Learning by Sharing: 
Monetary Policy and Common Knowledge

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Abstract

A common view states that central bank releases decrease central banks’ own information about the economy and are harmful if about inefficient disturbances, such as cost-push shocks. This paper shows how neither is true in a micro-founded macroeconomic model in which households and firms learn from central bank releases and the central bank learns from the observation of firm prices. Central bank releases make private sector and central bank expectations closer to common knowledge. This helps transmit dispersed information between the private sector and the central bank. As a result, the release of additional central bank information decreases the central bank’s own uncertainty and can be beneficial, irrespective of the efficacy of macroeconomic fluctuations. A calibrated example suggests that the benefits of disclosure are substantial.

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1 Introduction

A central question in macroeconomics is how to best set economy-wide policy. At a basic level, good policy requires accurate information about the state of the economy. To set interest rates correctly, a policymaker needs to know whether a demand or a supply shock has hit the economy, what the size of the shock was, and what the private sector thinks of it. All are important determinants of the policymaker’s choices. At the same time, modern-day

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policymakers also disclose a torrent of information. For example, members of the Board of the US Federal Reserve and Senior Treasury Staff speak, on average, nine times per week publicly about their own views about the state of the economy.\footnote{This is based on Bloomberg news summary data. The precise number of releases is 457 in 2016, or nine times per working week. These values include speeches, comments, and documents by the President, Federal Reserve Presidents, senior US Treasury officials, and members of the CEA and the CBOE.}

As a result of these disclosures, a two-sided flow of information arises. On the one hand, policymakers devote considerable resources to learning about the state of the economy from private sector actions, such as prices. On the other hand, such private sector actions themselves reflect people’s imperfect information about the economy, and are often informed and influenced by policymaker announcements.

In this paper, I study the consequences of this two-sided flow of information for the social value of policymaker releases.\footnote{A considerable debate has developed about the social value of public information, such as that from policymakers. This includes inter alia Morris and Shin (2002, 2005), Hellwig (2005), Svensson (2006), Angeletos and Pavan (2007), James and Lawler (2011), Paciello and Wiederholt (2013), and Angeletos et al. (2016). This debate has, however, abstracted from the two-sided information flow that is the focus of this paper.} To do so, I introduce imperfect information and a lack of common knowledge between households, firms, and a central bank in an otherwise standard macroeconomic model with monopolistic competition and nominal frictions. In the model, higher-order uncertainty arises from private information about common disturbances. But it could equally well arise from a behavioral friction that limits the amount of attention paid to economic factors (Sims, 2003). Crucially, both firms and the central bank learn from each other’s actions: The central bank learns about the private sectors’ information from the observation of firms’ prices, and the private sector from the central bank’s disclosures.

My contribution is to show that central bank disclosure enhances the efficacy of monetary policy within this framework. Because of the absence of common knowledge between the private sector and the central bank, disclosure not only provides more information to the private sector, but also increases the amount of common information between the private sector and the central bank. This increases what the central bank knows about private sector expectations, what the central bank knows about private sector expectations of its own beliefs, and so on. I detail how such decreases in higher-order uncertainty increase what the central bank knows about private sector responses to shocks and simplifies the central bank’s own inference problem when it learns from firms’ prices. As a result, I show that disclosure can improve monetary policy’s ability to replicate the first best outcome.

My results qualify two prevalent theories of the costs of central bank disclosure: (i) that disclosure can be socially costly since it increases firms’ responses to inefficient shocks, such as cost-push shocks (e.g. Angeletos and Pavan, 2007; Paciello and Wiederholt, 2013; Angeletos et al., 2016); and (ii) that disclosure decreases central banks’ own information about the state
of the economy by decreasing the information content of prices, and hence leads to worse monetary policy (e.g. Morris and Shin, 2005; Amador and Weill, 2007 and 2010).

By contrast, in my model, where central bank disclosure decreases higher-order uncertainty between the private sector and the central bank, these costs can be overcome. Indeed, full disclosure can become beneficial irrespective of the efficacy of macroeconomic fluctuations and can also increase the information content of prices. In fact, in a calibrated, extended version of my model that introduces a lack of common knowledge into the baseline New Keynesian framework, I find that disclosure decreases welfare losses by between 27 and 33 percent relative to the complete opacity baseline.\(^3\) This depends on whether cost-push or productivity shocks drive the economy, and it is in each case caused by an increase in the efficacy of monetary policy. Around 16 percentage points of these welfare benefits are brought about solely by the increase in the informativeness of prices leading to better monetary policy.

First, imagine the economy is driven only by an inefficient cost-push shock. On the one hand, additional central bank disclosure increases firms’ responses to the inefficient shock. But, on the other hand, it also increases the central bank’s information about firms’ expectations since firms will use the central bank’s disclosure when forming their own beliefs. This, in turn, allows the central bank to better offset firms’ responses to the shock, since it knows more about them. I show how the latter effect can dominate the former.

Second, my results also qualify the common concern about central bank disclosures that they crowd out private sector information from market outcomes, such as prices.\(^4\) All else equal, this would lead to less informed monetary policy and hence to potentially worse welfare outcomes. While the baseline model allows for this mechanism, in equilibrium its effect is overcome by the capacity of disclosure to alleviate a particular identification problem faced by the central bank when there is incomplete common knowledge.

Consider the case in which the central bank observes constant prices from one period to the next. This observation could be either due to firms receiving private information in line with their prior, or due to all firms receiving different information but expecting the central bank to alter monetary policy in response such that prices remain constant. Disclosure solves this identification problem. By making the central bank’s own information, and hence beliefs, common knowledge, disclosure offers the distinction between the two options. As a result, central bank disclosure can decrease uncertainty for everyone, even the central bank itself.

To keep the analysis tractable, the baseline model abstracts from household imperfect in-

\(^3\)I throughout measure welfare losses in terms of life-time consumption (Lucas, 1987).
\(^4\)Besides the aforementioned literature, see also, for example, Broadbent (2013), Kohn (2005), Issing (2005), and the discussion in Woodford (2005) and Reis (2013) for such concerns voiced by policymakers. The related literature section contains additional references on the associated learning externality of public information that exists in markets in which agents learn from prices.
formation, a signaling role of monetary policy, and limits higher-order uncertainty by assuming one-period perfect state verification. I relax these assumptions when I turn to a calibrated, extended version of my model that attempts to provide a quantitative first pass at the strength of the aforementioned benefits of disclosure. This, in effect, renders the extended model into an amended version of the dispersed information New Keynesian model studied in Lorenzoni (2009). Crucially, and in departure from Lorenzoni (2009), or the extensions considered by Nimark (2014) and Melosi (2016), the central bank and the private sector here lack common knowledge about each other’s beliefs.

The solution of the model poses technical difficulty due to the infinite regress of expectations that arises when agents need to “forecast the forecasts of others” (Townsend 1983). To address these difficulties, I extend the truncated state space solution method proposed in Nimark (2017) to the case with non-atomistic agents, such as a central bank. To calibrate the model, I rely on data on private sector and central bank forecast accuracy from the “Survey of Professional Forecasters” by the Philadelphia Federal Reserve Bank and the “Greenbook”, respectively. I use numerical simulations to explore the quantitative implications of the model.

The calibrated model shows considerable benefits of central bank disclosure. When the economy is driven only by unobserved cost-push shocks, full disclosure decreases welfare losses by 27 percent under the optimal policy. Of this decrease, around 50 percentage points are due to the improvement in monetary policy caused by a decrease in higher-order uncertainty between the private sector and the central bank. The direct increase in firms’ responses to the cost-push shock, by contrast, only increases welfare losses by 23 percentage points. The decrease in welfare losses is of a similar magnitude when the economy is instead driven only by productivity shocks. Specifically, disclosure decreases welfare losses by 33 percent under the optimal policy, of which 16 percentage points are due to improved monetary policy responses caused by an increase in the information content of prices.

My results and core informational assumptions are consistent with two salient empirical observations. First, despite substantial increases in central bank disclosure over the past two decades, there are no indications that central banks’ ability to forecast the economy has deteriorated. The root mean-square error of the US Federal Reserve’s one-quarter ahead inflation forecast is, for instance, 1.2 percent and 0.9 percent before and after it started to increase its transparency in February 1994 (see also Crowe, 2010). Second, as documented in Blinder...
et al. (2008), among others, the increase in central bank disclosure that has occurred since
the mid-1990s has substantially reduced private sector uncertainty about future interest rates.
This is consistent with disclosure leading to a broad-based increase in common knowledge
between the private sector and the central bank.

Combined, these results showcase the importance of a lack of common knowledge between
the private sector and the policymaker for an accurate picture of the social value of policymaker
releases. They, however, also hint at broader consequences of incomplete common knowledge
for several macroeconomic policies which success depends on private sector knowledge of future
policymaker actions. This includes among others the recent debate about the efficacy of central
bank forward guidance (Werning, 2015; McKay et al., 2016; Angeletos and Lian, 2018). As
shown by, for example, Weale (2013), rather than decrease future interest rate expectations
*per se*, forward guidance has often simply created common knowledge between the private sector
and the central bank. My results suggest that forward guidance through this mechanism
increased the efficacy of subsequent monetary policy choices.

Last, a common line of criticism of arguments that rest on the formation of higher-order
expectations is that people do not seem to form many of them in practice. My analytical
results, however, only strictly require individuals to engage in first or second-order thinking,
consistent with the experimental results in Nagel (1995). Moreover, although the quantitative
results weaken somewhat when I restrict people’s ability to compute higher-order expectations,
their sign and order of magnitude remain in all cases unchanged.

**Related Literature:** This paper is related to the recent debate about the social value of
public information that has followed Morris and Shin’s (2002) influential contribution. In
particular, Hellwig (2005), Angeletos and Pavan (2007), and Angeletos et al. (2016) show
that the social value of public releases depends critically on the efficacy of macroeconomic
fluctuations in models with incomplete common knowledge *among* private sector agents. By
contrast, this paper demonstrates that once we also account for a lack of common knowledge
*between* the private sector and the policymaker, an invariable benefit of disclosure can arise.
One that holds irrespective of the efficacy of macroeconomic fluctuations.

Morris and Shin (2005) and Amador and Weill (2010) have relatedly proposed a stark
“Paradox of Transparency”.\(^6\) This shows how central bank disclosure could be socially costly
because it decreases the informativeness of prices by crowding out private information. Para-
doxically, disclosure could thus end up increasing uncertainty for everyone, including the
outcome variable. All are available from the Federal Reserve Bank of Philadelphia’s website. The first sample
extends from Jan 1970 to Jan 1994; the second from Apr 1994 to Dec 2010.

\(^6\)See also Amato et al. (2002), Amato and Shin (2006), Wong (2008), Gaballo (2016), and the related work
on the *learning externality* of public information in markets where agents also observe and learn from prices
(see, for example, Vives, 1997; Amador and Weill, 2012; Vives, 2017; and the summary in Veldkamp, 2011).
central bank itself. This paper, by contrast, demonstrates how disclosure can increase the informativeness of prices by alleviating a particular identification problem.

Complementary to this paper, Gosselin et al. (2008) document a different mechanism for how this paradox could be resolved. They show that central bank disclosure could increase the informativeness of prices by turning firms “from Fed watchers to inflation watchers (Veldkamp, 2011)”. However, Gosselin et al. (2008) do not address the identification problem that is caused by the central bank’s own stabilization of prices, nor how disclosure affects it, which is the focus of this paper. Most importantly, Gosselin et al. (2008) do not share my explicit focus on disclosure’s role in modifying common knowledge between the private sector and the central bank and instead focus on the signaling role of interest rates.

My paper shares the emphasis on the importance of higher-order expectations for the effects of monetary policy with Wiederholt (2017) and Angeletos and Lian (2018). Central to their respective contributions is that an absence of common knowledge among households and firms dampens the effects of prospective monetary policy. By contrast, I focus on how an absence of common knowledge between the private sector and the central bank can cause central bank disclosure to boost the efficacy of monetary policy.

Finally, this paper is related to the literature that studies the combined optimal use of policymaker disclosure and the conditional use of policy instruments. Walsh (2007), Baeriswyl and Cornand (2010), and James and Lawler (2011) show how disclosure can be suboptimal since a policymaker can instead always condition his policy instrument on his information. By contrast, in this paper I show how the combined use of disclosure and instrument policy can arise as an optimal outcome. This occurs because information frictions exist alongside and interact with nominal frictions. Carlsson and Skans (2012) demonstrate the need for nominal frictions in imperfect information models to match the observed behavior of firms’ prices.

**Organization:** The rest of this paper proceeds as follows. Section 2 presents the baseline model. Section 3 characterizes the equilibrium and the limit-cases in which the central bank can replicate the first best outcome. Sections 4 and 5 contain the main results that illustrate the welfare benefits of disclosure. Section 6 describes the extended version of the benchmark model, and Section 7 the numerical results that I obtain after calibrating it. I conclude in Section 8. Additional extensions and all proofs are contained in the Appendix.
2 A Baseline Model

I start with a dynamic model with dispersed information and monopolistic competition. The model consists of a representative household, a continuum of firms, and a central bank. Each period is comprised of three stages. At the start of each period, firms pre-set prices based on imperfect information subject to a cost. After prices are set, the economy transitions to the second stage, where the central bank determines the money supply, in part based on its own imperfect information. The economy then transitions to the final stage, where all information that was previously unknown becomes publicly available. The representative household now meets with firms to produce what is demanded of firms’ goods at stated prices. The wage adjusts to clear the labor market. Commodity markets open and the household consumes.

Households: A representative household has preferences given by the utility function,

\[ U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(C_t) - \frac{1}{1 + \eta} L_t^{1+\eta} \right], \quad (2.1) \]

where \( \beta \) denotes the household discount factor, \( C_t \) the consumption index at time \( t \), \( L_t \) the number of hours worked by the household, and \( \eta \) parametrizes the Frisch elasticity of labor supply. The consumption index is comprised of

\[ C_t = \left( \int_0^1 C_{it}^{\rho-1} \, di \right)^{\frac{1}{\rho}}, \quad P_t = \left( \int_0^1 P_{it}^{1-\rho} \, di \right)^{\frac{1}{1-\rho}}, \quad (2.2) \]

where \( C_{it} \) is the quantity the household consumes of the goods produced by firm \( i \in [0, 1] \) and \( \rho > 1 \); \( P_t \) denotes the associated welfare-based price index and \( P_{it} \) the price set by firm \( i \).

Because the representative household receives all profits and labor income in the economy, its per-period budget constraint is

\[ \int_0^1 P_{it}C_{it} \, di + M_t^{d} \leq \int_0^1 \Pi_{it} \, di + W_t L_t + M_{t-1}^{d} + T_t^{h}, \quad (2.3) \]

where \( \Pi_{it} \) denotes the profits of firm \( i \in [0, 1] \), \( M_t^{d} \) the household’s demand for nominal balances, \( W_t \) the nominal wage, and \( T_t^{h} \) lump-sum nominal transfers. Household consumption is, in addition to (2.3), restricted by a cash-in-advance constraint after receiving nominal transfers,

\[ \int_0^1 P_{it}C_{it} \, di \leq M_{t-1} + T_t^{h}, \quad T_t^{h} = M_t - M_{t-1}. \quad (2.4) \]

The representative household seeks to maximize its utility (2.1) subject to the per-period budget constraint (2.3) and the cash-in-advance constraint (2.4).
**Firms:** The production sector consists of a continuum of imperfectly informed firms $i \in [0, 1]$ that specialize in the production of differentiated goods, also indexed by $i \in [0, 1]$. The production function used by firms is linear,

$$Y_{it} = A_t L_{it}, \quad A_t = A_{t-1} \exp (\theta_t),$$  \hspace{1cm} (2.5)

where $L_{it}$ denotes the amount of labor input used and $A_t$ common labor productivity with random innovation $\theta_t \sim \mathcal{N}(0, 1/\tau_{\theta})$.

An individual firm’s objective is to set its price $P_{it}$ to maximize its own expectation of the household’s valuation of its stream of profits, using the per-period discount factor $\beta (P_t C_t)^{-1}$. Profits at time $t$ are given by

$$\Pi_{it} = (1 + T^s_t) P_{it} Y_{it} - W_t L_{it} - \frac{\psi}{2} \left( \frac{P_{it}}{P_{it-1}} - 1 \right)^2 P_t Y_t,$$  \hspace{1cm} (2.6)

where $1 + T_t^s$ is stochastic with mean $\frac{\rho}{\rho - 1}$ such that, in a symmetric equilibrium $(P_{it} = P_t)$, a firm’s mark-up over marginal cost $\mathcal{M}_t = \frac{P_t}{W_t/\mathcal{A}_t}$ follows

$$\mathcal{M}_t = \frac{\rho}{\rho - 1} \frac{1}{1 + T^s_t} \mathcal{M}_{t-1} \exp (\xi_t), \quad \xi_t \sim \mathcal{N}(0, 1/\tau_{\xi}).$$  \hspace{1cm} (2.7)

I allow $\mathcal{M}_t$ to be random so as to accommodate mark-up (or cost-push) shocks.

Last, separate from the cost associated with physical production, firms in (2.6) face a quadratic price-adjustment cost, as in Rotemberg (1982), where $\psi > 0$ denotes a parameter which measures the severity of the nominal friction.

**Central Bank:** Similar to an individual firm, the central bank makes its choices based on imperfect information about the state of the economy. As a starting point, I assume that it sets its policy instrument, the money supply, directly based on its own expectation about the two fundamental shocks, the productivity and the mark-up disturbance,

$$M^*_t = M_{t-1}^* \exp \left\{ \phi_0 + \phi_\theta \mathbb{E}^c_{t-1} [\theta_t] + \phi_\xi \mathbb{E}^c_{t-1} [\xi_t] \right\},$$  \hspace{1cm} (2.8)

where $\phi_\theta$ and $\phi_\xi$ denote the publicly known levels of policy activism and $\mathbb{E}^c_t [\cdot]$ central bank expectations (described below). Monetary policy is thus characterized in terms of a commitment to a log-linear rule. This assumption by itself does not prevent policy from achieving the first best outcome because of the below log-quadratic specification of welfare. In fact, as I show in Section 3, the central bank can always attain the efficient outcome with (2.8)

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7See, for example, Steinsson (2003).
if it observes all shocks without error and firms do so as well. \(^8\) Sections 4 to 7 demonstrate how my results carry over to other policy rules that also allow the central bank to replicate the efficient outcome under full information, such as when it instead responds to deviations of output from flex-price levels, or to the price level itself. However, for the sake of brevity, and because it allows for cleaner exposition of the main results, I choose to adopt the simpler approach in (2.8) to start with.

**Information Structure:** At the start of each period, all firms observe noisy private and public information. As a first pass, I assume that all firms observe the same firm-specific information, unknown to the central bank, which I refer to as firms’ private information. I then later address the case in which each firm observes individual-specific signals.

Firms’ private information is summarized by the two noisy signals \(x_t^a\) and \(x_t^\mu\),

\[
x_t^a = a_t + \epsilon_{xt}^a : \quad \epsilon_{xt}^a \sim \mathcal{N}(0, 1/\tau_x^a), \quad x_t^\mu = \mu_t + \epsilon_{xt}^\mu : \quad \epsilon_{xt}^\mu \sim \mathcal{N}(0, 1/\tau_x^\mu),
\]

where lower-case letters denote the logarithm of their upper-case counterparts, \(\tau_x^a\) and \(\tau_x^\mu\) the precision of firms’ private information, and \(\epsilon_{xt}^a\) and \(\epsilon_{xt}^\mu\) are independent of all other disturbances.

In addition to their private information, firms observe two (potentially noisy) public signals \(\omega_t^a\) and \(\omega_t^\mu\) sent by the central bank of its own private information about labor productivity and price mark-ups,

\[
\omega_t^a = z_t^a + \epsilon_{zt}^a : \quad \epsilon_{zt}^a \sim \mathcal{N}(0, 1/\tau_x^a), \quad \omega_t^\mu = z_t^\mu + \epsilon_{zt}^\mu, \quad \epsilon_{zt}^\mu \sim \mathcal{N}(0, 1/\tau_x^\mu),
\]

where \(z_t^a\) and \(z_t^\mu\) denote the central bank’s noisy private signals,

\[
z_t^a = a_t + \epsilon_{zt}^a : \quad \epsilon_{zt}^a \sim \mathcal{N}(0, 1/\tau_x^a), \quad z_t^\mu = \mu_t + \epsilon_{zt}^\mu : \quad \epsilon_{zt}^\mu \sim \mathcal{N}(0, 1/\tau_x^\mu).
\]

The case of *full disclosure* here corresponds to the limit \(\tau_j^\omega \to \infty\) with \(j = \{a, \mu\}\), while *complete opacity* is equivalent to the situation where the central bank’s communication contains no valuable information, \(\tau_j^\omega \to 0\). *Partial disclosure* refers to the interim case, \(\tau_j^\omega \in \mathbb{R}_+\).\(^9\)

\(^8\)When the central bank can respond to all shocks within each period, then it can always accommodate (or offset) each shock perfectly. This, in turn, ensures that the economy in each period can track its flex-price, first best counterpart from a time-less perspective (see Section 3). A similar result would, of course, hold if the central bank were to respond directly to the level of the driving forces instead.

\(^9\)Thus, for example, the logarithm of firms’ mark-up becomes \(\mu_t = \log M_t\).

\(^10\)My chosen approach to model communication policy in (2.10) follows that of Cukierman and Meltzer (1986) and has been used extensively since (see, for instance, Faust and Svensson, 2001). An advantage of this approach is that it allows for a meaningful discussion of different, intermediate levels of disclosure. This advantage, of course, rests on the central bank committing to a disclosure rule such as (2.10). Absent this commitment, the central bank could communicate anything following the realization of its private information, and the only values that would be consistent with equilibrium would be full or zero disclosure. I demonstrate below how my main results still remain valid in this case.
Turning to the central bank, besides its own private information, the central bank also observes a noisy public signal of the economy-wide price level,

\[ \bar{p}_t = p_t + \epsilon_{pt}, \quad \epsilon_{pt} \sim \mathcal{N}(0, 1/\tau_p), \quad (2.12) \]

where \( \epsilon_{pt} \) is independent of all other disturbances for all \( t \).\(^{11}\)

We can summarize the information structure by the following information sets:

\[ \Omega^f_t = \{ x_{t-j}, \omega_{t-j}, a_{t-j-1}, \mu_{t-j-1}, m_{t-j-1} \}_{j=0}^{\infty} \quad (2.13) \]

\[ \Omega^{cb}_t = \{ z_{t-j}, \omega_{t-j}, \bar{p}_{t-j}, a_{t-j-1}, \mu_{t-j-1}, m_{t-j-1} \}_{j=0}^{\infty}, \quad (2.14) \]

where \( x_t' = \begin{bmatrix} x^a_t & x^\mu_t \end{bmatrix} \), \( z_t' = \begin{bmatrix} z^a_t & z^\mu_t \end{bmatrix} \), \( \omega_t' = \begin{bmatrix} \omega^a_t & \omega^\mu_t \end{bmatrix} \), and both firms and the central bank also publicly observe last period’s realization of the driving forces of the economy \( a_{t-1} \) and \( \mu_{t-1} \), in addition to last period’s money supply \( m_{t-1} \). I denote firm and central bank expectations based on \((2.13)\) and \((2.14)\) by \( \mathbb{E}^f_t[\cdot] \) and \( \mathbb{E}^{cb}_t[\cdot] \), respectively.

Although stylized, the above information structure displays the two-sided informational interaction between the private sector and the central bank that is central to my results. On the one hand, private sector firms learn from the central bank’s communication and use this information to set prices. But, on the other hand, the central bank also itself learns from the observation of firms’ prices and uses this information to set monetary policy.

Within this context, there are two notable features of \((2.13)\). First, as in Woodford (2002a), Hellwig (2005), Angeletos et al. (2016), and others, firms in \((2.13)\) do not observe the current value of the central bank’s policy instrument. Firm prices are pre-set and made before the realization of the money supply. Section 7 demonstrates that my results are robust towards this assumption. All that is required is that the central bank’s disclosure provides some truly new information about the central bank’s private signals beyond what firms can learn from the observation of firms’ prices and uses this information to set monetary policy.

Second, all firms in \((2.13)\) observe the same firm-specific information. However, what is central for my results is only that firms observe some information that the central bank cannot directly observe but instead needs to infer from firms’ prices. Equation \((2.13)\) operationalizes this idea in a tractable manner by condensing firms’ private information into one signal. This allows me to focus on the welfare consequences of an absence of common knowledge between firms and the central bank, rather than to conflate it with the welfare consequences of dispersed information among firms (e.g. Hellwig, 2005). Section 7 shows how my main results extend to the case in which each firm observes an individual-specific signal of the driving forces.

\(^{11}\)The introduction of the shock \( \epsilon_{pt} \) in \((2.12)\) serves a technical purpose. Suppose that there are no mark-up shocks, that is that \( \tau_p \rightarrow \infty \). The shock \( \epsilon_{pt} \) then prevents the central bank from perfectly inferring firm’s private information from the observation of the price level. The use of such “non-invertibility shocks” is standard in the literature on noisy rational expectations (see, for example, Hellwig, 1980).
We proceed to study the equilibrium of the economy. I define an equilibrium in a familiar manner as a sequence of prices, production levels, household labor supply, firm labor demand, and wage rates such that at each point in time: (i) the representative household maximizes utility and firms maximize profits subject to informational and other constraints, and (ii) all goods markets clear, $Y_{it} = C_{it}$ for all $i \in [0, 1]$, and so too does the money market, $M^d_t = M^s_t$. Below, I focus on two of these equilibrium objects: firms’ prices and the central bank’s money supply. These are the same two equilibrium objects for which imperfect information, and hence the presence of two-sided informational interactions, matters directly. The remaining quantities as well as wages are straightforward to derive and can be computed from (2.1), (2.2), (2.5), and (A.5) in the Appendix.

**Characterization of Prices:** To characterize firms’ prices, I first solve the representative household’s problem, imposing market clearing, to derive a relationship between the wage rate, output, and productivity in the economy. I then use this relationship to derive a simple expression for firms’ optimal prices. Throughout, I focus on log-linear approximations of agents’ decision rules around the full-information non-stochastic steady state.

**Lemma 1.** Let $\phi_0$ be set such that $\delta = \beta E_t M_{t+1} < 1$. Then, the cash-in-advance constraint always binds, $m_t - p_t = y_t$, and the symmetric linear equilibrium price for all $i \in [0, 1]$ is

$$p_t = \lambda_0 E_t [m_t - a_t] + \lambda_{-1} p_{t-1} + \lambda_1 E_t [p_{t+1}] + \lambda_2 E_t [\mu_t].$$

(3.1)

where $\{\lambda_{-1}, \lambda_0, \lambda_1, \lambda_2\} \in [0, 1]$ with $\sum_{i=-1}^{1} \lambda_i = 1$ and $\lambda_{-1} = \lambda_1 = 0$ if and only if $\psi = 0$.

Lemma 1 provides an intuitive result. On the one hand, because of nominal frictions, firms’ prices depend upon past and expected future prices. On the other hand, because of their direct influence on firms’ real marginal cost, firms’ prices also depend upon firms’ expectations about labor productivity, in addition to firms’ expectations about current nominal demand (because of its influence on output and wages through the cash-in-advance constraint).

We can further simplify (3.1) by solving the equation forward.

**Corollary 1.** The symmetric linear equilibrium firm price is given by

$$p_t = \nu_0 E_t [m_t - a_t] + \nu_{-1} p_{t-1} + \nu_1 E_t [\mu_t]$$

(3.2)

$$m_t = m_{t-1} + \phi_0 + \phi_0^{\text{prob}} [\theta_t] + \phi_\xi E_t^{\text{prob}} [\xi_t],$$

(3.3)

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Since the real resource cost of inflation is of second-order, the log-linearized resource constraint is simply $y_t = c_t$ (see Appendix A). As a result, the cash-in-advance constraint entails that $m_t - p_t = c_t = y_t$. 

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where \( \{ \nu_1, \nu_0 \} \in [0, 1] \) with \( \nu_1 = 0, \nu_0 = 1 \) if and only if \( \psi = 0 \), and \( \nu_1 \in \mathbb{R}_+ \).

Corollary 1 shows how the presence of nominal frictions \( \psi > 0 \) dulls firms’ responses to their own expectations about productivity (\( \nu_0 < 1 \)). This attenuation of firms’ responses will later be important for the optimal conduct of monetary policy.

**Social Welfare Loss:** We can use the above characterization of firms’ prices to study the normative properties of our economy. I take my criterion to be utilitarian welfare and analyze the ex-ante utility of the representative household before knowledge of period zero shocks.

A second-order approximation around the flex-price, full information steady state shows that the welfare losses obtained relative to the first best frictionless case can be approximated by \( \mathcal{W} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t (y_t - a_t)^2 \) (Appendix A). This shows how standard welfare expressions familiar from workhorse New Keynesian models with nominal frictions (e.g. Woodford, 2002b; Nistico, 2007; and Gali, 2008) extend almost immediately to the above economy.

We can further simplify \( \mathcal{W} \), using the Law of Iterated Expectations combined with that \( \mathbb{E}_{t-1} (y_t - a_t)^2 \) is constant over time.

**Lemma 2.** Equilibrium welfare losses relative to the first best, frictionless case can be approximated by \( \mathcal{W} = \frac{1}{1-\beta} \mathbb{E}_{-1} [y_t - a_t]^2 \), where \( y_t = m_t - p_t \) and \( a_t = a_{t-1} + \theta_t \).

A convenient benchmark to compare subsequent optimal policies to is the special case in which the central bank and firms observe all fundamental shocks without error. Combined, Corollary 1 and Lemma 2 show that welfare losses under full information are equal to

\[
\mathcal{W}^{full} = \frac{1}{1-\beta} \left\{ (1-\nu_0)^2 (\phi_\theta - 1)^2 \frac{1}{\tau_\theta} + [(1-\nu_0) \phi_\xi - \nu_1] \frac{1}{\tau_\xi} \right\}. \tag{3.4}
\]

It follows that the optimal policy under full information is to set \( \phi_\theta = \phi_\theta^{full} = 1 \) and \( \phi_\xi = \phi_\xi^{full} = \frac{\nu_1}{1-\nu_0} > 0 \), and that the central bank under this optimal policy replicates the first best, flex-price outcome (\( \mathcal{W}^{full} = 0 \)). This shows how one tenet of optimal monetary policy carries over to our economy: The central bank accommodates the efficient productivity shock and offsets the inefficient mark-up disturbance. However, unlike economies with price dispersion, due to for example dispersed information, the central bank here chooses to completely offset the mark-up shock under full information. By contrast, price dispersion would require the central bank to trade-off output and price stabilization, and thus to only partially offset the mark-up shock. Section 7 shows how my main results also extend to this case.

\textsuperscript{13}That is, the special case in which \( \tau_j^1 \rightarrow \infty \) and \( \tau_j^j \rightarrow \infty \) for both \( j = \{a, \mu\} \)

\textsuperscript{14}This follows immediately from the output gap being equal to \( y_t - a_t = m_t - p_t - a_t = (1 - \nu_0) (\phi_\theta \theta_t + \phi_\xi \xi_t - \theta_t) - \nu_1 \xi_t + l.p.t \), where \( l.p.t \) denotes last period’s terms.
4 Disclosure about Inefficient Disturbances

I now turn to the costs and benefits of central bank disclosure. I start with a much discussed cost that arises from increased responses to inefficient disturbances, such as cost-push (or mark-up) shocks (e.g. Angeletos and Pavan, 2007; Paciello and Wiederholt, 2013; and Angeletos et al., 2016). In this section, I show how a lack of common knowledge between firms and the central bank modifies this cost. Specifically, I show that it can make full disclosure about an otherwise inefficient mark-up shock invariably beneficial.

Preliminaries: I consider the special case in which $\tau_\theta \to \infty$ and $\tau_p \to 0$. The former assumption allows me to focus on the inefficient fluctuations caused by the mark-up disturbance without having to also account for the efficient productivity shock. The latter assumption ensures that the central bank does not learn about firms’ private information from the noisy observation of the price level. This simplifies the analysis. I extend my results to positive values of $\tau_p$ further below, while Section 5 deals with the productivity shock case.

A Cost of Disclosure: I start with the welfare cost of disclosure that arises from increased responses to inefficient disturbances. To see a stark example of this cost, suppose that the money supply is held fixed ($\sigma = 0$) so that central bank expectations about the mark-up shock do not affect nominal demand. Corollary 1 shows that the output gap, the determinant of social welfare, in this case takes a particularly simple form,

\[
y_t - a_t = m_t - p_t = \phi_\xi E^C_t [\xi_t] - \nu_0 E^f_t \left[ \phi_\xi E^C_t \xi_t + \frac{\nu_1}{\nu_0} \xi_t \right] + l.p.t. \quad (4.1)
\]

where I abstract from last period terms ($l.p.t.$) irrelevant to current welfare and productivity is held constant at its steady state value ($a_t = 0$).\footnote{I will henceforth abstract from last period terms in all derivations of welfare.} We conclude from Lemma 2 that the associated welfare losses are $W = \frac{1}{1-\theta(1)} \nu_1^2 V \left[ E^f_t \xi_t \right]$. Equation (4.2) illustrates how additional central bank disclosure can be harmful for social welfare. Increases to $\tau^\mu$ always increase \( V \left[ E^f_t \xi_t \right] \) and thus $W$. Additional central bank disclosure increases firms’ responses to the inefficient mark-up shock, causing further fluctuations in output, despite constant productivity.

Lemma 3. Suppose that only mark-up shocks drive the economy, $\tau_\theta \to \infty$, and that the money supply is held fixed, $\phi_\xi = 0$. Then, complete opacity $\tau^\mu \to 0$ is uniquely optimal.

The result in Lemma 3 essentially replicates that in Hellwig (2005) and Angeletos and Pavan (2007) for the case where price dispersion is muted since firms, among themselves,
have common knowledge. Because welfare losses in (4.2) increase monotonically in disclosure, complete opacity is uniquely optimal when the money supply is held fixed.

A Benefit of Disclosure: The result in Lemma 3 contrasts with those which arise when monetary policy responds more forcefully. In fact, because of the lack of common knowledge that exists between firms and the central bank, disclosure can become invariably beneficial once we allow for a more active monetary policy.

On the one hand, the more precisely the central bank discloses its information, the more firms will respond to the mark-up shock. But, on the other hand, the more precisely the central bank discloses its information, the more firms will also use the central bank’s disclosure to form their own expectations. This, in turn, aligns firms’ expectations more with the central bank’s own, which enables monetary policy to better counter firms’ responses.

To see this latter benefit of disclosure, consider the case in which monetary policy is set to its optimal full information value from Section 3. That is, consider the special case in which

\[ \phi_\xi = \phi_\xi^{*,\text{full}} = \frac{\nu_1}{1-\nu_0} > 0. \]

We can in this case write (4.1) as

\[
y_t - a_t = m_t - p_t = \phi_\xi E_t^{cb} [\xi_t] - \nu_0 \phi_\xi E_t^f [E_t^{cb} \xi_t] - \nu_1 E_t^f [\xi_t] + l.p.t.
\]

\[ = \nu_1 \left( E_t^{cb} [\xi_t] - E_t^f [\xi_t] \right) + \frac{\nu_0}{1-\nu_0} \nu_1 \left( E_t^{cb} [\xi_t] - E_t^f \left[ E_t^{cb} \xi_t \right] \right) + l.p.t. \tag{4.3}\]

Equation (4.3) shows that when \( \phi_\xi = \phi_\xi^{*,\text{full}} \) welfare losses only arise due to a lack of common knowledge between firms and the central bank \( (E_t^{cb} [\xi_t] \neq E_t^f [\xi_t] \neq E_t^f E_t^{cb} [\xi_t]) \).

Consider now the extreme case in which the central bank also has full information about the mark-up shock \( (\tau^*_z \to \infty) \). Full disclosure would then result in the worst possible outcome if monetary policy remained constant \( (\phi_\xi = 0) \). By contrast, when the central bank fully discloses its information when \( \phi_\xi = \phi_\xi^{*,\text{full}} \), it replicates the first best outcome \( (W = 0) \).

The more precisely the central bank discloses its information, the more firms will use it to form their own expectations, and the more firms’ expectations will resemble the central bank’s. In fact, when the central bank in this case fully discloses its information \( \tau^*_w \to \infty \), \( E_t^f [\xi_t] = E_t^{cb} [\xi_t] = \xi_t \) and \( E_t^f E_t^{cb} [\xi_t] = E_t^{cb} [\xi_t] = \xi_t \). Disclosure increases common knowledge and in this case eliminates all higher-order uncertainty. This, in turn, allows the central bank to perfectly counter firms’ responses to the mark-up shock.

Proposition 1 shows that this benefit of disclosure dominates the aforementioned cost whenever monetary policy attempts to offset firms’ responses to the mark-up sufficiently.

**Proposition 1.** Let \( \bar{\phi}_\xi = \frac{\nu_1}{2-\nu_0} \frac{\tau^*_z + \tau^*_x}{\tau^*_z + \tau^*_x + \tau^*_w} < \phi_\xi^{*,\text{full}} \). Then, depending on whether \( 0 < \phi_\xi \leq \bar{\phi}_\xi \), either complete opacity \( \tau^*_w \to 0 \) or full disclosure \( \tau^*_w \to \infty \) is uniquely optimal.
Optimal Use of Information: I now turn to the combined optimal use of disclosure and monetary policy. Proposition 1 shows that the overall benefit of central bank disclosure depends critically on the conditional use of monetary policy. In turn, this raises the question of whether the identified benefit dominates the aforementioned cost under the optimal monetary policy. Theorem 1 provides the answer to this question that arises after a straightforward application of the dual approach to optimal policy.

Theorem 1. The combined optimal policy with mark-up shocks is full disclosure, \( \tau_{w}^{cb} \to \infty \), and a monetary policy that undoes the nominal friction, \( \phi_{\xi} = \frac{\gamma_{\xi}}{1-\gamma_{\xi}} > \tilde{\phi}_{\xi} \). Increases in central bank disclosure increase common knowledge between firms and the central bank.

Although disclosure always increases firms’ responses to the mark-up shock, Theorem 1 shows that full disclosure is invariably beneficial at the optimal monetary policy. Intuitively, this generic benefit arises because monetary policy can only counter the portion of firms’ responses to the mark-up shock that aligns with its own expectations and that it knows about. Disclosure always helps in this respect by furthering common knowledge. This differentiates Theorem 1 from the results in Hellwig (2005), where disclosure about an inefficient (money supply) shock is optimal because firms’ prices are in equilibrium too dispersed.

To explore why disclosure is generically optimal, consider any policy pair \( (\phi_{\xi}, \tau_{w}^{cb}) \) in (4.3),

\[
y_{t} - a_{t} = \left( \phi_{\xi} (1 - \nu_{0}) \mathbb{E}_{t}^{cb} [\xi_{t}] - \nu_{1} \mathbb{E}_{t}^{f} [\xi_{t}] \right) + \nu_{0} \phi_{\xi} \left( \mathbb{E}_{t}^{cb} [\xi_{t}] - \mathbb{E}_{t}^{f} \left[ \mathbb{E}_{t}^{cb} [\xi_{t}] \right] \right) + l.p.t, \tag{4.4}
\]

where we can always decompose firms’ expectations \( \mathbb{E}_{t}^{f} [\xi_{t}] \) and \( \mathbb{E}_{t}^{f} \left[ \mathbb{E}_{t}^{cb} [\xi_{t}] \right] \) into\(^{16}\)

\[
\mathbb{E}_{t}^{f} [\xi_{t}] = w_{0} \mathbb{E}_{t}^{cb} [\xi_{t}] + w_{1} \epsilon_{t}^{f}, \quad \mathbb{E}_{t}^{f} \left[ \mathbb{E}_{t}^{cb} [\xi_{t}] \right] = v_{0} \mathbb{E}_{t}^{cb} [\xi_{t}] + v_{1} \epsilon_{t}^{cb}, \tag{4.5}
\]

in which \( \epsilon_{t}^{f} \sim \mathcal{N} (0, 1) \), \( j = \{ f, cb \} \) denotes a convex combination of shocks \( \epsilon_{t}^{cb}, \epsilon_{t}^{f}, \) and \( \epsilon_{t}^{w} \). Importantly, the coefficients on central bank expectations \( w_{0} \in (0, 1) \) and \( v_{0} \in (0, 1) \) in (4.5) increase in central bank disclosure \( \tau_{w}^{cb} \), while the coefficients \( w_{1} \) and \( v_{1} \) on \( \epsilon_{t}^{f} \) decrease in \( \tau_{w}^{cb} \).

One consequence of (4.5) is that increases in central bank disclosure make firm and central bank beliefs more similar. Both \( \mathbb{E}_{t}^{f} [\xi_{t}] \) and \( \mathbb{E}_{t}^{f} \left[ \mathbb{E}_{t}^{cb} [\xi_{t}] \right] \) become closer to \( \mathbb{E}_{t}^{cb} [\xi_{t}] \), on average,

\[^{16}\text{We have that (see Appendix B)}\]

\[
\mathbb{E}_{t}^{f} [\xi_{t}] = w_{x} x_{t} + w_{\omega} \omega_{t} = w_{x} x_{t} + w_{\omega} (z_{t} + \epsilon_{t}^{w}), \quad \mathbb{E}_{t}^{cb} [\xi_{t}] = \beta_{z} z_{t},
\]

where \( w_{x} = \frac{\tau_{x}^{cb} (\tau_{x}^{cb} + \tau_{\xi}^{cb})}{(\tau_{x}^{cb} + \tau_{\xi}^{cb}) (\tau_{x}^{cb} + \tau_{\xi}^{cb} + \tau_{w}^{cb} + \tau_{z}^{cb})} \), \( w_{\omega} = \frac{\tau_{\omega}^{cb} \tau_{z}^{cb}}{(\tau_{x}^{cb} + \tau_{\xi}^{cb} + \tau_{w}^{cb} + \tau_{z}^{cb}) \tau_{z}^{cb}} \), and \( \beta_{z} = \frac{\tau_{z}^{cb}}{\tau_{x}^{cb} + \tau_{z}^{cb}} \). It follows that

\[
\mathbb{E}_{t}^{f} [\xi_{t}] = w_{0} \mathbb{E}_{t}^{cb} [\xi_{t}] + w_{1} \epsilon_{t}^{f},
\]

in which \( w_{0} = \frac{w_{x} + w_{\omega}}{\beta_{z}} \) and \( w_{1} \epsilon_{t}^{f} = w_{x} (\epsilon_{t}^{w} - \epsilon_{t}^{cb}) + w_{\omega} \epsilon_{t}^{cb} \). A similar argument shows \( \mathbb{E}_{t}^{f} \left[ \mathbb{E}_{t}^{cb} [\xi_{t}] \right] \) in (4.5). Last, notice that \( \frac{\partial w_{1}}{\partial \tau_{w}^{cb}} > 0 \), while it also follows from the definition of \( w_{1} \) that \( \frac{\partial v_{1}}{\partial \tau_{w}^{cb}} < 0 \).
and resemble less the other disturbances captured in $\epsilon^j_t$. This, in turn, allows the central bank to set monetary policy to better offset firms’ responses to the mark-up shock.

Indeed, as (4.4) and (4.5) show, the central bank can always offset the increased responses to the mark-up shock that are caused by its own disclosure by correctly modifying the conditional use of monetary policy. Suppose the central bank sets $\phi_\xi = \frac{\nu}{1-\nu_0} w_0$ in (4.4). Then, all of firms’ increased responses to the mark-up shock are fully offset by monetary policy. Conversely, when $\phi_\xi = \frac{\nu}{1-\nu_0} w_0$, additional disclosure then only decreases the portion of firms’ responses to the mark-up shock that is caused by information that the central bank does not itself have access to, such as firms’ private information, and hence cannot offset. Additional disclosure as a result always improves welfare.

A stark feature of Theorem 1 is that the optimal monetary policy (which results from countering firms’ responses with $\phi_\xi = \frac{\nu}{1-\nu_0} w_0$ under full disclosure) is itself independent of imperfect information ($\phi_\xi = \frac{\nu}{1-\nu_0} w_0 \rightarrow \phi_\xi^* = \frac{\nu}{1-\nu_0}$ when $\tau^\mu_\omega \rightarrow \infty$). This follows from how monetary policy under full disclosure does not affect the prediction errors that firms make. Consider once more (4.4)

$$
\begin{align*}
y_t - a_t = m_t - p_t &= \mathbb{E}^b_t [\phi_\xi (1 - \nu_0) \xi_t - \nu_1 \xi_t] + \nu_0 \phi_\xi \left( \mathbb{E}^b_t \xi_t - \mathbb{E}^b_t \mathbb{E}^b_t \xi_t \right) \\
&\quad + \nu_1 \left( \mathbb{E}^b_t \xi_t - \mathbb{E}^b_t \xi_t \right) + l.p.t.,
\end{align*}
$$

(4.6)

where I have subtracted and added $\nu_1 \mathbb{E}^b_t [\xi_t]$ from the right-hand side. Equation (4.6) shows that for any partial disclosure $\tau^\mu_\omega \in \mathbb{R}_+$, the value of monetary policy $\phi_\xi$ that minimizes welfare losses $W = \frac{1}{1-\beta} \mathbb{E}_{t-1} \left[ y_t - a_t \right]^2$ differs from its full information value ($\phi_\xi^{full} = \frac{\nu}{1-\nu_0}$). This is because of the second term in (4.6), and is consistent with monetary policy responding to the extent of imperfect information in the economy. But now notice that, when the central bank fully discloses its information, $\mathbb{E}^b_t \mathbb{E}^b_t \xi_t$ equals $\mathbb{E}^b_t \xi_t$. With full disclosure, firms do not commit any prediction errors about future monetary policy. As a result, monetary policy no longer affects the errors that firms make due to imperfect information. Monetary policy, in effect, becomes divorced from the information friction. We can therefore apply the certainty-equivalence results in Svensson and Woodford (2004), which show that monetary policy should in such cases be set to its full information value.

Clearly, the results in Theorem 1 are sharp. Nevertheless, the underlying benefit of disclosure that leads to Theorem 1 extends to other cases in which monetary policy optimally responds to the extent of imperfect information (see Section 7). As long as there is an absence of common knowledge between the private sector and the central bank, disclosure will always

\footnote{Under this optimal policy, the output gap is simply: $y_t - a_t = -w_1 \epsilon^j_t + \nu_0 \phi_\xi ((1 - \nu_0) \mathbb{E}^b_t [\xi_t] - v_1 \epsilon^b_t)$. Notice that additional disclosure $\tau^\mu_\omega$ decreases the coefficients $w_1$, $1 - v_0$, as well as $v_1$.}
help better align private sector expectations with the central bank’s own. All else equal, this then allows the central bank to better steer the economy and improve welfare.

**Learning from Prices:** I have so far simplified the exposition by assuming that $p_\xi = 0$ such that the central bank does not learn about firms’ expectations from the noisy observation of the price level. None of the main insights, however, depend critically on this assumption. Appendix B shows how my results readily extend to the case in which the central bank learns about firms’ expectations from the price level; that is to the case where $\tau_p > 0$ and conditional on this value of $\phi_\xi$, full disclosure is once more optimal because it increases the efficacy of monetary policy.\(^{18}\) I postpone the discussion of how central bank disclosure also increases the information content of the price level in this case to the next section.

**Other Monetary Policy Rules:** I conclude this section with studying the consequences of an alternative monetary policy rule. While the monetary policy rule in (2.8) makes the analysis particularly convenient, it is not central to the main results from this section. Suppose that instead of (2.8) the central bank directly targets the variable that causes fluctuations in the output gap, the price level, in the case where $\tau_p$ is finite,

$$m_t = m_{t-1} + \phi_0 + \phi_p \mathbb{E}_t^{cb} [p_t], \quad (4.7)$$

and suppose moreover that the cash-in-advance constraint always binds.\(^{19}\) The central bank can with (4.7) still replicate the flex-price, first best outcome when it itself has full information about the mark-up shock (with $\phi_p^{full} = 1$ and $\tau_\mu \rightarrow \infty$).

Equilibrium prices from (3.2) can be combined with (4.7) to show that

$$p_t = \nu_{-1} p_{t-1} + \nu_0 \mathbb{E}_t^f [m_t] + \nu_1 \mathbb{E}_t^f [\mu_t]$$

$$= \nu_1 \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left( \mathbb{E}_t^{cb} \mathbb{E}_t^f \right)^j [\xi_t] + l.p.t., \quad (4.8)$$

where $\left( \mathbb{E}_t^{cb} \mathbb{E}_t^f \right)^j [\xi_t]$ is defined by the recursion $\left( \mathbb{E}_t^{cb} \mathbb{E}_t^f \right)^j [\xi_t] = \mathbb{E}_t^{cb} \mathbb{E}_t^f \left( \left( \mathbb{E}_t^{cb} \mathbb{E}_t^f \right)^{j-1} [\xi_t] \right)$ with $\left( \mathbb{E}_t^{cb} \mathbb{E}_t^f \right)^0 [\xi_t] = \xi_t$, and I abstract from irrelevant constant terms. Equilibrium prices can thus

---

\(^{18}\) One might think that monetary policy should differ when $\tau_p$ is finite, to account for how much the central bank learns from the price level about firms’ private information. But notice that the informativeness of the price level under full disclosure is independent of monetary policy. Even though the price level becomes more stable as we increase $\phi_\xi \in \left[ 0, \phi_\xi^{full} \right]$, the central bank can under full disclosure perfectly account for how much more stable since it knows firms’ expectations about its own beliefs (see Section 5). The informativeness of the price level therefore remains constant as we increase $\phi_\xi$, and thus $\phi_\xi = \phi_\xi^{full}$ when $\tau_\mu \rightarrow \infty$.

\(^{19}\) We can indeed always set $\phi_0$ such that this is the case.
be described by a weighted sum of an entire infinite sequence of higher-order expectations, comprised of firms’ expectations of central bank expectations and vice versa.

The corresponding output gap, in this case, becomes

\[ y_t - a_t = \phi_p \nu_1 \mathbb{E}_t^{cb \mathbb{E}_t^f} \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left( \mathbb{E}_t^{cb \mathbb{E}_t^f} \right)^j \mathbb{E}_t^f [\xi_t] - \nu_1 \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left( \mathbb{E}_t^{cb \mathbb{E}_t^f} \right)^j \mathbb{E}_t^f [\xi_t] + l.p.t. \]

\[ = \nu_1 \left\{ \phi_p \mathbb{E}_t^{cb \mathbb{E}_t^f} \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left( \mathbb{E}_t^{cb \mathbb{E}_t^f} \right)^j \mathbb{E}_t^f [\xi_t] - \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left( \mathbb{E}_t^{cb \mathbb{E}_t^f} \right)^j \mathbb{E}_t^f [\xi_t] \right\} + l.p.t. \] (4.9)

Equation (4.9) shows that when the central bank sets monetary policy to its optimal full information value (\( \phi_p = 1 \)) welfare losses once more only arise from a lack of common knowledge. When the central bank now also fully discloses its information and sets \( \tau_{\mu}^{\bullet} \to \infty \), the expression for the output gap in (4.9) collapses to

\[ y_t - a_t = m_t - p_t = \frac{\nu_1}{1 - \nu_0} \left( \mathbb{E}_t^{cb \mathbb{E}_t^f} [\xi_t] - \mathbb{E}_t^f [\xi_t] \right) + l.p.t. \] (4.10)

Indeed, following a similar approach to that above shows that \( \phi_p^* = 1 \) and \( \tau_{\mu}^{\bullet \bullet} \to \infty \) describe the combined optimal policy when the central bank targets the price level.

Combined, (4.9) and (4.10) demonstrate how the results from the simple monetary policy rule in (2.8) carry over with more force to the extended case studied in (4.7). Disclosure now decreases uncertainty about the entire infinite sequence of higher-order expectations that make up the price level. As before, this decrease in higher-order uncertainty alleviates the errors in monetary policy that the lack of common knowledge otherwise entails. The more precisely the central bank discloses its information, the more firms will use its disclosure to form their own expectations. This, in turn, allows the central bank to better predict (and hence offset) firms’ responses to the mark-up shock since it knows more about them.

In fact, (4.10) shows that welfare losses under the optimal policy are only due to the remaining errors in central bank beliefs about firms’ expectations. These still arise with full disclosure because the noisy signal of the price level does not perfectly reveal firms’ private information. There is thus a sense in which welfare losses under the optimal policy are only due to the remaining lack of common knowledge between the central bank and firms; that which arises because the central bank does not perfectly know firms’ private information. The next section demonstrates how central bank disclosure also decreases this residual uncertainty by increasing the information content of the price level.
5 Disclosure and the Paradox of Transparency

I now shift the focus from inefficient mark-up (cost-push) shocks to another influential cost of disclosure. This cost stipulates that one of the consequences of central bank disclosure is that the central bank has to rely on less informative prices to steer monetary policy (e.g. Morris and Shin, 2005 and Amador and Weill, 2010). In this section, I show that a lack of common knowledge between firms and the central bank also qualifies this second cost of disclosure.

Preliminaries: Since this cost does not depend on the precise source of economic fluctuations, I focus on the special case in which only productivity shocks drive the economy ($\tau_\xi \to \infty$). This allows me to cleanly separate the effects of disclosure from those discussed in the previous section. I solve for the set of symmetric linear Bayesian equilibria when $\tau_\xi \to \infty$ using the method of undetermined coefficients. I then use this solution to show how disclosure modifies the informativeness of prices. Proposition 2 details the outcome of the first step.

**Proposition 2.** The set of symmetric linear equilibria with productivity shocks is non-empty, and is comprised of firm prices and associated central bank money supply equal to

\begin{align*}
p_t &= \nu_{-1}p_{t-1} + \nu_0 (m_{t-1} - a_{t-1}) + k_0 x_t^a + k_1 \omega_t^a + k_2 p_t, \\
m_t &= m_{t-1} + q_0 z_t^a + q_1 p_t,
\end{align*}

where $p_t = \theta_t + \epsilon_{xt}^a + \frac{1}{k_0} \epsilon_{pt}$ and the coefficients $\{k_0, k_1, k_2, q_0, q_1\}$ are all constants.

Proposition 2 establishes the existence of a linear equilibrium. However, because of the potential for firms and the central bank to learn from each others actions, the economy can admit multiple linear equilibria (either one or three). This multiplicity introduces a well-known impediment to any welfare analysis. One has to decide on which equilibrium agents coordinate, and if so what the comparative statics are in each case. I circumvent this problem in Appendix C by focusing on the highest welfare equilibrium, in line with Harsanyi and Selten’s (1988) “Pay-off Dominance Argument”, and thus abstract from any possible coordination failures (see also Amador and Weill, 2010). Appendix C shows how none of my results depend crucially on the exact equilibrium selection device used. All hold in areas of the parameter space where the equilibrium is unique.

The coefficients $k_0$ and $q_0$ in Proposition 2 are central to the analysis in this section. Indeed, we can collect the various elements that make up the noisy signal of the price level

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\(^{20}\)See also Amato et al. (2002), Amato and Shin (2006), and the related work on the learning externality of additional public information in markets where agents learn from prices (see, for example, Vives, 1997; Amador and Weill, 2012; Vives, 2017; and the summary in Veldkamp, 2011.)
that are also observed by the central bank on the left-hand side of (2.12) using (5.1), and then divide by \( k_0 \) to arrive at,

\[
\frac{1}{k_0} \left[ \tilde{p}_t - \nu_{t-1} \tilde{p}_{t-1} - \nu_0 (m_{t-1} - a_{t-1}) - k_1 \omega^a_t - k_2 \tilde{p}_t \right] = x^a_t + \frac{1}{k_0} \epsilon_{pt}.
\]

This demonstrates how the observation of the noisy signal of the price level is equivalent to \( x^a_t + \frac{1}{k_0} \epsilon_{pt} \), or \( \theta_t + \epsilon^a_{xt} + \frac{1}{k_0} \epsilon_{pt} \) since the central bank can also condition out last period’s productivity \( a_{t-1} \).\(^{21}\) I equate this signal to \( \bar{p}_t \) in Proposition 2. When the variance of the noise term \( \frac{1}{k_0} \epsilon_{pt} \), equal to \( k_0^{-2} \tau^{-1}_p \), is small, the price level conveys a precise signal of firms’ private information to the central bank, and conversely when \( k_0^{-2} \tau^{-1}_p \) is large.

**Corollary 2.** The equilibrium coefficients \( k_0 \) and \( q_0 \) solve

\[
k_0 = \nu_0 \tau^a_t \frac{q_0 \tau^a_t - (\tau^a_t + \tau^a_z)}{(\tau^a_z + \tau^a_x)(\tau^a_z + \tau^a_t) + \tau^a_z \tau^a_t}, \quad q_0 = \phi^o \frac{\tau^a_z (\tau^a_x + \tau^a_k)}{(\tau^a_z + \tau^a_z)(\tau^a_z + \tau^a_t) + \tau^a_z \tau^a_k},
\]

where all solutions of \( k_0 \in (-\nu_0, 0) \) and \( q_0 \in (0, 1) \) when \( \phi^o \in (0, 1) \).

The two coupled equations in Corollary 2 describe a fixed point problem, which connects the equilibrium weight that firms attach to their private information \( k_0 \) in Proposition 2, to the weight that the central bank itself accords to its own private information \( q_0 \), and to the variance of the noise in the price level \( k_0^{-2} \tau^{-1}_p \). The more weight firms attach to their private information (that is, the more negative \( k_0 \) is), the more informative the price level, all else equal, becomes to the central bank.\(^{22}\)

**Disclosure and the Price Level:** We are now ready to characterize how disclosure \( \tau^a_\omega \) affects the informativeness of the price level \( k_0^{-2} \tau^{-1}_p \). Indeed, it follows from Corollary 2 that disclosure can either decrease or increase the informativeness of the price level, depending on monetary policy \( \phi^o \). Disclosure can therefore either decrease or increase the central bank’s own information about the economy. Proposition 3 details the conditions for either case.

**Proposition 3.** Increases in central bank disclosure, \( \tau^a_\omega \in \mathbb{R}_+ \), either decrease or increase the informativeness of the price level, \( \tau^a_p k_0^2 \), and hence the central bank’s own information about the productivity shock, \( \theta_t \). This depends on whether \( \phi^o \leq \hat{\phi}^o = \frac{\tau^a_\omega + \tau^a_\theta}{\tau^a_\theta + \tau^a_\omega + \tau^a_x} < 1 \), where \( \alpha = \frac{\tau^a_\omega + \tau^a_\theta}{\tau^a_\theta + \tau^a_\omega + \tau^a_x} \).

We can discern the presence of the two offsetting effects that lead to Proposition 3 from

\(^{21}\)I will, for brevity, from now on refer to “the informativeness of the price level.” By that, I always mean the “informativeness of the noisy signal of the price level.”

\(^{22}\)See Amador and Weill (2010) and Vives (2017) for other expositions of the equilibrium link between the weight that agents attach to their private information and the informativeness of prices.
(2.12) and Corollary 2. Combined, these show that

\[
\bar{p}_t = \nu_0 \mathbb{E}_t^f \left[ \theta_t + l.p.t + \epsilon_{pt} \right] + \nu_0 (\phi_0 v_x - w_x) x_t^a + \text{public signals} + \epsilon_{pt} = k_0 x_t^a + \text{public signals} + \epsilon_{pt},
\]

where \( w_x \) and \( v_x \) denote the weight on private information \( x_t^a \) in firms’ expectations about productivity \( \mathbb{E}_t^f [\theta_t] \) and central bank expectations \( \mathbb{E}_t^f \left[ \mathbb{E}_t^{cb} \theta_t \right] \), respectively (\( w_x > v_x > 0 \)).

“Public signals” here also capture last period terms (\( l.p.t. \)). It follows from (5.5) that the total weight on private information \( k_0 \) in (5.3) can also be written as \( k_0 = \nu_0 (\phi_0 v_x - w_x) < 0 \).

A Learning Externality Cost: When \( \phi_0 \in (0, \hat{\phi}_0) \), Proposition 3 illustrates a “Paradox of Transparency” (Morris and Shin, 2005). Although disclosure provides firms with more information, it also decreases the central bank’s own information about productivity.

Suppose, for simplicity, that the money supply is once more held fixed (\( \phi_0 = 0 < \hat{\phi}_0 \)). Firms’ beliefs about central bank expectations, and therefore this aspect of the absence of common knowledge, thus becomes immaterial for firms’ prices. Now consider the case in which the central bank discloses additional public information. All else equal, this additional information decreases firms’ uncertainty about productivity. But the corollary of this decrease in uncertainty is that firms now rely less on their own private information when forming their expectations and more on the information from the central bank, which the central bank already knows. The weight \( w_x \) on \( x_t^a \) in (5.5) decreases in central bank disclosure \( \tau^a \), which makes \( k_0 \) less negative. As a result, the informativeness of the price level falls. Indeed, a simple total differentiation of the fixed point equation in Corollary 2 shows that \( \frac{d\mathbb{E}_t^f \theta_t}{d\tau^a} < 0 \) when \( \phi_0 = 0 \).

The price level, in this case, reflects less firms’ private information about productivity, the truly new information that the central bank could learn from firms. This, in turn, increases central bank uncertainty and the associated welfare losses from monetary policy being set based on worse information.

At the core of this adverse effect of disclosure lies a learning externality. When deciding on how much to respond to their own private information, firms do not internalize the informativeness of the price level, and hence how much the central bank is able to learn from it. Because of this externality, a fundamental trade-off arises between, on the one hand, firms’ uncertainty (and thus their ability to correctly set prices) and, on the other hand, the central

\[23\] Specifically, it follows that \( w_x = \frac{\tau^a \tau_0^*}{(\tau^a + \tau^*_0)(\tau^a + \tau^*_0 + \tau^*_0 + \tau^*_0)} \) and \( v_x = \frac{\tau^a \tau_0^*}{(\tau^a + \tau_0^*)(\tau^a + \tau_0^* + \tau^*_0 + \tau^*_0)} \).

Notice also that both \( \frac{\partial w_x}{\partial \tau^a} < 0 \) and \( \frac{\partial w_x}{\partial \tau_0^*} < 0 \).

\[24\] The derivative of the right-hand side of (5.3) with respect to \( q_0 \) equals:

\[ \frac{\partial \text{RHS}}{\partial \tau^a} = \nu_0 \left[ \tau^a + \tau^*_0 \left( \frac{\tau^a - q_0 (\tau^a + \tau^a + \tau^*_0)}{\tau^a + \tau^a + \tau^*_0} \right) \right] \]

Thus, \( \frac{\partial \text{RHS}}{\partial \tau^a} > 0 \) and hence \( \frac{\partial w_x}{\partial \tau^a} > 0 \) if \( q_0 < \frac{\tau^a}{\tau^a + \tau^a + \tau^*_0} \). The latter is equivalent to \( \phi_0 \leq \frac{\tau^a + \tau^a + \alpha}{\tau^a + \tau^a + \tau^*_0} \) from (5.3). This shows that \( \frac{d\mathbb{E}_t^f \theta_t}{d\tau^a} < 0 \) when \( \phi_0 = 0 \),
A Common Knowledge Benefit: However, as (5.5) also shows, the total effect of central bank disclosure also depends on the weight firms’ place on private information in their expectations about central bank beliefs $v_x$, in addition to monetary policy $\phi_\theta$. Indeed, when $\phi_\theta \in (\hat{\phi}_\theta, 1)$, Proposition 3 shows that the “Paradox of Transparency” is upturned. Disclosure in this case increases the informativeness of the price level.

On the one hand, the more precisely the central bank discloses its information, the less firms will use their own private information when forming their expectations about productivity. Disclosure decreases $w_x$, and thus makes $k_0 = \phi w v_x - w_x$ less negative when $\phi_\theta$ is small. However, on the other hand, the more precisely the central bank discloses its information, the less firms will also use their private information when forming their beliefs about central bank expectations; $v_x$ also decreases with central bank disclosure. As a result, although both weights on private information decrease, $k_0 = \phi_\theta v_x - w_x$ can become more negative when $\phi_\theta$ is sufficiently large. Disclosure in this case increases the informativeness of the price level.

The intuition behind this second, countervailing effect follows from how monetary policy itself affects the informativeness of the price level. Because monetary policy attempts to replicate the flex-price level of output, the money supply responds positively to new information about productivity ($\phi_\theta > 0$). In turn, this decreases forward-looking firms’ expected price responses to the productivity shock. Consequently, firms’ update their prices less in response to news about productivity, and therefore also less in response to new private information about the productivity shock. Equation (5.5) and Corollary 2 show that $k_0 = \phi_\theta v_x - w_x < 0$ increases in $\phi_\theta \in (0, 1)$.25 Thus, similar to early results in King (1982), prospective monetary policy, responding to information that is potentially unknown to firms, here stabilizes the price level, which decreases its informativeness.

This stabilization of the price level caused by monetary policy, in effect, creates an identification problem for the central bank, which disclosure helps resolve. Suppose monetary policy completely stabilizes the price level, and hence that the central bank observes a constant price level from one period to the next. All else equal, this observation could be because (a) firms received private information in line with their prior, or (b) because all firms received different private information but expect the central bank to alter the money supply in response such that prices remain constant. That is, in terms of (5.5), it could either be because $x_t^a = a_{t-1}$ or

25Conversely, (5.3) shows that increases to $\phi_\theta$ decrease the information content of the price level $k_0^2 \tau_p$. The partial derivates of the right-hand side (RHS) of (5.3) with respect to $\phi_\theta$ is

$$\frac{\partial \text{RHS}}{\partial \phi_\theta} = \nu_0 \left( \frac{\tau_2^2 \tau_z^2}{(\tau_0 + \tau_z^2)} \right) + \tau_0^2 \left( \frac{\tau_2^2 \tau_z^2}{(\tau_0 + \tau_z^2)} \right) + \tau_z^2 \left( \frac{\tau_2^2 \tau_z^2}{(\tau_0 + \tau_z^2)} \right) + \frac{\tau_2^2 \tau_z^2}{(\tau_0 + \tau_z^2)} \left( \tau_0 + \tau_z^2 \right)$$

Thus, $\frac{\partial \text{RHS}}{\partial \phi_\theta} > 0$, and hence $\frac{\partial k_0}{\partial \phi_\theta} > 0$. But since $k_0 < 0$, we arrive at $\frac{d|k_0|}{d\phi_\theta} < 0$.
be because \( x_t^a \neq a_{t-1} \) but \( k_0 = 0 \). Central bank disclosure solves this identification problem. By making the central bank’s own information, and hence beliefs, common knowledge, disclosure offers the distinction between (a) and (b). Central bank disclosure increases firms’ knowledge about central bank expectations (and \textit{vice versa}). This manifests itself as a decrease in the weight \( v_x \) firms place on private information when predicting central bank expectations (full disclosure pushes \( v_x \) to zero). Thus, disclosure makes \( k_0 = \phi_\theta v_x - w_x \) all else equal more negative, which increases the informativeness of the price level, despite its stability. Indeed, when monetary policy stabilizes the price level sufficiently \( \phi_\theta > (\hat{\phi}_\theta, 1) \), this increase in common knowledge between firms and the central bank dominates the learning externality. Central bank disclosure then decreases uncertainty for everyone, even the central bank itself.

This potential for central bank disclosure to increase the information content of the price level contrasts with earlier results in Morris and Shin (2005), Amador and Weill (2010), and others, where disclosure invariably decreases the informativeness of prices. Unlike in those papers, because of the absence of common knowledge between firms and the central bank, disclosure here decreases firms’ use of private information when predicting future central bank actions. This, in turn, increases the informativeness of price level in (5.5).

More generally, the increase in the informativeness of the price level follows from two characteristics of our model. First, that the central bank uses its own information to set nominal demand to replicate the flex-price level of output. This makes forward-looking firms’ prices depend positively on expected central bank information about productivity. And second, that the central bank both learns new information from firms (through the observation of the price level) but also provides new information to them (through its public disclosures). Combined, these characteristics create the identification problem for the central bank, which disclosure helps to resolve. This also illustrates how similar results to Proposition 3 could extend to other circumstances in which agents learn from market outcomes and their expectations about other’ choices offset their expectations about unobserved fundamentals.

Optimal Use of Information: I now turn to the combined optimal disclosure and monetary policy. To accommodate for the endogenous informativeness of the price level, I solve for the combined optimal use of central bank information with a mix of the primal and dual approach (Appendix C). I then address an important question Proposition 3 raises: Does disclosure optimally increase or decrease the informativeness of prices?

**Theorem 2.** When productivity shocks drive the economy, the unique optimal policy is full disclosure, \( \tau_0^{*,*} \rightarrow \infty \), combined with monetary policy that undoes the nominal friction, \( \phi_\theta^* = 1 \). Increases in central bank disclosure globally increase the informativeness of the price level.
Theorem 2 shows that monetary policy should once more be set to undo the nominal friction, to its optimal full information value from Section 3; and that conditional on this value, full disclosure maximizes the informativeness of the price level. At the optimal monetary policy, the increase in common knowledge dominates the learning externality. As a result, full disclosure decreases both firm and central bank uncertainty by the largest possible amount.

Central bank disclosure has several benefits which combine to make full disclosure optimal in Theorem 2. To see these, consider the output gap that arises from Corollary 1 when only productivity shocks drive the economy

\[
y_t - a_t = \phi_\theta E_t^{cb} [\theta_t] - \nu_0 E_t^{f} \left[ \phi_\theta E_t^{cb} \theta_t - \theta_t \right] - \theta_t + l.p.t. \tag{5.6}
\]

\[
= \nu_0 \left( E_t^{f} \theta_t - \theta_t \right) + (1 - \nu_0) \left( E_t^{cb} \theta_t - \theta_t \right) + \nu_0 \left( \phi_\theta E_t^{cb} \theta_t - E_t^{f} \left[ E_t^{cb} \theta_t \right] \right) + l.p.t, \tag{5.7}
\]

where I once more abstract from last period terms irrelevant to current welfare. Public disclosure of central bank information has three distinct effects on this expression, the first two of which are identical to those discussed in Section 4. Central bank disclosure (i) increases firms’ responses to fundamental shocks (\(E_t^{f} \theta_t\) is closer to \(\theta_t\), on average); and (ii) increases the commonality of expectations (\(E_t^{f} \theta_t\) and \(E_t^{cb} \theta_t\) are both closer to \(E_t^{cb} \theta_t\), on average). Both effects improve welfare in (5.7), either by allowing firms’ prices to better reflect unobserved productivity or by increasing the efficacy of monetary policy.

The third effect (iii) is, however, new to the setup with productivity shocks. It arises from how disclosure, by increasing the commonality of beliefs, also increases the informativeness of the price level at the optimal monetary policy (\(\phi_\theta^* = 1 > \hat{\phi}_\theta\)). This, in turn, makes the central bank’s own expectation of the productivity shock more accurate (\(E_t^{cb} \theta_t\) is closer to \(\theta_t\), on average). Similar to (ii), this third effect increases the ability of monetary policy to replicate the full-information first best outcome. The more the central bank knows about the productivity shock, the lower the associated welfare losses are.\(^{26}\)

Combined, these benefits of disclosure result in a stark feature of Theorem 2. Similar to Theorem 1, Theorem 2 shows that monetary policy should optimally be set to its full information value. This equivalence follows from the optimality of full disclosure. In fact, it is relatively straightforward to show that even when \(\phi_\theta < \hat{\phi}_\theta\) disclosure improves welfare. Even in this case, the benefit from allowing firms to set their prices more in line with productivity, in addition to the partially improved conduct of monetary policy, dominates the decrease in the informativeness of the price level. One consequence of this dominance of full disclosure

\(^{26}\)The reason that this third effect did not arise in Section 4 is that only firms’ expectations of the mark-up shock, in addition to the central bank’s expectations and firms’ expectations of the central bank’s expectations, mattered in equilibrium for the output gap (see 4.3). The realization of the shock itself did not matter. By contrast, the realization of the productivity shock here directly matters for (5.7).
is that monetary should optimally be set to its full information value ($\phi_\theta^* = \phi_\theta^{\text{full}}$). The reason is that full disclosure once more separates monetary policy from any informational consequences. Notice, for example, that the weight on private information $k_0$ in Corollary 2 becomes completely independent of monetary policy $\phi_\theta$ when $\tau_{\omega}^a \to \infty$. As a result, we can again apply the results from Svensson and Woodford (2004), which show that monetary policy should in such cases be set to its full information value. Clearly, that the optimal full information value of monetary policy always falls within the range where $\phi_\theta^{\text{full}} = 1 > \hat{\phi}_\theta$ is a particular feature of the model. But, as Section 7 shows, this result (and the related benefit of disclosure) provides an appropriate departure point for natural extensions of our framework.

**Other Monetary Policy Rules:** As in Section 4, neither of the above results depend on the specificities of the monetary policy rule in (2.8). Suppose that the central bank instead of (2.8) directly targets what causes changes to the output gap in (5.6), the price level and labor productivity,

$$m_t = m_{t-1} + \phi_0 + \phi_{\theta}E_t^b[a_t] + \phi_{p}E_t^{cb}[p_t]$$

$$= \phi_{\theta}E_t^{cb}[\theta_t] + \phi_{p}E_t^{cb}[p_t] + l.p.t. \quad (5.8)$$

Moreover, suppose that the central bank considers to set $\phi_\theta = \phi_p = 1$ and to fully disclose its information $\tau_{\omega}^a \to \infty$. This is, in fact, the optimal policy when the central bank follows (5.8) (Appendix C) and replicates the first best outcome under full information.

Equilibrium prices with (5.8) then become

$$p_t = \nu_0E_t^f [E_t^{cb}\theta_t + E_t^{cb}p_t - \theta_t] + l.p.t.$$ 

$$= \nu_0 \left(E_t^{cb}\theta_t - E_t^f\theta_t\right) + \frac{\nu_0^2}{1 - \nu_0} \left(E_t^{cb}\theta_t - E_t^{cb}E_t^f\theta_t\right) + l.p.t., \quad (5.9)$$

such that the corresponding expression for the output gap is

$$y_t - a_t = m_t - p_t - a_t = \left(E_t^{cb}[\theta_t] - \theta_t\right) - \nu_0 \left(E_t^{cb}E_t^f\theta_t - E_t^f\theta_t\right) + l.p.t. \quad (5.10)$$

Similar to (5.7), all that matters for welfare in (5.10) is the extent to which the central bank’s expectation of the productivity shock is accurate, in addition to the extent to which there is common knowledge between firms and the central bank. Central bank disclosure increases the extent of common knowledge and with (5.8) eliminates an entire infinite sequence of higher-order expectations that would otherwise arise in (5.9) (see 4.8 for a comparison). By doing so,

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27 In particular, $k_0 \to -\frac{\tau_{\omega}^a}{\tau_{\omega}^a + \tau_{\omega}^b + \tau_{\omega}}$ when $\tau_{\omega}^a \to \infty$.

28 I once more set $\phi_0$ such that the cash-in-advance constraint binds.
disclosure increases the informativeness of the price level, and hence the accuracy of central bank expectations. This, in turn, contributes to the optimality of disclosure.

6 A Quantitative Extension

The analysis that I have covered has shown how an absence of common knowledge between firms and the central bank can alter the desirability of central bank disclosure. In this section, I solve an extended version of the above model, which resembles that of Lorenzoni (2009). Unlike the model described in Section 2, the extended model features an imperfectly informed household and a signaling role for monetary policy. It also allows for dispersed information among firms. All three features are potentially important for an accurate assessment of the social benefits of central bank disclosure. The next section then uses the extended model to take a first pass at two basic quantitative questions. First, do the benefits of disclosure that arise from an increase in common knowledge outweigh the aforementioned costs for plausible parameter values? And second, if so, are the welfare benefits substantial?

Amended Setup: I amend the setup from Section 2, while keeping the timing of events unchanged. The representative household’s preferences once more equal

\[ U = \mathbb{E}_0^h \sum_{t=0}^{\infty} \beta^t \left[ \log (C_t) - \frac{1}{1 + \eta} L_{t+1}^{1+\eta} \right], \]

(6.1)

where, unlike in Section 2, the household now also has imperfect information about the fundamentals of the economy and bases its expectations \( \mathbb{E}_t^h [\cdot] = \mathbb{E} [\cdot | \Omega_t^h] \) upon the information set \( \Omega_t^h \) (described below). I dispense with the cash-in-advance constraint (2.4) and instead assume that money is held in bank deposits, earning an interest rate of \( i_t \). The household’s budget constraint therefore in place of (2.3) becomes

\[ \int_0^1 P_t C_t di + (1 + i_t) M_t^d \leq \int_0^1 \Pi_t di + W_t L_t + M_{t-1}^d + T_t^h. \]

(6.2)

The central bank, which controls the interest rate on bank deposits, follows a simple Taylor Rule, in which it targets deviations of log-output from its full-information flex-price levels,

\[ i_t = \exp \left( \mathbb{E}_t^{ch} [y_t - a_t] \right) \phi \exp (\epsilon_{mt}), \]

(6.3)

where \( \epsilon_{mt} \sim \mathcal{N}(0, \tau_m^{-1}) \) denotes a monetary policy shock.\(^{29}\)

\(^{29}\)The introduction of the monetary policy shock \( \epsilon_{mt} \) in (6.3) here serves a technical purpose, similar to that of \( \epsilon_{pt} \) in Section 2. It prevents the private sector from perfectly inferring the central bank’s private information directly from observations of the central bank interest rate.
Last, I keep the production side of the economy unchanged with the exception of the stochastic processes for labor productivity and firms’ mark-ups. I assume that both of these follow stationary AR(1)s in logarithms,

\[ A_t = A_{t-1}^\rho \exp (\theta_t), \quad M_t = M_{t-1}^\mu \exp (\xi_t), \quad (6.4) \]

where \( \rho_\alpha \in (0, 1) \) and \( \rho_\mu \in (0, 1) \).

**Linearized Equilibrium Conditions:** I once more study a log-linear approximation to the rational expectation equilibria. Following well-known steps, Appendix D shows that the equilibrium conditions of the model reduce to three key log-linear equations. First, an Euler equation, which determines the optimal intertemporal allocation of consumption and output,

\[ y_t = E_t^h [y_{t+1}] - \left( i_t - E_t^h [\pi_{t+1}] \right), \quad (6.5) \]

where \( \pi_t \) denotes the inflation rate for the consumption index. Second, a New-Keynesian Phillips Curve, which relates firms’ expectations of mark-ups and marginal cost, proportional to the output gap, to expected future and current inflation,

\[ \pi_t = \beta \bar{E}_t^f [\pi_{t+1}] + \lambda \bar{E}_t^f [y_t - a_t] + \bar{E}_t^f [\mu_t] \quad (6.6) \]

in which \( \lambda = \frac{1+\rho}{\psi} \) and \( \bar{E}_t^f [\cdot] = \int_{0}^{1} E \left[ \cdot | \Omega_t^f \right] di \) denotes firms’ average expectation, where \( \Omega_t^f \) is defined further below. And third, a log-linear central bank Taylor Rule,

\[ i_t = \phi E_t^{cb} [y_t - a_t] + \epsilon_{rt}. \quad (6.7) \]

Combined, (6.3)-(6.6) closely resemble the equilibrium conditions of the dispersed information New Keynesian model in Lorenzoni (2009), extended to the case where the private sector and the central bank also lack common knowledge about each others’ beliefs. Nimark (2014) and Melosi (2016) consider other extensions of Lorenzoni’s (2009) framework. In particular, Melosi (2016) studies the case in which firms learn from movements in the central bank interest rate about the shocks that hit the economy. I accommodate for such signaling effects from changes to the interest rate in the information structure.

**Information Structure:** The information structure mirrors that from Section 2. The private sector, comprised of firms and the representative household, observes private information about the driving forces of the economy, as well as the central bank’s disclosure. The substantive differences are that (i) firms now also observe and learn about central bank expectations from the current value of the central bank interest rate, and that (ii) firms now observe
individual-specific private information,

\[ \Omega_{it}^f = \{x_{it-j}, \omega_{t-j}, \pi_{t-j}, \bar{i}_{t-j}\}_{j=0}^{\infty}, \] 

(6.8)

where \( x_{it}^\mu = [x_{it}^\mu, x_{it}^a] \) is comprised of

\[ x_{it}^\mu = \mu_t + \epsilon_{it}^\mu + \epsilon_{it}^\mu : \epsilon_{it}^\mu \sim \mathcal{N}(0, 1/\tau^\mu_x), \quad x_{it}^a = a_t + \epsilon_{it}^a + \epsilon_{it}^a : \epsilon_{it}^a \sim \mathcal{N}(0, 1/\tau^a_x). \] 

(6.9)

Equation (6.9) reduces to (2.9) in the case where \( \tau^j_x \to \infty, \ j = \{\mu, a\}. \) In the terminology of Myatt and Wallace (2011), firms’ private information in (6.9) is simultaneously subject to sender and receiver noise (see also e.g. Hellwig et al., 2012; Colombo et al., 2014; and Yang, 2015). In (6.8), I also dispense with the somewhat artificial assumption of one-period perfect state verification used in (2.13) and instead assume that firms (as well as the central bank) do not observe last period’s realization of the fundamental shocks.

Last, I assume that \( \Omega^h_t \) is set such that \( \bar{E}_t^h [\cdot] = E_t^h [\cdot]. \) The aggregate equations for output and inflation in (6.5) and (6.6), respectively, that result from this assumption are identical to those in Lorenzoni (2009), where households and firms inhabit an “island-structure”. It is also equivalent to the assumption used in Svensson and Woodford (2004).

The central bank’s information set is similarly close to that from Section 2. Indeed, besides that the central bank now uses the (stationary) inflation rate to infer firms’ private information instead of the (non-stationary) price level and the absence of one-period perfect state verification, it is almost identical to before,

\[ \Omega^cb_t = \{z_{t-j}, \omega_{t-j}, \pi_{t-j}, \bar{i}_{t-j}\}_{j=0}^{\infty}. \] 

(6.10)

where \( \pi_t = \pi_t + \epsilon_{pt} \) with \( \epsilon_{pt} \sim \mathcal{N}(0, \tau^{-1}_p) \) denotes the noisy realization of inflation observed by the central bank.

**Solution and Calibration:** Unlike the framework presented in Section 2, the equilibrium solution for the endogenous variables, such as output and inflation, can no longer be derived analytically. I therefore solve the model numerically instead.

The above model exhibits two features which render common solution methods inapplicable. First, the central bank learns about common disturbances from an endogenous market outcome (inflation). The combination of endogenous public information with the absence of perfect state verification has since Townsend (1983) been known to imply that standard state space representations of the equilibrium have infinite-dimensional state vectors. This is due to the infinite regress of expectations that arises when agents need to “forecast the forecast of others”. Second, since the non-atomistic central bank’s actions affect inflation, both inflation,
in addition to the other simultaneously determined variables, depend on current central bank estimates of the state; yet these also depend on what the central bank infers from inflation. This circularity problem, which was also present in Section 4 and 5, however, now telescopes due to the infinite state vector and further complicates the solution of the model.

In this subsection, I first describe how to adapt the solution method proposed in Nimark (2017) to the current setting with a non-atomistic agent. I keep details to a minimum and focus on the important role that higher-order expectations, influenced by central bank disclosure, play in the equilibrium solution. I then calibrate the model separately for the mark-up shock and the productivity shock case to match data on forecast accuracy from the “Survey of Professional Forecasters” (for the private sector) and the “Greenbook” (for the central bank). I calibrate for each persistent shock separately to avoid confounding the two distinct benefits of central bank disclosure discussed in Section 4 and 5.

Dynamic Models with Two-sided Information: Nimark (2017) shows that when all disturbances are stationary, an approximate solution can be found to linear rational expectations equilibria in models with endogenous information. This can be done by direct truncation of the state vector comprised of higher-order expectations, to achieve a finite-dimensional representation. I extend this solution method to deal with the additional complication of non-atomistic agents in Appendix D. When the model is solved, the approximate law of motion for the endogenous triplet \( q_t = [\pi_t \, y_t \, i_t] \)' admits the form

\[
q_t = \alpha_0 X_t^{(0,k)} + \alpha_1 u_t, \tag{6.11}
\]

where \( u_t = [\epsilon_t \, \epsilon_{zt}^j \, \epsilon_{yt}^j \, \epsilon_{pt} \, \epsilon_{mt}]' \) with \( j = \{a, \mu\} \) and \( \epsilon_t = \{\theta_t, \xi_t\} \), depending on which of the two persistent shocks drive the economy. \( X_t^{(0,k)} \) here denotes the expectational state vector comprised of the entire hierarchy of private sector and central bank higher-order expectations about the persistent fundamental \( X_t^{(0)} = \{a_t, \mu_t\} \) up to the \( k \)th order,

\[
X_t^{(0,k)} = \begin{bmatrix} X_t^{(0)} & X_t^{(1)} & \cdots & X_t^{(k)} \end{bmatrix}', \quad X_t^{(k)} = \begin{bmatrix} \mathbb{E}_{t}^{f} X_t^{(k-1)} \\ \mathbb{E}_{t}^{cb} X_t^{(k-1)} \end{bmatrix}, \quad k \in \{1, 2, \ldots, \tilde{k}\}. \tag{6.12}
\]

The true equilibrium law of motion has \( \tilde{k} \rightarrow \infty \). I truncate this expectational state vector at

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30 See also Nimark (2014), Melosi (2016), and Struby (2016) for other applications of this solution method.

31 The Greenbook contains forecasts computed by the Staff of the Federal Reserve. These forecasts are published a few days before the FOMC meeting and collected with a five year lag in “the Greenbook data set” (Reifschneider et al., 1997). The data for private sector forecasts comes from the Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia. The sample stretches from 1968Q1 to 1993Q4. In February 1994, the Federal Reserve Market Committee began a long process of increased disclosure, altering the informational assumption used to calibrate the model (Dincer and Eichengreen, 2009).
\( \bar{k} = 50 \). All impulse response functions are stable from around \( \bar{k} = 15 \).

An implicit assumption used to arrive at (6.11) is that it is common knowledge that all agents form model consistent expectations. Similar to Nimark (2017), Appendix D shows that this assumption, combined with the Kalman Filter extended to the case in which the central bank can back-out part of the noise component in the private sector’s signals, ensures that \( X_t^{(0,\bar{k})} \) follows a \( \text{VAR}(1) \),

\[
X_t^{(0,\bar{k})} = MX_{t-1}^{(0,\bar{k})} + Nu_t. \tag{6.13}
\]

Because the private sector and the central bank learn from the observation of each other’s actions, the matrices \( M \) and \( N \) here depend on the coefficients in \( \alpha_0 \) and \( \alpha_1 \), and vice versa. I solve for the fixed point \( \{M, N\} \leftrightarrow \{\alpha_0, \alpha_1\} \leftrightarrow \{M, N\} \) by repeated iteration.

The finite-dimensional representation in (6.11) to (6.13) is particularly convenient for our purposes. Central bank disclosure affects the commonality of beliefs, and hence the effect of the \( k \)th order of higher-order expectations on the equilibrium dynamics. Equations (6.11) to (6.13) allow us to directly study the impact of such changes to higher-order uncertainty.\(^{23}\)

**Calibration:** The parameter \( \beta \) is set to 0.99, such that the time period can be interpreted as one quarter. The inverse Frisch elasticity of labor supply \( \eta \) is set to one, and the value of the elasticity of substitution \( \rho \) to six, which implies a mark-up of 20%. The price adjustment parameter \( \psi \) is set such that the slope of the New Keynesian Phillip’s Curve is \( \lambda = 0.25 \), and the standard deviation of the idiosyncratic monetary policy shock to two. The parameter on the interest rule is set such that the model with productivity shocks is consistent with a Taylor Rule coefficient on inflation of one-and-a-half. This implies \( \phi = 1.81 \). These values are all in the range of those used in existing studies within the New Keynesian framework.

Next, I determine the parameters that control the mark-up and productivity shock, in addition to the noise in the private information about these. The persistence parameters are set to \( \rho_\mu = 0.70 \) and \( \rho_\alpha = 0.80 \), respectively, consistent with existing studies. For each persistent shock, the parameters which control the average precision of private information (\( \tau^a_x \) or \( \tau^\mu_x \)) are set under complete central bank opacity (\( \tau^j_\omega \to 0, j = \{a, \mu\} \)) to match pre-February 1994 data on the one-quarter ahead root-mean-square error of GNP/GDP forecasts

---

\(^{22}\)The central bank can from the observation of its own private information \( z_t \) and its own action \( i_t \) back out the value of the monetary policy shock \( \epsilon_{mt} \) from (6.7). This is the shock that otherwise prevents the private sector from perfectly inferring \( z_t \) from the observation of the interest rate.

\(^{23}\)While convenient, the solution in (6.11) to (6.13) yet merely provides one tractable solution method. Other approaches exist: Townsend (1983) pioneered the use of lagged perfect state verification and more recently Rondina and Walker (2012), Kasa et al. (2013), and Huo and Takayama (2015) have demonstrated the applicability of frequency-domain techniques to solving models with private information. Nevertheless, when applied to the above setup, these methods turn out not to lend themselves so immediately to the below calibration procedure. Besides, the direct solution for higher-order expectations available from the chosen method is particularly convenient for studying the consequences of incomplete common knowledge.
from the Survey of Professional Forecasters for the private sector and the Greenbook for the central bank. In February 1994, the Federal Reserve Open Market Committee commenced a long process of increased transparency. I therefore do not employ data after this quarter as it would conflict with the complete opacity assumption otherwise used in the calibration. To start, I set the dispersion in firms’ private information $(\bar{\sigma}_x^a)^{-1}$ and $(\bar{\sigma}_x^\mu)^{-1}$ equal to zero, to make my results comparable to those in Section 4 and 5. I then explore the robustness of my results to realistic amounts of dispersion in firms’ information.

Last, to set $\tau_p$, I follow Lorenzoni (2009) and interpret $\epsilon_p$ as measurement error in early releases of inflation data and match the signal-to-noise ratio in these. Specifically, I interpret $\pi_t$ as the first release and $\pi_t$ as the last. I then choose $\tau_p$ to match the ratio between the standard deviation of the measurement error and the standard deviation of the innovation to inflation. The latter is measured by running a simple univariate regression of final release inflation on two lags. Lorenzoni (2009) employs this approach an obtains a ratio of 1.97 for PCE inflation. Matching this value, I obtain the parameters listed in Table I. The noise in inflation is for both the mark-up and productivity shock case substantial, consistent with the evidence presented in, for example, Runkle (1998).

A feature that immediately stands out from Table I is that the central bank has superior private information. To match the data on forecast accuracy, central bank private information has to be around 38 to 50 percent more precise than that of the private sector. This is consistent with the empirical results in Romer and Romer (2000), which document a substantial information advantage for the US Federal Reserve relative to the mean forecast from the Survey of Professional Forecasters.

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Table I: Baseline Shock and Information Parameters

<table>
<thead>
<tr>
<th></th>
<th>Productivity Shock</th>
<th>Mark-up Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_a$</td>
<td>0.80</td>
<td>0.70</td>
</tr>
<tr>
<td>$\sigma_a^2$</td>
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<td>$\sigma_a^\mu$</td>
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</tr>
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</tr>
<tr>
<td>$\sigma_p^\mu$</td>
<td>$\rightarrow \infty$</td>
<td></td>
</tr>
</tbody>
</table>

(i) The mapping between standard deviation $\sigma$ and precision $\tau$ is $\tau = 1/\sigma^2$.
7 Quantitative Benefits of Learning by Sharing

In this section, I present estimates of the welfare benefits of central bank disclosure using the calibrated model. I demonstrate how disclosure increases the commonality of beliefs, and thereby central bank information, to such an extent that it can be beneficial, irrespective of the source of macroeconomic fluctuations. To make my results comparable to those in Section 4 and 5, I start with quantitative results under the baseline calibration in which all firms observe the same information. I then end the section with a breakdown of the sensitivity of the quantitative results to the presence of dispersed private sector information, as well as to the importance of higher-order expectations and central bank signaling.

A First Best Benchmark: I once more take my welfare criterion to be the \textit{ex-ante} utility of the representative household.\footnote{Arguably, with both an imperfectly informed household and central bank one could instead consider the household’s own expectation or the central bank’s expectation about future welfare losses. Instead of either, I choose to adopt the Rawlsian approach at the centre of an \textit{ex-ante} expression for welfare. The substantive conclusions from this section remain the same irrespective of which of the three definitions of welfare is used.} Similar to Sections 3 to 5, the central bank can in the extended model replicate the first best outcome in several limit cases under the baseline calibration. For example, when both the central bank and the private sector have full information about all shocks, the central bank can achieve the full information flex-price outcome by letting $\phi \rightarrow \infty$ (Appendix D). This ability to replicate the first best extends to the case where the private sector has imperfect information but the central bank has full information, both about all shocks as well as about the private sector’s beliefs about them. The central bank can in this case still replicate the first best by letting $\phi \rightarrow \infty$ (Appendix D).\footnote{One may think that firms will always learn the central bank’s private information when $\phi \rightarrow \infty$ from the observation of the interest rate. This is, however, not the case because central bank expectations of the size of the output gap also decrease as we increase $\phi$. Define the central bank’s forecast error of the output gap as $\lambda_{y_{t-1}, a_t} = (y_t - a_t) - E^{cb}_t [y_t - a_t]$. Then,}

$$y_t = E^{h}_t [y_{t+1} + \pi_{t+1}] - i_t = E^{h}_t [y_{t+1}] + E^{h}_t [\pi_{t+1}] - \phi E^{cb}_t [y_t - a_t]$$

$$= E^{h}_t [y_{t+1}] + E^{h}_t [\pi_{t+1}] - \phi (y_t - a_t) - \phi \lambda_{y_{t-1}, a_t},$$

and hence $\lim_{\phi \rightarrow \infty} y_t = a_t + \lim_{\phi \rightarrow \infty} \lambda_{y_{t-1}, a_t}$. But then it follows from

$$i_t = \phi E^{cb}_t [y_t - a_t] = \phi (y_t - a_t) - \phi \lambda_{y_{t-1}, a_t},$$

that $\lim_{\phi \rightarrow \infty} i_t = \lim_{\phi \rightarrow \infty} \phi \lambda_{y_{t-1}, a_t} - \lim_{\phi \rightarrow \infty} \phi \lambda_{y_{t-1}, a_t} = 0$. In the limit where $\phi \rightarrow \infty$, the interest rate becomes completely uninformative.
The figure illustrates the root mean-squared error of private sector estimates of the first four orders of expectations in $X_t^{(0:k)}$. The panel is plotted for the calibrated values in Table I. Subscripts indicate order of expectation in $X_t^{(0:k)}$. For instance, the line for $X_1^{cb}$ represents the root mean-squared error of the private sector’s estimate of the central bank’s expectation of the mark-up shock.

orders of the expectational state vector $X_t^{(0:k)}$ as we increase the precision of central bank disclosure. The remaining orders follow a similar pattern and are omitted for clarity. By introspection, the private sector cannot under the baseline calibration be uncertain about its own expectations. The figure therefore only depicts elements that pertain to central bank expectations, in addition to the mark-up shock. The line for $X_1^{cb}$ thus, for example, represents private sector uncertainty about central bank expectations of the mark-up shock (and similarly for the second-order expectation in $X_2^{cb}$). Figure 2 illustrates the associated changes to central bank uncertainty about the vector of higher-order expectations $X_t^{(1:k)}$.

The results in Figure 1 and 2 mirror those from Section 4. First, disclosure decreases private sector uncertainty about the mark-up shock, visible from the $X_0$-line in Figure 1. This, all else equal, leads to larger private sector responses. Second, disclosure also decreases central bank uncertainty about private sector expectations and vice versa, which is evident from the rest of the lines in Figures 1 and 2. As Section 5 shows, this in turn allows the central bank to better counter private sector responses to the mark-up shock. Which of these two effects dominates depends on the forcefulness by which monetary policy attempts to offset private sector responses. The immediate question then becomes which of these two countervailing

\[ \text{The first four orders of expectations in } X_t^{(0:k)} \text{ are } \mu_t, E_t^f \mu_t, E_t^{cb} \mu_t, E_t^f E_t^{cb} \mu_t, E_t^f E_t^{cb} E_t^f \mu_t, \text{ and last } E_t^{cb} E_t^{f cb} E_t^{cb} \mu_t. \]
Figure 2: Central Bank Uncertainty about $X^{(1;k)}_t$ with Mark-up Shocks

The figure illustrates the root mean-squared error of central bank estimates of the first four orders of expectations in $X^{(1;k)}_t$. The panel is plotted for the calibrated values in Table I. Subscripts indicate the orders of expectation in $X^{(0;k)}_t$. For instance, the line for $X^f_1$ represents the root mean-squared error of the central bank’s estimate of the private sector’s expectation of the mark-up shock.

effects dominates for different values of monetary policy?

Table II shows that for the calibrated value of monetary policy the latter effect dominates the former. Public disclosure of information about the mark-up shock decreases welfare losses measured in life-time consumption by around 58 percent.\footnote{With full disclosure, I here in practice mean $\tau^u = 1e + 5$. I then cross-check all results with $\tau^u = 1e + 7$.} We can decompose this welfare benefit from central bank disclosure into its two constituent components. Table II illustrates the breakdown that I obtain when I let the central bank disclose its information but fix private sector and central bank higher-order uncertainty to that from the (complete opacity) baseline. All else equal, the increase in private sector responses to the mark-up shock increases welfare losses by around 56 percentage points. But this increase is more than offset by a substantial fall in higher-order uncertainty. Indeed, for the calibrated parameters, the fall in higher-order uncertainty is around twice as important in welfare terms as the direct increase in private sector responses to the mark-up shock.

Consistent with Theorem 1, this benefit of central bank disclosure carries over from the baseline value of monetary policy to its optimal combination with communication policy, which I find to be $\phi \to \infty$ and $\tau^u \to \infty$.\footnote{In practice, I allow for values of $\phi$ and $\tau^u$ up to to $1e + 5$ and cross-check with values equal to $1e + 7$. To be precise, whenever I write, for instance, $\phi \to \infty$ in the below I mean $\phi = 1e + 5$. All welfare results are constant to the sixth decimal place in Table II and III for values above $1e + 4$.} Once more the optimal monetary policy equals that under
Table II: Welfare Effects of Disclosure with Mark-up Shocks

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameters</th>
<th>$% \Delta W_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>$\phi = 1.81$</td>
<td>$\tau_\mu \to 0$</td>
</tr>
</tbody>
</table>

Breakdown of Benefits from Disclosure

A. Benchmark with full disclosure | $\phi = 1.81$ | $\tau_\mu \to \infty$ | -58.54 |
B. Benchmark with constant h.o. unc.† | $\phi = 1.81$ | $\tau_\mu \to \infty$ | +56.10 |
A-B. Benefit from decrease in h.o. unc. | | | -114.63 |

Breakdown of Benefits from Optimal Policy

A. Optimal policy | $\phi \to \infty$ | $\tau_\mu \to \infty$ | -96.46 |
B. Benefit from optimal mon. policy | $\phi \to \infty$ | $\tau_\mu \to 0$ | -69.82 |
A-B. Benefit from central bank disclosure | | | -26.64 |

(i) $W_C$ denotes the life-time consumption equivalent of $W$.
(ii) $\% \Delta W_C$ denotes the $\%$ change in $W_C$ relative to the calibrated benchmark.
(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

full information. At the optimal value of monetary policy, central bank disclosure decreases welfare losses by around 27 percentage points, due to decreases in central bank uncertainty about private sector expectations and *vice versa*. This, in turn, contributes to an overall welfare benefit of moving from the calibrated complete opacity policy to the optimal policy that is as large as to almost eliminate all welfare losses. This is clearly a forceful implication of the extended model paired with the calibrated parameters and is in part driven by the central bank’s quite precise private information about mark-up shocks (see Table I). Admittedly, a large share of the benefits from moving to the combined optimal policy derive from the optimal use of monetary policy. But the conclusion remains that disclosure contributes a healthy share to the total (around one-quarter of the total welfare gains).

**Productivity Shock Case:** I now turn to the productivity shock case. Figure 3 illustrates the two competing effects of central bank disclosure on the central bank’s own information about productivity discussed in Section 5. On the one hand, disclosure decreases the private sector’s weight on its own private information in its expectation about productivity, decreasing the information content of inflation (lower left-hand panel). This, all else equal, *increases* central bank uncertainty. However, on the other hand, disclosure also decreases the central bank’s uncertainty about private sector expectations of its own beliefs, and so on (lower right-hand panel). This, in turn, allows the central bank to better back out changes in private sector information about productivity from changes to inflation, *decreasing* central bank uncertainty.
Figure 3: Central Bank Uncertainty about $X_t^{(0:k)}$ with Productivity Shocks

The top panel shows the root mean-squared error of central bank estimates of productivity as we increase the precision of central bank disclosure. The panel is plotted for $\phi = 1.81$ (low, left-hand scale) and for $\phi \to \infty$ (high, right-hand scale). The lower left-hand panel depicts the private sector’s weight on private information ($w_x$) in its expectation of productivity when $\phi = 1.81$. The lower right-hand panel shows the root mean-squared error of central bank estimates of $X_t^{(0:k)}$ when $\phi = 1.81$. 
Table III: Welfare Effects of Disclosure with Productivity Shocks

<table>
<thead>
<tr>
<th>Parameters</th>
<th>%∆WC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>φ = 1.81 τω → 0</td>
</tr>
</tbody>
</table>

Breakdown of Benefits from Disclosure

A. Benchmark with disclosure | φ = 1.81 τω → ∞ | −2.95 |
B. Private sector benefit of disclosure† | φ = 1.81 τω → ∞ | −14.31 |
A-B. Central bank cost of disclosure | | +11.36 |

Breakdown of Benefits from Optimal Policy

A. Optimal policy | φ → ∞ τω → ∞ | −32.72 |
B. Benefit from optimal mon. policy | φ → ∞ τω → 0 | +8.61 |
C. Private sector benefit of disclosure† | φ → ∞ τω → ∞ | −12.84 |
A-B-C. Central bank benefit of disclosure | | −28.49 |

(i) WC denotes the life-time consumption equivalent of W.
(ii) %∆WC denotes the %change in WC relative to the calibrated benchmark.
(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.

Similar to the results in Section 5, Figure 3 (top panel) shows that for “small values” of monetary policy φ, here consistent with the baseline value, the former effect dominates the latter. Disclosure on balance increases central bank uncertainty about productivity by crowding out private sector information from inflation. Although disclosure decreases welfare losses (by three percent), the increase in central bank uncertainty, all else equal, increases welfare losses by 11 percentage points. I show this in Table III, where I breakdown the effect of disclosure on the private sector and the central bank by fixing central bank higher-order uncertainty to its (complete opacity) baseline value.

However, consistent with Proposition 3 and Theorem 2, for “larger values” of monetary policy, disclosure instead decreases central bank uncertainty. Figure 3 (top panel) illustrates this for φ → ∞, which I once more find to be optimal. At the optimal monetary policy, disclosure decreases central bank uncertainty about productivity by alleviating the identification problem that arises when the central bank attempts to infer private sector information from relatively stable inflation. We can see the welfare benefits from this learning by sharing in Table III. Moving from the calibrated benchmark to the optimal policy decreases welfare losses by around 33 percent. A large share of this decrease is due to more informed private sector choices caused by more information (around 13 percentage points). But a more substantial share is, in fact, driven by the decrease in central bank uncertainty and the associated
improvement in monetary policy (around 28 percentage points).\footnote{Interestingly, moving to the optimal monetary policy ($\phi \rightarrow \infty$) without at the same time disclosing the private information that monetary policy is based on ($\tau^a \rightarrow 0$) is socially costly. It increases welfare losses by around nine percentage points relative to the benchmark case. This provides a stark example of the interdependence of monetary and communication policy discussed in this paper.} Importantly, disclosure increases the informativeness of inflation also for other values of monetary policy $\phi$ that we could have considered as our baseline value, such as that, for example, calibrated to match the post-Great Moderation estimates in Clarida et al. (2000).

**Alternative Specifications:** I conclude this section by exploring the sensitivity of the quantitative results. Specifically, I re-compute the welfare benefits of disclosure in the following four cases: (i) when the parameters that control the dispersion in firms’ private information are set to match the pre-February 1994 dispersion in one-quarter ahead GNP/GDP forecasts from the Survey of Professional Forecasters, consistent with our baseline calibration ($\bar{\sigma}_x = (\bar{\tau}^G)^{-0.5} = 0.11$ and $\bar{\sigma}_a = (\bar{\tau}^a)^{-0.5} = 0.20$);\footnote{I measure the dispersion in individual forecasts by their average cross-sectional standard deviation. This provides me with a target equal to 0.33 percentage points. Because individual-specific error terms add additional noise to the private sector’s information in (6.9), I also recalibrate the values of $\tau^G$ and $\tau^a$ to once more match the observed one-quarter ahead root-mean squared error of the average pre-February 1994 GNP/GDP forecast from Survey of Professional Forecasters ($\sigma_x = (\tau^G)^{-0.5} = 0.50$ and $\sigma_a = (\tau^a)^{-0.5} = 0.60$).} (ii) when both firms and the central bank compute only two higher-order expectations ($\bar{k} = 3$), consistent with the experimental evidence on higher-order reasoning in Nagel (1995); (iii) when the discount factor decreases from $\beta = 0.99$ to $\beta = 0.75$, decreasing the extent to which firms’ prices depend upon expectations of future firm and central bank actions and hence on higher-order expectations; and last (iv) when the signaling role of monetary policy is absent ($i_t \notin \Omega^f_t$). Table IV summarizes the results, while Appendices E.1 to E.4 document in detail how in all four cases the main insights from Table II and III continue to hold.

**On the Importance of Dispersed Information:** As discussed in, for example, Hellwig (2005) and Angeletos and Pavan (2007), dispersed information among firms can have important consequences for the benefits of central bank disclosure. In particular, because of strategic complementarities between firms’ prices, the presence of dispersed information may cause public signals to receive either too little or too much weight. This depends in part on how monetary policy is set (Angeletos et al., 2016). Thus, the welfare effects of additional central bank disclosure may differ with dispersed private sector information from those reported in Tables II and III. However, as Table IV and Appendix E.1 illustrate, the main insights from my analysis extend to the case in which firms observe dispersed private information.

For both the mark-up and productivity shock case, I re-compute the optimal monetary and disclosure policy, taking into account the welfare consequences of the price dispersion that now exists. I find that the optimal policy remains unaltered in both cases: The optimal
monetary policy remains $\phi^* \to \infty$, while full disclosure is still optimal in both the mark-up and the productivity shock case ($\tau_\mu^{\nu*} \to \infty$ and $\tau_\omega^{\nu*} \to \infty$).

Table IV and Appendix E.1 show the breakdown of the quantitative results when only mark-up shocks drive the economy. Consistent with the results in Table II, full disclosure improves welfare, both at the calibrated pre-February 1994 benchmark and at the optimal value of monetary policy. The benefit from the central bank being able to better predict (and hence offset) private sector responses to the mark-up shock once more dominates the increase in private sector responses for the calibrated parameters. Furthermore, relative to the results in Table II, central bank disclosure is somewhat more beneficial under the optimal monetary policy (welfare losses decrease by -50 percent vs. -26 percent previously), while somewhat less so at the calibrated benchmark (-26 percent now vs -115 percent before). This demonstrates how the dispersion in private sector information modifies our quantitative results, while upholding our main conclusions.

Turning to the productivity shock case, Table IV and Appendix E.1 show that the results with dispersed information are remarkably similar to those in Table III. For example, going from complete opacity to full disclosure now decreases welfare losses by around 32 percent at the optimal monetary policy (Appendix E.1). Around 30 percentage points of this decrease is due to the increase in central bank information about productivity (compared to 28 percentage points in Table III). The benefit from the central bank being able to better back out information from inflation once more dominates the learning externality. This, in turn, makes disclosure more beneficial and the overall effects similar to those in Table III.

An interesting exercise is to re-calibrate the noise in private information about productivity to target double the amount of dispersion in forecasts to that observed in the SPF data. This shows how the above benefit of central bank disclosure extends to circumstances for which full disclosure is no longer optimal. In fact, in this case, maintaining modest amount of private sector uncertainty about central bank expectations is preferable ($\tau_\omega^{\nu*} = 1.98$). The optimal monetary policy, by contrast, remains the same. However, although full disclosure is no longer optimal, moving from the complete opacity baseline to the combined optimal monetary and disclosure policy reduces welfare losses still in part because of the increase in central bank knowledge about productivity. (A similar exercise for the mark-up shock case shows that for double the amount of the dispersion in forecasts full disclosure is still optimal).

On the Importance of Higher-Order Expectations: Decreases to the amount of higher-order expectations computed and to the discount factor decrease the importance of higher-order expectations for the equilibrium dynamics of the model. Consequently, both the costs as well

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44Recall that the standard deviation of the productivity process $a_t$ is one. Hence, $\tau_\omega^{\nu*} = 1.98$ equates to a noise-to-signal ratio of only around one half.
as the benefits of disclosure fall. The costs decrease because of the decrease in amplification that arises from, for example, firms discounting future firms’ responses to the inefficient mark-up shock relatively more. By contrast, the benefits decrease because of the direct reduction in the importance of firms’ expectations of central bank beliefs, and so on, for example through the decrease in the importance of future interest rates for firms’ prices. Yet, because of the relative symmetry of these decreases (Appendix E.2 and E.3), the quantitative benefits of disclosure still on balance resemble those reported in Table II and III (Table IV).45

On the Importance of Signaling: Under opacity or partial disclosure, movements in the interest rate provide firms with a noisy signal of the central bank’s private information. By contrast, full disclosure separates the interest rate from its signaling effect. A concern could therefore be that the lion-share of the quantitative benefits of disclosure reported in Table II and III arises from the separation of monetary policy from its informational consequences rather than from a decrease in higher-order uncertainty. Table IV and Appendix E.4 shows that this is not the case. In fact, the resulting separation of monetary policy contributes at most one percentage point to the quantitative benefits of disclosure. This is because the interest rate, both in the benchmark calibration and at the optimal value of monetary policy, provides a rather dim indicator of the central bank’s private information.46 This is consistent with the substantial impact of central bank disclosure on financial markets and on private sector uncertainty about future interest rates documented in, for example, Blinder et al. (2008).

Summary of Quantitative Results: Combined, the quantitative results illustrate the importance of the lack of common knowledge between the private sector and the central bank for an accurate picture of the social value of central bank information. Specifically, the quantitative results have shown how for realistic parameter values additional disclosure can provide the central bank with more information, both indirectly about private sector expectations and by directly simplifying the central bank’s own inference problem when it learns from market outcomes. And although the extended model merely provides a first pass at a full quantitative assessment, the results suggest that the welfare benefits that arise from an improved conduct of monetary policy could be substantial. Indeed, they could make disclosure beneficial even in cases when other prominent forces push towards opacity.

45One noteworthy difference is that (when we restrict the amount of higher-order expectations computed) central disclosure decreases central bank uncertainty in the productivity shock case even under the pre-February 1994 baseline value of monetary policy.

46See also the derivations in footnote 34.
Table IV: Welfare Effects of Disclosure: Alternative Specifications

<table>
<thead>
<tr>
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<th>A. Mark-up Shock Case</th>
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<td>Discount rate</td>
<td>Limited k</td>
<td>Signaling</td>
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<td>-62.33</td>
<td>-62.95</td>
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<table>
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<td>Disp. Information</td>
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<td>-28.49</td>
<td>-29.61</td>
<td>-25.21</td>
<td>-2.98</td>
<td>-29.23</td>
</tr>
</tbody>
</table>

(i) The first row in Panel A and B shows the %change in life-time consumption due to full disclosure. The second row, by contrast, shows the %change in life-time consumption due to changes in central bank uncertainty that are caused by full disclosure (evaluated at the optimal monetary policy). (ii) The productivity shock case nets out firm benefits of disclosure (see Table III). (iii) Dispersed information (\( \sigma_x^u = 0.50, \sigma_{x,f}^u = 0.11; \sigma_x^a = 0.60, \sigma_{x,f}^a = 0.20 \)); discount rate (\( \beta = 0.75 \)); limited higher-order expectations (\( k = 3 \)); and no signaling (\( i_t \notin \Omega_f^t \)).

8 Concluding Remarks

In this paper, I have explored the consequences of policymakers’ desire to learn additional information for the social value of policymaker disclosures. To do so, I have focused on a model with monopolistic competition, nominal frictions, and incomplete common knowledge between a private sector and a central bank. At the heart of my results has been that communication decreases higher-order uncertainty. A central bank’s disclosure not only provides more information to the private sector, but also increases common knowledge between the private sector and the central bank. This has important consequences for what the central bank knows about private sector expectations, what it can learn from private sector actions, and hence for the set of potential outcomes that monetary policy can attain.

Specifically, I have shown that the increase in common knowledge that disclosure entails can overcome two otherwise standard costs of central bank disclosure: (i) the decrease in the central bank’s own information, caused by a decrease in the informativeness of market outcomes, like inflation; and (ii) the increased responses to inefficient disturbances, such as cost-push shocks. In particular, because of the decrease in higher-order uncertainty, disclosure increases what the central bank knows about private sector responses to shocks and simplifies the central bank’s own inference problem when it learns from market outcomes. Combined, I have shown that the resultant decrease in central bank uncertainty can increase the efficacy of monetary policy to such an extent that disclosure becomes invariably beneficial.

Finally, my results speak to the current debate about the efficacy of central bank forward guidance. Recent work by Wiederholt (2017) and Angeletos and Lian (2018) has shown how
incomplete common knowledge among households and firms dampens general equilibrium multipliers of expected monetary policy. Yet, as argued by Weale (2013) and others, rather than change future interest rate expectations *per se*, forward guidance often simply creates less dispersed expectations; expectations which are also more closely aligned with the central banks’ own. My results suggest that forward guidance by increasing common knowledge between the private sector and the central bank increases the potential efficacy of subsequent monetary policy choices. I further conjecture that forward guidance also increases dampened general equilibrium multipliers of expected monetary policy by increasing common knowledge among private sector agents themselves. Combined, such increases in common knowledge could more generally have important consequences for how public disclosures interact with the preponderance of macroeconomic puzzles that rest on powerful effects of policy.\footnote{See, for example, the “Forward Guidance Puzzle” (McKay et al., 2016; Werning, 2015; Angeletos and Lian, 2018), or “The Paradox of Toil” with decreases in labor taxes (Eggertsson, 2010; Mulligan, 2010).}
References


Appendix A: A Baseline Model

This Appendix details the proofs of the Lemmas and Corollaries in Sections 2 to 3.

Proof of Lemma 1: I proceed in two steps. I first solve the representative household’s problem, imposing market clearing, to derive a relationship between the real wage rate, output, and productivity in the economy. Specifically, I show that if \( \phi_0 \) is set such that \( \delta = \beta \mathbb{E}_t \left[ \frac{M_t}{M_{t+1}} \right] < 1 \), then the cash-in-advance constraint always binds \( M_t = P_t C_t, W_t/L_t^\eta = M_t^\delta/\delta \), and the real wage rate follows a simple relationship.\(^1\) I then use this relationship to derive an expression for firms’ optimal prices.

Step 1: The Lagrangian of the household’s problem is

\[
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \frac{1}{1+\eta} L_t^{1+\eta} - \mu_t (P_t C_t - M_t^d - T_t^h) \right] - \lambda_t \left[ P_t C_t + M_t^d - \int_0^1 \Pi_{it} di - W_t L_t - M_t^d + T_t^h \right],
\]

with associated sufficient first-order conditions

\[
\begin{align*}
C_t : & \quad \frac{1}{C_t} = P_t (\lambda_t + \mu_t) = 0, \quad \frac{1}{P_t C_t} = \lambda_t + \mu_t, \quad (A1) \\
L_t : & \quad -L_t^\eta + \lambda_t W_t = 0, \quad L_t^\eta = \lambda_t W_t, \quad (A2) \\
M_t^d : & \quad -\lambda_t + \beta \mathbb{E}_t [\lambda_{t+1} + \mu_{t+1}] = 0, \quad \lambda_t = \beta \mathbb{E}_t [\lambda_{t+1} + \mu_{t+1}] \quad (A3)
\end{align*}
\]

We check that the proposed solution \( M_t = P_t C_t \) and \( W_t/L_t^\eta = M_t^\delta/\delta \) with \( \delta = \beta \mathbb{E}_t \left[ \frac{M_t}{M_{t+1}} \right] < 1 \) satisfies the relevant first-order conditions. The combination of (A1) and (A2) shows that

\[
\mu_t = \frac{1}{P_t C_t} - \frac{L_t^\eta}{W_t} = (1 - \delta) \frac{1}{M_t^\delta} > 0,
\]

where I have used that \( \lambda = L_t^\eta/W_t = \delta/M_t \). This proves the first part of the first step.

To find \( \delta \), combine (A3) with \( W_t/L_t^\eta = M_t^\delta/\delta \) to arrive at

\[
\frac{\delta}{M_t^\delta} = \beta \mathbb{E}_t \left[ \frac{\delta}{M_{t+1}^\delta} + 1 - \frac{\delta}{M_{t+1}^\delta} \right] = \beta \mathbb{E}_t \left[ \frac{1}{M_{t+1}^\delta} \right].
\]

This in turn shows that \( \delta = \beta \mathbb{E}_t \left[ \frac{M_t}{M_{t+1}} \right] = \beta \exp \left( -\phi_0 + \frac{1}{2} \phi_\theta \mathbb{V} \left[ E_{\omega t} \theta_t \right] + \frac{1}{2} \phi_\xi \mathbb{V} \left[ E_{\omega t} \xi_t \right] \right) > 0 \), and thus that all three first-order conditions are satisfied at the candidate solution.

We now use \( W_t/L_t^\eta = M_t^\delta/\delta \) and the binding cash-in-advance constraint to find the labor market-clearing real wage. Equating labor supply \( L_t^\delta = \delta^{1/\delta} W_t^{1/\delta} (P_t Y_t)^{-\frac{\delta}{\eta}} \) with labor demand \( L_t^\delta = \int_0^1 \frac{Y_t}{A_t} di = \int_0^1 \left( \frac{P_t}{P_t^*} \right)^{\phi/\gamma} \frac{Y_t}{A_t} di = \frac{Y_t}{A_t} \), since all firms set the same price in equilibrium, shows that\(^2\)

\[
\frac{W_t}{A_t P_t} = \delta^{-1} \left( \frac{Y_t}{A_t} \right)^{1+\eta} \quad (A4)
\]

A log-linear approximation of (A4) then completes the first step and shows that

\[
w_t - p_t - a_t = (1 + \eta) (y_t - a_t) \quad (A5)
\]

\(^1\)The proof of the first step follows that of Lemma 1 in Hellwig (2005).

\(^2\)I here set \( C_t \) equal to \( Y_t \) (see the last section of this Appendix). This is because the real resource cost of inflation is of second-order, \( Y_t = C_t + O(2) \), and I subsequently linearize all resultant expressions.
Step 2: The demand for firm $i \in [0, 1]$ goods is

$$Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\rho} Y_t.$$  

The representative firm’s problem is therefore

$$\max_{P_{it}} E_{t0} \sum_{t=0}^{\infty} \beta^t \left[ (1 + T_i^\rho) P_{it} Y_{it} - W_t L_{it} - \frac{\psi}{2} \left( \frac{P_{it}}{P_{it-1}} - 1 \right) P_{it} \right] =$$

$$\max_{P_{it}} E_{t0} \sum_{t=0}^{\infty} \beta^t \left[ (1 + T_i^\rho) \left( \frac{P_{it}}{P_t} \right)^{1-\rho} - \left( \frac{W_t}{A_t P_t} \right) \left( \frac{P_{it}}{P_t} \right)^{-\rho} - \frac{\psi}{2} \left( \frac{P_{it}}{P_{it-1}} - 1 \right) \right].$$

where $1 + T_i^\rho$ has mean $\frac{\rho}{\rho - 1}$. The sufficient first-order condition to this problem is

$$E_{ft} \left[ (1 + T_i^\rho) (1 - \rho) \left( \frac{P_{it}}{P_t} \right)^{-\rho} \frac{1}{P_t} + \rho \left( \frac{W_t}{A_t P_t} \right) \left( \frac{P_{it}}{P_t} \right)^{-\rho} \frac{1}{P_{it}} \right] =$$

$$E_{ft} \left[ \frac{1}{M_t} + \rho \left( \frac{P_{it}}{P_{it-1}} - 1 \right) \left( \frac{P_{it}}{P_{it-1}} \right) + \frac{\beta\psi}{\rho} \left( \frac{P_{it+1}}{P_{it}} - 1 \right) \left( \frac{P_{it+1}}{P_t} \right) \right] = 0. \quad (A6)$$

Equation (A6) represents the New Keynesian Phillips Curve for the economy described in Section 2.

A straightforward log-linearization of (A6) then shows that

$$p_t = \gamma_1 p_{t-1} + \gamma_0 E_{ft} [w_t - p_t - a_t] + \gamma_1 E_{ft} [p_{t+1}] + \gamma_2 E_{ft} [\mu_t], \quad (A7)$$

where $\gamma_1 = \frac{1}{1 + \gamma_2}, \gamma_1 = \frac{\beta}{\gamma_2}, \gamma_2 = \frac{\rho}{\gamma_2 (1 + \beta)}, \gamma_0 = \frac{\rho W^{**}}{\gamma_2 (1 + \beta)}$, and $W^{**}$ denotes the steady state value of $\frac{W_t}{A_t}$.

We can now use (A5) and that $y_t = m_t - p_t$ to re-write (A7) as

$$p_t = \lambda_1 p_{t-1} + \lambda_0 E_{ft} [m_t - a_t] + \lambda_1 E_{ft} [p_{t+1}] + \lambda_2 E_{ft} [\mu_t], \quad (A8)$$

where $\lambda_2 = [W^{**}(1 + \eta)]^{-1} \lambda_0$ and

$$\lambda_0 = \frac{W^{**}(1 + \eta)}{\beta (1 + \beta) + W^{**}(1 + \eta)}, \quad \lambda_1 = \frac{\psi}{\beta} (1 + \beta),$$

$$\lambda_2 = \frac{\psi}{\beta} (1 + \beta). \quad (A9)$$

**Proof of Corollary 1:** We start with the conjecture that (A8) collapses to

$$p_t = \nu_1 p_{t-1} + \nu_0 E_{ft} [m_t - a_t] + \nu_1 E_{ft} [\mu_t]. \quad (A11)$$
Inserting this conjecture into (A8) and matching terms terms shows that the conjecture is true iff

\[ \nu_{-1} = \frac{\lambda_{-1}}{1 - \lambda_1 \nu_{-1}}, \quad \nu_0 = \frac{\lambda_0}{1 - \lambda_1 \nu_{-1}} + \frac{\lambda_1}{1 - \lambda_1 \nu_{-1}} \nu_0, \quad \nu_1 = \frac{\lambda_2}{1 - \lambda_1 \nu_{-1}} + \frac{\lambda_1}{1 - \lambda_1 \nu_{-1}} \nu_1. \]  

(A12)

Let us proceed with a closer look at this system of equations, and let us start with \( \nu_{-1} \). The fixed point equation for this coefficient equals from (A12)

\[ g_{(-1)}(\nu_{-1}) = -\lambda_1 \nu_{-1}^2 + \nu_{-1} - \lambda_{-1} = 0. \]

Now, since \( g_{(-1)} \) is globally concave, \( g_{(-1)}(0) = -\lambda_{-1} < 0 \), and \( g_{(-1)}(1) = 1 - \lambda_1 - \lambda_{-1} > 0 \), we conclude that there exist two positive solutions for \( \nu_{-1} \), one of which is stable \( \nu_{-1} \in (0, 1) \).

Consider now the remaining two fixed point equations in (A12)

\[ \begin{align*}
g_0(\nu_0) &= \frac{\lambda_0}{1 - \lambda_1 \nu_{-1}} + \frac{\lambda_1}{1 - \lambda_1 \nu_{-1}} \nu_0 = \nu_0, \\
g_1(\nu_1) &= \frac{\lambda_2}{1 - \lambda_1 \nu_{-1}} + \frac{\lambda_1}{1 - \lambda_1 \nu_{-1}} \nu_1 = \nu_1.
\end{align*} \]

Because \( g_0(0) = \frac{\lambda_0}{1 - \lambda_1 \nu_{-1}} > 0, \ 0 < g_0(1) < \frac{\lambda_0 + \lambda_1}{1 - \lambda_1 \nu_{-1}} = \frac{1 - \lambda_{-1}}{1 - \lambda_1 \nu_{-1}} < 1 \) and \( \frac{dg_0}{d\nu_0} > 0 \), it follows that there exists a unique \( \nu_0 \in (0, 1) \). A similar argument then shows that there exists a unique, positive solution for \( \nu_1 \). Last, it immediately follows from (A9) and (A12) that when \( \psi = 0 \),

\[ \nu_{-1} = 0, \quad \nu_0 = 1, \quad \nu_1 = \left[ W^{ss}(1 + \eta) \right]^{-1}. \]

Proof of Lemma 2: The steps are well-known (see, for instance, Gali 2008, or Nistico 2007 for the case with Rotemberg 1982 quadratic nominal cost). Define

\[ U_t = \log C_t - \frac{1}{1 + \eta} l_t^{1+\eta}. \]

A second-order approximation around the non-stochastic full information steady state then shows that

\[ U_t \approx U_t^{C^{ss}} \left[ c_t + \frac{U_t^{C}}{U_t^{L}} L^{ss} \left( l_t + \frac{1 + \eta}{2} l_t^2 \right) \right], \]

where all derivates are evaluated at their steady state values (ss). Now, employing the resource constraint in its log-linear form (see the next subsection in this Appendix) \( y_t = c_t \), the economy-wide production function \( y_t = a_t + l_t \), and using that at the first best allocation \( \frac{U_t}{U_t^{C}} = -A^{ss} \), we arrive at

\[ W_t = \frac{U_t}{U_t^{C^{ss}}} \approx y_t - \left[ y_t - a_t + \frac{1 + \eta}{2} (y_t - a_t)^2 \right] = \frac{1 + \eta}{2} (y_t - a_t)^2 + \text{t.i.p}, \]

where t.i.p denotes terms independent of policy. Since \( W = \frac{1}{1-\phi} E_{t-1} [ W_t ] \), this completes the proof.

[\Box]

Real Resource Cost of Inflation: The aggregate resource constraint is in levels

\[ Y_t = C_t + \frac{\bar{\pi}^2}{2} Y_t, \]  

(A13)

where \( \bar{\pi} \) denotes the (non-log-linearized) level of inflation.
Log-linearizing (A13) around the full information steady state immediately shows that

\[ y_t = c_t + \frac{3}{2} \psi [\tilde{\pi}_t^2 Y_t]_{ss} (\pi_t + y_t), \]

where \([\cdot]_{ss}\) denotes an expression evaluated at its steady state level. But since the steady state rate of inflation is zero \((\tilde{\pi}_t)_{ss} = 0\), it follows that, to a first order,

\[ y_t = c_t, \quad (A14) \]

precisely as I have assumed throughout the paper. Consequently, the real resource cost of inflation is immaterial for the linear economies considered in Sections 2 to 7 of the main text.

**Appendix B: Inefficient Disturbances**

I detail the proofs of the results in Section 4 in this Appendix.\(^3\)

**Equilibrium Prices and Money Supply:** I immediately consider the more general case in which \(\tau_p \in \mathbb{R}_+\). I first solve follow for the equilibrium price level and money supply. I then later (in the proof of Theorem 1) derive the optimal policy. To start with, I conjecture that

\[
m_t = m_{t-1} + \phi_t \mathbb{E}_{cbt} [\xi_t] \quad (A15)
\]

\[
= m_{t-1} + q_0 z_t + q_1 p_t \quad (A16)
\]

\[
p_t = \nu_{-1} p_{t-1} + \nu_0 \mathbb{E}_{ft} [m_t] + \nu_1 \mathbb{E}_{ft} [\mu_t] \quad (A17)
\]

\[
= \nu_{-1} p_{t-1} + \nu_0 m_{t-1} + \nu_1 m_{t-1} + \nu_0 \phi_t \mathbb{E}_{ft} [\mathbb{E}_{cbt} \xi_t] + \nu_1 \mathbb{E}_{ft} [\xi_t]
\]

\[
= \nu_{-1} p_{t-1} + \nu_0 m_{t-1} + \nu_1 m_{t-1} + k_0 x_t + k_1 \omega_t + k_2 p_t, \quad (A18)
\]

The noisy signal of the economy-wide price level \(\hat{p}_t\) is thus equivalent to the observation of

\[
\hat{p}_t = \frac{1}{k_0} \left[ \hat{p}_t - \nu_{-1} p_{t-1} - \nu_0 m_{t-1} - \nu_1 m_{t-1} - k_1 \omega_t - k_2 p_t \right] - \mu_{t-1}
\]

\[
= x_t + \frac{1}{k_0} \epsilon_{pt} - \mu_{t-1} = \xi_t + \epsilon_{xt} + \frac{1}{k_0} \epsilon_{pt}.
\]

I note that \(p_t\) by design is independent of the other signals in \(\Omega_{cbt}\) and \(\Omega_{ft}\) conditional on \(\xi_t\).

To verify the conjecture in (A16) and (A18), we need to derive expressions for firm and central bank expectations of the unobserved mark-up shock, in addition to firm expectations of the central bank’s private information about the mark-up shock. Due to the linear-normal assumption on the information structure, these are given by

\[
\mathbb{E}_{ft} [\xi_t] = w_x x_t + w_\omega \omega_t, \quad w_x = \frac{\tau_x (\tau_\omega + \tau_z)}{(\tau_\omega + \tau_z) (\tau_\xi + \tau_x) + \tau_\omega \tau_z}
\]

\[
\mathbb{E}_{ft} [\zeta_t] = v_x x_t + v_\omega \omega_t, \quad v_x = \frac{\tau_x \tau_\zeta}{(\tau_\omega + \tau_z) (\tau_\xi + \tau_x) + \tau_\omega \tau_z}
\]

\[
\mathbb{E}_{cbt} [\xi_t] = \beta_z z_t + \beta_p p_t, \quad \beta_z = \frac{\tau_z (\tau_x + \tau_p k_0)}{(\tau_x + \tau_p k_0) (\tau_\xi + \tau_z) + \tau_x \tau_p k_0^2}.
\]

\(^3\)I below dispense with all superscripts to ease notation when it does not cause confusion. Therefore, the variable \(x_t\), for example, refers to \(x_t^1\) in this appendix.
where \( w, v \) and \( \beta_p \) are implicitly defined and follow the standard expressions.

Inserting these expressions into (A15) and (A17) then demonstrates that

\[
\begin{align*}
\begin{alignat}{3}
\text{Proof of Lemma 3:} & \quad \text{The economy-wide output gap is} \\
&m_t &= m_{t-1} + \phi_\xi \left( \beta_z z_t + \beta_p p_t \right) \\
p_t &= \nu_{-1} m_{t-1} + \nu_0 m_{t-1} + \nu_1 \mu_{t-1} + \nu_0 \left( q_0 E_{ft} [z_t] + q_1 p_t \right) + \nu_1 E_{ft} [\xi_t] \\
&= \nu_{-1} m_{t-1} + \nu_0 m_{t-1} + \nu_1 \mu_{t-1} + \nu_0 \left( q_0 v_x + \frac{\nu_1}{\nu_0} w_x \right) x_t + \nu_0 \left( q_0 v_\omega + \frac{\nu_1}{\nu_0} w_\omega \right) \omega_t + \nu_0 q_1 p_t \tag{A22}
\end{alignat}
\end{align}
\]

which verifies our conjecture iff, there exists a solution to the system of equations

\[
\begin{alignat}{3}
q_0 &= \phi_\xi \beta_z, \quad q_1 = \phi_\xi \beta_p \\
k_0 &= \nu_0 \left( q_0 v_x + \frac{\nu_1}{\nu_0} w_x \right), \quad k_1 = \nu_0 \left( \frac{\nu_1}{\nu_0} w_\omega \right), \quad k_2 = \nu_0 q_1,
\end{alignat}
\] 

where \( q_h \in \mathbb{R} \) and \( k_j \in \mathbb{R} \), \( h = \{0, 1\} \) and \( j = \{0, 1, 2\} \).

Since all fixed point equations in (A23) and (A24) ultimately depend only on \( k_0 \), all that is needed to show is that the equation for \( k_0 \) has a solution. We can re-write this equation as

\[
Q(k_0) = (\tau_\xi + \tau_x + \tau_z) \tau_p k_0^2 - \nu_0 \left[ \phi_\xi \tau_x v_x + \frac{\nu_1}{\nu_0} \left( \tau_\xi + \tau_x + \tau_z \right) w_x \right] \tau_p k_0^2 + \tau_x (\tau_\xi + \tau_x) k_0 - \nu_0 \tau_x \left[ \phi_\xi \tau_x v_x + \frac{\nu_1}{\nu_0} \left( \tau_\xi + \tau_x \right) w_x \right] = 0. \tag{A25}
\]

Because \( \phi_\xi > 0 \), Descartes’ Rule of Signs then establishes that there exists either one or three positive solutions for \( k_0 \) to (A25), depending on parameter values.

\[ \square \]

**Proof of Lemma 3:** The economy-wide output gap is

\[
\begin{alignat}{3}
\begin{alignat}{3}
y_t &= m_t - p_t \\
&= \phi_\xi E_{cht} [\xi_t] - \nu_0 E_{ft} \left[ \phi_\xi E_{cht} \xi_t + \frac{\nu_1}{\nu_0} \xi_t \right] + t.l.p.
\end{alignat}
\end{align}
\]

where \( t.l.p \) denotes terms from last period irrelevant to current welfare. Thus, when \( \phi_\xi = 0 \)

\[
y_t = -\nu_1 E_{ft} [\xi_t] + t.l.p,
\]

and therefore

\[
(1 - \beta) W = \nu_1^2 \left( 1 - \frac{1}{\tau_\xi} \right) - \frac{1}{(\tau_\omega + \tau_z) (\tau_\xi + \tau_x + \tau_\omega \tau_z)},
\]

where I have used the expression for \( E_{ft} [\xi_t] \) from (A19). It follows that \( \frac{dW}{d_\omega} > 0 \).

\[ \square \]
Proof of Theorem 1: I consider the case in which $\tau_p \to 0$. The economy-wide output gap is

$$y_t = m_t - p_t = \phi_\xi \mathbb{E}_t^b [\xi_t] - \nu_0 \phi_\xi \mathbb{E}_t^f \mathbb{E}_t^b [\xi_t] - \nu_1 \mathbb{E}_t^f [\xi_t] + t.l.p$$

$$= [\phi_\xi \beta_z - \nu_0 \phi_\xi \beta_z (v_x + w) - \nu_1 (w_x + w_w)] \xi_t - [\nu_0 \phi_\xi \beta_z v_x + \nu_1 w_x] \epsilon_{xt}$$

$$+ [\phi_\xi \beta_z - \nu_0 \phi_\xi \beta_z w - \nu_1 w_w] \epsilon_{zt} - [\nu_0 \phi_\xi \beta_z v_w + \nu_1 w_w] \epsilon_{w}.$$

Thus, it follows from Lemma 2 that

$$(1 - \beta) W = \left[ \frac{\tau_x \tau_z + \tau_w (\tau_x + \tau_z)}{(\tau_w + \tau_x)(\tau_x + \tau_z)} - \frac{\tau_x (\tau_z + \tau_w)}{(\tau_w + \tau_x)(\tau_x + \tau_z)} + \frac{\tau_x (\tau_z + \tau_w)}{(\tau_w + \tau_x)(\tau_x + \tau_z)} \right] \frac{1}{\tau_x}$$

In turn, this shows after a few simple but tedious derivations that $\frac{\partial W}{\partial \tau_z} \geq 0$ iff:

$$\nu_0 (\nu_0 - 2) [\tau_x + \tau_z + \tau_w] \phi_\xi \beta_z + 2 \nu_1 \tau_x (\nu_0 - 1) [(\tau_x + \tau_z + \tau_w) \phi_\xi \beta_z] + \nu_1^2 \tau_z \geq 0.$$  \hfill (A26)

Equation (A26) is second-degree polynomial in $(\tau_x + \tau_z + \tau_w) \phi_\xi \beta_z$ with a unique positive solution, which follows from Descartes’ Rule of Signs. We can now use this solution to show that $\frac{\partial W}{\partial \tau_z} \geq 0$ iff.

$$\phi_\xi \leq \frac{\nu_1}{2 - \nu_0 \tau_x + \tau_z + \tau_w} \equiv \bar{\phi}_\xi.$$  \hfill □

Proof of Theorem 1: I once more consider the general case in which $\tau_p \in \mathbb{R}_+$. The proof has three steps: The first step uses the equilibrium price level (A18) and Lemma 2 to derive a convenient expression for $W$ as a function of $k_0$, $q_0$, and $\tau_x$. The second step then uses that expression to find the unique optimal values for these variables. Last, I use the expression for $q_0$ from (A23) to translate the optimal $q_0$ coefficients back into the level of $\phi_\xi$ that it entails.

Step 1: Equilibrium welfare.

The economy-wide output gap is

$$y_t = m_t - p_t = [q_0 + (1 - \nu_0) q_1 - k_1] \xi_t + [(1 - \nu_0) q_1 - k_0] \epsilon_{xt} + (1 - \nu_0) q_1 \frac{1}{k_0} \epsilon_{pt} + t.l.p$$

$$+ (q_0 - k_1) \epsilon_{zt} - k_1 \epsilon_{w} + t.l.p.$$
Thus, after a few, simple derivations

\[
(1 - \beta)W = [\nu_1 + (1 - \nu_0) q_0 \left( \tau_x + \alpha + \frac{\tau_x}{\tau_s} k_0 \right) \frac{1}{\tau_s} + (1 - \nu_0) q_0 \left( \frac{\alpha}{\tau_s} \right) - k_0] \frac{2}{\tau_s} + \left(1 - \nu_0 \right) q_0 \left( \frac{\alpha}{\tau_s} \right) - k_0 \right) \frac{1}{\tau_s}
\]

(A27)

where \( \alpha = \frac{\tau_x \tau_s k_0^2}{\tau_x + \tau_s k_0} \) and \( k_1 = \nu_0 r \omega \left( \frac{\alpha}{\tau_s} \right) \left( \frac{\tau_x + \tau_s}{\tau + \tau_s} + \frac{\tau_x}{\tau_s} \right)

Step 2: The unique optimal values of \( k_0, \tau_s, \) and \( q_0 \).

Equations (A23) and (A24) show that there exists a one-to-one relationship between \( \phi_s \) and \( q_0 = (1 - \nu_0) q_0 \left( \frac{\alpha}{\tau_s} \right) = (1 - \nu_0) \phi_s \beta_s \left( \frac{\alpha}{\tau_s} \right) \) for given \( \tau_s \) and \( k_0 \). Instead of optimally choosing \( \phi_s > 0 \), I therefore choose \( q_0 \) instead, implicitly defining the associated optimal \( q_0 \) and \( \phi_s \). This simplifies the derivations.

Consider the Lagrangian associated with the optimal policy problem:

\[
\mathcal{L} = W + \lambda \left[ k_0 - \nu_0 r \omega \left( \frac{q_0 \tau_x + \nu_0 (\tau_x + \tau) \left( \tau_x + \tau_s \right)}{(\tau_s + \tau_s) (\tau_s + \tau_s) + \tau_s \tau_s} \right) \right]
\]

Thus,

\[
\frac{\partial \mathcal{L}}{\partial \tau_s} = 0 : \left[ \nu_0 \left( \frac{\tau_x + \tau}{\tau_s + \tau_s} \right) + \lambda \left( \frac{2 k_1 \tau_x + \tau_s}{\tau_s + \tau_s} + \tau_s \right) \right] q_0 \left( \frac{\tau + \tau_s}{\tau_s + \tau_s} + \frac{\lambda \tau_s}{\tau_s + \tau_s} \right) = 0.
\]

(A28)

This equation is satisfied for \( \tau_s \rightarrow \infty \). Notice that \( q_0 = -\nu_0 \frac{\tau_s}{\tau_s + \tau_s + \tau_s} \) is not an option in (A28) since that would imply that \( \phi_s < 0 \), and neither is \( k_1 = -\lambda \frac{\tau_s}{\tau_s + \tau_s + \tau_s} \) since that would imply that \( k_1 < 0 \).

We can now return to (A27) with \( \tau_s \rightarrow \infty \). Minimizing (A27) with respect to \( q_0 \) when \( k_0 = \nu_1 \frac{\tau_x}{\tau_x + \tau_s} \geq 0 \) and \( k_1 = 0 \) then yields

\[
\frac{\partial W}{\partial q_0} = 0 : \left[ -\nu_1 + \frac{\tau_x}{\tau_s + \tau_s} \left( \frac{\tau + \alpha}{\alpha} \right) + \frac{\tau_s}{\tau_s} k_0 \right] \frac{\tau_x + \alpha}{\alpha} + \left( q_0 - k_0 \right) \frac{\tau_s}{\tau_s} + q_0 \frac{1}{\tau_s} k_0
\]

(A29)

which is affine in \( q_0 \). Thus, after a few simple, derivations

\[
q_0^* = \nu_1 \frac{\alpha^*}{\tau_s + \alpha^* + \tau_s} > 0,
\]

where \( \alpha^* = \frac{\tau_s \tau_x k_0^2}{\tau_x + \tau_s k_0^2} \), \( \tau_s \rightarrow \infty \) and \( k_0^* = \nu_1 \frac{\tau_x}{\tau_x + \tau_s + \tau_s} \).

Step 3: The optimal level of instrument policy, \( \phi_s^* \).

In turn, the optimal value of \( q_0 \) can uniquely be implemented with

\[
\phi_s^* = \frac{\nu_1}{1 - \nu_0} > 0,
\]

(A30)

where I have used the definition of \( q_0 \) in addition to (A23).  

\[ \square \]

\( \text{The social welfare loss function in (A27) is strictly pseudo-convex. When combined with the expression for } k_0 \text{ in (A24), we thus have that an interior solution } \frac{\partial W}{\partial q_0} = 0, \text{ where } x = \{q_0, \tau_s, k_0\}, \text{ is the unique global minimum.} \]
Alternative Monetary Policy Rule: I follow a condensed version of the steps used to derive (i) the equilibrium of the model with mark-up shocks, and (ii) the proof of Theorem 1. Similar steps to those used to derive the equilibrium conditions with the monetary policy rule from Section 2 show that

\[ \begin{align*}
    p_t &= \nu_1 p_{t-1} + \nu_0 m_{t-1} + \nu_1 \mu_{t-1} + k_0 x_t + k_1 \omega_t + k_2 \beta_t^x, \\
    m_t &= m_{t-1} + q_0 z_t + q_1 \beta_t^x + q_2 \omega_t, 
\end{align*} \tag{A31} \tag{A32} \]

where the key coefficients \( k_0 \) and \( q_0 \) now solve

\[ \begin{align*}
    k_0 &= \nu_0 \left( q_0 v_x + \frac{\nu_1}{\nu_0} w_x \right), \\
    q_0 &= \phi_p k_0 \beta_t^x, \tag{A33} \]

where \( \beta_t^x \) denotes the signal extraction coefficient on \( z_t \) in the central bank’s expectation of \( x_t \).

We can now proceed to derive the optimal policy. The economy-wide output gap is

\[ y_t = m_t - p_t = \phi_p \mathbb{E}_{c,b,t} [p_t] - p_t = \mathbb{E}_{c,b,t} [p_t] - p_t + (\phi_p - 1) \mathbb{E}_{c,b,t} [p_t]. \]

Thus,

\[ \mathcal{W} = \frac{1}{1 - \beta} \left\{ MSE_{cb} [p_t] + (\phi_p - 1)^2 \mathbb{V} [\mathbb{E}_{c,b,t} [p_t]] \right\} = \frac{1}{1 - \beta} \left\{ MSE_{cb} [k_0 x_t] + (\phi_p - 1)^2 \mathbb{V} [\mathbb{E}_{c,b,t} [p_t]] \right\}, \tag{A34} \]

since only \( x_t \) is unknown to the central bank in (A31). First-order conditions are then

\[ \begin{align*}
    \phi_p : \quad & \frac{\partial MSE_{cb} [k_0 x_t]}{\partial \phi_p} + 2(\phi_p - 1) \mathbb{V} [\mathbb{E}_{c,b,t} [p_t]] + (\phi_p - 1)^2 \frac{\partial \mathbb{V} [\mathbb{E}_{c,b,t} [p_t]]}{\partial \phi_p} = 0, \tag{A35} \\
    \tau_\omega : \quad & \frac{\partial MSE_{cb} [k_0 x_t]}{\partial \tau_\omega} + (\phi_p - 1)^2 \frac{\partial \mathbb{V} [\mathbb{E}_{c,b,t} [p_t]]}{\partial \tau_\omega} = 0. \tag{A36} 
\end{align*} \]

Now, since \( k_0 \) in (A33) is independent of \( \phi_p \) and \( \tau_\omega \) when \( \tau_\omega \rightarrow \infty \) (and \( v_x \rightarrow 0 \)), it follows that \( \frac{\partial MSE_{cb} [k_0 x_t]}{\partial \phi_p} \rightarrow 0 \) and \( \frac{\partial MSE_{cb} [k_0 x_t]}{\partial \tau_\omega} \rightarrow 0 \) when \( \tau_\omega \rightarrow \infty \). This is because only the informativeness of \( \beta_t^x \) in the central bank’s information set, controlled by \( k_0 \), is modified by different policy actions. Thus, (A35) and (A36) are satisfied for \( \phi_p = 1 \) and \( \tau_\omega \rightarrow \infty. \tag{A37} \tag{A38} \tag{A39} \tag{A40} \)

\section*{Appendix C: Efficient Disturbances}

This Appendix details the proofs of the Propositions and Theorems in Sections 5.

\textbf{Proof of Proposition 2:} I use the same steps as those used for mark-up shock case. To solve for the symmetric, linear rational expectations equilibria, I conjecture that \( m_t \) and \( p_t \) equal

\[ \begin{align*}
    m_t &= m_{t-1} + \phi_0 \mathbb{E}_{c,b,t} [\theta_t] \tag{A37} \\
    &= m_{t-1} + q_0 z_t + q_1 \beta_t^x, \tag{A38} \\
    p_t &= \nu_1 p_{t-1} + \nu_0 \mathbb{E}_{f,t} [m_t - a_t] \tag{A39} \\
    &= \nu_1 p_{t-1} + \nu_0 (m_{t-1} - a_{t-1}) + \nu_0 \mathbb{E}_{f,t} [\phi_0 \mathbb{E}_{c,b,t} (\theta_t) - \theta_t] \\
    &= \nu_1 p_{t-1} + \nu_0 (m_{t-1} - a_{t-1}) + k_0 x_t + k_1 \omega_t + k_2 \beta_t^x. \tag{A40} 
\end{align*} \]

\footnote{Uniqueness once more follows from the the strict pseudo-convexity of the social welfare loss function.}
The noisy signal of the economy-wide price level is thus equivalent to the observation of
\[
P_t = \frac{1}{k_0} \left[ p_t - \nu_{-1}p_{t-1} - \nu_0 (m_{t-1} - a_{t-1}) - k_1 \omega_t - k_2 p_t \right] - a_{t-1}
= x_t + \frac{1}{k_0} \epsilon_{pt} - a_{t-1} = \theta_t + \epsilon_{xt} + \frac{1}{k_0} \epsilon_{pt}.
\]

We need to check the conjectures in (A38) and (A40). To do so, we first compute expressions for firm and central bank expectations of the productivity shock as well as firm expectations of the central bank’s private information. Because of the linear-normal information structure:
\[
\begin{align*}
\mathbb{E}_{ft} [\theta_t] &= w_x x_t + w_\omega \omega_t, \quad w_x = \frac{\tau_x (\tau_\omega + \tau_z)}{(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z} \\
\mathbb{E}_{ft} [z_t] &= v_x x_t + v_\omega \omega_t, \quad v_x = \frac{\tau_\omega + \tau_z}{(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z} \\
\mathbb{E}_{cht} [\theta_t] &= \beta_z z_t + \beta_p p_t, \quad \beta_z = \frac{\tau_z (\tau_\theta + \tau_p k_0^2)}{\tau_x + \tau_p k_0^2 (\tau_\theta + \tau_z) + \tau_x \tau_p k_0^2},
\end{align*}
\]
where \(w_x, v_\omega, \) and \(\beta_p\) are implicitly (re-)defined in accordance with the standard expressions.

Inserting these expressions into (A37) and (A39) shows that
\[
m_t = m_{t-1} + \phi_0 \left( \beta_z z_t + \beta_p p_t \right)
\]
\[
p_t = \nu_{-1}p_{t-1} + \nu_0 (m_{t-1} - a_{t-1}) + \nu_0 \left( q_0 \mathbb{E}_{ft} [z_t] + q_1 p_t - \mathbb{E}_{ft} [z_t] \right)
\]
\[
= \nu_{-1}p_{t-1} + \nu_0 (m_{t-1} - a_{t-1}) + \nu_0 (q_0 v_x - w_x) x_t + \nu_0 (q_0 v_\omega - w_\omega) \omega_t + \nu_0 q_1 x_t,
\]
which verifies the conjecture iff there exists a solution to the system of equations
\[
q_0 = \phi_0 \beta_z, \quad q_1 = \phi_0 \beta_p
\]
\[
k_0 = \nu_0 (q_0 v_\omega - w_\omega), \quad k_1 = \nu_0 (q_0 v_x - w_x), \quad k_2 = \nu_0 q_1,
\]
in which \(q_h \in \mathbb{R} \) and \(k_j \in \mathbb{R}, \ h = \{0, 1\} \) and \(j = \{0, 1, 2\} \).

Because all fixed point equations in (A44) and (A45) only depend on \(k_0\), all that however needs to be shown is that the equation for \(k_0\) has a solution. We can re-write the equation for \(k_0\) as
\[
Q (k_0) = (\tau_\theta + \tau_x + \tau_z) \tau_p k_0^2 + \nu_0 \left[ (\tau_\theta + \tau_x + \tau_z) w_x - \phi_0 \tau_z v_x \right] \tau_p k_0^2
\]
\[
+ \tau_z (\tau_\theta + \tau_x) k_0 + \nu_0 \tau_x \left[ (\tau_\theta + \tau_z) w_x - \phi_0 \tau_z v_x \right] = 0.
\]
Since \(\phi_0 \in [0, 1]\) by assumption and \(v_x \leq w_x\), Descartes’ Rule of Signs then establishes that there exists either one or three negative solutions for \(k_0\) to (A46), depending on parameter values.

Proof of Proposition 3: The proof has two steps:

(i) The first is to show how disclosure decreases the informativeness of the noisy signal of the price level \(k_0^2 \tau_p\), and hence the central bank’s own information about productivity, when \(\phi_0 \leq \hat{\phi}_0\).

The equation determining \(k_0\) is
\[
k_0 = \nu_0 \tau_x \frac{q_0 \tau_z - (\tau_\omega + \tau_z)}{(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z} < 0, \quad q_0 = \phi_0 \frac{\tau_z (\tau_x + \tau_0 k_0^2)}{\tau_x + \tau_0 k_0^2 (\tau_\theta + \tau_x) + \tau_x \tau_0 k_0^2}.
\]
The derivative of the right-hand side (RHS) of \( k_0 \) in (A47) with respect to \( \tau_\omega \) equals

\[
\frac{\partial \text{RHS}}{\partial \tau_\omega} = \nu_0 \frac{\tau_x \tau_z}{[(\tau_\omega + \tau_z)(\tau_\theta + \tau_x) + \tau_\omega \tau_z]^2} \left[ \tau_z - q_0 (\tau_\theta + \tau_x + \tau_\omega) \right].
\]

Thus, \( \frac{\partial \text{RHS}}{\partial \tau_\omega} \geq 0 \) and hence \( \frac{dk_0}{d\tau_\omega} \geq 0 \) iff \( q_0 \leq \frac{\tau_z}{\tau_\theta + \tau_x + \tau_\omega} \). From (A47), the latter is equivalent to

\[
\phi_0 \leq \frac{\tau_\theta + \alpha + \tau_z}{\tau_\theta + \tau_x + \tau_\omega} = \hat{\phi}_0,
\]

where \( \alpha = \frac{\tau_x \tau_z^2}{\tau_x + \tau_z k_0} \) denotes the precision of \( p_x \). Conversely, \( \frac{dk_0}{d\tau_\omega} < 0 \) when \( \phi_0 > \hat{\phi}_0 \).

As a result, we arrive at \( \frac{dk_0}{\tau_\omega} \leq 0 \) if and only if \( \phi_0 \leq \hat{\phi}_0 \).

(ii) The second step is to demonstrate how increases in central bank uncertainty about productivity shocks, all else equal, increase welfare losses.

It follows from the definition of the output gap, the cash-in-advance constraint, and (3.4) and (3.5) that

\[
W = \frac{1}{1 - \beta} \left\{ \mathbb{V}[X_t - \mathbb{E}_t X_t] + (1 - \nu_0)^2 \mathbb{V}[\mathbb{E}_t X_t] \right\}, \quad X_t = \phi_0 \mathbb{E}_{cbt}[\theta_t] - \theta_t.
\]

Now, since both the variance of the forecast error and the variance of the expectation of \( X_t \) increase in the variance of \( X_t \),

\[
\mathbb{V}[X_t] = \mathbb{V}[\mathbb{E}_{cbt}\theta_t - \theta_t] + (\phi_0 - 1)^2 \mathbb{V}[\mathbb{E}_{cb}\theta_t],
\]

which itself increases in the central bank’s variance of \( \theta_t \), we immediately arrive at the result.\(^6\) Increases in central bank mean-squared uncertainty, all else equal, increase welfare losses.

\[\square\]

**Proof of Theorem 2:** I use the same three step procedure as in the proof of Theorem 1.

**Step 1:** Equilibrium welfare.

The economy-wide output gap is

\[
y_t - a_t = m_{t} - m_t - a_t = [q_0 + (1 - \nu_0) q_1 - k_0 - k_1 - 1] \theta_t + [(1 - \nu_0) q_1 - k_0] \epsilon_x t + (1 - \nu_0) q_1 \frac{1}{k_0} \epsilon_p t + t.t.p \]

\[+ (q_0 - k_1) \epsilon_x t + k_1 \epsilon_w t + t.t.p.\]

Thus, after a few, simple derivations

\[
(1 - \beta)W = \left[ - (1 - \nu_0) + (1 - \nu_0) q_0 \frac{\tau_x + \alpha}{\tau_x} + \frac{\tau_\theta}{\tau_x} k_0 \right]^2 \frac{1}{\tau_\theta} + \left[ (1 - \nu_0) q_0 \left( \frac{\alpha}{\tau_z} \right) - k_0 \right]^2 \frac{1}{\tau_z} (A48)
\]

\[+ (1 - \nu_0)^2 q_0^2 \left( \frac{\alpha}{\tau_z} \right)^2 \frac{1}{\tau_p k_0^2} + \left[ \nu_0 + (1 - \nu_0) q_0 + \frac{\tau_\theta + \tau_x}{\tau_x} k_0 \right]^2 \frac{1}{\tau_x} + k_1^2 \frac{1}{\tau_\omega},\]

where \( \alpha = \frac{\tau_x \tau_z^2}{\tau_x + \tau_z k_0} > 0 \) and \( k_1 = \nu_0 \tau_\omega \frac{q_0 (\tau_\theta + \tau_x + \tau_\omega) - \tau_z}{(\tau_\theta + \tau_x + \tau_\omega - \tau_z) \tau_\omega} \).

**Step 2:** The unique optimal values of \( k_0, \tau_\omega \) and \( q_0 \).

Corollary 2 demonstrates that there is a one-to-one relationship between \( q_0 \) and \( \phi_0 \) for given \( \tau_\omega \) and

\[\text{This follows from } \mathbb{V}[X_t] = (\phi_0 - 1)^2 \frac{1}{\tau_\theta} + [1 - (\phi_0 - 1)^2] \frac{1}{\tau_\theta + \tau_x + \alpha}, \text{ where } 1 - (\phi_0 - 1)^2 > 0 \text{ and } \frac{1}{\tau_\theta + \tau_x + \alpha} \text{ is the central bank’s mean-squared uncertainty about } \theta_t.\]
that an interior solution
where

where the key coeﬃcients

Similar steps to those used to derive Proposition

In turn, the optimal value of

Hence,

which is linear in

Thus,

This equation is satisfied for

We can now return to (A48). Minimizing (A48) with respect to

when

and

which is linear in

After a few simple derivations, (A49) then shows that

Hence, \( \tau_0^* \to \infty \), \( k_0^* = -\nu_0 \frac{\tau_x}{\tau_0 + \tau_x + \tau_z} \), and \( q_0^* \) is given by (A50).

Step 3: The optimal level of instrument policy, \( \phi_0^* \). In turn, the optimal value of \( q_0 \) can uniquely be implemented by,

where I have once more used Corollary 2.

Alternative Monetary Policy Rule: I follow an analogous approach to that used in Appendix B. Similar steps to those used to derive Proposition 2 then show that

where the key coefficients \( k_0 \) and \( q_0 \) now solve

where \( \beta_z^2 \) again denotes the signal extraction coefficient on \( z_t \) in the central bank’s expectation of \( x_t \).

\( ^7 \)The social welfare loss function in (A48) is strictly pseudo-convex. When combined with Corollary 2, we thus have that an interior solution \( \frac{dL}{dx} = 0 \), where \( x = \{ \tilde{q}_0, \tau_0, k_0 \} \), is the unique global minimum.
We can now continue to derive the optimal policy. The economy-wide output gap equals
\[
y_t = m_t - p_t - \theta_t = \phi_0 E_{cbt} [\theta_t] - \theta_t + \phi_p E_{cbt} [p_t] - p_t + t.l.p.
\]
where \( \theta_t' = [\theta_t \ p_t] \) and \( \phi = [\phi_0 \ \phi_p] \). Thus,
\[
y_t = \theta_t' (E_{cbt} [\theta_t] - \theta_t) + (\phi - \theta_t') E_{cbt} [\theta_t].
\]
It follows that
\[
W = \frac{1}{1 - \beta} \left\{ 1'MSE_{cb} [\theta_t] 1 + (\phi - 1') V [E_{cbt} [\theta_t]] (\phi - 1')' \right\}.
\]
Hence, the sufficient first order conditions are:\(^8\)
\[
\phi_0 : \quad \frac{\partial 1'MSE_{cb} [\theta_t] 1}{\partial \phi_0} + \frac{\partial (\phi - 1') V [E_{cbt} [\theta_t]] (\phi - 1')'}{\partial \phi_0} = 0 \quad \text{(A56)}
\]
\[
\phi_p : \quad \frac{\partial 1'MSE_{cb} [\theta_t] 1}{\partial \phi_p} + \frac{\partial (\phi - 1') V [E_{cbt} [\theta_t]] (\phi - 1')'}{\partial \phi_p} = 0 \quad \text{(A57)}
\]
\[
\tau_\omega : \quad \frac{\partial 1'MSE_{cb} [\theta_t] 1}{\partial \tau_\omega} + \frac{\partial (\phi - 1') V [E_{cbt} [\theta_t]] (\phi - 1')'}{\partial \tau_\omega} = 0. \quad \text{(A58)}
\]
Now, notice that since \( k_0 \) in (A54) is independent of \( \phi_p \) and \( \tau_\omega \) when \( \tau_\omega \to \infty \) (and \( v_x \to 0 \)), it follows that \( \frac{\partial 1'MSE_{cb} [\theta_t] 1}{\partial \phi_p} \to 0 \), \( \frac{\partial 1'MSE_{cb} [\theta_t] 1}{\partial \tau_\omega} \to 0 \), and \( \frac{\partial 1'MSE_{cb} [\theta_t] 1}{\partial \tau_\omega} \to 0 \) when \( \tau_\omega \to \infty \). This is because only the informativeness of \( p_t \) in the central bank’s information set, controlled by \( k_0 \), is modified by different policy actions. We therefore have that (A56) to (A58) are satisfied for \( \phi_0 = \phi_p = 1 \) when \( \tau_\omega \to \infty \).

\(^8\)Sufficiency once more follows from the the strict pseudo-convexity of the social welfare loss function in (A55).
Appendix D: A Quantitative Extension

This Appendix describes the derivations and solutions of the equilibrium conditions in Section 6. I also provide detail on the calibration as well as the optimal policy in the central bank full information limit.

Linearized Equilibrium Conditions: The are three main log-linear equilibrium conditions: the Euler equation in (6.5), the Taylor Rule in (6.7), and the New Keynesian Phillip’s Curve in (6.6). First, the maximization of (6.1) subject to (6.2) leads to

\[ C_t = \beta(1 + i_t)E_{ht} \left[ C_{t+1} \frac{P_{t+1}}{p_t} \right]. \]

A simple log-linearization around the full information steady-state then immediately yields the Euler equation in (6.5), after imposing market clearing,

\[ y_t = E_{ht} [y_{t+1}] - (i_t - E_{ht} [\pi_{t+1}]). \]

Second, direct log-linearization of (6.3) provides us with the Taylor Rule in (6.7)

\[ i_t = \phi E_{cht} [y_t - a_t] + \epsilon_{mt}. \]

Last, to arrive at the New Keynesian Phillip’s Curve in (6.6), start with (A6)

\[ \mathbb{E}_{ft} \left[ -\frac{1}{M_t} + \left( \frac{W_t}{A_t P_t} \right) - \psi \left( \frac{P_t}{P_{t-1}} - 1 \right) \left( \frac{P_t}{P_{t-1}} \right) + \beta \psi \frac{P_{t+1}}{P_t} - 1 \right] \left( \frac{P_{t+1}}{P_t} \right) = 0. \]

Log-linearizing this equation yields

\[ \pi_t = \beta \mathbb{E}_{ft} [\pi_{t+1}] + \phi W^{ss} \mathbb{E}_{ft} [w_t - p_t - a_t] - \frac{1}{M^{ss}} \mathbb{E}_{ft} [\mu_t], \]

where \( W^{ss} \) and \( M^{ss} \) denote the steady state values of \( \frac{W_t}{P_t A_t} \) and \( M_t \), respectively. Using that \( w_t - p_t - a_t = (1 + \eta)(y_t - a_t) \) from the start of Appendix A, redefining the mark-up shock, and normalizing the steady state price level to equal the steady state wage rate then results in (6.6),

\[ \pi_t = \beta \mathbb{E}_{ft} [\pi_{t+1}] + \lambda \mathbb{E}_{ft} [y_t - a_t] + \mathbb{E}_{ft} [\mu_t]. \]

Solution of the Extended Business Cycle Model: I extend the solution method proposed in Nimark (2017) to the two-sided learning case, to solve the three-equation model described by (6.5), (6.6), (6.7), and the information sets in (6.8) and (6.10) under the baseline calibration in which \( \tau^x, \tau^s \rightarrow \infty \). There are two steps to the solution procedure: First, I start by conjecturing a solution to the model, and then use this conjecture to derive an expression for the endogenous triplet \( q_t = [\pi_t y_t i_t]^t \) as a function of the state, taking as given the equation of motion for the state. I then solve agents’ signal extraction problem to find the equation of motion for the state, taking as given the function mapping the endogenous triplet into the state. Iterating across these two steps to find the fixed point yields an arbitrarily precise approximation to the rational expectations solution.

(1) Conjecture and Endogenous Variables as a Function of the State: I conjecture (and later verify) that
\( q_t \) can be written as a linear function of the expectational state vector \( X_t \) and the vector of shocks \( u_t \),\(^9\)
\[
q_t = \alpha_0 X_t + \alpha_1 u_t,
\]  
(A63)
and that the expectational state vector itself follows a \( VAR(1) \)
\[
X_t = MX_{t-1} + Nu_t.
\]  
(A64)
Solving the model implies finding expressions for the matrices \( \alpha_0, \alpha_1, M, \) and \( N \).
Using (A63) and that \( \Omega_{ft} = \Omega_{ht} \), we can stack (6.5), (6.6), (6.7) to arrive at
\[
A_0 q_t = A_1 \mathbb{E}_{ft} [q_{t+1}] + A_2^j X_t + A_u u_t, \quad j = \{a, \mu\}
\]  
(A65)
where
\[
A_0 = \begin{bmatrix} 1 & 0 & 1 \\ -\lambda & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_u = \begin{bmatrix} 0 \\ 0 \\ \sqrt{\tau_m} s_l' \end{bmatrix},
\]
in which \( s_l \) denotes the \( 1 \times 6 \) selector vector with an entry equal to one in the \( l \)th column and \( A_2^j \) is either
\[
A_x^a = \begin{bmatrix} 0 \\ -\lambda e_2' \\ \phi (a_0^{-1} - e_1') H_{cb} \end{bmatrix} \quad \text{or} \quad A_x^\mu = \begin{bmatrix} 0 \\ e_2' \\ \phi a_0^{-1} H_{cb} \end{bmatrix},
\]
depending on which of the structural shocks \( j = \{a, \mu\} \) drive the economy. The vector \( e_1' \) is the \( 1 \times 2k+1 \) selector vector with an entry equal to one in the \( l \)th column and the matrix \( H_{cb} \) is defined so that
\[
\mathbb{E}_{cbt} [X_t] = H_{cb} X_t.
\]
That is, \( H_{cb} \) selects the central bank’s expectations in \( X_t \) and moves the hierarchy of expectations one step up. Equation (A65) implies that
\[
q_t = F_1 \mathbb{E}_{ft} [q_{t+1}] + F_2^j X_t + F_u u_t,
\]  
(A66)
where
\[
F_1 = A_0^{-1} A_1, \quad F_2 = A_0^{-1} A_2^j, \quad F_u = A_0^{-1} A_u.
\]
Inserting the conjecture in (A63) and (A64) into (A66) then shows that
\[
q_t = F_1 \alpha_0 ME_{ft} [X_t] + F_2^j X_t + F_u u_t = (F_1 \alpha_0 M H_f + F_2^j) X_t + F_u u_t,
\]  
(A67)
where \( H_f \) is defined analogously to \( H_{cb} \) so that
\[
\mathbb{E}_{ft} [X_t] = H_f X_t.
\]
\(^9\)To ease notation, I refer to \( X_t^{(0:k)} \) simply as \( X_t \) when it does not cause confusion.
Equating coefficients in \( (A67) \) with those from the conjecture now shows that
\[
\alpha_0 = F_0 \alpha_0 MH_f + F_0^F, \quad \alpha_1 = F_u. \tag{A68}
\]

The solution to \( (A68) \) provides, for given \( M \), the coefficients that determine the triplet \( q_t = [\pi_t y_t i_t]' \) as a function of the state. This completes the first step of the solution procedure.

(2) State Evolution as a Function of the Endogenous Variables: We still need to determine the equation of motion for the state \( X_t \). To do so, I proceed in two steps. First, I solve the private sector’s and the central bank’s respective signal extraction problems under the baseline calibration, taking as given \( (A63) \). I then stack these expressions and match to the conjecture in \( (A64) \).

I start with the private sector. Its measurement equation equals
\[
\begin{bmatrix}
Z_{ft} = \begin{bmatrix}
x_t \\
\omega_t \\
\pi_t \\
i_t
\end{bmatrix} = \begin{bmatrix}
e_1' \\
e_1' \\
a_0' \\
a_0'
\end{bmatrix} X_t + \begin{bmatrix}
\sqrt{\tau_x} s_2' + \sqrt{\tau_\omega} s_4' \\
\alpha_1^2 + \sqrt{\tau_p} s_5'
\end{bmatrix} u_t = L X_t + Qu_t. \tag{A69}
\end{bmatrix}
\]

By the properties of linear projection, the private sector’s expectation of \( X_t \) can be written as
\[
\mathbb{E}_{ft} [X_t] = M \mathbb{E}_{ft-1} [X_{t-1}] + K (Z_{ft} - \mathbb{E}_{ft-1} [Z_{ft}]), \tag{A70}
\]
where the matrix of Kalman Gains is determined by
\[
K = (PL' + NQ') (LPL' + QQ' + LNQ' + QN'L')^{-1},
\]
and the one-step ahead mean squared error \( P \) solves the Ricatti equation
\[
P = M \left[ P - K (PL' + NQ') \right] M' + NN'.
\]

We can then rewrite \( (A70) \) as, using \( (A69) \),
\[
\mathbb{E}_{ft} [X_t] = (I - KL) M \mathbb{E}_{ft-1} [X_{t-1}] + KLM X_{t-1} + K (LN + Q) u_t. \tag{A71}
\]

Moving on analogously to the central bank, its measurement equation is
\[
\begin{bmatrix}
Z_{cbt} = \begin{bmatrix}
z_t \\
\omega_t \\
\pi_t \\
i_t
\end{bmatrix} = \begin{bmatrix}
e_1' \\
e_1' \\
a_0' \\
a_0'
\end{bmatrix} X_t + \begin{bmatrix}
\sqrt{\tau_z} s_3' \\
\alpha_1^2 + \sqrt{\tau_p} s_5'
\end{bmatrix} u_t = L_{cb} X_t + Q_{cb} u_t. \tag{A72}
\end{bmatrix}
\]

such that the central bank’s expectation equals
\[
\mathbb{E}_{cbt} [X_t] = M \mathbb{E}_{cbt-1} [X_{t-1}] + K_{cb} (Z_{cbt} - \mathbb{E}_{cbt-1} [Z_{cbt}]), \tag{A73}
\]

where
\[
K_{cb} = (P_{cb} L_{cb}' + NQ_{cb}') (L_{cb} P_{cb} L_{cb}' + Q_{cb} Q_{cb}' + L_{cb} N Q_{cb}' + Q N L_{cb}')^{-1},
\]

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and \( P_{cb} \) solves
\[
P_{cb} = M \left[ P_{cb} - K_{cb} (P_{cb} L_{cb} + N Q_{cb})' \right] M' + NN'.
\]
Combining (A73) with (A72) then shows that
\[
\mathbb{E}_{cbt} [X_t] = (I - K_{cb} L_{cb}) M \mathbb{E}_{cbt-1} [X_{t-1}] + K_{cb} L_{cb} M X_{t-1} + K_{cb} (L_{cb} N + Q_{cb}) u_t.
\] (A74)

Equations (A71) and (A74) describe the evolution of private sector and central bank expectations, taking as given the mapping between the endogenous variables and the state in (A63).

We are now ready to stack these expressions and match them to the conjecture in (A64). To do so, first notice that
\[
X_t^{(0,k+1)} = \begin{bmatrix} X_t^{(0)} \\ \bar{H} \begin{bmatrix} X_t^{(0)} \\ \mathbb{E}_{ct} X_t^{(0,k)} \end{bmatrix} \end{bmatrix},
\]
where \( \bar{H} \) reorders the elements in \( \begin{bmatrix} \mathbb{E}_{ct} X_t^{(0,k)} \\ \mathbb{E}_{ct} X_t^{(0,k)} \end{bmatrix} \). The last steps to arrive at the conjectured form (A64) are to stack (A71) and (A74) and append the underlying fundamental. Stacking provides us with
\[
\bar{H} \begin{bmatrix} \mathbb{E}_{ct} X_t^{(0,k)} \\ \mathbb{E}_{ct} X_t^{(0,k)} \end{bmatrix} = \bar{H} \begin{bmatrix} (I - KL) M \\ (I - K_{cb} L_{cb}) M \end{bmatrix} \begin{bmatrix} \mathbb{E}_{ct} X_t^{(0,k)} \\ \mathbb{E}_{ct} X_t^{(0,k)} \end{bmatrix} + \bar{H} \begin{bmatrix} KLM \\ K_{cb} L_{cb} M \end{bmatrix} X_{t-1}^{(0,k)}
\]
\[
+ \bar{H} \begin{bmatrix} K (LN + Q) \\ K_{cb} (L_{cb} N + Q_{cb}) \end{bmatrix} u_t = \bar{M}_0 \begin{bmatrix} \mathbb{E}_{ct} X_t^{(0,k)} \\ \mathbb{E}_{ct} X_t^{(0,k)} \end{bmatrix} + \bar{M}_1 X_{t-1}^{(0,k)} + \bar{N} u_t,
\]
where \( \bar{M}_0, \bar{M}_1, \) and \( \bar{N} \) are implicitly defined. Appending the AR(1) process for the fundamental \( X_t^{(0)} = \{a_t, \mu_t\} \) verifies the conjectured VAR(1) form
\[
X_t^{(0,k+1)} = \begin{bmatrix} X_t^{(0)} \\ \bar{H} \begin{bmatrix} X_t^{(0)} \\ \mathbb{E}_{ct} X_t^{(0,k)} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \rho_j & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_t^{(0)} \\ X_{t-1}^{(1,k+1)} \end{bmatrix}
\]
\[
+ \begin{bmatrix} 0 & 0 \\ 0 & \bar{M}_0 \end{bmatrix} \begin{bmatrix} X_t^{(0)} \\ X_{t-1}^{(1,k+1)} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{M}_1 \end{bmatrix} \begin{bmatrix} X_t^{(0)} \\ X_{t-1}^{(1,k+1)} \end{bmatrix} + \begin{bmatrix} \sqrt{\tau_j^{-1} s_1} \\ N \end{bmatrix} u_t,
\]
where \( j = \{a, \mu\} \). Finally, equating coefficients with those in the conjecture in (A64) gives the solution to the coefficient matrices in the law of motion for the state
\[
M = \begin{bmatrix} \rho_j & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{M}_0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{M}_1 \end{bmatrix}
\] (A75)
\[
N = \begin{bmatrix} \sqrt{\tau_j^{-1} s_1} \\ N \end{bmatrix}
\] (A76)

where the last two rows/columns have been cropped to make the matrices conformable; i.e. implementing the approximation that for all orders of expectation where \( k > k, X_t^{(k)} = 0 \).

\textbf{Fixed Point Problem:} Equations (A68), (A75), and (A76) present a mapping from \( \{M, N\} \mapsto \{a_0, \alpha_1\} \mapsto \{M, N\} \), the fixed point of which provides the approximate rational expectations equilibrium to the ex-
tended business cycle model. To find the coefficient matrices, I iterate on the described two steps until convergence. To initialize the algorithm, I use the solution to the model with only exogenous private information; that is where \( \Omega_{ft} = \{\pi_{t-j}\}_{j=0}^{\infty} \) and \( \Omega_{cbt} = \{z_{t-j}\}_{j=0}^{\infty} \).

**Calibration:** The match between the moments and the model outcomes is listed in Table 1.

### Table 1: Model Calibration

<table>
<thead>
<tr>
<th>Data/Moment</th>
<th>Model Outcome</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root MSE Output</td>
<td>1.96</td>
<td>Survey of Prof. Forecasters</td>
</tr>
<tr>
<td>Root MSE Output</td>
<td>1.98</td>
<td>Central Bank</td>
</tr>
<tr>
<td>Central Bank</td>
<td>1.80</td>
<td>Greenbook Data</td>
</tr>
<tr>
<td>Central Bank</td>
<td>1.86</td>
<td></td>
</tr>
<tr>
<td>Runkle Statistic</td>
<td>1.97</td>
<td>Lorenzoni (2009)</td>
</tr>
<tr>
<td>Runkle Statistic</td>
<td>2.00</td>
<td></td>
</tr>
</tbody>
</table>

(i) One-period ahead output forecasts for both the private sector and the central bank.
(ii) Both private sector and central bank data pre-Feb 1994.

**Optimal Policy Limits:** Suppose the central bank has full information about \( y_t \) and \( a_t \). In this case, we can re-write the Euler equation in (A59) as

\[
y_t = \mathbb{E}_{ht} [y_{t+1}] + \mathbb{E}_{ht} [\pi_{t+1}] - i_t
\]

\[
= \mathbb{E}_{ht} [y_{t+1}] + \mathbb{E}_{ht} [\pi_{t+1}] - \phi (y_t - a_t),
\]

such that

\[
y_t = \frac{1}{1+\phi} (\mathbb{E}_{ht} [y_{t+1}] + \mathbb{E}_{ht} [\pi_{t+1}]) + \frac{\phi}{1+\phi} a_t. \tag{A77}
\]

It follows that

\[
\lim_{\phi \to \infty} y_t = a_t,
\]

and hence that \( \phi^* \to \infty \) obtains the first best outcome under full information.

Now, suppose instead that the central bank has imperfect information about \( y_t \) and \( a_t \), and define its forecast error of the output gap as

\[\xi_{y_t-a_t} = (y_t - a_t) - \mathbb{E}_{cbt} [y_t - a_t].\]

Then,

\[
y_t = \mathbb{E}_{ht} [y_{t+1}] + \mathbb{E}_{ht} [\pi_{t+1}] - i_t
\]

\[
= \mathbb{E}_{ht} [y_{t+1}] + \mathbb{E}_{ht} [\pi_{t+1}] - \phi \xi_{y_t-a_t}
\]

\[
= \mathbb{E}_{ht} [y_{t+1}] + \mathbb{E}_{ht} [\pi_{t+1}] - \phi (y_t - a_t) - \phi \xi_{y_t-a_t},
\]

and hence

\[
\lim_{\phi \to \infty} y_t = a_t + \lim_{\phi \to \infty} \xi_{y_t-a_t}, \tag{A78}
\]

This shows that when \( \phi \to \infty \) social welfare losses only arise from the presence of central bank imperfect information. This is precisely as in the baseline model from Section 2.
Appendix E: Robustness Checks

Appendix E.1: Dispersed Information

A substantial debate has arisen about the social value of public information in models with incomplete common knowledge among private sector agents. Because of strategic complementarities, public signals may namely in such models receive either too little or too much weight (Angeletos and Pavan, 2007). This depends in part on how monetary policy is set (Angeletos et al, 2016).

To explore how the welfare effects of central disclosure differ with dispersed private sector information, I in this appendix solve the extended model with dispersed private sector information. In particular, consistent with the benchmark calibration, I set the private information parameters in (6.9) to jointly match the observed pre-February 1994 dispersion in one-quarter ahead GNP/GDP forecasts in the Survey of Professional Forecasters (equal to 0.33 percentage points), in addition to the one-quarter ahead mean-squared error of the average forecast from the same survey. I find that to match these targets $\mu_x = 0.50$, $\sigma_{x,f}^2 = 0.11$, $\sigma_z^2 = 0.60$, $\sigma_{x,f}^2 = 0.20$. I then re-compute the optimal policy for both the mark-up and productivity shock case. Table 2 and 3 show that the main insights from my analysis extend to the case with dispersed private sector information.

The introduction of dispersed private sector information further complicates the solution of the model. Because of dispersed information, the Law of Iterated Expectations does not hold for private sector expectations: Average private sector expectations of average private sector expectations, and so on, do not simply equal average private sector expectations. One consequence of this failure of the Law of Iterated Expectations is that the number of higher-order expectations in each order of expectations $k$ in (6.12) follows the Fibonacci sequence rather than simply increases by two, as under the baseline calibration where $\sigma_{x,f}^2 = 0$. Already, with $k = 15$ we therefore have to keep a track of 4,179 different expectations. I solve the model for $k = 15$ and re-compute the optimal policies.

Table 2 shows the breakdown of the quantitative results when only mark-up shocks drive the economy. As in Section 7, I find that the combined optimal policy is to set $\tau^\mu \rightarrow \infty$ and $\phi \rightarrow \infty$. Consistent with my previous results, Table 2 shows that disclosure improves welfare – both at the calibrated pre-February 1994 benchmark and at the optimal monetary policy. The benefit from the central bank being able to better predict (and counter) private sector actions once more dominates the increase in private sector responses to the mark-up shock. Moreover, compared to the results in Section 7, the benefit from central bank disclosure is somewhat larger at the optimal monetary policy (c. -50 percent now vs -26 percent previously), while slightly smaller at the calibrated benchmark (c. -26 percent now vs -115 percent before). This shows how the dispersion of private sector information modifies our previous estimates.

Table 3 shows the corresponding breakdown when productivity instead drive the economy. As in Section 7, I find that the optimal monetary policy is again to set $\tau^\alpha \rightarrow \infty$ and $\phi \rightarrow \infty$. The results in Table 3 are remarkably close to those in Table III. Going from complete opacity to full disclosure decreases welfare losses by around 32 percent at the optimal monetary policy. Around 30 percentage points of this decrease is alone due to the increase in central bank information about productivity (compared to 28 percentage points before). The benefit from the central bank being able to back out more information once more dominates the learning externality. In turn, this makes disclosure more beneficial and the overall effect similar to those reported in Table III.

In sum, the main insights from Section 7 are robust to the introduction of realistic amounts of dispersed private sector information and the associated absence of common knowledge.

---

10Recall that the relationship between the precision $\tau$ and the standard deviation $\sigma$ is $\tau = \sigma^{-2}$. 
Table 2: Dispersed Information (with Mark-up Shocks)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$%\Delta W_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>$\phi = 1.81$, $\tau^m_w \to 0$</td>
</tr>
</tbody>
</table>

_Breakdown of Benefits from Disclosure_

A. Benchmark with full disclosure: $\phi = 1.81$, $\tau^m_w \to \infty$ $-62.33$
B. Benchmark with constant h.o. unc.†: $\phi = 1.81$, $\tau^m_w \to \infty$ $-36.80$
A-B. Cost from decrease in h.o. unc.: $-25.53$

_Breakdown of Benefits from Optimal Policy_

A. Optimal policy: $\phi \to \infty$, $\tau^m_w \to \infty$ $-99.35$
B. Benefit from optimal mon. policy: $\phi \to \infty$, $\tau^m_w \to 0$ $-48.97$
A-B. Benefit from central bank disclosure: $-50.38$

(i) $W_C$ denotes the life-time consumption equivalent of $W$.
(ii) $\%\Delta W_C$ denotes the % change in $W_C$ relative to the calibrated benchmark.
(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 3: Dispersed Information (with Productivity Shocks)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$%\Delta W_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>$\phi = 1.81$, $\tau^a_w \to 0$</td>
</tr>
</tbody>
</table>

_Breakdown of Benefits from Disclosure_

A. Benchmark with disclosure: $\phi = 1.81$, $\tau^a_w \to \infty$ $-1.15$
B. Private sector benefit of disclosure†: $\phi = 1.81$, $\tau^a_w \to \infty$ $-8.93$
A-B. Central bank cost of disclosure: $+7.78$

_Breakdown of Benefits from Optimal Policy_

A. Optimal policy: $\phi \to \infty$, $\tau^a_w \to \infty$ $-31.74$
B. Benefit from optimal mon. policy: $\phi \to \infty$, $\tau^a_w \to 0$ $+1.02$
C. Private sector benefit of disclosure†: $\phi \to \infty$, $\tau^a_w \to \infty$ $-3.16$
A-B-C. Central bank benefit of disclosure: $-29.61$

(i) $W_C$ denotes the life-time consumption equivalent of $W$.
(ii) $\%\Delta W_C$ denotes the % change in $W_C$ relative to the calibrated benchmark.
(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.
Appendix E.2: Limited Number of Higher-Order Expectations

A substantial literature in experimental economics has demonstrated people's limited capacity to form higher-order expectations (see, for instance, Nagel, 1995). One advantage of the computational approach taken in Section 6 and 7 is that it directly allows one to study the consequences of such behavioral limits for the benefits of central bank disclosure. I demonstrate below how the main quantitative insights extend to cases in which firms and the central bank compute only a few higher-order expectations. I for brevity consider the case where $k = 3$, consistent with the upper-bound in Nagel (1995) The main findings, however, straightforwardly extend to other limits that one could consider, such as $k = 1, 2$.

Table 4 shows the breakdown of the welfare benefits of disclosure when only mark-up shocks drive the economy. The table corresponds to Table II in the main text. Compared to the case reported in the main text in which firms compute $k = 50$ higher-order expectations, we see that the benefit of disclosure at the calibrated benchmark is somewhat reduced. This is consistent with fewer higher-order expectations contributing to the equilibrium dynamics of the model. That said, the first $k = 3$ higher-order expectations still account for the lion-share of the welfare benefit that follows from disclosure in Table II. Indeed, the welfare benefit with $k = 3$ is over 83 percent of that with $k = 50$.\footnote{This follows from a comparison of the -95.65% decrease in Table 4 with the -114.63% decrease in Table II.}

Table 5 shows the corresponding breakdown of the welfare benefits of disclosure when productivity shocks instead drive the economy. This table corresponds to Table III in the main text. Compared to Table III, the welfare benefit of disclosure under the optimal monetary policy is smaller, although the decrease in central bank uncertainty is still clearly present: It contributes three percentage points to the overall decrease in welfare losses. Furthermore, with $k = 3$ the decrease in higher-order uncertainty already dominates the standard learning externality at the calibrated value of monetary policy. As a result, increases in central bank disclosure from this calibrated value would decrease central bank uncertainty, leading to better monetary policy.

In sum, the main quantitative insights from Section 7 are robust to decreases in the number of higher-order expectations that firms and the central bank compute. Although plausible limits to the amount of higher-order expectations somewhat dampen the magnitude of the quantitative results, in all cases central bank disclosure is unequivocally beneficial.
Table 4: Limited Higher-Order Expectations (with Mark-up Shocks)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>%ΔWC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>φ = 1.81 τω → 0 ...</td>
</tr>
</tbody>
</table>

Breakdown of Benefits from Disclosure

A. Benchmark with full disclosure  φ = 1.81 τω → ∞ -41.06
B. Benchmark with constant h.o. unc.† φ = 1.81 τω → ∞ +54.58
A-B. Benefit from decrease in h.o. unc. -95.64

Breakdown of Benefits from Optimal Policy

A. Optimal policy φ → ∞ τω → ∞ -74.42
B. Benefit from optimal mon. policy φ → ∞ τω → 0 -36.27
A-B. Benefit from central bank disclosure -38.15

(i) WC denotes the life-time consumption certainty equivalent of W.
(ii) %ΔWC denotes the %change in Wc relative to the calibrated benchmark.
(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 5: Limited Higher-Order Expectations (with Productivity Shocks)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>%ΔWC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>φ = 1.81 τω → 0 ...</td>
</tr>
</tbody>
</table>

Breakdown of Benefits from Disclosure

A. Benchmark with disclosure  φ = 1.81 τω → ∞ -31.93
B. Private sector benefit of disclosure† φ = 1.81 τω → ∞ -13.21
A-B. Central bank cost of disclosure -18.72

Breakdown of Benefits from Optimal Policy

A. Optimal policy φ → ∞ τω → ∞ -47.96
B. Benefit from optimal mon. policy φ → ∞ τω → 0 -17.77
C. Private sector benefit of disclosure† φ → ∞ τω → ∞ -27.31
A-B-C. Central bank cost of disclosure -2.87

(i) WC denotes the life-time consumption certainty equivalent of W.
(ii) %ΔWC denotes the %change in Wc relative to the calibrated benchmark.
(†) Central bank mean-squared error of the state X₁^{(0,4)} fixed at calibrated benchmark value.

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Appendix E.3: Strategic Interactions and Higher-Order Expectations

An important driver of the costs and benefits of central bank disclosure is the extent to which households and firms are forward-looking. The less forward-looking households and firms are, the less their expectations about the future actions of others matter for equilibrium outcomes. Since the degree of such dynamic strategic complementarity is tied intimately to the importance of higher-order expectations, changes to the extent to which households and firms are forward-looking could matter for the costs and benefits of disclosure.

To understand the consequences of a decrease in the discount factor, for example, consider to start the *New Keynesian Phillips Curve* in (6.6) from the main text,

\[
\pi_t = \beta E_t^F [\pi_{t+1}] + \lambda E_t^F [y_t - a_t] + E_t^F [\mu_t] = E_t^F \sum_{l} \beta^l \{ \lambda [y_{t+l} - a_{t+l}] + \mu_{t+l} \}. \tag{A79}
\]

Inserting the forward-solution for output from the Euler equation into (A79) then shows that

\[
\pi_t = \lambda E_t^F \sum_{l=0}^\infty \beta^l \left[ \sum_{j=0}^{\infty} \pi_{t+l+1+j} - i_{t+l+j} \right] - \frac{\lambda}{1 - \beta \rho_a} E_t^F [a_t] + \frac{1}{1 - \beta \rho_{\mu}} E_t^F [\mu_t]. \tag{A80}
\]

Thus, the smaller \( \beta \) is the less inflation at time \( t \) depends upon firms’ expectations of future inflation and interest rates via the output equation. As a result, the less inflation depends upon firms’ higher-order expectations of future firm and central bank actions. This, in turn, decreases both the costs and benefits of central bank disclosure. The costs decrease because of the decrease in amplification that arises from firms discounting future firms’ responses to, for instance, the inefficient mark-up shock relatively more. By contrast, the benefits decrease because of the reduction in the importance of firms’ expectations of central bank beliefs through the decrease in the importance of future interest rates. Table 6 and 7 show the reduction in the quantitative costs and benefits of disclosure when we decrease \( \beta \) from 0.99 to 0.75. Crucially, in both cases the benefits of disclosure still outweigh the costs at the optimal value of monetary policy, and by a comparable amount to that reported in Table II and III of the main text.

At the calibrated benchmark, the decrease in \( \beta \) scales down the costs and benefits of disclosure about the inefficient mark-up shock by 4 and 14 percentage points, respectively. The costs of disclosure about the efficient productivity shock, by contrast, decrease by around 27 percentage points at the calibrated benchmark, while the benefits at the optimal value for monetary policy decrease by 11 percentage points. As a consequence, the benefits of disclosure still dominate the costs, on balance. This holds true both at the calibrated benchmark value and at the optimal value of monetary policy for disclosure of information about the mark-up and the productivity shock. Thus, while decreases to the discount factor decrease the costs and benefits of central bank disclosure, this happens in a symmetric manner, keeping the magnitude of the relative advantage of the benefits over the costs from Table II and III in the main text.
### Table 6: Strategic Interactions and Higher-Order Expectations (with Mark-up Shocks)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$% \Delta W_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>$\phi = 1.81 \quad \tau_\omega \to 0 \quad \ldots$</td>
</tr>
</tbody>
</table>

**Breakdown of Benefits from Disclosure**

A. Benchmark with full disclosure  
\[ \phi = 1.81 \quad \tau_\omega \to \infty \quad -62.95 \]

B. Benchmark with constant h.o. unc.†  
\[ \phi = 1.81 \quad \tau_\omega \to \infty \quad +47.88 \]

A-B. Benefit from decrease in h.o. unc.  
\[ -110.83 \]

**Breakdown of Benefits from Optimal Policy**

A. Optimal policy  
\[ \varphi \to \infty \quad \tau_\omega \to \infty \quad -95.98 \]

B. Benefit from optimal mon. policy  
\[ \varphi \to \infty \quad \tau_\omega \to 0 \quad -63.60 \]

A-B. Benefit from central bank disclosure  
\[ -32.39 \]

(i) $W_C$ denotes the life-time consumption certainty equivalent of $W$.

(ii) $\% \Delta W_C$ denotes the $\%$ change in $W_c$ relative to the calibrated benchmark.

(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

### Table 7: Strategic Interactions and Higher-Order Expectations (with Productivity Shocks)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$% \Delta W_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>$\phi = 1.81 \quad \tau_\omega \to 0 \quad \ldots$</td>
</tr>
</tbody>
</table>

**Breakdown of Benefits from Disclosure**

A. Benchmark with disclosure  
\[ \phi = 1.81 \quad \tau_\omega \to \infty \quad -18.64 \]

B. Private sector benefit of disclosure†  
\[ \phi = 1.81 \quad \tau_\omega \to \infty \quad -26.88 \]

A-B. Central bank cost of disclosure  
\[ +8.23 \]

**Breakdown of Benefits from Optimal Policy**

A. Optimal policy  
\[ \varphi \to \infty \quad \tau_\omega \to \infty \quad -32.49 \]

B. Benefit from optimal mon. policy  
\[ \varphi \to \infty \quad \tau_\omega \to 0 \quad +8.59 \]

C. Private sector benefit of disclosure†  
\[ \varphi \to \infty \quad \tau_\omega \to \infty \quad -15.87 \]

A-B-C. Central bank cost of disclosure  
\[ -25.21 \]

(i) $W_C$ denotes the life-time consumption certainty equivalent of $W$.

(ii) $\% \Delta W_C$ denotes the $\%$ change in $W_c$ relative to the calibrated benchmark.

(†) Central bank mean-squared error of the state $X_t^{(\bar{k})}$ fixed at calibrated benchmark value.
Appendix E.4: The Signaling Channel of Monetary Policy

In the model described in Section 6, under complete opacity or partial disclosure, changes to the the interest rate provide firms with a noisy signal of the central bank’s private information. By contrast, full disclosure separates the interest rate from the signaling channel of monetary policy. Full disclosure thus allows the central bank to set monetary policy without having to account for the informational consequences of such policy steps. A concern could therefore be that the bulk of the quantitative benefits of disclosure reported in Table II and III arise from the resulting freedom of monetary policy rather than from the mechanisms described in the paper. Table 8 and 9 show that this is not the case.

Table 8 and 9 show the equivalent results to those reported in Table II and III of the main text when one excludes the central bank interest rate from firms’ information set; that is, when \( i_t \notin \Omega^f_{it} \), which eliminates any signaling effects of monetary policy. For both the mark-up and productivity shock case, the differences in results relative to Table II and III are minor. The welfare effects of disclosure increase by between a couple of tenths of a percentage point (to 114.79% from 114.60% at the benchmark calibration for the mark-up shock case) to somewhat over one percentage point (to 29.23% from 28.40% at the optimal value of monetary policy for the productivity shock case). On balance, both at the calibrated benchmark and at the optimal value of monetary policy, the signaling channel of monetary policy has a relatively minor influence on the benefits of central bank disclosure. This is because in both cases the interest rate provides a rather dim indicator of the central bank’s private information. This is consistent with the substantial impact of central bank disclosure on financial markets and on private sector uncertainty about future interest rates documented in, for example, Blinder et al (2008). In sum, the signaling channel of monetary policy comprises a relatively minor component of the welfare benefits of central bank disclosure documented in Table II and III of the main text.

12See also footnote 34 in the main text for more on this topic.
Table 8: Signaling Channel of Monetary Policy (with Mark-up Shocks)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>%ΔWC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>( \phi = 1.81 ) ( \tau_w \rightarrow 0 )</td>
</tr>
</tbody>
</table>

**Breakdown of Benefits from Disclosure**

A. Benchmark with full disclosure \( \phi = 1.81 \) \( \tau_w \rightarrow \infty \) -58.94
B. Benchmark with constant h.o. unc.† \( \phi = 1.81 \) \( \tau_w \rightarrow \infty \) +55.60
A-B. Benefit from decrease in h.o. unc. -114.60

Breakdown of Benefits from Optimal Policy

A. Optimal policy \( \phi \rightarrow \infty \) \( \tau_w \rightarrow \infty \) -96.52
B. Benefit from optimal mon. policy \( \phi \rightarrow \infty \) \( \tau_w \rightarrow 0 \) -69.78
A-B. Benefit from central bank disclosure -26.75

(i) \( W_C \) denotes the life-time consumption certainty equivalent of \( W \).
(ii) %ΔWC denotes the %change in \( W_c \) relative to the calibrated benchmark.
(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

Table 9: Signaling Channel of Monetary Policy (with Productivity Shocks)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>%ΔWC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated benchmark</td>
<td>( \phi = 1.81 ) ( \tau_w \rightarrow 0 )</td>
</tr>
</tbody>
</table>

**Breakdown of Benefits from Disclosure**

A. Benchmark with disclosure \( \phi = 1.81 \) \( \tau_w \rightarrow \infty \) -2.94
B. Private sector benefit of disclosure† \( \phi = 1.81 \) \( \tau_w \rightarrow \infty \) -14.40
A-B. Central bank cost of disclosure +11.46

Breakdown of Benefits from Optimal Policy

A. Optimal policy \( \phi \rightarrow \infty \) \( \tau_w \rightarrow \infty \) -32.71
B. Benefit from optimal mon. policy \( \phi \rightarrow \infty \) \( \tau_w \rightarrow 0 \) +8.91
C. Private sector benefit of disclosure† \( \phi \rightarrow \infty \) \( \tau_w \rightarrow \infty \) -12.40
A-B-C. Central bank cost of disclosure -29.23

(i) \( W_C \) denotes the life-time consumption certainty equivalent of \( W \).
(ii) %ΔWC denotes the %change in \( W_c \) relative to the calibrated benchmark.
(†) Central bank mean-squared error of the state \( X_1^{(n,k)} \) fixed at calibrated benchmark value.