_0 \sim \mathcal{N}(0, \tau_m^{-1})$. If a household does not acquire information, it observes the noisy signal $s_i = \tilde{m}_0 + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \tau_\epsilon^{-1})$ and $\mathbb{E}[\epsilon_i \epsilon_j] = 0$. By contrast, if a household does acquire information, it knows $\tilde{m}_0$ with certainty.[4] The case in which all households have the same cash-at-hand $\tilde{m}_0$, and are all on their Euler equation, resembles the classical case studied in Grossman and Stiglitz (1980).[5] As in Grossman-Stiglitz' analysis, agents' actions are strategic substitutes: the Euler equation in (3.2) directly shows that

$$\frac{\partial k_1}{\partial K_1} = -\beta \frac{\alpha}{1-\alpha} \frac{\mathbb{E}\left[\frac{1}{\left(k' + \frac{1-\alpha}{\alpha} K\right)^2}\right]}{\frac{1}{(y-k')^2} + \beta \mathbb{E}\left[\frac{1}{\left(k' + \frac{1-\alpha}{\alpha} K\right)^2}\right]} < 0. \tag{3.3}$$

This substitutability of savings choices, in turn, decreases the dispersion of the capital stock in equilibrium, and hence decreases the value of additional information. In fact, as in Grossman and Stiglitz (1980) and Hellwig and Veldkamp (2009), strategic substitutability in actions leads to strategic substitutability in information choice. Consequently, a range of values of the information cost parameter $\kappa \in \mathbb{R}_+$ exists for which there is no pure strategy equilibrium.

Proposition 3. *If households cash-at-hand $m_{i0} = \tilde{m}$ for all i, a range of values for $\kappa \in \mathbb{R}_+$ exists for which there is no equilibrium with a representative household with cash-at-hand $m_{i0} = \tilde{m}$: if all (other) households purchase information about $\bar{m}_0$, the change in expected utility from information $\mathbb{E}\Delta\mathcal{U} - \kappa$ is negative for small uncertainties, and vice versa.*

The results in Proposition 3 connect the neoclassical environment laid out in Section 2 with the class of quadratic games studied in Hellwig and Veldkamp (2009) and Colombo *et al.* (2014), in addition to the CARA-Gaussian asset pricing models analyzed in Grossman and Stiglitz (1980), Hellwig (1980), and Veldkamp (2011). But, although in each case, strategic substitutability of actions leads to strategic substitutability in information choice, the mechanism by which this occurs differs in our model from that in the previous literature.

The impact of (other) households' information choices does, namely, not arise through the observation of *endogenous information*, nor through a *direct payoff externality*. Instead, other

[4] We further follow convention and assume that $\int_0^1 \epsilon_i di = 0$.

[5] In this subsection, we also assume that there is no borrowing constraint ($k_1 \geq 0$ is absent).

households' information choices here matter because of a *pecuniary effect*. The more other households' purchase information, the less volatile the aggregate capital stock becomes, all else equal. The strength of the strategic substitutability in (3.3) is stronger when the average household is more informed, and does not dampen its responses due to imperfect information. This, in turn, lessens the incentive for an individual household to acquire information about capital, and hence the net-present value of wages $\frac{1-\alpha}{\alpha}K$ in the first place.

Finally, because of the strategic substitutability of information choices, a representative agent equilibrium, in which all households make the same information choice, may not exist for our economy. This is even though all households have the same cash-at-hand.

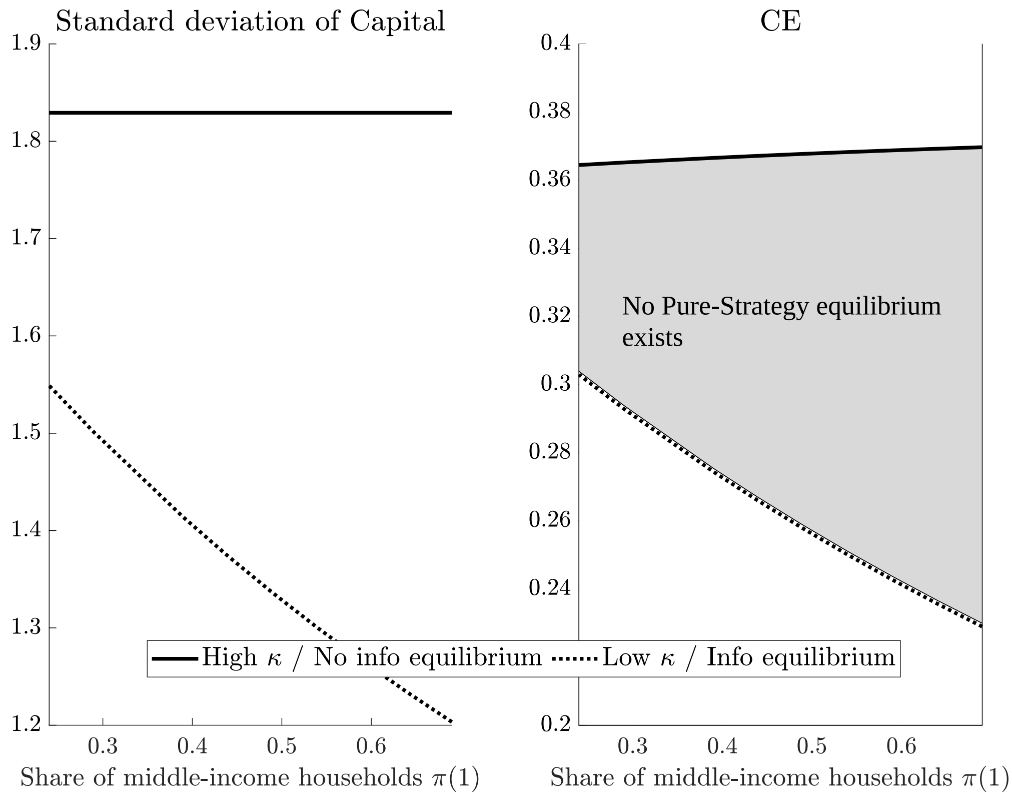
Heterogenous Agent Case: The introduction of heterogeneity in the cash-at-hand distribution into our above simplified setup in effect merges the insights of Proposition 1 with those of Proposition 3. To illustrate this result, we assume that m_0 takes on three values $m_0 \in \mathcal{M} = \{\epsilon, 1, \hat{m}\}$, and denote the mass of agents at each point as $\pi(x), x \in \{\epsilon, 1, \hat{m}\}$. We choose the points such that households at $m = \epsilon$ are always constrained, and hence have no incentives to acquire information. By contrast, households at $m = \hat{m}$ face second-period wages that are a negligible fraction of $\hat{m}$. Consequently, only middle-income households make non-trivial information choices. We choose a simple source of uncertainty about the cash-at-hand distribution: households know $\pi(1)$ and the aggregate cash-at-hand, but have a 50-50 prior about how many of the remaining households have cash-at-hand $m = \hat{m}$, where ϵ adjusts to keep the aggregate endowment constant. Because of the higher marginal propensity to save of high cash-at-hand households, this assumption translates into a 50-50 prior over aggregate savings K_1, and hence over future wages and returns.

Proposition 4. *Consider the three-point distribution for cash-at-hand $(m_{i0})_i$, where $m_0 \in \mathcal{M} = \{\epsilon, 1, \hat{m}\}$. Then, there exists for each $\pi(1) \in (0, 1)$ a fixed cost $\kappa > 0$ such that no equilibrium with homogeneous, pure-strategy information choices about K_1 exists.*

Proposition 4 shows that the non-existence of pure strategy equilibria, where all households make the same information choice, identified in Proposition 3, extends to the case with wealth inequality. To see how wealth inequality, nevertheless, modifies our previous insights, consider the extreme case in which only there are only low- and high-cash-at-hand households.

Low-cash-at-hand households are constrained, and in all states choose to save zero. As a result, the savings choices of these households do not feature the strategic substitutability highlighted in the previous subsection. This, all else equal, decreases the range of fixed cost parameters for which the non-existence of equilibria is present. High-cash-at-hand households are through this lens similar. Proposition 1 shows that high cash-at-hand households also have a smaller incentive to acquire information, all else equal, than medium wealth households.

Figure 2: Equilibrium information choice in the two-period model



The figure depicts the equilibrium standard deviation of capital in period 1 (left panel) and the ex ante expected consumption equivalent loss of not acquiring information (right panel, in percent), both evaluated at the prior distribution of uninformed agents, as a function of the mass of households at middle income $\pi(1)$ (along the bottom axis) when these middle-income households either acquire (dashed lines), or do not acquire information (solid lines). The remaining mass of households is split between high-income households (whose income equals 6500 and whose mass is either $\pi^h(\overline{y}) = 0.08$ or $\pi^h(\overline{y}) = 0.06$ percent. For every $\pi(1)$ we choose ϵ to normalize aggregate endowments to 1 (checking that the resulting value is consistent with zero savings). Also, we use illustrative parameters $\beta = 0.99$ and $\alpha = 0.4$.

These households are, however, still their Euler equation. Hence, their savings' choices are still strategic substitutes with those of other households. Yet, because of their smaller incentive to acquire information, the presence of a large mass of high wealth households once more narrows the range of fixed cost parameters for which the non-existence of equilibria is present.

Figure 2 illustrates the insights of Proposition 4. The left-hand panel of the figure shows that the expected standard deviation of capital is small in a low-κ economy, where all middle-income households choose to be informed, and hence save more when aggregate savings is low (and vice versa). This is compared to a high-κ economy, where no household buys information, and aggregate savings are independent of the true state of the economy. Because the expected benefit of information is higher the more uncertain the capital stock is, there exists a range of fixed costs κ that are too high for all middle-wealth households to buy information, but too low for no-one to buy it (the gray area in the right-hand panel). Finally, as the mass of medium-wealth households increases, the standard deviation of the equilibrium capital stock falls. This, in turn, causes a broader range of fixed costs parameters $\kappa > 0$ for which no homogenous pure strategy equilibrium exists. Combined, Proposition 4 and Figure 2 demonstrate the crucial interaction that exists between wealth inequality and information choice.

3.3 Summary and Discussion

The simple model that we have studied in this section has shown that the incentives to acquire information are heterogeneous across the wealth distribution. It has also shown that there is strategic substitutability in information choice at the heart of our neoclassical environment. The benefits from informed savings choices are lower the more other households are informed. Combined, our results imply that homogeneous-information equilibria, where all households make the same information choice, may not exist. The next section shows how our results extend to the general model, with an endogenous, continuous wealth distribution, endogenous dynamics of the capital stock, and an infinite planning horizon (that makes information acquisition beneficial even when it does not affect savings choices in the current period). We then conclude by discussing how our results cast doubt on the motivation behind standard solution methods used to solve dynamic macroeconomic models with wealth inequality.

4 Information Choice in the General Model

This section considers household information choice in the infinite-horizon version of our economy. We compute homogeneous-information equilibria, where all households acquire the same information $\widetilde{\mathcal{I}}$ in each period, and study individual households' incentives to deviate. As in the two-period model, the expected loss from uninformed choices varies strongly with household wealth, and savings choices are strategic substitutes. Because of these dependencies, we show that homogeneous-information equilibria do not exist for plausible information costs.

4.1 Quantitative Strategy

To quantify the expected losses associated with different information choices, we first solve for an equilibrium in which all households use the same information $\widetilde{\mathcal{I}}$ in each period. We compute the equilibrium using the iterative algorithm proposed in Krusell and Smith (1998):

1. Choose the information set $\widetilde{\mathcal{I}}$, and postulate a law of motion for the aggregate state H.[6]

[6] As in Krusell and Smith (1998), we assume a log-linear law of motion.

2. Solve the household problem conditional on $\widetilde{\mathcal{I}}$ and H.

3. Using the resulting decision rules, simulate a large number of households for a large number of periods. From this simulation, calculate time series for the elements in $\widetilde{\mathcal{I}}$, and use these to estimate a new law of motion H'.

4. Compare H' to H used in (2). If different, update conjecture for H and return to (1).[7]

With such an homogeneous-information equilibrium at hand, we then calculate optimal decision rules for consumption and savings when a household uses *a different information set* $\mathcal{I}$ in each period. Consistent with its information choice, the household also considers an alternative law of motion $H(\mathcal{I})$ for the endogenous variables in $\mathcal{I}$. We calculate the expected utility associated with both information choices, conditional on aggregate and individual states at time zero.[8] Finally, we transform the relative utility benefit from using $\mathcal{I}$ instead $\tilde{\mathcal{I}}$ into units of permanent consumption:

$$CE_{\mathcal{I}}\left(k,\epsilon;\tilde{\mathcal{I}}\right)=\left[\frac{V_{\tilde{\mathcal{I}}}+\frac{1}{1-\beta}\frac{1}{1-\gamma}}{V_{\mathcal{I}}+\frac{1}{1-\beta}\frac{1}{1-\gamma}}\right]^{\frac{1}{1-\gamma}}-1, \tag{4.1}$$

where $V_{\mathcal{I}}$ equals the discounted utility that a household with capital k and labor market status ϵ expects when they use the information set $\mathcal{I}$ in each period and the aggregate state of the economy is described by particular values of the elements in $\tilde{\mathcal{I}}$ (see also 2.5). Note that all terms in (4.1) also depend on the equilibrium law of motion for $\widetilde{\mathcal{I}}$, suppressed for simplicity.

As a baseline for our analysis, we consider the information set studied in Krusell and Smith (1998): each household observes economy-wide productivity and the capital stock in each period ($\widetilde{\mathcal{I}}=\{z,K\}\equiv\tilde{\mathcal{I}}_{\max}$). Krusell and Smith (1998) find that this information set allows for extremely accurate predictions of future wages and interest rates. We then study households incentives to acquire strictly less information in each period; that is to acquire instead either (i) $\mathcal{I}=\{z\}$, (ii) $\mathcal{I}=\{K\}$, or (iii) $\mathcal{I}=\emptyset$.[9] At the end of this section, we briefly comment on the potential benefits from observing additional information.

We characterize the relative utility loss $CE_{\mathcal{I}}(k,\epsilon;\mathcal{I}_{\max})$ of using $\mathcal{I}$ rather than the most comprehensive $\mathcal{I}_{\max}$, and how it depends on individual and aggregate states, as well as the average level of information in the economy $\widetilde{\mathcal{I}}$. This allows us to discuss the existence of homogeneous-information equilibria for different values of the fixed information cost κ.

[7]Given the log-linear nature of H, we use a simple regression to update the parameters.

[8]We evaluate the expectations in both cases using the law of motion H associated with the more comprehensive information set $\widetilde{\mathcal{I}}$.

[9]When $\mathcal{I}=\emptyset$, we assume that households use only average transitions and the unconditional mean of capital in their forecasts of future wages and interest rates.

Table I: Benchmark parameters

	β	γ	α	δ	$\bar{l}$	μ	Z_l	Z_h
Values	0.99	5.00	0.36	0.025	1/0.90	0.40	0.98	1.01

Table II: Transition probabilities

	$0\|Z_l$	$1\|Z_l$	$0\|Z_h$	$1\|Z_h$
$0\|Z_l$	0.55	0.45	0.55	0.45
$1\|Z_l$	0.050	0.95	0.0056	0.99
$0\|Z_h$	0.45	0.55	0.45	0.55
$1\|Z_h$	0.078	0.92	0.035	0.96

4.2 Parameters and Calibration

Table I summarizes the parameters we use in our quantitative analysis. We interpret a time period as a quarter, and choose a period utility function u with constant relative risk aversion equal to γ. Given evidence that prediction errors are declining in wealth in US micro data (Broer *et al.*, 2021), we choose a value of γ equal to 5, which is consistent with incentives to acquire information that are increasing in wealth over some range (see Figure 1). We choose standard parameters for the discount factor β (0.99), the capital share α (0.36), and the depreciation rate δ (0.025). We calibrate the structure of aggregate and idiosyncratic uncertainty to capture key features of the dynamics of unemployment and job-finding rates in the post-world war II US economy, in the spirit of Krusell and Smith (1998). Specifically, we specify transitions in aggregate productivity to capture good and bad times, defined as periods when unemployment is below and above trend, respectively.[10] The productivity then captures the difference in average US total factor productivity during the periods thus identified. The resulting persistence of good and bad times is 0.88 and 0.82, respectively, similar to that in Krusell and Smith (1998). The resulting values for Z_l, Z_h are 0.98 and 1.01, respectively. The parameters governing individual transition probabilities are specified to be similar to those observed in the US labor market. In particular, we choose an unemployment rate in booms and recessions equal to 6 and 10 percent, respectively. Job-finding rates are set such that unemployment spells are relatively short, as in US data, equal to 55 and 45 percent in booms and recessions, respectively. The remaining transition probabilities are then pinned down by the requirement that the unemployment rate depend only on current productivity, and are reported in Table II. Finally, we normalize the labor endowment $\bar{l}$ to have unit labor supply in the bad aggregate state, and set the replacement rate μ equal to 0.40.

[10] We use an hp filter with smoothing parameter 14400 to construct the trend in the unemployment rate.

4.3 Quantitative Results

4.3.1 Benefits of Information Acquisition

Figures 3 and 4 show the utility losses incurred by an *employed* household taking savings choices *without* any knowledge of the current state of the economy (so that $\mathcal{I} = \emptyset$). Figure 3 depicts these losses for a *high-information economy*, where all (other) households use $\widetilde{\mathcal{I}} = \{z, \overline{k}_t\} = \tilde{\mathcal{I}}_{\max}$. This is the economy considered in Krusell and Smith (1998).

We note that relative losses are markedly different at different levels of cash-at-hand, and the overall pattern is similar to that observed in our simple, two-period economy (the right-hand panel of Figure 1): Expected losses are highest for low but positive values of wealth, where savings choices are unconstrained but dominated by the difference between current and future labour income. Such high losses result because predictions of future labor income are, in this case, substantially improved by information about the current capital stock (which determines the level of future wage), as well as current aggregate productivity (whose persistence makes it a good predictor for future separation rates). As wealth rises, the difference between informed and uninformed savings policies falls, as do the expected losses from uninformed choices. Overall, however, losses are small in the high-information environment underlying Figure 3, with modest differences between periods of high and low productivity (in the left and right column, respectively), and across values of aggregate capital corresponding to the first and third quartile of its ergodic distribution (in the top and bottom row, respectively).

Given the calibrated high job-finding rates (of 50 percent on average), the (forward-looking) expected losses for the *unemployed* are similar to those of the *employed* shown in Figure 3. The main difference is that the expected losses are lower at low cash-at-hand levels, where the unemployed's current savings choice is constrained and expected losses only arise from uninformed choices in future unconstrained periods.

Finally, one crucial, closely related difference between Figure 1 and 3 lies in the losses of the wealth-poorest households in the economy, who did not incur any losses in our two-period economy, but who loose from uninformed savings choices in later periods in the more general version. This shows the importance of the dynamic nature of households' savings for households' information acquisition strategies.

Figure 4 reports the same utility losses of uninformed savings decisions as in Figure 3, but instead considers a *low-information economy*, in which $\widetilde{\mathcal{I}} = \emptyset$ so that all (other) households only take unconditional decisions. While the relationship between expected losses and individual wealth maintains its previous shape, expected losses are now substantially larger on average, and differ by up to a factor of two across the four panels.

Figure 5 shows the reason for why expected losses from uninformed savings choices are

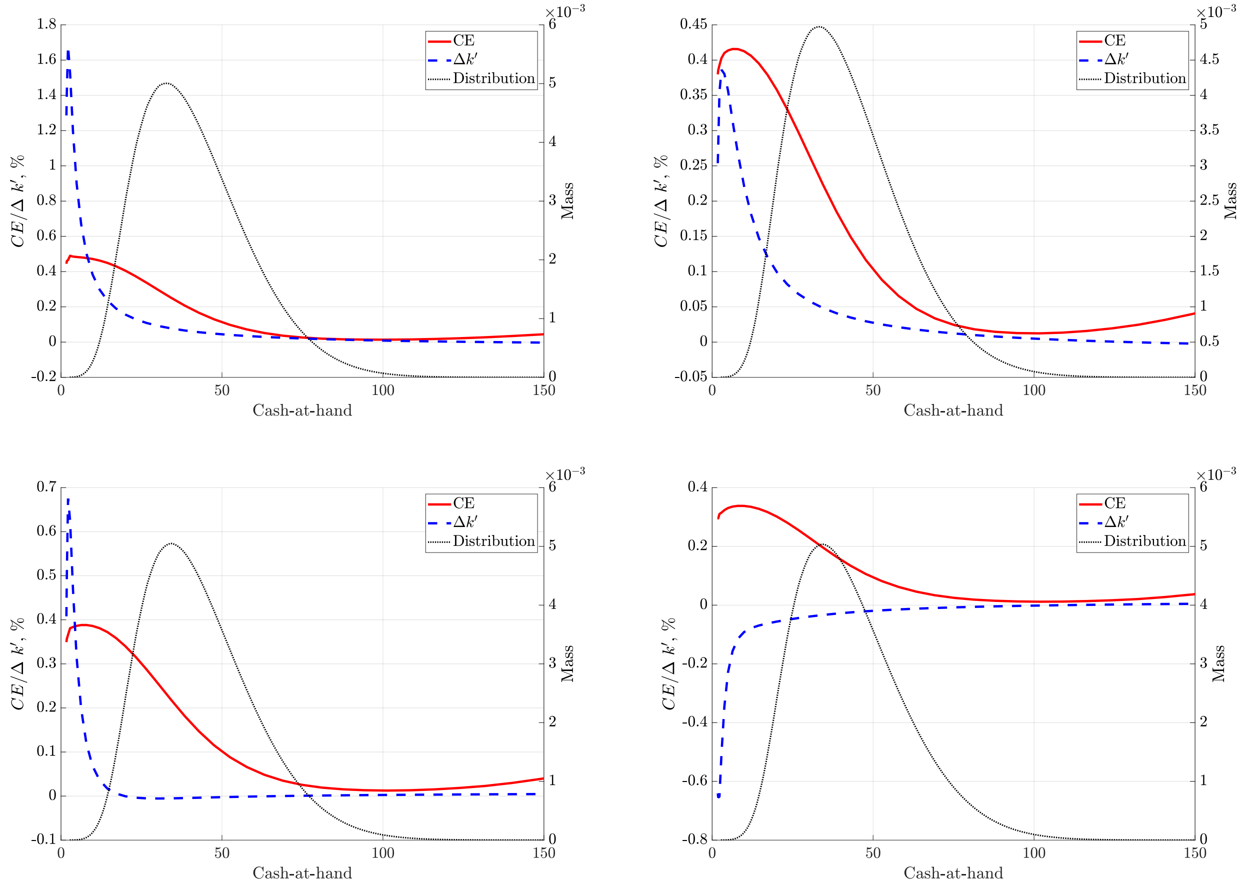


Figure 3: For the high-information economy ($\widetilde{\mathcal{I}} = \{z, \bar{k}\}$), the figure presents utility losses $CE_{\mathcal{I}}(k, \epsilon; \mathcal{I}_{max})$ (solid lines), differences in policy functions $k'(k, \epsilon; \mathcal{I} = \emptyset)$ relative to $k'(k, \epsilon; \mathcal{I} = \{z, \bar{k}\})$ (dashed lines), and the average cross-sectional distribution of households, conditional high and low aggregate productivity (in the left and right column, respectively) and averaged within a 2.5-percentile band around high and low $\bar{k}$, corresponding to the first and third quartile of the distribution of mean capital (in the top and bottom row), respectively. The figure concentrates on the employed ($\epsilon = 1$).

larger in the low-information economy. Indeed, the reason is identical to that identified in Section 3: the capital stock is substantially more volatile and persistent when households do not condition their savings choices on current productivity and the current level of the capital stock. In equilibrium, uninformed savings choices thus strongly dampen the mean reversion inherent in neoclassical economies, whereby higher returns implied by a lower capital stock increase savings in bad times, and vice versa for good times. The implied widening of the capital distribution around its average level makes average, uninformed savings choices more costly. And the increase in persistence makes information about the current level of capital even more valuable to predict the future.

Table III summarizes the expected and maximum gains or losses implied by deviations

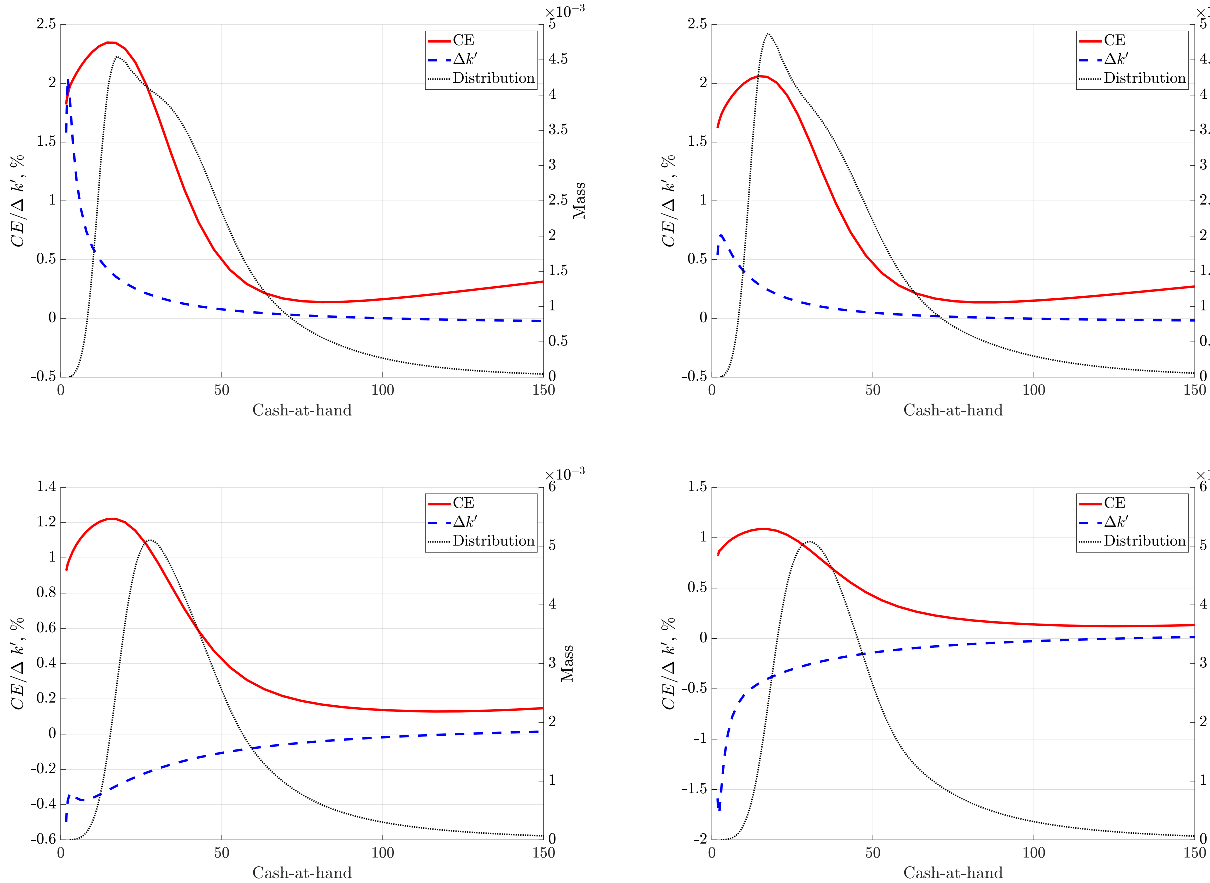


Figure 4: For the low-information economy ($\widetilde{\mathcal{I}} = \emptyset$), the figure presents utility losses $CE_{\mathcal{I}}(k, \epsilon; \mathcal{I}_{max})$ (solid lines), differences in policy functions $k'(k, \epsilon; \mathcal{I} = \emptyset)$ relative to $k'(k, \epsilon; \mathcal{I} = \{z, \bar{k}\})$ (dashed lines), and the average cross-sectional distribution of households, conditional on aggregate productivity and averaged within a 2.5-percentile band around high and low $\bar{k}$, corresponding to the first and third quartile of the distribution of mean capital, respectively. See the description of Figure 3. The figure concentrates on the employed ($\epsilon = 1$).

from the equilibrium Information set $\widetilde{I}$ in the high- and low-information economies, respectively. The table shows (i) *average* ex-ante expected losses or benefits (columns 1 and 3), and (ii) *maximum* ex-ante expected losses or benefits (columns 2 and 4). We compute averages and maxima over the whole ergodic distribution of individual and aggregate state variables. Consistent with our earlier results, additional information affects ex-ante welfare substantially more in the low-information economy ($\widetilde{\mathcal{I}} = \emptyset$), where average benefits from additional information are large. Consistent with Figure 5, most of these benefits can be reaped by knowing the capital stock alone ($\tilde{\mathcal{I}} = \{K_t\}$), while the incremental benefit of information about productivity z_t is small. Average losses from foregoing information in the high-information ($\widetilde{\mathcal{I}} = \{z_t, \bar{k}_t\}$) economy are, by contrast, much smaller. Information about capital K_t is still more costly to forego than that about productivity z_t, but the difference is considerable smaller.

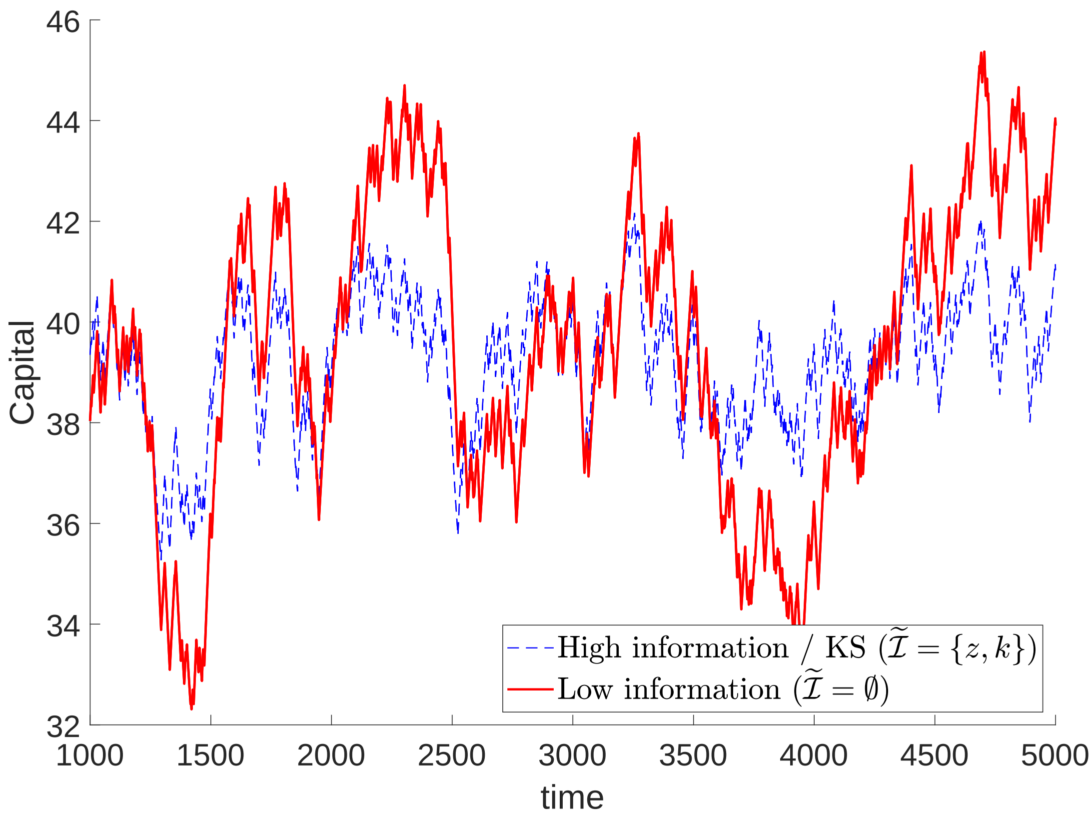


Figure 5: The figure shows the time series of aggregate capital from a simulation of the Krusell-Smith economy ($\widetilde{\mathcal{I}} = \{z_t, \bar{k}_t\}$, black line) and the economy with uninformed savings ($\widetilde{\mathcal{I}} = \emptyset$, gray line).

Combined, the results in this subsection confirm the main insight from Section 3. First, there is substantial heterogeneity in benefits of information acquisition along the wealth distribution. Second, the incentives to acquire information depend crucially on other households' information choices. All else equal, the more informed other households are, the less incentive there is for any given household to acquire information about future prices. The next subsection turns to the implications of the resulting strategic substitutability in information choice for the existence of homogeneous-information equilibria.

4.3.2 Non-existence of Homogeneous-information Equilibria

Because the benefits of information are low when average information is high, and conversely, our results in the previous subsection can be used to show that homogeneous-information equilibria may not exist for moderate-but-positive information costs. The maximum losses depicted in columns 2 and 4 of Table III show this more formally: As the maximum gain from acquiring comprehensive information ($\mathcal{I} = \{z_t, \bar{k}_t\}$) in the low-information economy

Table III: Expected losses in percent CE

	$\widetilde{\mathcal{I}} = \{z, \bar{k}_t\}$		$\widetilde{\mathcal{I}} = \emptyset$	
	Average Loss	Maximum Loss	Average Gain	Maximum Gain
$\mathcal{I} = \emptyset$	-0.1798	-0.7130	·	·
$\mathcal{I} = \{z\}$	-0.1566	-0.5575	0.0726	0.3496
$\mathcal{I} = \{\bar{k}_t\}$	-0.0851	-0.1348	0.9724	4.5259
$\mathcal{I} = \{z, \bar{k}_t\}$	·	·	1.0262	4.5431

The Table presents ex-ante expected average and maximum losses from using different individual information sets $\mathcal{I}$ (indicated in the first column) rather than a comprehensive information set $\mathcal{I}_{\max} = \{z, \bar{k}\}$, for different specifications of the information used by all other households $\widetilde{\mathcal{I}}$ (indicated in the top row). Expectations are taken across the 2.5-97.5 percentile range of the ergodic distribution of aggregate and individual states.

strongly exceeds the maximum loss of foregoing all information ($\mathcal{I} = \emptyset$) in the the low-information version, they together define a range of fixed costs for which no homogeneous high- or low-information equilibrium exists. Further, since the maximum loss from choosing $I = \emptyset$ in the high-information economy is small, and the gain from choosing $\mathcal{I} = \{z_t, \bar{k}_t\}$ in the low-information economy is large, non-existence happens for reasonably moderate costs of information when measured in consumption-equivalent units. Other similar examples can be constructed for interim information sets from Table III. On balance, we conclude that pure strategy equilibrium, where all households acquire the same information, do not exists for our baseline calibration and moderate costs of acquiring information.

Finally, we have above considered welfare losses relative to the “comprehensive" Krusell and Smith (1998) information set $\mathcal{I}_{max} = \{z_t, K_t\}$. This leaves the question of whether equilibria exists in which households acquire more information. However, Krusell and Smith (1998) show that, if all agents use $\mathcal{I}_{max} = \{z_t, K_t\}$, considering more information (in the form of additional moments of capital) has a “vanishingly small" (p. 878) effect on welfare. We confirmed this result in several exercises: The maximum welfare gains from increasing the information set to also contain the variance of individual capital, i.e. $\mathcal{I} = \{z_t, K_t, var(k)_t\}$, is less than one hundredth of a percent of permanent consumption.[11] In other words, the relevant information choice in our standard neoclassical economy is about giving up information relative to the Krusell and Smith (1998) benchmark, not about adding more.

Overall, our results in this section cast doubt on the homogenous information choice assumption $\widetilde{\mathcal{I}} = \mathcal{I}_{\max}$ used in much of modern macroeconomics to solve heterogenous-agents models with aggregate risk. Although such information choice allows for accurate predictions of future wages and rates of returns, the incentives for households to acquire such information conditional on others' behavior is small. In this sense, the classical dictum that “I am not

[11] We once more take maximum is take the maximum over all individual and aggregate states.

worried if you are" seems appropriate for our neoclassical environment. Instead, our analysis suggests that if informational choice assumptions are to be invoked, to simplify household problems in heterogenous-agents models, one needs to consider either mixed-strategy equilibria, or state-dependent information choice rules. In ongoing work (Broer *et al.*, 2021), we take the latter approach, and show how to compute an equilibria in the Krusell and Smith (1998) model that is consistent with households incentive to acquire information. Importantly, such an equilibrium features different output and consumption dynamics, and substantially more wealth inequality. The introduction of state-dependent information choice further substantially modifies the economy's responses to shocks.

5 Conclusion

In this paper, we have shown that a standard behavioral assumption invoked to solve a a broad class of heterogenous-agents models in macroeconomics appears inconsistent with rational information choice. We illustrated this result in the baseline framework of Krusell and Smith (1998), but our insights apply more broadly to settings in which agents' information choices interacts with general-equilibrium dampening. While our findings may sound negative in nature, we see our contribution as opening up a fruiting and exciting avenue of research on household information choice, and how it interacts with household inequality. In particular, it would be interesting to consider more general, dynamic information-choice strategies. In Broer *et al.* (2021), we take an initial step in this direction, but it would also be particularly fruitful to cases with optimal information design. Finally, it would be interesting to consider additional choices that information acquisition may improve, such as labour supply or the portfolio composition of savings.

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Appendix A: Analytical Results

Proof of Proposition 1: The proof proceeds in three steps: First, we show that households for which $m_0 \le \underline{m} \equiv \underline{w}\,(\beta \bar{r})^{-1} > 0$ have zero benefits of acquiring information. Second, we show that there exists another threshold $\bar{m} > \underline{m}$ such that households for which $m_0 > \bar{m}$ have a strictly positive benefit of acquiring information. Finally, we show that there exists cash-at-hand values $m_0 \in (\underline{m}, \bar{m})$ for which the benefit of acquiring information is strictly positive and increases in cash-at-hand m_0.

Step 1: Let $u(c) \equiv \log c$. Then, $m_0 \le \underline{m}$ implies that $u'(m_0) > \max_{r,w}\left[r_1 u'(w_1)\right]$. Hence, $u'(m_0) > \mathbb{E}[r_1 u'(w)]$. So the household would not choose a positive k_1 for any value of r_1 and w_1, and its choice would therefore be unchanged by information.

Step 2: Let k_1 and $k_1^\star$ denote the optimal savings choice without and with information, respectively. The expected utility differential of acquiring information is therefore

$$\begin{aligned}
\mathbb{E}\Delta\mathcal{U} &= \mathbb{E}\left[\log\left(m_0 - k_1\right) + \beta \log\left(r_1 k_1 + w_1\right) - \log\left(m_0 - k_1^\star\right) - \beta \log\left(r_1 k_1^\star + w_1\right)\right] && \text{(A1)}\\
&= \mathbb{E}\log\left(\frac{m_0 - k_1}{m_0 - k_1^\star}\right) + \beta\mathbb{E}\log\left(\frac{\alpha K_1^{\alpha-1} k_1 + (1-\alpha) K_1^\alpha}{\alpha K_1^{\alpha-1} k_1^\star + (1-\alpha) K_1^\alpha}\right)\\
&= \mathbb{E}\log\left(\frac{m_0 - k_1}{m_0 - k_1^\star}\right) + \beta\mathbb{E}\log\left(\frac{\alpha K_1 k_1 + 1 - \alpha}{\alpha K_1 k_1^\star + 1 - \alpha}\right) < 0, && \text{(A2)}
\end{aligned}$$

where the last inequality follows from the concavity of the utility function, and the fact that $k_1^\star$ is the optimal choice under perfect foresight. The envelope theorem then shows that

$$\frac{\partial \mathbb{E}\Delta\mathcal{U}}{\partial m_0} = \mathbb{E}\left[\frac{1}{m_0 - k_1} - \frac{1}{m_0 - k_1^\star}\right] = \frac{1}{m_0 - k_1} - \mathbb{E}\left[\frac{1}{m_0 - k_1^\star}\right] > \frac{1}{m_0 - k_1} - \frac{1}{m_0 - \mathbb{E}k_1^\star} > 0,$$

where the last inequality follows from the Euler equation showing that $k_1 > \mathbb{E}k_1^\star$, and that $k_1 \le m_0$.[1]

Step 3: Follows from the continuity of (A2), and the results in *Step 1* and *2*. □

Proof of Proposition 2: There are two types of households: First, households that are *on their Euler equation*. They set k_1 in accordance with

$$\frac{1}{m_0 - k_1} = \beta\mathbb{E}\left[\frac{1}{k_1 + \frac{1-\alpha}{\alpha} K_1}\right],$$

which is independent of z_t. Second, households that are *off their Euler equation*. They $k_1 = 0$, which is also independent of z_t. We conclude that households never have any incentive to acquire information about z_t, because their decisions are in all states of the world unaffected by realizations in z_t. The rest of the statement follows from information costs $\kappa > 0$ being strictly positive. □

Proof of Proposition 3: The proof is simplified by first defining some additional notation. Let $\mathbb{E}\mathcal{U}(a, a_{-1})$ denote the expected utility of a household, who either buys information ($a = b$) or not ($a = nb$), conditional on other households' information acquisition strategy. The condition for the non-existence of a pure strategy equilibria can then be stated as follows: there exists a $\kappa \in \mathbb{R}_+$ such that

$$\mathbb{E}\mathcal{U}(b, nb) - \mathbb{E}\mathcal{U}(nb, nb) > \kappa > \mathbb{E}\mathcal{U}(b, b) - \mathbb{E}\mathcal{U}(nb, b).$$

Let $\Delta(\cdot) \equiv \mathbb{E}\mathcal{U}(b, \cdot) - \mathbb{E}\mathcal{U}(nb, \cdot)$ denote the expected utility difference between buying information and

[1] Notice that Jensen's inequality is here strict because w_1 and r_1 are defined over a bounded support.

not. A few simple derivations, using the household utility function, show that

$$\Delta(\cdot) = \mathbb{E}\left[\log\left(\frac{m-k_1^b}{m-k_1^{nb}}\right) + \beta\log\left(\frac{\alpha k_1^b/K_1+1-\alpha}{\alpha k_1^{nb}/K_1+1-\alpha}\right)\right],$$

As each household makes the same savings choice as everyone else when $a = a_{-1}$, we have that

$$\Delta(b) = \mathbb{E}\left[\log\left(\frac{m-K_1^b}{m-k_1^{nb}}\right) - \beta\log\left(\alpha k_1^{nb}/K_1^b+1-\alpha\right)\right] \tag{A3}$$

$$\Delta(nb) = \mathbb{E}\left[\log\left(\frac{m-k_1^b}{m-K_1^{nb}}\right) + \beta\log\left(\alpha k_1^b/K_1^{nb}+1-\alpha\right)\right]. \tag{A4}$$

Now, notice that to a first order a household's Euler equation shows that[2]

$$k_{1i} = \frac{1}{1+\beta}m_i - \frac{1}{1+\beta}\bar{\alpha}\mathbb{E}_i\left[K_1\right], \quad \bar{\alpha} \equiv \frac{1-\alpha}{\alpha}, \tag{A5}$$

where we once more use subscript i to denote the ith household.

Thus, in a symmetric equilibrium, $K_1 = \int_0^1 k_{1i}di$ equals:

$$K_1 = \frac{1}{1+\beta}\sum_{j=0}^{\infty}\left(-\frac{\bar{\alpha}}{1+\beta}\right)^j \bar{\mathbb{E}}^j\left[m\right], \tag{A6}$$

where $\bar{\mathbb{E}}\left[\cdot\right] \equiv \int_0^1 \mathbb{E}_i[\cdot]di$ and $\bar{\mathbb{E}}^j\left[\cdot\right] \equiv \int_0^1 \mathbb{E}_i\left[\bar{\mathbb{E}}^{j-1}\left[\cdot\right]\right]di$ with $\bar{\mathbb{E}}^0\left[m\right] = m$.

Solving (A6) now shows that

$$K_1 = \frac{m}{1+\beta}\frac{1}{1+\frac{\bar{\alpha}}{1+\beta}w}, \tag{A7}$$

where $w = 1$ if all households acquire information, or $w \in (0,1)$ if all households do not.

Using (A7), the best response of the ith household in (A5), irrespective of information acquisition strategy, becomes

$$k_{1i} = \frac{1}{1+\beta}\left(1-\frac{\bar{\alpha}w_i}{1+\beta+\bar{\alpha}w}\right)m - \frac{\bar{\alpha}w_i}{(1+\beta)(1+\beta+\bar{\alpha}w)}\epsilon_i, \tag{A8}$$

where $w_i = 1$ and $\epsilon_i = 0$ if the ith household acquires information, and $w_i \in (0,1)$ and $\epsilon_i \sim \mathcal{N}\left(0,\tau_\epsilon^{-1}\right)$ if the ith household does not. With the help of (A7) and (A8), we can compute the expected utility differentials in (A3) and (A4). Specifically, evaluating and inserting(A7) and (A8) into (A3) shows that

$$\Delta(b) = \mathbb{E}\left[\log\left(\frac{(\beta+\bar{\alpha})\,m}{\left(\beta+\bar{\alpha}+\bar{\alpha}\frac{1-w}{1+\beta}\right)m+\frac{\bar{\alpha}}{1+\beta}w\epsilon}\right) - \beta\log\left(1-\alpha+\alpha\frac{(1+\bar{\alpha}\frac{1-w}{1+\beta})m-\frac{\bar{\alpha}}{1+\beta}w\epsilon}{m}\right)\right],$$

while inserting into (A4) provides us with

$$\Delta(nb) = \mathbb{E}\left[\log\left(\frac{(\beta+\bar{\alpha}w)m}{\left(\beta+\bar{\alpha}-\bar{\alpha}\frac{1-w}{1+\beta}\right)m+\frac{\bar{\alpha}}{1+\beta}\epsilon}\right) + \beta\log\left(1-\alpha+\alpha\frac{\left(1-\bar{\alpha}\frac{1-w}{1+\beta}\right)m-\frac{\bar{\alpha}}{1+\beta}\epsilon}{m}\right)\right].$$

Because $w \in (0,1)$, the formula for the mean of log-normal random variable, then shows that

$$\Delta(nb) > \Delta(b).$$

This completes the proof. □

[2] To a first order here implies that $\mathbb{E}\left[f(X)\right] = f(\mathbb{E}X)$ for some random variable X and some continuous function $f:\mathbb{R}\to\mathbb{R}$.

Proof of Proposition 4: The proof has two steps. The first shows how the value of acquiring information is decreasing in the mass of informed (middle-wealth) households. This follows immediately from the previous proposition. The second shows how this implies that for every $\pi(1)$, there is a nonempty range of values for κ such that no pure-strategy equilibrium exists.

Clearly, for any mass of middle-income households $\pi(1)$, at $\kappa = 0$ all middle-income households choose to buy information, while for high enough κ none do. Now suppose that there was a pure-strategy equilibrium for all values of κ. This would imply that, for all $\pi(1)$, there is a cutoff value $\kappa(\pi(1))$ where $\Delta(\cdot) = \kappa$, and around which an infinitesimal increase in κ makes all middle-income individuals change from acquiring to not acquiring information. Since the net benefit of acquiring information $\Delta(\cdot)$ is strictly higher in the no-information equilibrium, however, this cannot be the case. In other words, for all $\pi(1)$, there must be a range of values for κ for which there are only mixed-strategy equilibria. □

Appendix B: Quantitative Results

`To be done....`