Learning by Sharing: Monetary Policy and Common Knowledge

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Abstract

A common view states that central bank releases decrease central banks' own information about the economy and are harmful if about inefficient disturbances, such as cost-push shocks. This paper shows how neither is true in a micro-founded macroeconomic model in which households and firms learn from central bank releases and the central bank learns from the observation of firm prices. Central bank releases make private sector and central bank expectations closer to common knowledge. This helps transmit dispersed information between the private sector and the central bank. As a result, the release of additional central bank information decreases the central bank's own uncertainty and can be beneficial, irrespective of the efficacy of macroeconomic fluctuations. A calibrated example suggests that the benefits of disclosure are substantial.

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1 Introduction

Good policy requires accurate information about the state of the economy. To set interest rates correctly, for example, a policymaker needs to know whether a demand or a supply shock has hit the economy, what the size of the shock was, and what the private sector thinks of it. All are important determinants of the policymaker's choices. At the same time, modern-day policymakers also disclose a torrent of information. On average, members of the Board of the

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US Federal Reserve and Senior Treasury Staff, for example, speak nine times per week on the record about their own views about the state of the economy.[1]

As a result of these disclosures, a two-sided flow of information arises. On the one hand, policymakers devote considerable resources to learning about the state of the economy from private sector actions, such as prices. On the other hand, such private sector actions themselves reflect people's imperfect information about the economy, and are often informed and influenced by policymaker announcements.[2]

In this paper, I study the consequences of this two-sided flow of information for the social value of policymaker releases.[3] To do so, I introduce imperfect information and a lack of common knowledge between households, firms, and a central bank in an otherwise standard macroeconomic model. In the model, higher-order uncertainty arises from private information about common disturbances. But it could equally well arise from a behavioral friction that limits the amount of attention paid to economic factors (Sims, 2003).

My contribution is to show how central bank disclosures enhance the efficacy of monetary policy by decreasing higher-order uncertainty. A central bank's disclosure not only provides more information to the private sector, but also increases the amount of common information between the private sector and the central bank. This increases what the central bank knows about private sector expectations, what the central bank knows about private sector expectations of its own beliefs, and so on. I detail how such decreases in higher-order uncertainty increase what the central bank knows about private sector responses to shocks and simplifies the central bank's inference problem when it learns from private sector actions, such as prices. As a result, disclosure improves monetary policy's capacity to replicate the first best outcome.

My results qualify two prevalent theories of the costs of central bank disclosure: (*i*) that disclosure can be socially costly since it increases firms' responses to inefficient shocks, such as cost-push shocks (Hellwig, 2005; Angeletos and Pavan, 2007; Angeletos *et al.*, 2016); and (*ii*) that disclosure decreases the central bank's own information about the state of the economy by decreasing the information content of prices, and hence leads to worse monetary policy (Morris and Shin, 2005; Amador and Weill, 2007 and 2010).

In contrast, in my model, where central bank disclosures decrease higher-order uncertainty, these costs can be overcome. Because of the lack of common knowledge between the private

[1]This is based on Bloomberg news summary data. The precise number of releases is 457 in 2016, or nine times per working week. These values include speeches, comments, and documents by the President, Federal Reserve Presidents, senior US Treasury officials, and members of the CEA and the CBOE.

[2]See, for example, Blinder et al (2008) and Broadbent (2013).

[3]A considerable debate has developed about the social value of public information, such as that from policymakers. This includes *inter alia* Morris and Shin (2002, 2005), Hellwig (2005), Svensson (2006), Angeletos and Pavan (2007), James and Lawler (2011), Paciello and Wiederholt (2013), and Angeletos *et al.* (2016). This debate has, however, abstracted from the two-sided information flow that is the focus of this paper.

sector and the central bank, full disclosure becomes beneficial, irrespective of the efficacy of macroeconomic fluctuations. Furthermore, disclosure increases the central bank's own information about the state of the economy by increasing the informativeness of prices.

In fact, in a calibrated, extended version of my model that introduces a lack of common knowledge into the baseline New Keynesian framework, I show that disclosure decreases welfare losses by between 27 and 33 percent relative to the complete opacity baseline.[4] This depends on whether cost-push or productivity shocks drive the economy. In both cases, welfare improvements arise due to better monetary policy responses caused by the decrease in higher-order uncertainty. Around 16 percentage points of these welfare benefits are caused solely by the increase in the informativeness of prices leading to better monetary policy.

My results offer an explanation for two empirical facts that are otherwise at odds with the supposed costs of central bank disclosure. First, despite a considerable increase in the amount of disclosure over the past two decades, central banks have on balance not chosen to adopt state-dependent disclosure rules. Central banks regularly provide information about the state of the economy, irrespective of whether they believe changes are driven by efficient or inefficient shocks.[5] Second, despite substantial increases in central bank disclosures, there are no indications that their ability to forecast the economy has been compromised. The root mean-square error of the US Federal Reserve's one-quarter ahead inflation forecast is, for instance, 1.2 percent and 0.9 percent before and after it started to increase its transparency in February 1993.[6] My results provide a step towards explaining these observations. At the same time, they also hint at a broader consequence; that to accurately assess the consequences of changes to any communication policy one has to first account for how such changes affect the policymaker's ability to effectively use his own policy instrument.

My analysis first centers on a simple macroeconomic model with the standard features of monopolistic competition, nominal frictions, and a central bank that can control nominal demand. As in Lucas (1972), firms are assumed to have private information about common disturbances -- here, the economy-wide level of total factor productivity and price markups. The central bank is also assumed to possess private information about the common disturbances. Crucially, both firms and the central bank learn from each other's actions: the central bank from the observation of firms' prices and firms from the central bank's disclosures.

I derive the baseline model's implications for central bank policy and use it to show how disclosure can overcome the two aforementioned costs.

[4] I throughout measure welfare losses in terms of life-time consumption (Lucas, 1987).

[5] See, for example, Dincer and Eichengreen (2009) and Eichengreen and Dincer (2014).

[6] A similar decrease occurred in the US Federal Reserve's one-quarter ahead GDP forecasts: the root-mean squared error fell from 1.9 to 1.6. To compute these numbers, I use forecast data from the Greenbook and first release realizations of the outcome variable. All are available from the Federal Reserve Bank of Philadelphia's website. The first sample extends from Jan 1970 to Jan 1993; the second from Apr 1993 to Dec 2010.

First, conditional on the optimal monetary policy, central bank disclosure is generically optimal because of the decrease in higher-order uncertainty. This holds true even if the economy is driven only by inefficient cost-push (mark-up) shocks. Suppose that the central bank, in this case, discloses additional information. On the one hand, such disclosure would increase firms' responses to the inefficient shock. On the other hand, it would also increase the central bank's information about firms' expectations, and so on, since firms will use the central bank's disclosure when forming their own beliefs. This, in turn, allows the central bank to better counter firms' responses to the shock. I show how the latter effect dominates the former for the optimal monetary policy.

Second, a common concern about central bank disclosures has been that that they crowd out private sector information from market outcomes, such as prices (see Morris and Shin, 2005). All else equal, this leads to less informed monetary policy, and hence potentially worse welfare outcomes. While the baseline model allows for this mechanism, in equilibrium its effect is overcome by the capacity of disclosure to alleviate a particular identification problem faced by the central bank when there is incomplete common knowledge.

Consider the case in which the central bank observes constant prices from one period to the next. This observation could be either due to firms receiving private information in line with their prior, or due to all firms receiving different information but expecting the central bank to alter nominal demand in response such that prices remain constant. Disclosure solves this identification problem. By making the central bank's own information, and hence beliefs, common knowledge, disclosure offers the distinction between the two options. As a result, central bank disclosure can decrease uncertainty for everyone, even the central bank itself. This, in turn, allows the central bank to better replicate the first best outcome.

To keep the analysis tractable, the baseline model abstracts from households' imperfect information, a signaling role of monetary policy, and limits higher-order uncertainty by assuming one-period perfect state verification. I relax these assumptions when I turn to the extended model. This, in effect, renders the extended model into an amended version of the dispersed information New Keynesian model studied in Lorenzoni (2009). Crucially, and in departure from Lorenzoni (2009), or the extensions considered by Nimark (2014) and Melosi (2016), the central bank and the private sector here lack common knowledge about each other's beliefs.

The solution of the model poses technical difficulty due to the infinite regress of expectations that arises when agents need to "forecast the forecasts of others" (Townsend 1983). To address these difficulties, I extend the truncated state space solution method proposed in Nimark (2017) to the case with non-atomistic players, such as a central bank. To calibrate the model, I rely on data on private sector and central bank forecast accuracy from the "Survey of Professional Forecasters" by the Philadelphia Federal Reserve Bank and the "Greenbook", respectively. I use numerical simulations to explore the quantitative implications of the model.

The calibrated model shows considerable benefits of central bank disclosure. When the economy is driven only by unobserved cost-push shocks, disclosure decreases welfare losses by 27 percent under the optimal policy. Of this decrease, around 50 percentage points are due to the improvement in monetary policy caused by a decrease in higher-order uncertainty between the private sector and the central bank. The direct increase in firms' responses to the cost-push shock, by contrast, only increases welfare losses by 23 percentage points. The decrease in welfare losses is of a similar magnitude when the economy is instead driven only by productivity shocks. Specifically, disclosure decreases welfare losses by 33 percent under the optimal policy, of which 16 percentage points are now due to improved monetary policy responses caused by an increase in the information content of prices.

Combined, these results showcase the importance of a lack of common knowledge between the private sector and the policymaker for an accurate picture of the social value of policymaker releases. They, however, also hint at broader consequences of incomplete common knowledge for several macroeconomic policies which success depends on private sector knowledge of future policymaker actions. This includes among others the debate about the efficacy of central bank forward guidance (Werning, 2015; McKay *et al.*, 2016; Angeletos and Lian, 2018) and the front- versus back-loading of tax cuts (Barro, 1974; Heathcote, 2005).

Last, a common line of criticism of arguments that rest on the formation of higher-order expectations is that people do not seem to form many of them in practice. My analytical results, however, only strictly require individuals to engage in first or second-order thinking, consistent with the experimental results in Nagel (1995). Moreover, although the quantitative results weaken somewhat when I restrict people's ability to compute higher-order expectations, their sign and order of magnitude remain in all cases unchanged.

Related Literature: This paper is related to the recent debate about the social value of public information that has followed Morris and Shin's (2002) and Angeletos and Pavan's (2007) influential contributions. In particular, Hellwig (2005), Paciello and Wiederholt (2013), and Angeletos *et al.* (2016) show how the social value of public releases depends critically on the efficacy of macroeconomic fluctuations. By contrast, this paper demonstrates that once we also account for a lack of common knowledge between the private sector and the policymaker, an invariable benefit of disclosure can arise. One that holds irrespective of the efficacy of macroeconomic fluctuations.

Morris and Shin (2005) and Amador and Weill (2010) have relatedly proposed a stark "Paradox of Transparency".[7] This shows how central bank disclosure could be socially costly

[7] See also Amato *et al.* (2002), Amato and Shin (2006), Wong (2008), Gaballo (2016), and the related work on the *learning externality* of public information in markets where agents also observe and learn from prices (see, for instance, Vives, 1997; Amador and Weill, 2012; Vives, 2017; and the summary in Veldkamp, 2011).

because it decreases the informativeness of prices by crowding out private information. Paradoxically, disclosure could thus end up increasing uncertainty for everyone, even the central bank itself. This paper, by contrast, demonstrates how disclosure can increase the informativeness of prices by alleviating a particular identification problem.

Complementary to this paper, Gosselin *et al.* (2008) document a different mechanism for how this paradox could be resolved. They show that central bank disclosure could increase the informativeness of prices by turning firms "from Fed watchers to inflation watchers (Veldkamp, 2011)". However, Gosselin *et al.* (2008) do not address the identification problem that is caused by the central bank's own stabilization of prices, nor how disclosure affects it, which is the focus of this paper. Most importantly, Gosselin *et al.* (2008) do not share my explicit focus on disclosure's role in modifying common knowledge between the private sector and the central bank, and instead focus on the signaling role of interest rates.

My paper shares the emphasis on the importance of higher-order expectations for the effects of monetary policy with Wiederholt (2017) and Angeletos and Lian (2018). Central to their respective contributions is that a lack of common knowledge among households and firms dampens the effects of prospective monetary policy. By contrast, I below abstract completely from any lack of common knowledge among private sector agents and instead focus on its absence between the private sector and the central bank. This allows me to cleanly demonstrate how disclosure alleviates the errors in monetary policy that a lack of common knowledge between the private sector and the central bank otherwise entails.

Finally, this paper is related to the literature that studies the combined optimal use of policymaker disclosure and the conditional use of policy instruments. Walsh (2007), Baeriswyl and Cornand (2010), and James and Lawler (2011) show, in this context, how disclosure can be suboptimal since the policymaker can instead always condition his policy instrument on the news. Kohlhas (2017) extends these results and demonstrates how they depend centrally on the noise in the policymaker's partial disclosure. By contrast, in this paper I show how the combined use of disclosure and conditional instrument policy can arise as an optimal outcome. This occurs because information frictions exist alongside and interact with nominal frictions. Carlsson and Skans (2012) demonstrate the need for nominal frictions in imperfect information models to match the observed behavior of firm prices (see also Nimark, 2008).

Organization: The rest of this paper proceeds as follows. Section 2 presents the baseline model. Section 3 characterizes the equilibrium and the limit-cases in which the central bank can replicate the first best outcome. Sections 4 and 5 contain the main results that illustrate the welfare benefits of disclosure. Section 6 describes the extended version of the benchmark model, and Section 7 the numerical results that I obtain after a calibration of it. I conclude in Section 8. Additional extensions and all proofs are contained in the Appendix.

2 A Baseline Model

I start with a dynamic model with dispersed information and monopolistic competition. The model consists of a representative household, a continuum of firms, and a central bank. Each period is comprised of three stages. At the start of each period, firms pre-set prices based on imperfect information subject to a cost. After prices are set, the economy transitions to the second stage, where the central bank determines the money supply, in part based on its own imperfect information. The economy then transitions to the final stage, where all information that was previously dispersed becomes publically known. The representative household now meets with firms to produce what is demanded of firms' goods at stated prices. The wage adjusts to clear the labor market. Commodity markets open and the household consumes.

Households: A representative household has preferences given by the utility function,

$$\mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{1}{1+\eta} L_t^{1+\eta} \right], \tag{2.1}$$

where β denotes the household discount factor, C_t the consumption index at time t, L_t the number of hours worked by the household, and η parametrizes the Frisch elasticity of labor supply. The consumption index is comprised of

$$C_t = \left(\int_0^1 C_{it}^{\frac{\rho-1}{\rho}} di \right)^{\frac{\rho}{\rho-1}}, \quad P_t = \left(\int_0^1 P_{it}^{1-\rho} di \right)^{\frac{1}{1-\rho}}, \tag{2.2}$$

where C_{it} is the quantity the household consumes of the goods produced by firm $i \in [0, 1]$ and $\rho > 1$. P_t denotes the associated welfare-based price index and P_{it} the price set by firm i.

Because the representative household receives all profit and labor income in the economy, its per-period budget constraint is

$$\int_0^1 P_{it} C_{it} di + M_t^d \leq \int_0^1 \Pi_{it} di + W_t L_t + M_{t-1}^d + T_t^h, \tag{2.3}$$

where Π_{it} denotes the profits of firm $i \in [0, 1]$, M_t^d the household's demand for nominal balances, W_t the nominal wage, and T_t^h lump-sum nominal transfers. Household consumption is, in addition to (2.3), restricted by a cash-in-advance constraint after receiving nominal transfers,

$$\int_0^1 P_{it} C_{it} di \leq M_{t-1} + T_t^h, \quad T_t^h = M_t - M_{t-1}. \tag{2.4}$$

The representative household seeks to maximize its utility (2.1) subject to the per-period budget constraint (2.3) and the cash-in-advance constraint (2.4).

Firms: The production sector consists of a continuum of firms $i \in [0, 1]$ that specialize in the production of differentiated goods, also indexed by $i \in [0, 1]$. The production function used by firms is linear,

$$Y_{it} = A_t L_{it}, \quad A_t = A_{t-1} \exp(\theta_t), \tag{2.5}$$

where L_{it} denotes the amount of labor input used and A_t common total factor productivity with random innovation $\theta_t \sim \mathcal{WN}(0, 1/\tau_\theta)$.

An individual firm's objective is to set its price P_{it} to maximize its own expectation of the household's valuation of its stream of profits, using the per-period discount factor $\beta (P_t C_t)^{-1}$. Profits at time t are given by

$$\Pi_{it} = (1 + T_t^s) P_{it} Y_{it} - W_t L_{it} - \frac{\psi}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2 P_t Y_t, \tag{2.6}$$

where $1 + T_t^s$ is stochastic with mean $\frac{\rho}{\rho-1}$ such that, in a symmetric equilibrium ($P_{it} = P_t$), a firm's mark-up over marginal cost $\mathcal{M}_t = \frac{P_t}{W_t/A_t}$ follows:[8]

$$\mathcal{M}_t = \frac{\rho}{\rho - 1} \frac{1}{1 + T_t^s} = \mathcal{M}_{t-1} \exp(\xi_t), \quad \xi_t \sim \mathcal{WN}(0, 1/\tau_\xi). \tag{2.7}$$

I allow $\mathcal{M}_t$ to be random so as to accommodate mark-up (or cost-push) shocks.

Last, separate from the cost associated with physical production, firms in (2.6) face a quadratic price-adjustment cost, as in Rotemberg (1982), where $\psi > 0$ denotes a parameter which measures the severity of the nominal friction.

Central Bank: Similar to an individual firm, the central bank makes its choices based on imperfect information about the state of the economy. As a starting point, I assume that it sets its policy instrument, the money supply, directly based on its own expectation about the two fundamental shocks, the productivity and the mark-up disturbance,

$$M_t^s = M_{t-1}^s \exp\left\{ \phi_0 + \phi_\theta \mathbb{E}_t^{cb}[\theta_t] + \phi_\xi \mathbb{E}_t^{cb}[\xi_t] \right\}, \tag{2.8}$$

where lower-case letters denote the logarithm of their upper-case counterparts; ϕ_θ and ϕ_ξ the publically known levels of policy activism; and $\mathbb{E}_t^{cb}[\cdot]$ central bank expectations (described below). I thus characterize monetary policy in terms of a commitment to a log-linear rule. This assumption by itself does not prevent policy from achieving the first best outcome because of the below log-quadratic specification of welfare. In fact, as I show in Section 3, the central bank can always attain the efficient outcome with (2.8) if it observe all shocks without error

[8] See, for example, Steinsson (2003).

and firms do so as well.[9] Sections 4 to 7 demonstrate how my results carry over to other policy rules that also allow the central bank to replicate the first best under full information, such as when it instead responds to deviations of output from flex-price levels, or to the price level. However, for the sake of brevity, and because it allows for cleaner exposition of the main results, I choose to adopt the simpler approach in (2.8) to start with.

Information Structure: At the start of each period, all firms observe a combination of private and public information about the unobserved fundamentals of the economy. All firms observe the same private information, unknown to the central bank. Public information, by contrast, is observed by all agents in the economy.

Firms private information is summarized by the two noisy signals x_t^a and x_t^μ,

$$x_t^a = a_t + \epsilon_{xt}^a : \quad \epsilon_{xt}^a \sim \mathcal{WN}\left(0, 1/\tau_x^a\right), \qquad x_t^\mu = \mu_t + \epsilon_{xt}^\mu : \quad \epsilon_{xt}^\mu \sim \mathcal{WN}\left(0, 1/\tau_x^\mu\right), \tag{2.9}$$

where μ_t denotes the logarithm of firms' mark-up $\mathcal{M}_t$, τ_x^a and τ_x^μ the precision of firms' private information, and ϵ_{xt}^a and ϵ_{xt}^μ are independent of θ_t and ξ_t and all other random disturbances.

In addition to their private information, firms observe five distinct public signals, which are also observed by the central bank. These are: last period's productivity a_{t-1}, last period's mark-up μ_{t-1}, last period's money supply m_{t-1}, and the two potentially noisy signals ω_t^a and ω_t^μ sent by the central bank of its own private information,

$$\omega_t^a = z_t^a + \epsilon_{\omega t}^a : \quad \epsilon_{\omega t}^a \sim \mathcal{WN}\left(0, 1/\tau_\omega^a\right), \qquad \omega_t^\mu = z_t^\mu + \epsilon_{\omega t}^\mu, \quad \epsilon_{\omega t}^\mu \sim \mathcal{WN}\left(0, 1/\tau_\omega^\mu\right), \tag{2.10}$$

where z_t^a and z_t^μ denote the central bank's private information,

$$z_t^a = a_t + \epsilon_{zt}^a : \quad \epsilon_{zt}^a \sim \mathcal{WN}\left(0, 1/\tau_z^a\right), \qquad z_t^\mu = \mu_t + \epsilon_{zt}^\mu : \quad \epsilon_{zt}^\mu \sim \mathcal{WN}\left(0, 1/\tau_z^\mu\right). \tag{2.11}$$

The case of *full disclosure* here corresponds to the limit $\tau_\omega^j \to \infty$ with $j = \{a, \mu\}$, while *complete opacity* is equivalent to the situation where the central bank's communication contains no valuable information, $\tau_\omega^j \to 0$. *Partial disclosure* refers to the interim case, $\tau_\omega^j \in \mathbb{R}_+$.[10]

[9] When the central bank can respond to all shocks within each period, then it can always accommodate (or offset) each shock perfectly. This, in turn, ensures that the economy in each period can track its flex-price, first best counterpart from a time-less perspective (see Section 3). A similar result would, of course, hold if the central bank were to respond directly to the level of the driving forces instead.

[10] My chosen approach to model communication policy in (2.10) follows that of Cukierman and Meltzer (1986) and has been used extensively since (see, for instance, Faust and Svensson, 2001). An advantage of this approach is that it allows for a meaningful discussion of different, intermediate levels of disclosure. This advantage, of course, rests on the central bank committing to a disclosure rule such as (2.10). Absent this commitment, the central bank could communicate anything following the realization of its private information, and the only values that would be consistent with equilibrium would be full or zero disclosure. I demonstrate below how my main results still remain valid in this case.

Turning to the central bank, besides its own private information and the aforementioned public information, it also observes a noisy signal of the economy-wide price level,

$$\bar{p}_t = \log(P_t) + \epsilon_{pt}, \quad \epsilon_{pt} \sim \mathcal{WN}(0, 1/\tau_p), \tag{2.12}$$

where ϵ_{pt} is independent of all other disturbances for all t.[11]

We can summarize the information structure by the following information sets:

$$\Omega_t^f = \{x_{t-j}, \omega_{t-j}, \bar{p}_{t-j}, a_{t-j-1}, \mu_{t-j-1}, m_{t-j-1}\}_{j=0}^{\infty} \tag{2.13}$$

$$\Omega_t^{cb} = \{z_{t-j}, \omega_{t-j}, \bar{p}_{t-j}, a_{t-j-1}, \mu_{t-j-1}, m_{t-j-1}\}_{j=0}^{\infty}, \tag{2.14}$$

where $x_t' = \begin{bmatrix} x_t^a & x_t^\mu \end{bmatrix}$, $z_t' = \begin{bmatrix} z_t^a & z_t^\mu \end{bmatrix}$, and $\omega_t' = \begin{bmatrix} \omega_t^a & \omega_t^\mu \end{bmatrix}$. I denote firm and central bank expectations based on (2.13) and (2.14) by $\mathbb{E}_t^f[\cdot]$ and $\mathbb{E}_t^{cb}[\cdot]$, respectively.

A stark feature of (2.13) is that firms *do not* observe the current value of the central bank's policy instrument. Firm prices are pre-set and made before the realization of the money supply. This is identical to the assumption used in, for example, Woodford (2002a), Hellwig (2005), Adam (2007), and Angeletos *et al.* (2016). Sections 6 and 7 below demonstrate how my results are robust to relaxing this assumption. All that is required is that the central bank's disclosure provides some truly new information about the central bank's private information beyond that which firms can learn from the observation of the central bank's policy instrument.

Clearly, the information structure in (2.13) and (2.14) is stylized. Yet, at its heart it displays the two-sided informational interaction between the private sector and the central bank that is central to my main results. One the one hand, private sector firms learn from the central bank's communication and use this information to better set prices. But, on the other hand, the central bank itself also learns from the observation of firms' prices and uses this information to better set monetary policy.

3 Equilibrium, Prices, and Social Welfare

We can now proceed to study the equilibrium of the above economy. An equilibrium is defined in a familiar manner as a sequence of prices, production levels, household labor supply, firm labor demand, and wage rates such that at each point in time: *(i)* the representative household maximizes utility and firms maximize profits subject to informational and other constraints, and *(ii)* all goods markets clear, $Y_{it} = C_{it}$ for all $i \in [0, 1]$, and so too does the money

[11]The introduction of the shock ϵ_{pt} in (2.12) serves a technical purpose. Suppose that there are no mark-up shocks, that is that $\tau_\xi \to \infty$. The shock ϵ_{pt} then prevents the central bank from perfectly inferring firm's private information from the observation of the price level. The use of such "non-invertibility shocks" is standard in the literature on noisy rational expectations (see, for instance, Hellwig, 1980).

market, $M_t^d = M_t^s$. Below, I focus on two of these equilibrium objects: firms' prices and the central bank's money supply. These are the same two equilibrium objects for which imperfect information, and hence the presence of two-sided informational interactions, matters directly. The remaining quantities and wages are straightforward to derive and can be computed from (2.1), (2.2), (2.5), and Lemma 1 below.

The main obstacle to studying the role of imperfect information in the above economy is the non-linearity of the implied decision rules. I approach this problem by analyzing a log-linear approximation of firm, household, and central bank choices around the full information, non-stochastic steady state. This does not eliminate the crucial role that imperfect information plays in the economy, but it does render the optimal decision rules analytically tractable.

Characterization of Prices: Let me start with the characterization of firms' prices. To do so, I first solve the representative household's problem, imposing market clearing, to derive a relationship between the wage rate, output, and productivity in the economy. Then, I use this relationship to derive a simple expression for firms' prices.

Lemma 1 details the first step.[12]

Lemma 1. *Assume that ϕ_0 is set such that $\delta = \beta \mathbb{E}_t \left[\frac{M_t}{M_{t+1}}\right] < 1$. Then, the cash-in-advance constraint always binds, $m_t - p_t = y_t$, and the real wage rate satisfies,*[13]

$$w_t - p_t - a_t = (1+\eta)(y_t - a_t). \tag{3.1}$$

Equation (3.1) is a common expression that relates firms' real marginal cost to the level of output and productivity in the economy. We can now use this expression, where ϕ_0 is set such that $\delta < 1$, to derive the solution to a firm's problem.

Lemma 2. *The symmetric, linear equilibrium firm price for all $i \in [0,\, 1]$ is*

$$\begin{aligned} p_t &= \gamma_0 \mathbb{E}_t^f \left[w_t - p_t - a_t\right] + \frac{1}{1+\beta} p_{t-1} + \frac{\beta}{1+\beta} \mathbb{E}_t^f \left[p_{t+1}\right] + \mathbb{E}_t^f \left[\mu_t\right] &(3.2)\\ &= \lambda_0 \mathbb{E}_t^f \left[m_t - a_t\right] + \lambda_{-1} p_{t-1} + \lambda_1 \mathbb{E}_t^f \left[p_{t+1}\right] + \lambda_2 \mathbb{E}_t^f \left[\mu_t\right]. &(3.3)\end{aligned}$$

where $\gamma_0 \in \mathbb{R}_+$, $\{\lambda_{-1},\, \lambda_0,\, \lambda_1,\, \lambda_2\} \in [0,\, 1]$ with $\sum_{i=-1}^{1} \lambda_i = 1$ and $\lambda_{-1} = \lambda_1 = 0$ iff. $\psi = 0$.

Lemma 2 provides an intuitive result. Equilibrium prices in (3.2) are determined by firms' expectations about a convex combination of firms' real marginal cost and mark-up, on the one hand, and past and expected future prices, on the other hand. Nominal demand and labor

[12] The proof of Lemma 1 follows the steps in Hellwig (2005).

[13] Since the real resource cost of inflation is of second-order, the log-linearized resource constraint is simply $y_t = c_t$ (see Appendix A). As a result, the cash-in-advance constraint entails that $m_t - p_t = c_t = y_t$.

productivity appear in (3.3) because of their immediate influence on firms' real marginal cost in (3.1), in part through the cash-in-advance constraint. Past and future prices, by contrast, appear in (3.2) and (3.3) due to the nominal friction.

We can further simplify (3.3) by solving the equation forward.

Corollary 1. *The symmetric, linear equilibrium firm price and the associated central bank nominal demand are given by*

$$p_t = \nu_0 \mathbb{E}_t^f [m_t - a_t] + \nu_{-1} p_{t-1} + \nu_1 \mathbb{E}_t^f [\mu_t] \tag{3.4}$$

$$m_t = m_{t-1} + \phi_0 + \phi_\theta \mathbb{E}_t^{cb} [\theta_t] + \phi_\xi \mathbb{E}_t^{cb} [\xi_t] , \tag{3.5}$$

where $\{\nu_{-1},\, \nu_0\} \in [0,\, 1]$ *with* $\nu_{-1} = 0$*,* $\nu_0 = 1$ *iff.* $\psi = 0$ *and* $\nu_1 \in \mathbb{R}_+$*.*

Corollary 1 shows how the presence of nominal frictions $\psi > 0$ dulls firms' responses to their own expectations about nominal demand, $\nu_0 < 1$. This attenuation of firms' responses will, in turn, be important for the optimal conduct of monetary policy.

Social Welfare Loss: We can use the characterization of firms' prices to study the *normative* properties of the economy. I take my criterion to be utilitarian welfare and analyze the *ex-ante* utility of the representative household before knowledge of period zero shocks.

A second-order approximation around the flex-price, full information steady state then shows that the welfare losses obtained relative to the first best frictionless case are proportional to $\mathcal{W} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t (y_t - a_t)^2$ (see Appendix A). This shows how benchmark welfare expressions familiar from standard *New Keynesian* models with nominal frictions (see Woodford, 2002b; Nistico, 2007; and Galí, 2008) extend almost immediately to the above economy with imperfect information and a lack of common knowledge.

We can further simplify $\mathcal{W}$ using the law of iterated expectations combined with that $\mathbb{E}_{t-1} (y_t - a_t)^2$ is constant over time.

Lemma 3. *Equilibrium social welfare loss relative to the first best, frictionless case can be approximated by* $\mathcal{W} = \frac{1}{1-\beta} \mathbb{E}_{t-1} [y_t - a_t]^2$*, where* $y_t = m_t - p_t$ *and* $a_t = a_{t-1} + \theta_t$*.*

A Full Information Benchmark: A convenient welfare benchmark to compare subsequent optimal policies to is the special case in which the central bank and firms observe all fundamental shocks without error. That is the special case in which $\tau_x^j \to \infty$ and $\tau_z^j \to \infty$ for $j = \{a,\, \mu\}$. This benchmark will also later help expose the mechanism behind my results.

Combined, Lemma 1, Corollary 1, and Lemma 3 show that welfare losses under full information are[14]

$$\mathcal{W}^{full} = \frac{1}{1-\beta}\left\{(1-\nu_0)^2(\phi_\theta - 1)^2\frac{1}{\tau_\theta} + [(1-\nu_0)\,\phi_\xi - \nu_1]\frac{1}{\tau_\xi}\right\}. \tag{3.6}$$

It follows that the optimal policy under full information is to set $\phi_\theta = \phi_\theta^{\star,full} = 1$ and $\phi_\xi = \phi_\xi^{\star,full} = \frac{\nu_1}{1-\nu_0} > 0$, and that the central bank under this optimal policy replicates the first best, flex-price outcome, $\mathcal{W}^{full} = 0$. This shows how one tenet of optimal monetary policy under the New Keynesian framework carries over to our economy: The central bank accommodates the efficient productivity shock and offsets the inefficient mark-up disturbance.[15]

4 Disclosure about Inefficient Disturbances

I now turn to the costs and benefits of central bank disclosure. I start with a much discussed cost that arises from the increased responses to *inefficient disturbances*, such as cost-push (or mark-up) shocks, that disclosure entails (see Hellwig, 2005; Angeletos and Pavan, 2007; and Angeletos *et al.*, 2016).[16] In this section, I show how a lack of common knowledge between firms and the central bank modifies this cost. Specifically, I show how it can make full disclosure about an otherwise inefficient mark-up shock invariably beneficial.

Equilibrium with Mark-up Shocks: I consider the special case in which $\tau_\theta \to \infty$ and $\tau_p \to 0$. The former assumption allows me to focus on the inefficient fluctuations caused by the mark-up disturbance without having to also account for the efficient productivity shock. The latter assumption ensures that the central bank does not learn about firms' private information from the noisy observation of the price level. This simplifies the analysis. I extend my results to positive values of τ_p further below, while Section 5 deals with the productivity shock case.

I restrict myself to symmetric linear Bayesian equilibria. To find the equilibrium, I use the *method of undetermined coefficients*. Here, that involves three steps: First, one conjectures that p_t and m_t are linear in the elements of Ω_t^f and Ω_t^{cb}, respectively. Then, one computes firm and central bank expectations, in addition to firm expectations about central bank beliefs. Last, one inserts these expectations into the equilibrium expressions for p_t and m_t from Corollary 1. Consistent with the initial conjecture, these will be linear in Ω_t^f and Ω_t^{cb}, but with coefficients that now rest on those from the conjecture. Clearly, in equilibrium, the two

[14]This follows immediately from the output gap being equal to $y_t - a_t = m_t - p_t - a_t = (1-\nu_0)(\phi_\theta\theta_t + \phi_\xi\xi_t - \theta_t) - \nu_1\xi_t + l.p.t$, where $l.p.t$ denotes last period's terms.

[15]In fact, because of the quadratic nominal friction, the central bank here chooses to completely offset the mark-up shock. With a Calvo (1983) friction instead of a quadratic nominal cost, the central bank would in contrast choose to only partially offset the shock (Woodford, 2002b). This is because of the trade-off that arises from price dispersion. Neither of my main results depend critically on the exact nominal friction used.

[16]See also, for instance, Baeriswyl and Cornand (2010) and Paciello and Wiederholt (2013).

sets of coefficients have to be the same. Solving this fixed point problem yields Proposition 1.

Proposition 1. *The symmetric, linear equilibrium with mark-up shocks when $\tau_p \to 0$ is comprised of firm prices and a central bank money supply in which*

$$p_t = \nu_{-1} p_{t-1} + \nu_0 m_{t-1} + \nu_1 \mu_{t-1} + \kappa_0 x_t^\mu + \kappa_1 \omega_t^\mu \tag{4.1}$$

$$m_t = m_{t-1} + q_0 z_t^\mu, \tag{4.2}$$

and where κ_0, κ_1, and q_0 are all constants in $\mathbb{R}_+$.

A Cost of Disclosure with Common Knowledge: Suppose that the money supply is held fixed, $\phi_\xi = 0$. This ensures that central bank expectations about the mark-up shock do not affect nominal demand. Firms' beliefs about central bank expectations, and therefore this aspect of the absence of common knowledge, therefore becomes immaterial for firms' prices. Corollary 1 then entails that the economy-wide output gap, the determinant of social welfare, takes a particularly simple form,

$$y_t - a_t = m_t - p_t = \phi_\xi \mathbb{E}_t^{cb}[\xi_t] - \nu_0 \mathbb{E}_t^f\left[\phi_\xi \mathbb{E}_t^{cb} \xi_t + \frac{\nu_1}{\nu_0}\xi_t\right] + l.p.t. \tag{4.3}$$

$$= -\nu_1 \mathbb{E}_t^f[\xi_t] + l.p.t., \tag{4.4}$$

where I abstract from last period terms (*l.p.t.*) irrelevant to current welfare and productivity is constant at its steady state value ($a_t = 0$).[17] We conclude from Lemma 3 that the associated welfare losses are $\mathcal{W} = \frac{1}{1-\beta}\nu_1^2 \mathbb{V}\left[\mathbb{E}_t^f \xi_t\right]$. Equation (4.4) illustrates how additional central bank disclosure can be harmful for social welfare. Increases to τ_ω^μ always increase $\mathbb{V}\left[\mathbb{E}_t^f \xi_t\right]$ and thus $\mathcal{W}$. Additional central bank disclosure increases firms' responses to the inefficient mark-up shock, causing further fluctuations in output, despite constant productivity.

Proposition 2. *Suppose that only mark-up shocks drive the economy, $\tau_\theta \to \infty$, and that the money supply is held fixed, $\phi_\xi = 0$. Then, complete opacity $\tau_\omega^\mu \to 0$ is uniquely optimal.*

The result in Proposition 2 essentially replicates that in Hellwig (2005) and Angeletos and Pavan (2007) for the case where price dispersion is muted since firms, among themselves, have common knowledge. Because welfare losses from (4.4) increase monotonically in central bank disclosure, complete opacity is uniquely optimal when the money supply is held fixed.

A Benefit of Disclosure with Incomplete Common Knowledge: Proposition 2 contrasts with that which arises when monetary policy responds actively. In fact, disclosure can

[17] I will henceforth abstract from last period terms in all derivations of welfare.

become invariably beneficial once we allow for an active response of monetary policy. The reason is that to set monetary policy correctly the central bank needs to know firms' responses, and thus their expectations of the mark-up shock. Disclosure helps in this respect.

To see why, consider the case in which $\phi_\xi = \phi_\xi^{\star,full} = \frac{\nu_1}{1-\nu_0} > 0$. That is, consider the special case in which monetary policy undoes the nominal frictions and is set to its optimal full information value from Section 3. We can in this case write (4.3) as

$$\begin{aligned} y_t - a_t = m_t - p_t &= \frac{1}{1-\nu_0}\nu_1 \mathbb{E}_t^{cb}\left[\xi_t\right] - \frac{\nu_0}{1-\nu_0}\nu_1 \mathbb{E}_t^{f}\left[\mathbb{E}_t^{cb}\xi_t\right] - \nu_1 \mathbb{E}_t^{f}\left[\xi_t\right] + l.p.t. \\ &= \nu_1\left(\mathbb{E}_t^{cb}\left[\xi_t\right] - \mathbb{E}_t^{f}\left[\xi_t\right]\right) + \frac{\nu_0}{1-\nu_0}\nu_1\left(\mathbb{E}_t^{cb}\left[\xi_t\right] - \mathbb{E}_t^{f}\left[\mathbb{E}_t^{cb}\xi_t\right]\right) + l.p.t. \quad (4.5) \end{aligned}$$

Equation (4.5) shows that welfare losses only arise due to a lack of common knowledge between firms and the central bank, $\mathbb{E}_t^{cb}\left[\xi_t\right] \neq \mathbb{E}_t^{f}\left[\xi_t\right] \neq \mathbb{E}_t^{f}\mathbb{E}_t^{cb}\left[\xi_t\right]$, when $\phi_\xi = \phi_\xi^{\star,full}$. With common knowledge, by contrast, $\mathbb{E}_t^{cb}\left[\xi_t\right] = \mathbb{E}_t^{f}\left[\xi_t\right] = \mathbb{E}_t^{f}\mathbb{E}_t^{cb}\left[\xi_t\right]$, and the central bank would with $\phi_\xi = \phi_\xi^{\star,full}$ perfectly replicate the first best outcome by correctly offsetting firms' responses to the mark-up shock. This dependence of welfare on the extent of common knowledge illustrates how disclosure helps increase the efficacy of monetary policy.

Consider the extreme case in which the central bank has full information about the mark-up shock, $\tau_z^\mu \to \infty$. Full disclosure would then result in the worst possible outcome if monetary policy remained constant, $\phi_\xi = 0$. By contrast, when the central bank fully discloses its information when $\phi_\xi = \phi_\xi^{\star,full} > 0$, it replicates the first best outcome.

The more precisely the central bank discloses its information, the more firms will use it to form their own expectations, and the more firms' expectations will resemble the central bank's. In fact, when the central bank in this case sets $\tau_\omega^\mu \to \infty$, $\mathbb{E}_t^{f}\left[\xi_t\right] = \mathbb{E}_t^{cb}\left[\xi_t\right] = \xi_t$ and $\mathbb{E}_t^{f}\mathbb{E}_t^{cb}\left[\xi_t\right] = \mathbb{E}_t^{cb}\left[\xi_t\right] = \xi_t$. Disclosure increases common knowledge, which here makes firms' expectations about the mark-up shock equal to the central bank's and eliminates higher-order uncertainty. Full disclosure creates full common knowledge. This, in turn, allows the central bank to perfectly counter firms' responses to the mark-up shock since it knows all about them.

A straightforward application of the *dual* approach to optimal policy shows that this reasoning extends to the case in which the central bank has imperfect information, $\tau_z^\mu \in \mathbb{R}_+$.

Theorem 1. *The unique optimal policy with mark-up shocks is full disclosure, $\tau_\omega^{\mu,\star} \to \infty$, combined with monetary policy that undoes the nominal friction, $\phi_\xi^\star = \frac{\nu_1}{1-\nu_0} > 0$. Increases in central bank disclosure increase common knowledge between firms and the central bank.*

We can separate the benefit of disclosure in Theorem 1 into two distinct components: Disclosure does not only allow the central bank to better counter firms' response to the mark-up shock, but also decreases firms' responses to start with.

We can see this additional benefit of central bank disclosure from the second term in (4.5). The more precisely the central bank discloses its information, the more firms' also know about central bank expectations ($\mathbb{E}_t^f \mathbb{E}_t^{cb}[\xi_t]$ is closer to $\mathbb{E}_t^{cb}[\xi_t]$, on average). Disclosure therefore also increases firms' knowledge about prospective central bank responses to the mark-up shock already at the stage where firms pre-set prices. Any attempted stabilization of firms' prices by the central bank will therefore be better anticipated, and hence more effective. Disclosure thus also increases the effectiveness of prospective monetary policy.

In sum, although firms know more about the mark-up shock under full disclosure, combined these two benefits alleviate the errors in monetary policy that the lack of common knowledge otherwise entails to such an extent that full disclosure becomes optimal.

Learning from Prices: I have so far simplified the exposition by assuming that $\tau_p \to 0$ such that the central bank does not learn about firms' expectations from the noisy observation of the price level. None of the main insights, however, depend critically on this assumption. Appendix B shows how Proposition 1, 2, and Theorem 1 readily extend to the case in which the central bank learns about firms' expectations from the noisy observation of the price level; that is to the case where τ_p is positive. The central bank still optimally uses monetary policy to undo the nominal friction, $\phi_\xi^\star = \frac{\nu_1}{1-\nu_0}$; and conditional on this value of ϕ_ξ, full disclosure is once more optimal because it increases the efficacy of monetary policy.[18]

I postpone the discussion of how central bank disclosure also increases the information content of the price level in this case to the next section.

Other Monetary Policy Rules: I conclude this section with studying the consequences of an alternative monetary policy rule. While the monetary policy rule in (2.8) makes the analysis leading up to Theorem 1 particularly convenient, it is not central to the conclusions from Theorem 1. Suppose that instead of (2.8) the central bank directly targets the variable that causes fluctuations in the output gap, the price level, in the case where τ_p is finite,

$$m_t = m_{t-1} + \phi_0 + \phi_p \mathbb{E}_t^{cb}[p_t], \tag{4.6}$$

and suppose moreover that the cash-in-advance constraint always binds.[19] The central bank can with (4.6) still replicate the flex-price, first best outcome when it itself has full information

[18]One might think that monetary policy should differ when τ_p is finite, to account for how much the central bank learns from the price level about firms' private information. But notice that the informativeness of the price level under full disclosure is independent of monetary policy. Even though the price level becomes more stable as we increase $\phi_\xi \in \left[0, \phi_\xi^{\star,full}\right]$, the central bank can under full disclosure perfectly account for how much more stable since it knows firms' expectations about its own beliefs (see Section 5). The informativeness of the price level therefore remains constant, and thus $\phi_\xi^\star = \phi_\xi^{\star,full}$ when $\tau_\omega^\mu \to \infty$.

[19]We can indeed always set ϕ_0 such that this is the case.

about the mark-up shock (with $\phi_p^{\star,full} = 1$ and $\tau_\omega^\mu \to \infty$).

Equilibrium prices from (3.4) can be combined with (4.6) to show that

$$\begin{aligned} p_t &= \nu_{-1} p_{t-1} + \nu_0 \mathbb{E}_t^f [m_t] + \nu_1 \mathbb{E}_t^f [\mu_t] \\ &= \nu_1 \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^j [\xi_t] + l.p.t., \end{aligned} \tag{4.7}$$

where $\left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^j [\xi_t]$ is defined by the recursion $\left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^j [\xi_t] = \mathbb{E}_t^{cb} \mathbb{E}_t^f \left\{ \left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^{j-1} [\xi_t] \right\}$ with $\left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^0 [\xi_t] = \xi_t$, and I abstract from irrelevant constant terms. Equilibrium prices can thus be described by a weighted sum of an entire infinite sequence of higher-order expectations, comprised of firms' expectations of central bank expectations and *vice versa*.

The corresponding output gap, in this case, becomes

$$\begin{aligned} y_t - a_t &= \phi_p \nu_1 \mathbb{E}_t^{cb} \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^j [\xi_t] - \nu_1 \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^j [\xi_t] + l.p.t. \\ &= \nu_1 \left\{ \phi_p \mathbb{E}_t^{cb} \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^j [\xi_t] - \mathbb{E}_t^f \sum_{j=0}^{\infty} (\nu_0 \phi_p)^j \left(\mathbb{E}_t^{cb} \mathbb{E}_t^f\right)^j [\xi_t] \right\} + l.p.t. \end{aligned} \tag{4.8}$$

Equation (4.8) shows that when the central bank undoes the nominal frictions and sets ϕ_p to its optimal full information value, $\phi_p = 1$, welfare losses once more only arise from a lack of common knowledge. When the central bank then also fully discloses its information and sets $\tau_\omega^\mu \to \infty$, the expression for the output gap in (4.8) collapses to

$$y_t - a_t = m_t - p_t = \frac{\nu_1}{1 - v_0} \left(\mathbb{E}_t^{cb} \mathbb{E}_t^f [\xi_t] - \mathbb{E}_t^f [\xi_t] \right) + l.p.t. \tag{4.9}$$

Indeed, following a similar approach to that above shows that $\phi_p^\star = 1$ and $\tau_\omega^{\mu,\star} \to \infty$ describe the optimal policy when the central bank targets the price level.

Combined, (4.8) and (4.9) demonstrate how the results from the simple monetary policy rule in (2.8) carry over with more force to the extended case studied in (4.6). Disclosure now decreases uncertainty about the entire infinite sequence of higher-order expectations that make up the price level. As before, this decrease in higher-order uncertainty alleviates the errors in current and prospective monetary policy that the lack of common knowledge otherwise entails: It allows the central bank to better counter firms' responses to the mark-up shock since it knows more about them. And conversely, it also allows firms to better anticipate central bank actions, and hence decreases firms' responses to the mark-up shock already at the stage where they pre-set prices. Combined, this once more makes full disclosure optimal.

In fact, (4.9) shows that welfare losses under the optimal policy are only due to the remaining errors in central bank beliefs about firms' expectations. These still arise with full disclosure because the noisy signal of the price level does not perfectly reveal firms' private information when τ_p is finite. There is thus a sense in which welfare losses under the optimal policy are only due to the remaining lack of common knowledge between the central bank and firms; that which arises because the central bank does not perfectly know firms' private information. The next section demonstrates how central bank disclosure also decreases this residual uncertainty by increasing the information content of the price level.

5 Disclosure and the Paradox of Transparency

I now shift the focus from inefficient mark-up (cost-push) shocks to another influential cost of disclosure. This cost, occasionally referred to as the "*Paradox of Transparency*", stipulates how one of the consequences of central bank disclosure is that the central bank has to rely on less informative prices to steer monetary policy (see Morris and Shin, 2005 and Amador and Weill, 2010).[20] In this section, I show how a lack of common knowledge between firms and the central bank can also qualify this second perceived cost of disclosure.

Equilibrium with Productivity Shocks: Since this cost does not depend on the precise source of economic fluctuations, I focus on the special case in which only productivity shocks drive the economy, $\tau_\xi \to \infty$. This allows me to cleanly separate the effects of disclosure from those discussed in the previous Section. I solve for the set of symmetric linear Bayesian equilibria when $\tau_\xi \to \infty$ using the three-step approach outlined in Section 4.

Proposition 3. *The set of symmetric, linear equilibria with productivity shocks is non-empty, and is comprised of firm prices and associated central bank money supply,*

$$p_t = \nu_{-1} p_{t-1} + \nu_0 \left(m_{t-1} - a_{t-1}\right) + \mathtt{k}_0 x_t^a + \mathtt{k}_1 \omega_t^a + \mathtt{k}_2 \underline{p}_t \tag{5.1}$$

$$m_t = m_{t-1} + \mathtt{q}_0 z_t^a + \mathtt{q}_1 \underline{p}_t \tag{5.2}$$

where $\underline{p}_t = \theta_t + \epsilon_{xt}^a + \frac{1}{\mathtt{k}_0}\epsilon_{pt}$, $\mathtt{k}_0 \in (-1,\,0)$, $\mathtt{q}_0 \in (0,\,1)$ *and all remaining coefficients* $\{\mathtt{k}_1,\,\mathtt{k}_2,\,\mathtt{q}_1\}$ *are constants that depend only on* $\mathtt{k}_0$, $\mathtt{q}_0$ *and the parameters of the model.*

Equilibrium Selection: While Proposition 3 establishes the existence of a linear equilibrium when τ_p is finite, the economy can, however, admit multiple, linear equilibria (either one or three), because of the potential for firms and the central bank to learn from each others actions. This multiplicity introduces a well-known impediment to any welfare analysis. One

[20] See also Amato *et al.* (2002), Amato and Shin (2006), and the related work on the *learning externality* of additional public information in markets where agents learn from prices (see, for instance, Vives, 1997; Amador and Weill, 2012; Vives, 2017; and the summary in Veldkamp, 2011.)

has to decide on which equilibrium agents coordinate, and if so what the comparative statics are in each case. I circumvent this problem in Appendix C by focusing on *the highest welfare equilibrium,* in line with Harsanyi and Selten's (1988) *Pay-off Dominance Argument*, and thus abstract from any possible coordination failures (see also Amador and Weill, 2010). Appendix C shows how none of my results depend crucially on the exact equilibrium selection device used. All hold in areas of the parameter space where the equilibrium is unique.

The Informativeness of Prices: The coefficients $\mathtt{k}_0$ and $\mathtt{q}_0$ in Proposition 3 are central to the analysis in this section. Indeed, we can collect the various elements that make up the noisy signal of the price level observed by the central bank on the left-hand side of (2.12) using (5.1) and divide by $\mathtt{k}_0$ to arrive at,

$$\frac{1}{\mathtt{k}_0}\left[\bar{p}_t - \nu_{-1}p_{t-1} - \nu_0\left(m_{t-1} - a_{t-1}\right) - \mathtt{k}_1\omega_t^a - \mathtt{k}_2\underline{p}_t\right] = x_t^a + \frac{1}{\mathtt{k}_0}\epsilon_{pt}.$$

This demonstrates how the observation of the price level is equivalent to $x_t^a + \frac{1}{\mathtt{k}_0}\epsilon_{pt}$, or $\theta_t + \epsilon_{xt}^a + \frac{1}{\mathtt{k}_0}\epsilon_{pt}$ since the central bank can also condition out last period's productivity a_{t-1}.[21] I equate this signal to $\underline{p}_t$ in Proposition 3. When $\mathtt{k}_0^2\tau_p$ is large, the price level conveys a precise signal of firms' private information to the central bank, and conversely when $\mathtt{k}_0^2\tau_p$ is small.

Corollary 2. *The equilibrium coefficients* $\mathtt{k}_0$ *and* $\mathtt{q}_0$ *solve*

$$\mathtt{k}_0 = \nu_0\tau_x^a\frac{\mathtt{q}_0\tau_z^a - \left(\tau_\omega^a + \tau_z^a\right)}{\left(\tau_\omega^a + \tau_z^a\right)\left(\tau_\theta + \tau_x^a\right) + \tau_\omega^a\tau_z^a}, \quad \mathtt{q}_0 = \phi_\theta\frac{\tau_z^a\left(\tau_x^a + \tau_p\mathtt{k}_0^2\right)}{\left(\tau_x^a + \tau_p\mathtt{k}_0^2\right)\left(\tau_\theta + \tau_z^a\right) + \tau_x^a\tau_p\mathtt{k}_0^2}, \tag{5.3}$$

where all solutions of $\mathtt{k}_0 \in (-1,\, 0)$ *and* $\mathtt{q}_0 \in (0,\, 1)$.

The two coupled equations in Corollary 2 describe a fixed point problem, which connects the equilibrium weight that firms attach to their private information $\mathtt{k}_0$ in Proposition 3, to the weight that the central bank itself accords to its own private information $\mathtt{q}_0$, and to the variance of the noise in the price level $\mathtt{k}_0^{-2}\tau_p^{-1}$. The more weight firms attach to their private information, the more informative the price level, all else equal, becomes to the central bank.[22]

A Learning Externality of Disclosure: Central bank disclosure comes at a notable cost within this setting, besides the obvious benefit of providing firms with more information.

Proposition 4. *Suppose* $\phi_\theta \leq \hat{\phi}_\theta = \frac{\tau_\theta + \tau_z^a + \alpha}{\tau_\theta + \tau_z^a + \tau_x^a} < 1$, *where* $\alpha = \frac{\tau_x^a\tau_p\mathtt{k}_0^2}{\tau_x^a + \tau_p\mathtt{k}_0^2}$. *Then, increases to central bank disclosure* $\tau_\omega^a \in \mathbb{R}_+$ *decrease the information content of the price level* $\mathtt{k}_0^2\tau_p$, *and hence the central bank's own information about the productivity shock,* θ_t.

[21] I will, for brevity, from now on refer to "the informativeness of the price level". By that, I always mean the "informativeness of the *noisy signal* of the price level".

[22] See Amador and Weill (2010) and Vives (2017) for particularly clear expositions of the equilibrium link between the weight that agents attach to their private information and the informativeness of prices.

Proposition 4 demonstrates a "*Paradox of Transparency*"; that central bank disclosure decreases the central bank's own information about shocks to the economy. This, all else equal, increases welfare losses because monetary policy becomes set based on worse information.

Suppose the central bank discloses additional public information. All else equal, this additional information decreases firms' uncertainty about productivity. But the corollary of this decrease in uncertainty is that firms now rely less on their own private information when forming their expectations and more on the information from the central bank, which the central bank already knows. A simple total differentiation of the fixed point equation in Corollary 2 shows that $\frac{d|\mathtt{k}_0|}{d\tau_\omega^a} < 0$ whenever $\phi_\theta < \frac{\tau_\theta+\tau_z^a+\alpha}{\tau_\theta+\tau_z^a+\tau_x^a}$.[23] As a result, the informativeness of the price level falls. It reflects less firms' private information, the truly new information that the central bank could learn from firms. This, in turn, increases central bank uncertainty and the associated welfare losses from monetary policy being set based on worse information.

At the core of this adverse effect lies a learning externality. When deciding on how much to respond to their own private information, firms do not internalize the informativeness of the price level, and hence how much the central bank is able to learn from it. Because of this externality, a fundamental trade-off arises between, on the one hand, firms' uncertainty and thus their ability to correctly set prices and, on the other hand, the central bank's own information and hence its own capacity to correctly set monetary policy.

A Sharing Benefit of Disclosure: I now turn to how the presence of higher-order uncertainty can qualify this cost of central bank disclosure. To accommodate for the endogenous informativeness of the price level, I solve for the optimal use of central bank information with a mix of the *primal* and *dual* approach (see Appendix C).

Theorem 2. *When productivity shocks drive the economy, the unique optimal policy is full disclosure, $\tau_\omega^{a,\star} \to \infty$, combined with monetary policy that undoes the nominal friction, $\phi_\theta^\star = 1$. Increases in central bank disclosure globally increase the informativeness of the price level.*

Theorem 2 shows how monetary policy should once more be set to undo the nominal friction, to its full information value from Section 3; and that conditional on this value, full disclosure maximizes the informativeness of the price level. As a result, full disclosure decreases both firm and central bank uncertainty by the largest possible amount. The "*Paradox of Transparency*" becomes a simple "*Benefit of Clarity*".

The Commonality of Beliefs and Welfare: To see the effect that counteracts the standard learning externality, consider to start the output gap that arises from Corollary 1 when only

[23] The derivative of *the right-hand side* of (5.3) with respect to $\mathtt{q}_0$ equals: $\frac{\partial RHS}{\partial \tau_\omega^a} = \nu_0 \frac{\tau_x^a \tau_z^a}{[(\tau_\omega^a+\tau_z^a)(\tau_\theta+\tau_x^a)+\tau_\omega^a\tau_z^a]^2}\left[\tau_z^a - \mathtt{q}_0\left(\tau_\theta+\tau_x^a+\tau_z^a\right)\right]$. Thus, $\frac{\partial RHS}{\partial \tau_\omega^a} > 0$ and hence $\frac{d\mathtt{k}_0}{d\tau_\omega^a} > 0$ *iff.* $\mathtt{q}_0 < \frac{\tau_z^a}{\tau_\theta+\tau_x^a+\tau_z^a}$. The latter is equivalent to $\phi_\theta \leq \frac{\tau_\theta+\tau_z^a+\alpha}{\tau_\theta+\tau_z^a+\tau_x^a}$ from (5.3).

productivity shocks drive the economy,

$$y_t - a_t = m_t - p_t - a_t \;=\; \phi_\theta \mathbb{E}_t^{cb}[\theta_t] - \nu_0 \mathbb{E}_t^f\left[\phi_\theta \mathbb{E}_t^{cb}\theta_t - \theta_t\right] - \theta_t + l.p.t. \tag{5.4}$$

$$= \mathbb{E}_t^{cb}[\theta_t] + \nu_0 \mathbb{E}_t^f[\theta_t] - \nu_0 \mathbb{E}_t^f \mathbb{E}_t^{cb}[\theta_t] - \theta_t + l.p.t., \tag{5.5}$$

where I have set $\phi_\theta = \phi_\theta^\star = 1$ and once more abstract from last period terms irrelevant to current welfare. Public disclosure of central bank information has three distinct effects on this expression, the first two of which are identical to those discussed in Section 4. Central bank disclosure increases the commonality of beliefs since both firms and the central bank use the disclosed information. This, in turn, (*i*) increases the extent to which firms' expectations of the productivity shock resemble the central bank's ($\mathbb{E}_t^f\theta_t$ is closer to $\mathbb{E}_t^{cb}\theta_t$, on average); and (*ii*) decreases higher-order uncertainty such that firms' beliefs about the central bank's expectations more closely align with the central bank's own expectations ($\mathbb{E}_t^f\mathbb{E}_t^{cb}\theta_t$ is closer to $\mathbb{E}_t^{cb}\theta_t$, on average). Both effects once more increase the efficacy of monetary policy.

The third effect (*iii*) is, however, new to the setup with productivity shocks. It arises from how disclosure, by increasing the commonality of beliefs, also increases the informativeness of the price level. This, in turn, makes the central bank's own expectation of the productivity shock more accurate ($\mathbb{E}_t^{cb}\theta_t$ is closer to θ_t, on average). Similar to (*i*) and (*ii*), this third effect increases the ability of monetary policy to replicate the full information first best outcome. The more the central bank knows about the productivity shock, the lower the associated welfare losses are when $\phi_\theta = \phi_\theta^\star = 1$.[24] But why does disclosure increase the informativeness of the price level, and hence the central bank's information to start with?

The Commonality of Beliefs and the Informativeness of Prices: We can discern this third effect from the equation for $\mathtt{k}_0$ in Corollary 2 that determines the informativeness of the price level. Total differentiation of this equation shows, after some elementary but tedious algebra, that $\frac{d\mathtt{k}_0}{d\tau_\omega^a} \gtreqless 0$ when $\psi > 0$ and $\phi_\theta > 0$, depending on the precise value of $\mathtt{q}_0$, and hence ϕ_θ chosen by the central bank.[25] I summarize this extension of Proposition 4 in Proposition 5.

[24] The reason that this third effect did not arise in Section 4 is that only firms' expectations of the mark-up shock, in addition to the central bank's expectations and firms' expectations of the central bank's expectations, mattered in equilibrium for the output gap (see 4.5). The realization of the shock itself did not matter. By contrast, the realization of the productivity shock here directly matters for (5.5).

[25] Specifically, the partial derivative of the *right-hand side (RHS)* of (5.3) with respect to τ_ω^a equals

$$\frac{\partial RHS}{\partial \tau_\omega^a} = \nu_0 \frac{\tau_x^a \tau_z^a}{\left[(\tau_\omega^a + \tau_z^a)(\tau_\theta + \tau_x^a) + \tau_\omega^a \tau_z^a\right]^2}\left[\tau_z^a - \mathtt{q}_0\left(\tau_\theta + \tau_x^a + \tau_z^a\right)\right].$$

Hence, $\frac{\partial RHS}{\partial \tau_\omega^a} \gtreqless 0$, and thus $\frac{dk_0}{d\tau_\omega^a} \gtreqless 0$ *iff.* $q_0 \lesseqgtr \frac{\tau_z^a}{\tau_\theta + \tau_x^a + \tau_z^a}$. From (5.3), the latter is equivalent to $\phi_\theta \lesseqgtr \frac{\tau_\theta + \alpha + \tau_z^a}{\tau_\theta + \tau_x^a + \tau_z^a} = \hat{\phi}_\theta$, where $\alpha = \frac{\tau_x^a \tau_p \mathtt{k}_0^2}{\tau_x^a + \tau_p \mathtt{k}_0^2}$ denotes the precision of $\underline{p}_t$.

Proposition 5. *Increases in central bank disclosure, $\tau^a_\omega \in \mathbb{R}_+$, either decrease or increase the informativeness of the price level, $\tau_p \mathtt{k}_0^2$. This depends on whether $\phi_\theta \lesseqgtr \hat{\phi}_\theta$.*

The intuition behind the mechanism that counters the learning externality, and dominates it when $\phi_\theta > \hat{\phi}_\theta$, follows from the stabilization of the price level that accommodative monetary policy entails, $\phi_\theta \in (0,\, 1)$. We can see this stabilization immediately from Corollary 1,

$$p_t = \nu_0 \mathbb{E}^f_t \left[\phi_\theta \mathbb{E}^{cb}_t \theta_t - \theta_t\right] + l.p.t. \tag{5.6}$$

As we increase $\phi_\theta \in (0,\, 1)$, the volatility of the price level in (5.6), all else equal, declines. But when the central bank commits to use monetary policy to stabilize firms' demand and hence prices, it also directly decreases firms' uncertainty. Yet, as with the learning externality, this decrease in firms' uncertainty comes at a cost; the cost of a less informative price level since firms now update their beliefs less with their own private information. Equation (5.3) shows how increases to ϕ_θ decrease $|\mathtt{k}_0|$.[26]

This stabilization of the price level creates an *identification problem* for the central bank. Suppose the central bank observes a constant price level from one period to the next. All else equal, this observation could be because *(a)* firms received private information in line with their prior, or *(b)* because all firms received different information but expect the central bank to alter the money supply in response such that prices remain constant (see 5.6). *Central bank disclosure solves this identification problem.* By making the central bank's own information, and hence beliefs, common knowledge, disclosure offers the distinction between *(a)* and *(b)*. Central bank disclosure increases the central bank's knowledge about what firms expect of its own beliefs. This, in turn, increases the informativeness of the price level, despite it being less volatile. Indeed, when monetary policy stabilizes the price level sufficiently $\phi_\theta > \hat{\phi}_\theta$, as it chooses to do under the optimal policy, this increase in central bank knowledge about firms' higher-order expectations dominates the learning externality. Central bank disclosure then decreases uncertainty for everyone, even the central bank itself.

Other Monetary Policy Rules: As in Section 4, neither of the above results depend on the specificities of the monetary policy in (2.8). Suppose that the central bank instead of (2.8) directly targets what causes changes to the output gap in (5.4), the price level and labor

[26] The partial derivate of the *right-hand side (RHS)* of (5.3) with respect to ϕ_θ is

$$\frac{\partial RHS}{\partial \phi_\theta} = \nu_0 \frac{\tau^a_x \tau^a_z}{\left(\tau^a_\omega + \tau^a_z\right)\left(\tau_\theta + \tau^a_x\right) + \tau^a_\omega \tau^a_z} \frac{\tau^a_z \left(\tau^a_x + \tau_p \mathtt{k}_0^2\right)}{\left(\tau^a_x + \tau_p \mathtt{k}_0^2\right)\left(\tau_\theta + \tau^a_z\right) + \tau^a_x \tau_p \mathtt{k}_0^2}.$$

Thus, $\frac{\partial RHS}{\partial \phi_\theta} > 0$, and hence $\frac{d\mathtt{k}_0}{d\phi_\theta} > 0$. But since $\mathtt{k}_0 < 0$, we arrive at $\frac{d|\mathtt{k}_0|}{d\phi_\theta} < 0$.

productivity,

$$\begin{aligned} m_t &= m_{t-1} + \phi_0 + \phi_\theta \mathbb{E}_t^{cb}\left[a_t\right] + \phi_p \mathbb{E}_t^{cb}\left[p_t\right] \\ &= \phi_\theta \mathbb{E}_t^{cb}\left[\theta_t\right] + \phi_p \mathbb{E}_t^{cb}\left[p_t\right] + l.p.t. \end{aligned} \tag{5.7}$$

Moreover, suppose that the central bank considers to set $\phi_\theta = \phi_p = 1$ and to fully disclose its information $\tau_\omega^a \to \infty$.[27] This is, in fact, the optimal policy when the central bank follows (5.7) (see Appendix C) and also replicates the first best outcome under full information.

Equilibrium prices with (5.7) then become

$$\begin{aligned} p_t &= \nu_0 \mathbb{E}_t^f\left[\mathbb{E}_t^{cb}\theta_t + \mathbb{E}_t^{cb} p_t - \theta_t\right] + l.p.t. \\ &= \nu_0\left(\mathbb{E}_t^{cb}\theta_t - \mathbb{E}_t^f\theta_t\right) + \frac{\nu_0^2}{1-\nu_0}\left(\mathbb{E}_t^{cb}\theta_t - \mathbb{E}_t^{cb}\mathbb{E}_t^f\theta_t\right) + l.p.t., \end{aligned} \tag{5.8}$$

such that the corresponding expression for the output gap is

$$y_t - a_t = m_t - p_t - a_t = \left(\mathbb{E}_t^{cb}\left[\theta_t\right] - \theta_t\right) - \nu_0\left(\mathbb{E}_t^{cb}\mathbb{E}_t^f\theta_t - \mathbb{E}_t^f\theta_t\right) + l.p.t. \tag{5.9}$$

Similar to (5.5), all that matters for welfare in (5.9) is the extent to which the central bank's expectation of the productivity shock is accurate, in addition to the extent to which there is common knowledge between firms and the central bank. Central bank disclosure increases the extent of common knowledge and with (5.7) eliminates an entire infinite sequence of higher-order expectations that would otherwise arise in (5.8) (see 4.7 for a comparison). By doing so, disclosure increases the informativeness of the price level, and hence the accuracy of central bank expectations. This, in turn, contributes to the optimality of disclosure.

Discussion: The mechanism behind Theorem 2 is at its core the same as that which underlined Theorem 1: that the truthful disclosure of private information from one side to another informs both sides about each other's beliefs. In the previous section, I showed how this decrease in higher-order uncertainty combines with the active role for monetary policy that nominal frictions entail to make full disclosure about inefficient mark-up shocks beneficial. By contrast, in this section I have considered one further implication of the decrease in higher-order uncertainty. I have illustrated how the decrease in central bank uncertainty about firms' expectations of the central bank's own information, and so on, can overcome a standard learning externality and increase the informativeness of prices. This, in turn, provides a new role for central bank disclosure: Disclosure not only decreases private sector uncertainty but also the central bank's, which allows the central bank to more accurately set monetary policy.

[27] I once more set ϕ_0 such that the cash-in-advance constraint binds.

6 A Quantitative Extension

The analysis that I have covered has shown how an absence of common knowledge between firms and the central bank can alter the desirability of central bank disclosure. The results were arrived at, however, only for a very particular model intended to make the exposition as simple as possible. The next question must be to what extent these results generalize.

To this end, I solve in this section an extended version of the above model, which resembles that of Lorenzoni (2009). Unlike the model described in Section 2, the extended model features an imperfectly informed household and a potent signaling role for monetary policy. Both features are important for an accurate assessment of the social benefits of central bank disclosure.[28] The next Section then uses the extended model to take a first pass at two basic quantitative questions. First, is disclosure indeed beneficial within this extended framework? In particular, does the increase in common knowledge brought on by central bank disclosure outweigh the two aforementioned costs for plausible parameter values. And second, if so, are the welfare benefits substantial?

Preferences and Technology: I amend the setup from Section 2, while keeping the timing of events unchanged. The representative household's preferences once more equal

$$\mathcal{U} = \mathbb{E}_0^h \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \frac{1}{1+\eta} L_t^{1+\eta} \right], \tag{6.1}$$

where, unlike in Section 2, the household now also has imperfect information about the fundamentals of the economy and bases its expectations $\mathbb{E}_t^h\left[\cdot\right] = \mathbb{E}\left[\cdot \mid \Omega_t^h\right]$ upon the information set Ω_t^h (described below). Similar to Svensson and Woodford (2004), the household and firms are assumed to share their information, such that $\Omega_t^h = \Omega_t^f$.[29] The consumption index C_t and the associated welfare-based price index P_t are identical to those from Section 2.

I dispense with the cash-in-advance constraint (2.4) and instead assume that money is held in bank deposits, earning an interest rate of i_t, as in Lorenzoni (2009). The household's budget constraint therefore in place of (2.3) becomes

$$\int_0^1 P_{it} C_{it} di + \left(1 + i_t\right) M_t^d \leq \int_0^1 \Pi_{it} di + W_t L_t + M_{t-1}^d + T_t^h. \tag{6.2}$$

The central bank, which controls the interest rate on bank deposits, follows a simple *Taylor*

[28] See, for instance, Walsh (2007) and Baeriswyl and Cornand (2010).

[29] This assumption can, in turn, be based on the presumption that the household and firms observe each others' information when firms set prices, since the household is employed by firms.

Rule, in which it targets deviations of log-output from its full information flex-price levels,

$$i_t = \exp\left(\mathbb{E}_t^{cb}\left[y_t - a_t\right]\right)^{\phi} \exp\left(\epsilon_{mt}\right), \tag{6.3}$$

where $\epsilon_{mt} \sim \mathcal{WN}\left(0,\, \tau_m^{-1}\right)$ denotes a monetary policy shock.[30]

Last, I keep the production side of the economy unchanged with the exception of the stochastic processes for productivity and mark-ups. I now assume that both of these follow stationary *AR(1)s* in logarithms, in which $\rho_a \in (0,\, 1)$ and $\rho_\mu \in (0,\, 1)$,

$$A_t = A_{t-1}^{\rho_a} \exp\left(\theta_t\right), \quad \mathcal{M}_t = \mathcal{M}_{t-1}^{\rho_\mu} \exp\left(\xi_t\right). \tag{6.4}$$

Linearized Equilibrium Conditions: I once more study a log-linear approximation to the rational expectation equilibria. Following well-known steps, Appendix D shows that the equilibrium conditions of the model reduce to three key log-linear equations. First, an *Euler equation,* which determines the optimal intertemporal allocation of consumption and output,

$$y_t = \mathbb{E}_t^{h}\left[y_{t+1}\right] - \left(i_t - \mathbb{E}_t^{h}\left[\pi_{t+1}\right]\right), \tag{6.5}$$

where π_t denotes the inflation rate for the consumption index. Second, a *New-Keynesian Phillips Curve*, which relates firms' expectations of mark-ups and marginal cost, proportional to the output gap, to expected future and current inflation,

$$\pi_t = \beta\mathbb{E}_t^{f}\left[\pi_{t+1}\right] + \lambda\mathbb{E}_t^{f}\left[y_t - a_t\right] + \mathbb{E}_t^{f}\left[\mu_t\right] \tag{6.6}$$

where $\lambda = \frac{1+\eta}{\psi}\rho$. And third, a log-linear central bank Taylor Rule,

$$i_t = \phi\mathbb{E}_t^{cb}\left[y_t - a_t\right] + \epsilon_{mt}. \tag{6.7}$$

Combined, (6.3) to (6.6) closely resemble the equilibrium conditions of the dispersed information New Keynesian model in Lorenzoni (2009), extended to the case where the private sector and the central bank lack common knowledge about each others' beliefs. Nimark (2014) and Melosi (2016) consider other extensions of Lorenzoni's (2009) framework. In particular, Melosi (2016) studies the case in which firms learn from movements in the central bank interest rate about the shocks that hit the economy. I accommodate for such signaling effects from changes to the interest rate in the below information structure.

[30] The introduction of the monetary policy shock ϵ_{mt} in (6.3) here serves a technical purpose, similar to that of ϵ_{pt} in Section 2. It prevents the private sector from perfectly inferring the central bank's private information directly from observations of the central bank interest rate.

Information Structure: The information structure mirrors that from Section 2. The private sector, comprised of firms and the representative household, observes its own private information about productivity and mark-ups, in addition to the central bank's disclosure. The only addition is that the private sector now also observes and learns about central bank beliefs from the current value of the central bank interest rate,

$$\Omega_t^f = \{x_{t-j},\, \omega_{t-j},\, \bar{\pi}_{t-j},\, i_{t-j}\}_{j=0}^{\infty} = \Omega_t^h. \tag{6.8}$$

The central bank's information set is similarly close to that from Section 2. Indeed, it is almost identical besides that the central bank now uses the (stationary) inflation rate to infer firms' private information instead of the (non-stationary) price level,

$$\Omega_t^{cb} = \{z_{t-j},\, \omega_{t-j},\, \bar{\pi}_{t-j},\, i_{t-j}\}_{j=0}^{\infty}. \tag{6.9}$$

where $\bar{\pi}_t = \pi_t + \epsilon_{pt}$ with $\epsilon_{pt} \sim \mathcal{N}\left(0,\, \tau_p^{-1}\right)$ denotes the noisy realization of the true rate of inflation observed by the central bank.

The information sets in (6.8) and (6.9) capture the two-sided flow of information between the private sector and the central bank that is central to Theorems 1 and 2. Finally, (6.8) and (6.9) dispense with the somewhat artificial assumption of one-period perfect state verification used in (2.13) and (2.14) and instead assume that neither the household, firms, nor the central bank observe last period's realization of the persistent shocks.

Solution and Calibration: Unlike the framework presented in Section 2, the equilibrium solution for the endogenous variables, such as output and inflation, can no longer be derived analytically. I therefore solve the model numerically instead.

The above model exhibits two features which render common solution methods for linear equilibria inapplicable. First, both the private sector and the central bank have private information about common disturbances. The combination of private information about common disturbances with the assumption of no-perfect state verification has since Townsend (1983) been known to imply that standard state space representations of the equilibrium have infinite-dimensional state vectors. This is due to the infinite regress of expectations that arises when agents need to "forecast the forecast of others". Second, the non-atomistic private sector and central bank learn about these common disturbances from market outcomes, such as inflation. Since inflation is forward-looking both it, in addition to the other simultaneously determined variables, depend on current private sector and central bank estimates of the state; yet the latter also depends on the dynamics of inflation. This circularity problem, which was also present in Sections 4 and 5, however, now telescopes due to the infinite state vector and further complicates the solution of the model.

In this subsection, I first describe how to adapt the solution method proposed in Nimark (2017) to the current setting, to find linear rational expectations equilibria in models with private information and non-atomistic agents.[31] I keep details to a minimum and focus on the important role that higher-order expectations, influenced by central bank disclosure, play in the equilibrium solution. I then calibrate the model separately for the mark-up shock and the productivity shock case to match data on forecast accuracy from the "*Survey of Professional Forecasters*" (for the private sector) and the "*Greenbook*" (for the central bank).[32] I calibrate for each persistent shock separately to avoid confounding the two distinct benefits of central bank disclosure discussed in Sections 4 and 5.

Dynamic Models with Two-sided Information: Nimark (2017) shows that when all disturbances are stationary, an arbitrarily precise approximate solution can be found to linear rational expectations equilibria in models with private information. This can be done by direct truncation of the state vector comprised of higher-order expectations, to achieve a finite-dimensional representation. I extend this solution method to deal with the additional complication of non-atomistic agents in Appendix D. When the model is solved, the approximate law of motion for the endogenous triplet $q_t = \begin{bmatrix} \pi_t & y_t & i_t \end{bmatrix}'$ admits the form

$$q_t = \alpha_0 X_t^{(0:\bar{k})} + \alpha_1 u_t, \tag{6.10}$$

where $u_t = \begin{bmatrix} \epsilon_t & \epsilon_{xt}^j & \epsilon_{zt}^j & \epsilon_{\omega t}^j & \epsilon_{pt} & \epsilon_{mt} \end{bmatrix}'$ with $j = \{a, \mu\}$ and $\epsilon_t = \{\theta_t, \xi_t\}$, depending on which of the two persistent shocks drives the economy. $X_t^{(0:\bar{k})}$ here denotes the expectational state vector comprised of the entire hierarchy of private sector and central bank higher-order expectations about the persistent fundamental $X_t^{(0)} = \{a_t, \mu_t\}$ up to the kth order,

$$X_t^{(0:\bar{k})} = \begin{bmatrix} X_t^{(0)} & X_t^{(1)\prime} & \dots & X_t^{(\bar{k})\prime} \end{bmatrix}', \quad X_t^{(k)} = \begin{bmatrix} \mathbb{E}_t^f X_t^{(k-1)} \\ \mathbb{E}_t^{cb} X_t^{(k-1)} \end{bmatrix}, \quad k \in \left[1,\, 2,\, \dots, \bar{k}\right]. \tag{6.11}$$

The true equilibrium law of motion has $\bar{k} \to \infty$. I truncate this expectational state vector at $\bar{k} = 50$. All impulse response functions are stable from around $\bar{k} = 15$.

An implicit assumption used to arrive at (6.10) is that it is common knowledge that all

[31] See also Nimark (2014), Melosi (2016), and Struby (2016) for other applications of this solution method.

[32] The Greenbook contains forecasts computed by the Staff of the Federal Reserve. These forecasts are published a few days before the FOMC meeting and collected with a five year lag in "*the Greenbook data set*" (Reifschneider *et al.*, 1997). The data for private sector forecasts comes from the *Survey of Professional Forecasters* conducted by the Federal Reserve Bank of Philadelphia. The sample stretches from 1968Q1 to 1993Q4. In February 1994, the Federal Reserve Market Committee began a long process of increased disclosure, altering the informational assumption used to calibrate the model (Dincer and Eichengreen, 2009).

agents form model consistent expectations. Similar to Nimark (2017), Appendix D shows that this assumption, combined with the Kalman Filter extended to the case in which both the private sector and the central bank can back-out part of the noise components of each others' signals, ensures that $X_t^{(0:\bar{k})}$ follows a *VAR(1)*,[33]

$$X_t^{(0:\bar{k})} = MX_{t-1}^{(0:\bar{k})} + Nu_t. \tag{6.12}$$

Because the private sector and the central bank learn from the observation of each other's actions, the matrices M and N here depend on the coefficients in α_0 and α_1, and *vice versa.* I solve for the fixed point $\{M,\, N\} \mapsto \{\alpha_0,\, \alpha_1\} \mapsto \{M,\, N\}$ by repeated iteration.

The finite-dimensional representation in (6.10) to (6.12) is particularly convenient for our purposes. Central bank disclosure affects the commonality of beliefs, and hence the effect of the kth order of higher-order expectations on the equilibrium dynamics. Equations (6.10) to (6.12) allow us to directly study the impact of such changes to higher-order uncertainty.

While convenient, the solution in (6.10) to (6.12) yet merely provides one tractable solution method. Other approaches exist: Townsend (1983) pioneered the use of lagged perfect state verification,[34] and more recently Rondina and Walker (2012), Kasa *et al.* (2013), and Huo and Takayama (2015) have demonstrated the applicability of frequency-domain techniques to solving models with private information. Nevertheless, when applied to the above setup, these methods turn out not to lend themselves so immediately to the below calibration procedure. Besides, the direct solution for higher-order expectations available from the chosen method is beneficial for studying the consequences of incomplete common knowledge.

Calibration: The parameter β is set to 0.99, such that the time period can be interpreted as one quarter. The inverse Frisch elasticity of labor supply η is set to one, and the value of the elasticity of substitution ρ to six, which implies a mark-up of 20%. The price adjustment parameter ψ is set such that the slope of the New Keynesian Phillip's Curve is $\lambda = 0.25$, and the standard deviation of the idiosyncratic monetary policy shock to two. The parameter on the interest rule is set such that the model with productivity shocks is consistent with a Taylor Rule coefficient on inflation of one-and-a–half. This implies $\phi = 1.81$. These values are all in the range of those used in existing studies within the New Keynesian framework.[35]

Next, I determine the parameters that control the mark-up and productivity shock, in addition to the noise in the private information about these. The persistence parameters for

[33] The central bank can, for example, from the observation of its own private information z_t and its own action i_t back out the value of the monetary policy shock ϵ_{mt} from (6.7). This is the shock that otherwise prevents the private sector from perfectly inferring z_t from the observation of i_t.

[34] See also, for instance, Hellwig and Venkateswaran (2009) and Lorenzoni (2009).

[35] See, for instance, the estimated business cycle model in Uhlig (2006) and Nimark (2014).

Table I: Baseline Shock and Information Parameters

Productivity Shock				Mark-up Shock			
ρ_a	0.80	σ_θ	0.60	ρ_μ	0.70	σ_ξ	0.16
σ_x^a	0.65	σ_z^a	0.40	σ_x^μ	0.20	σ_z^μ	0.10
σ_p	0.28	σ_ω^a	$\rightarrow \infty$	σ_p	1.30	σ_ω^μ	$\rightarrow \infty$

(i) The mapping between standard deviation σ_l^m and precision τ_l^m is $\tau_l^m = 1/(\sigma_l^m)^2$

the mark-up and productivity shock are set to $\rho_\mu = 0.70$ and $\rho_a = 0.80$, respectively, consistent with existing studies. For each persistent shock, the parameters which control the precision of private information are set under complete central bank opacity ($\tau_\omega^j \rightarrow 0, j = \{a, \mu\}$) to match pre-February 1994 data on one-quarter ahead GNP/GDP forecast accuracy from the *Survey of Professional Forecasters* for the private sector and the *Greenbook* for the central bank. In February 1994, the Federal Reserve Open Market Committee commenced a long process of increased transparency. I therefore do not employ data after this quarter as it would conflict with the complete opacity assumption otherwise used in the calibration.

Last, to set τ_p, I follow Lorenzoni (2009) and interpret ϵ_{pt} as measurement error in early releases of inflation data and match the signal-to-noise ratio in these.[36] Specifically, I interpret $\bar{\pi}_t$ as the first release and π_t as the last. I then choose τ_p to match the ratio between the standard deviation of the measurement error and the standard deviation of the innovation to inflation. The latter is measured by running a simple univariate regression of final release inflation on two lags. Lorenzoni (2009) employs this approach an obtains a ratio of 1.97 for PCE inflation. Matching this value, I obtain the parameters listed in Table I. The noise in inflation is for both the mark-up and productivity shock case substantial, consistent with the evidence presented in, for example, Runkle (1998).

A feature that immediately stands out from Table I is that the central bank has superior private information. To match the data on forecast accuracy, central bank private information has to be around 38 to 50 percent more precise than that of the private sector, as measured by the standard deviation of the noise.[37] This is consistent with the empirical results in Romer and Romer (2000), which document a substantial information advantage for the US Federal Reserve relative to the mean forecast from the Survey of Professional Forecasters.[38]

[36]This is a conservative assumption since the final release will undoubtedly still contain some noise.

[37]This follows, for example, from a comparison of $\sigma_x^a = 0.65$ and $\sigma_z^a = 0.40$ for the productivity shock case.

[38]In principle, because of the two-sided learning between the private sector and the central bank, the calibrated model could exhibit multiple stationary equilibria for the calibrated parameters (see Section 5). In practice, however, I seeded the algorithm with 1,000 randomly drawn initial values for M and N. In all cases, the recursion $\{M, N\} \mapsto \{\alpha_0, \alpha_1\} \mapsto \{M, N\}$ converged to the same fixed point.

7 Quantitative Benefits of Learning by Sharing

In this Section, I present estimates of the welfare benefits of central bank disclosure using the calibrated model. I demonstrate how disclosure increases the commonality of beliefs, and thereby central bank information, to such an extent that it is beneficial, irrespective of the source of macroeconomic fluctuations. I end the Section with a breakdown of the sensitivity of the quantitative results to the importance of higher-order expectations.

A First Best Benchmark: I once more take my welfare criterion to be the *ex-ante* utility of the representative household. Welfare losses obtained relative to the full information first best are therefore proportional to $\mathcal{W} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (y_t - a_t)^2$ (see Section 3).[39]

Similar to Sections 3 to 5, the central bank can in the extended model replicate the first best outcome of $\mathcal{W} = 0$ in several limit cases. For instance, when both the central bank and the private sector have full information about all shocks, the central bank can achieve the full information flex-price outcome by letting $\phi \to \infty$ (see Appendix D). This ability to replicate the first best extends to the case where the private sector has imperfect information but the central bank has full information, both about all shocks as well as about the private sector's beliefs about them. The central bank can in this case still replicate the first best by letting $\phi \to \infty$ (see Appendix D).[40] Thus, similar to the results in Sections 4 to 5, welfare losses are only necessarily a feature of the equilibrium under the optimal policy when the central bank itself has imperfect information about the state of the economy.

Mark-up Shock Case: I start with the case in which the economy is driven by mark-up shocks. Figure 1 illustrates the changes to *private sector uncertainty* about the first four orders of the expectational state vector $X_t^{(0:\bar{k})}$ as we increase the precision of central bank

[39] Arguably, with both an imperfectly informed household and central bank one could instead of $\mathcal{W}$ consider the household's own expectation or the central bank's expectation about future welfare losses. That is, one could in place of $\mathcal{W}$ consider $\mathcal{W}^h = \mathbb{E}_0^h \sum_{t=0}^{\infty} \beta^t (y_t - a_t)^2$ or $\mathcal{W}^{cb} = \mathbb{E}_0^{cb} \sum_{t=0}^{\infty} \beta^t (y_t - a_t)^2$. Instead of either, I choose to adopt the *Rawlsian* approach at the centre of the expression for $\mathcal{W}$. The substantive conclusions from this Section remain the same irrespective of which of the three definitions of welfare is used.

[40] One may think that firms will always learn the central bank's private information when $\phi \to \infty$ from the observation of the interest rate. This is, however, not the case because central bank expectations of the size of the output gap also decrease as we increase ϕ. Define the central bank's forecast error of the output gap as $\varsigma^{cb}_{y_t - a_t} = (y_t - a_t) - \mathbb{E}_t^{cb}[y_t - a_t]$. Then,

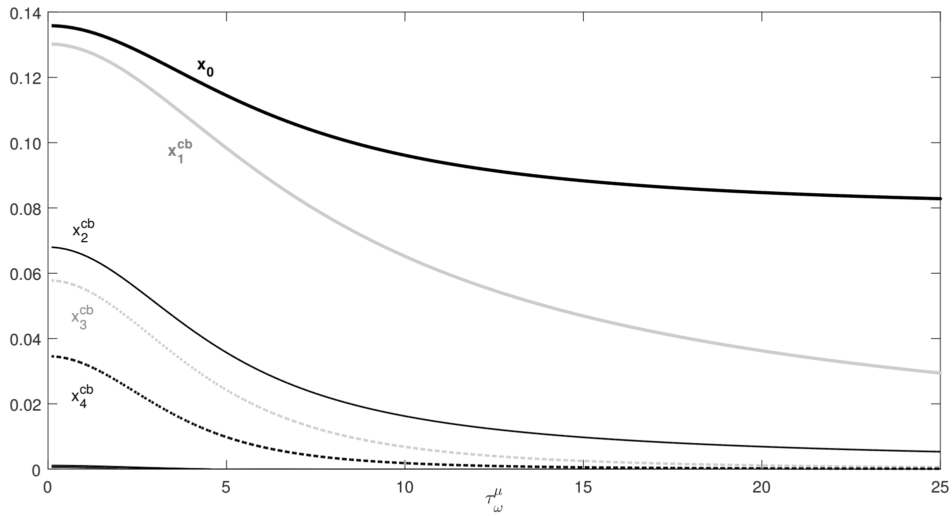
$$\begin{aligned} y_t &= \mathbb{E}_t^h[y_{t+1}] + \mathbb{E}_t^h[\pi_{t+1}] - i_t = \mathbb{E}_t^h[y_{t+1}] + \mathbb{E}_t^h[\pi_{t+1}] - \phi \mathbb{E}_t^{cb}[y_t - a_t] \\ &= \mathbb{E}_t^h[y_{t+1}] + \mathbb{E}_t^h[\pi_{t+1}] - \phi (y_t - a_t) - \phi \varsigma^{cb}_{y_t - a_t}, \end{aligned}$$

and hence $\lim_{\phi\to\infty} y_t = a_t + \lim_{\phi\to\infty} \varsigma^{cb}_{y_t - a_t}$. But then it follows from

$$i_t = \phi \mathbb{E}_t^{cb}[y_t - a_t] = \phi (y_t - a_t) - \phi \varsigma^{cb}_{y_t - a_t}$$

that $\lim_{\phi\to\infty} i_t = \lim_{\phi\to\infty} \phi \varsigma^{cb}_{y_t - a_t} - \lim_{\phi\to\infty} \phi \varsigma^{cb}_{y_t - a_t} = 0$. In the limit where $\phi \to \infty$, the interest rate becomes completely uninformative.

Figure 1: Private Sector Uncertainty about $X_t^{(0:\bar{k})}$ with Mark-up Shocks



The figure illustrates the root mean-squared error of *private sector* estimates of the first four orders of expectations in $X_t^{(0:\bar{k})}$. The panel is plotted for the calibrated values in Table I. Subscripts indicate order of expectation in $X_t^{(0:\bar{k})}$. For instance, the line for X_1^{cb} represents the root mean-squared error of the private sector's estimate of the central bank's expectation of the mark-up shock.

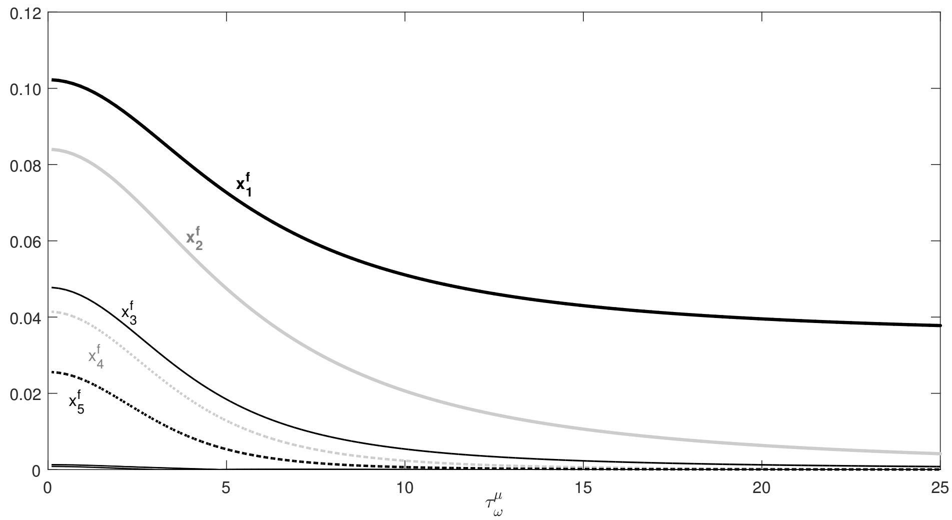
disclosure.[41] The remaining orders follow a similar a pattern and are omitted for clarity. By introspection, the private sector cannot be uncertain about its own expectations. The figure therefore only depicts elements that pertain to central bank expectations, in addition to the mark-up shock. The line for X_1^{cb} thus, for instance, represents private sector uncertainty about central bank expectations of the mark-up shock. Figure 2 illustrates the associated changes to *central bank uncertainty* about the vector of higher-order expectations $X_t^{(1:\bar{k})}$.

The results in Figure 1 and 2 mirror those from Section 4. First, disclosure decreases private sector uncertainty about the mark-up shock, visible from the X_0-line in Figure 1. This, all else equal, leads to larger private sector responses. Second, yet disclosure also decreases private sector uncertainty about central bank expectations and *vice versa*, which is evident from the rest of the lines in Figure 1 and 2. As Theorem 1 shows, this in turn allows the central bank to better counter private sector responses to the mark-up shock. The immediate question then becomes which of these two countervailing effects prevails?

Table II shows that for the baseline value of monetary policy the latter effect dominates the former. Disclosure of information about the mark-up shock *decreases* welfare losses, measured

[41] The first four orders of expectations in $X_t^{(0:\bar{k})}$ are μ_t, $\mathbb{E}_t^f \mu_t$, $\mathbb{E}_t^{cb} \mu_t$, $\mathbb{E}_t^f \mathbb{E}_t^{cb} \mu_t$, $\mathbb{E}_t^{cb} \mathbb{E}_t^f \mu_t$, $\mathbb{E}_t^f \mathbb{E}_t^{cb} \mathbb{E}_t^f \mu_t$, $\mathbb{E}_t^{cb} \mathbb{E}_t^f \mathbb{E}_t^{cb} \mu_t$, $\mathbb{E}_t^f \mathbb{E}_t^{cb} \mathbb{E}_t^f \mathbb{E}_t^{cb} \mu_t$, and last $\mathbb{E}_t^{cb} \mathbb{E}_t^f \mathbb{E}_t^{cb} \mathbb{E}_t^f \mu_t$.

Figure 2: Central Bank Uncertainty about $X_t^{(1:\bar{k})}$ with Mark-up Shocks



The figure illustrates the root mean-squared error of *central bank* estimates of the first four orders of expectations in $X_t^{(1:\bar{k})}$. The panel is plotted for the calibrated values in Table I. Subscripts indicate the orders of expectation in $X_t^{(0:\bar{k})}$. For instance, the line for X_1^f represents the root mean-squared error of the central bank's estimate of the private sector's expectation of the mark-up shock.

in life-time consumption, by around one-half.[42]

We can decompose this welfare benefit from central bank disclosure into its two constituent components. Table II illustrates the breakdown that I obtain when I let the central bank disclose its information but fix private sector and central bank higher-order uncertainty to that from the benchmark case. All else equal, the increase in private sector responses to the mark-up shock increases welfare losses by around 56 percent. But this increase is more than offset by a substantial fall in higher-order uncertainty. Indeed, for the calibrated parameters the fall in higher-order uncertainty is around twice as important in welfare terms as the direct increase in private sector responses to the mark-up shock.

This benefit of central bank disclosure carries over from the baseline value of monetary and communication policy to its optimal combination, which I find to be $\phi \to \infty$ and $\tau_\omega^\mu \to \infty$.[43] Once more the optimal monetary policy equals that under full information. At the optimal value of monetary policy, disclosure decreases welfare losses by around 27 percentage points, due to decreases in central bank uncertainty about private sector expectations and *vice versa*. This, in turn, contributes to an overall welfare benefit of moving from the calibrated complete

[42] With full disclosure, I here in practice mean $\tau_\omega^\mu = 1e+5$. I then cross-check all results with $\tau_\omega^\mu = 1e+7$.

[43] In practice, I allow for values of ϕ and τ_ω^μ up to to $1e+5$ and cross-check with values equal to $1e+7$. To be precise, whenever I write, for instance, $\phi \to \infty$ in the below I mean $\phi = 1e+5$. All welfare results are constant to the sixth decimal place in Table II and III for values above $1e+4$.

Table II: Welfare Effects of Disclosure with Mark-up Shocks

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega^\mu \to 0$	...
	Breakdown of Benefits from Disclosure		
A. Benchmark with full disclosure	$\phi = 1.81$	$\tau_\omega^\mu \to \infty$	-58.54
B. Benchmark with constant h.o. unc.[†]	$\phi = 1.81$	$\tau_\omega^\mu \to \infty$	$+56.10$
A-B. Benefit from decrease in h.o. unc.			-114.63
	Breakdown of Benefits from Optimal Policy		
A. Optimal policy	$\phi \to \infty$	$\tau_\omega^\mu \to \infty$	-96.46
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega^\mu \to 0$	-69.82
A-B. Benefit from central bank disclosure			-26.64

(i) W_C denotes the life-time consumption equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_C relative to the calibrated benchmark.

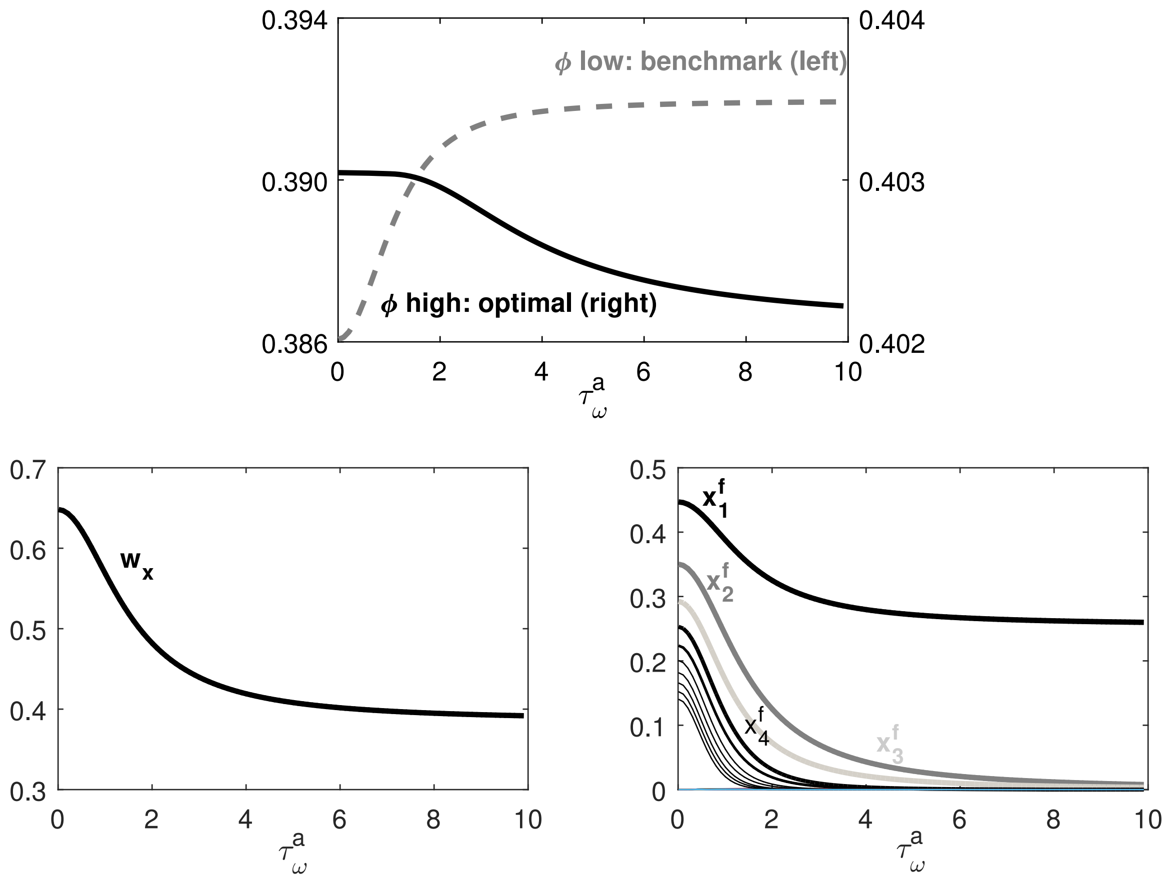
(†) Private sector and central bank higher-order uncertainty fixed at benchmark values.

opacity policy to the optimal one that is as large as to almost eliminate all welfare losses. This is clearly a forceful implication of the extended model paired with the calibrated parameters, and is in part driven by the central bank's quite precise private information about mark-up shocks (see Table I). Admittedly, a large share of the benefits from moving to the optimal policy derive from the optimal use of monetary policy itself. But the conclusion remains that disclosure contributes a healthy share to the total (around one-quarter of the welfare gains).

Productivity Shock Case: I now turn to the productivity shock case. Figure 3 illustrates the two competing effects discussed in Section 5 of central bank disclosure on the central bank's own information about productivity. On the one hand, disclosure decreases the private sector's weight on its own private information, decreasing the information content of inflation (lower left-hand panel). This increases central bank uncertainty. On the other hand, disclosure also decreases the central bank's uncertainty about private sector expectations of its own beliefs, and so on (lower right-hand panel). This, in turn, allows the central bank to better back out changes in private sector information about productivity from changes to inflation, decreasing the central bank's uncertainty.

Similar to the results in Section 5, Figure 3 (top panel) shows that for "l*ow values*" of monetary policy activism, here consistent with the benchmark value of ϕ, the former effect dominates the latter. Disclosure *increases* central bank uncertainty about productivity by crowding out the private sector's own information from inflation. This, in turn, increases

Figure 3: Central Bank Uncertainty about $X_t^{(0:\bar{k})}$ with Productivity Shocks



The top panel shows the root mean-squared error of *central bank* estimates of productivity as we increase the precision of central bank disclosure. The panel is plotted for $\phi = 1.81$ (low, left-hand scale) and for $\phi \to \infty$ (high, right-hand scale). The lower left-hand panel depicts the private sector's weight on private information (w_x) in its expectation of productivity when $\phi = 1.81$. The lower right-hand panel shows the root mean-squared error of *central bank* estimates of $X_t^{(0:\bar{k})}$ when $\phi = 1.81$.

Table III: Welfare Effects of Disclosure with Productivity Shocks

	Parameters		$\%\Delta W_C$
Calibrated benchmark	$\phi = 1.81$	$\tau_\omega^a \to 0$	...
Breakdown of Benefits from Disclosure			
A. Benchmark with disclosure	$\phi = 1.81$	$\tau_\omega^a \to \infty$	-2.95
B. Private sector benefit of disclosure†	$\phi = 1.81$	$\tau_\omega^a \to \infty$	-14.31
A-B. Central bank cost of disclosure			$+11.36$
Breakdown of Benefits from Optimal Policy			
A. Optimal policy	$\phi \to \infty$	$\tau_\omega^a \to \infty$	-32.72
B. Benefit from optimal mon. policy	$\phi \to \infty$	$\tau_\omega^a \to 0$	$+8.61$
C. Private sector benefit of disclosure†	$\phi \to \infty$	$\tau_\omega^a \to \infty$	-12.84
A-B-C. Central bank benefit of disclosure			-28.49

(i) W_C denotes the life-time consumption equivalent of W.

(ii) $\%\Delta W_C$ denotes the %change in W_C relative to the calibrated benchmark.

(†) Central bank higher-order uncertainty fixed at calibrated benchmark value.

welfare losses by around 11 percentage points. I show this in Table III, where I breakdown the effect of disclosure between the private sector and the central bank by fixing the central bank's higher-order uncertainty to its benchmark value.

That said, for "*high values*" of monetary policy activism the benefits of disclosure instead dominate the costs. Figure 3 (top panel) illustrates this case for $\phi \to \infty$, which I once more find to be optimal. Disclosure now *decreases* central bank uncertainty about productivity by alleviating the identification problem that arises when the central bank attempts to infer private sector information from relatively stable inflation. We can see the welfare benefits from this learning by sharing decrease in central bank uncertainty in Table III. Moving from the calibrated benchmark to the optimal policy decreases welfare losses by around one-third. A large share of this decrease is due to more informed private sector choices caused by more information (around 13 percentage points). But a more substantial share is, in fact, driven by the decrease in central bank uncertainty and the associated improvement in the conduct of monetary policy (around 28 percentage points).[44]

[44]Interestingly, moving to the optimal monetary policy ($\phi \to \infty$) without at the same time disclosing the private information that monetary policy is based on ($\tau_\omega^a \to 0$) is socially costly. It increases welfare losses by around nice percent relative to the benchmark case. This provides a stark example of the complementarity between monetary and communication policy discussed in this paper.

Alternative Specifications: The documented benefits of central bank disclosure arise from the decrease in higher-order uncertainty that disclosure entails. I conclude this Section by exploring the sensitivity of the quantitative results in Table II and III to changes in the importance of higher-order expectations. Specifically, I re-compute the welfare benefits of disclosure in the following three cases: (*i*) when both firms and the central bank compute only two higher-order expectations ($k = 3$), consistent with the experimental evidence on higher-order reasoning in Nagel (1995); (*ii*) when the discount factor decreases from $\beta = 0.99$ to $\beta = 0.75$, decreasing the extent to which firm prices depend upon expectations of future firm and central bank actions, and hence on higher-order expectations; and last (*iii*) when the signaling role of monetary policy is absent ($i_t \notin \Omega_t^f$). Table IV summarizes the results, while Appendix E.1 to E.3 documents in detail how in all three cases the main insights from Table II and III continue to hold.

On the Importance of Higher-Order Expectations: Both decreases to the amount of higher-order expectations computed and to the discount factor decrease the importance of higher-order expectations for the equilibrium dynamics of the model. Consequently, both the costs as well as the benefits of disclosure fall. The costs decrease because of the decrease in amplification that arises from, for example, firms discounting future firms' responses to the inefficient mark-up shock relatively more. By contrast, the benefits decrease because of the direct reduction in the importance of firms' expectations of central bank beliefs, and so on, for example through the decrease in the importance of future interest rates for firms' prices. Yet, because of the relative symmetry of these decreases (Appendix E.1 and E.2), the quantitative benefits of disclosure still on balance resemble those reported in Table II and III (Table IV).

On the Importance of Signaling: Under opacity or partial disclosure, movements in the interest rate provide firms with a noisy signal of the central bank's private information. By contrast, full disclosure separates the interest rate from its signaling effect. A concern could therefore be that the lion-share of the quantitative benefits of disclosure reported in Table II and III arises from the separation of monetary policy from its informational consequences rather than from the decrease in higher-order uncertainty. Table IV and Appendix E.3 shows that this is not the case. In fact, the resulting separation of monetary policy contributes at most one percentage point to the quantitative benefits of disclosure. This is because the interest rate, both in the benchmark calibration and at the optimal value of monetary policy, provides a rather dim indicator of the central bank's private information.[45] This is consistent with the substantial impact of central bank disclosure on financial markets and on private sector uncertainty about future interest rates documented in, for instance, Blinder *et al.* (2008).

[45] See also the derivations in footnote 34.

Table IV: Welfare Effects of Disclosure: Alternative Specifications

	Mark-up Shock Case			
	Baseline	*Discount rate*	*Limited k*	*Signalling*
$\phi = 1.81$	-58.51	-62.95	-41.06	-58.94
$\phi \to \infty$	-26.64	-32.39	-38.15	-26.75
	Productivity Shock Case			
	Baseline	*Discount rate*	*Limited k*	*Signalling*
$\phi = 1.81$	$+11.36$	$+8.23$	-18.72	$+11.46$
$\phi \to \infty$	-28.49	-25.21	-2.98	-29.23

(i) Table shows the %change in life-time consumption due to disclosure

(ii) The productivity shock case nets out firm benefits of disclosure (see Table III)

(iii) Discount rate ($\beta = 0.75$); Limited k ($k = 3$); Signalling ($i_t \notin \Omega_t^f$)

In sum, across all specifications the extended model shows material benefits of central bank disclosure. This is especially the case at the optimal value of monetary policy, where full disclosure is in all instances beneficial to social welfare.

8 Conclusion

Policymakers care deeply about knowing the state of the economy. To set interest rates correctly, for example, a policymaker needs to know which shock has hit the economy, what the size of the shocks was, and what the private sector thinks of it. All are important for the policymaker to make the correct choice. As a result, policymakers devote considerable resources to learning additional information from the observation of private sector actions, such as realizations of inflation. Actions that themselves reflect the private sector's own imperfect beliefs about the economy, and are often informed by policymaker announcements.

In this paper, I have explored the consequences of this two-sided learning for the social value of central bank disclosure. At the heart of my results has been that communication decreases higher-order uncertainty. A central bank's disclosure, for example, not only provides more information to the private sector, but also increases common knowledge between the private sector and the central bank. This has important implications for what the central bank knows about private sector expectations, what it can in turn learn from private sector actions, and hence for the set of potential outcomes that monetary policy can attain.

Specifically, I have shown how the decrease in higher-order uncertainty can overcome two otherwise standard costs of central bank disclosure: (*i*) the decrease in the central bank's own information caused by a decrease in the informativeness of endogenous market outcomes, like inflation; and (*ii*) the increased responses to inefficient disturbances, such as cost-push shocks. In particular, I have demonstrated how, once one accounts for a lack of common knowledge, disclosure increases central bank information about private sector expectations as well as the information content of market outcomes, such as inflation. I have then showed how the resulting decrease in central bank uncertainty can increase the effectiveness of monetary policy to such an extent that disclosure becomes beneficial, irrespective of the efficacy of macroeconomic fluctuations.

To keep the exposition clear, I initially focused on a simple framework that included the core features of monopolistic competition, nominal frictions, and two-sided learning about private sector and central bank beliefs in a tractable manner. I then showed how my results carried over for calibrated parameters to an extension of the baseline model that introduced incomplete common knowledge and two-sided learning into the standard New Keynesian framework. Within this extended model, central bank disclosure about an otherwise inefficient mark-up shock decreased welfare losses by 27 percent under the optimal monetary policy. Disclosure about an efficient productivity shock, meanwhile, decreased welfare losses by 33 percent, 28 percentage points of which were due to the increase in the informativeness of inflation.

My results have focused on the consequences of a lack of common knowledge between the private sector and the central bank for the social value of central bank disclosure. To do so, I have abstracted from a lack of common knowledge among private sector agents themselves. Recent work by Wiederholt (2017) and Angeletos and Lian (2018) has stressed how incomplete common knowledge among households and firms can dampen general equilibrium multipliers of prospective monetary policy. An interesting extension would be to explore how the combined introduction of incomplete common knowledge among private sector agents, in addition to between the private sector and the central bank, alters the overall efficacy of macroeconomic policy. I conjecture that this modification would reinforce my main conclusion that disclosure increases the efficacy of policy, as disclosure would also increase common knowledge among private sector agents. This, in turn, could have important ramifications for how policymaker disclosure interacts with the preponderance of macroeconomic puzzles that rest on powerful general equilibrium effects of policy.[46]

[46]See, for example, the "Forward Guidance Puzzle" (McKay *et al.*, 2016; Werning, 2015; Angeletos and Lian, 2018), or "The Paradox of Toil" with decreases in labor taxes (Eggertsson, 2010; Mulligan, 2010).

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