Forecaster (Mis-)Behavior*

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Abstract

We document two stylized facts in expectational survey data. First, professional forecasters overrevise their macroeconomic expectations. Second, such overrevisions mask evidence of both over- and underreactions to public signals. We show that the first fact is inconsistent with standard models of noisy rational expectations, but consistent with behavioral and strategic models of forecasters. The second fact, in contrast, presents a puzzle for existing theories. To explain this evidence, we propose a simple extension of noisy rational expectations that allows forecasters to be overconfident in their information. We show that this feature, when combined with the endogeneity of public signals, leads to over- and undereaction to public information consistent with the data.

JEL codes: C53, D83, D84, E31   *Keywords:* Expectations, forecasters, information

1 Introduction

Expectations are central to economics. Because individual expectations are typically unobserved, however, it is often difficult to discriminate between alternative models of expectation formation. One exception are professional forecaster surveys, which regularly publish individual expectations about macroeconomic and financial variables. Indeed, Muth (1961) proposed the rational expectations theory in part to explain the perceived sluggishness of average survey expectations as a rational response to noisy information (p. 316 Muth, 1961).

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Although the full-information variant of rational expectations later became the benchmark of modern macroeconomics, the work of Woodford (2002), Sims (2003), and others,[1] has revived interest in noisy information models of rational expectations. In turn, this has rekindled interest in the use of survey data to better discipline and test such models. In line with a central prediction of noisy rational expectations, Coibion and Gorodnichenko (2012, 2015) recently document that the average of survey expectations underreacts to new information relative to what a full-information framework would prescribe.[2] However, such underreactions of *average* expectations are not only consistent with noisy rational expectations, but also with a host of other both rational and behavioral theories of *individual* expectation formation.

In this paper, we provide new evidence on the statistical properties of individual survey expectations of macroeconomic variables. We document two stylized facts that present a challenge for noisy rational expectations. First, individual forecasters *overrevise* their macroeconomic expectations. Second, such overrevisions mask both *over- and underreactions* to salient public signals. We show that the first fact is inconsistent with standard models of noisy rational expectations, but in line with, e.g., models of strategic forecaster behavior. The second fact, in contrast, presents a puzzle for existing theories of expectation formation.

Our main contribution is to explain this co-existence of over- and underreactions: We propose a simple extension of noisy rational expectations that allows forecasters to be overconfident in the precision of their own information (both relative to the truth and relative to their perception of others). We show that such overconfidence causes forecasters to overrevise their expectations and misperceive others' responses to information. Importantly, such misperception leads forecasters to misinterpret public signals that aggregate others' information and actions, and results in over- or underreactions that are consistent with the data.

A well-known consequence of rational (mean-squared optimal) expectations is that individual forecast errors should be unpredictable based on known information. The two stylized facts that motivate our theory test this prediction. Our first test relates individual forecast errors to individual revisions in fixed-date forecasts.[3] Basic introspection by rational forecasters requires these two to be uncorrelated, even in the presence of noisy information. Our second test instead exploits the survey data to relate individual forecast errors directly to elements of public information that are salient to professional forecasters. Once more, individual rationality requires individual forecast errors to be uncorrelated with these.

[1] See, for example, Mankiw and Reis (2002), Angeletos and Pavan (2007), Nimark (2008), Lorenzoni (2009), Maćkowiak and Wiederholt (2009), and Angeletos *et al.* (2016).

[2] Coibion and Gorodnichenko (2012), Andrade and Le Bihan (2013), Ryngaert (2017), Fuhrer (2018), Bordalo *et al.* (2020), and Kohlhas and Walther (2021) document related evidence.

[3] In contemporaneous and independent work, Bordalo *et al.* (2020) propose a similar test. The working paper version of Bordalo *et al.* (2020) [Bordalo *et al.*, 2018a, p.7] acknowledges this simultaneity. We discuss the similarities and differences between the two approaches in the related literature section.

As a benchmark, we first consider inflation forecasts from the Survey of Professional Forecasters (SPF). We use the outcomes of our tests to document two stylized facts.

First, individual forecast revisions are systematically too large. This manifests itself in a pronounced negative relationship between individual forecast errors, on the one hand, and individual forecast revisions, on the other hand. Second, such observed overrevisions mask evidence of both over- and underreactions to salient public signals that are both predictive about future inflation, relevant, and observed in real-time (e.g., previous consensus forecasts or changes in the unemployment rate). We document that these patterns extend to forecasts of macroeconomic variables other than inflation, different forecast horizons, to the euro area, as well as to other forecasters than those that label themselves professional.

Combined, our empirical results present a challenge for existing models of expectation formation. While simple models of noisy rational expectations are consistent with an underrevision of average expectations, they are *prima facie* inconsistent with the overrevision documented at the individual level. Alternative theories of forecaster behavior that incorporate (i) the specific strategic considerations faced by professional forecasters (e.g., Laster *et al.*, 1999; Ottaviani and Sørensen, 2006; Ehrbeck and Waldmann, 1996); (ii) common behavioral biases (e.g., Daniel *et al.*, 1998; Bordalo *et al.*, 2020); or (iii) trembling-hand noise can explain such overrevisions. However, as we show, these theories either all predict optimal use of public information (conditional on private information), or that forecasters overreact to *all* new information, irrespective of its source. Both predictions are inconsistent with the simultaneous over- and underreactions to public information that we document in the data.[4]

To account for our empirical results, we propose a simple extension of noisy rational expectations that allows forecasters to be overconfident in their own information. Specifically, we allow forecasters to both perceive their private information to be more precise than it actually is ("*absolute overconfidence*"; Alpert and Raiffa, 1982; Soll and Klayman, 2004; and others), and to be more precise than the information of others' ("*relative overconfidence*"; Alicke and Govorun, 2005; Larrick *et al.*, 2007). Both dimensions of overconfidence are commonly used in the psychology literature (Moore and Healy, 2008), and we find direct evidence for them in the survey data. We show that, taken together, absolute and relative overconfidence can explain our empirical results when combined with the central feature that most public signals reflect the aggregate outcome of others' choices, and hence their information. The combination of overconfidence with the endogeneity of public signals further distinguishes our theory from previous models of overconfidence (e.g., Daniel *et al.*, 1998).

All else equal, absolute overconfidence makes forecasters overreact to private information,

[4] Angeletos and Huo (2021) show that the overrevisions that we document are also inconsistent with two common alternatives to noisy rational expectations: "cognitive discounting" and "level-k thinking".

and hence makes individual forecast revisions too large. In contrast, relative overconfidence makes forecasters underestimate the precision of others' information. This is important.

Any equilibrium model of expectation formation that internalizes that most public signals reflect others' choices requires an assumption about people's views about others' information. Rational expectations solves this problem by imposing the "symmetry assumption": that others' information is equal in quality to an individual agent's. Relative overconfidence, by contrast, imposes the empirically motivated "better than others" perception. By underestimating the precision of others' information, relative overconfidence causes forecasters to expect public signals to respond less to new information and to be less precise.

We show that such misperceptions have two offsetting effects: Underestimating the precision of public signals, all else equal, leads forecasters to dismiss them, and underreact to their realizations. However, underestimating the responsiveness of public signals, by contrast, leads forecasters to over-infer information from any given signal realization, and hence to overreact.

We demonstrate these results within the context of a workhorse noisy information model with mean-squared error preferences (e.g., Veldkamp, 2011). Although our model is simple, we quantitatively validate it along three dimensions.

First, we show that our model can match the estimated overrevision of inflation forecasts at the same time as the estimated overreaction to a particular public signal, previous period's consensus forecast. We focus on consensus forecasts because it reflects a public signal that only aggregates other's information. This allows us to focus on overconfidence's role in creating a friction between forecasters' perception of a public signal and that which arises in equilibrium. As argued in Ottaviani and Sørensen (2006), consensus outcomes also represent a particularly salient public signal for professional forecasters, such as those in the SPF. Second, an attractive feature of the survey data on professional forecasters is that respondents also report forecast densities, in addition to point estimates. We show that this additional information, when combined with auxiliary data on higher-order expectations from Coibion *et al.* (2021), allows us to validate our assumptions of absolute and relative overconfidence in the survey data. Third, two key implications of our model are that (i) forecasters should underreact more to public signals that are less precise, and (ii) that the magnitude of over- and underreactions should change with the volatility of the forecasted variable. We demonstrate that both predictions are in line with the patterns of responses in the data. We conclude the paper by studying the implications of our model for the distribution of forecast errors.

Finally, professional forecasters may admittedly differ from other economic agents in their incentives and information about the state of the economy. In this paper, we confront this issue head-on by directly contrasting the ability of agency-based models to explain the observed under- and overreactions with simple behavioral alternatives. To the extent that the evidence we uncover below speaks in favor of widely documented behavioral biases, rather than particular strategic incentives, we think that our results should carry over to other contexts. Indeed, we provide some evidence to this effect later in the paper.

Related Literature: Our paper is related to several strands of research. We review these in order of proximity, starting with the most closely related.

First, our paper relates to studies that use expectational survey data to test models of noisy rational expectations.[5] Recently, Coibion and Gorodnichenko (2012, 2015) document that *average forecasts* of several macroeconomic variables, across different surveys, underreact to new information. Our study departs from this observation, and studies both average and individual-level forecasts within a unified framework. Complementary to our paper, in contemporaneous and independent work, Bordalo *et al.* (2020) demonstrate similar overrevisions of *individual-level* forecasts to those that we document. In contrast to their paper, we show that these overrevisions mask evidence of both over- and underreactions to salient public signals. We further show that such simultaneous over- and underreactions present a challenge for existing models, including Bordalo et al.'s (2020) theory of "diagnostic expectations". Closely related, Kohlhas and Walther (2021) also depart from Coibion and Gorodnichenko's (2015) observation, but demonstrate that individual forecasters simultaneously extrapolate from recent events. Such extrapolation can be viewed as an overreaction to a specific public signal: that of the past outcome of the forecasted variable. Our paper elaborates on this observation and studies forecasters' responses to a wide set of salient public signals. Importantly, we demonstrate that forecasters also occasionally *underreact* to publicly available information. Finally, building on the above work and our present study, Angeletos *et al.* (2020) propose a model that combines absolute overconfidence with an additional behavior friction (overpersistence). This model tractably speaks to the above evidence, as well as several auxiliary results.[6] We view the above strand of literature as presenting complementary and related steps towards a unified model of expectations that is consistent with the survey data.

Second, although forecaster information is sometimes acknowledged to be an upper bound of that held by the population at large (Marinovic *et al.*, 2013), most studies abstract from the particular characteristics that separate professional forecasters from the rest of the population. This has attracted criticism (e.g., Scharfstein and Stein, 1990 and Lamont, 2002) and given

[5] Apart from the implications discussed here, broad aspects of survey forecasts are clearly consistent with noisy rational expectations. First, survey forecasts are dispersed and differ across forecasters (Zarnowitz, 1985). Second, forecasts are often smoother, with lower volatility, than the variable that is being forecasted (Ottaviani and Sørensen, 2006). In fact, one of Muth's (1961) aims in proposing the rational expectations hypothesis was to explain these two stylized facts (p. 316 in Muth, 1961).

[6] Angeletos and Huo (2021) show that the approach proposed in this paper also has the advantage that the *as-if* myopia and anchoring that are consequences of noisy rational expectations directly extend to our model of overconfidence.

rise to a literature that looks at forecasters' incentives to distort their stated predictions (e.g., Laster *et al.*, 1999; Ehrbeck and Waldmann, 1996).[7] Our contribution in this context is to show, within a common framework, how several of the most prominent of such agency-based models are inconsistent with individual-level forecasts from a variety of professional surveys.

Third, our paper relates to the substantial body of work that links over- and underreaction of expectations to asset price anomalies. For example, Daniel *et al.* (1998) show how a model of overprecision (leading to overreactions) and self-attribution of skill (leading to underreactions) is consistent with the excess volatility and short-run momentum often found in financial markets. Barberis *et al.* (1998), in contrast, show how a model of conservatism (underreaction) and "representativeness" (overreaction) can explain the underreaction of stock prices to earnings announcements jointly with the overreaction of stock prices to extreme events. Lastly, and closely related to our notion of relative overconfidence, Eyster *et al.* (2019) show how "cursedness" (the failure to extract information from market prices) may explain momentum in asset prices. In contrast to this work, our evidence of over- and underreactions is based directly on forecasters' stated predictions rather than the behavior of equilibrium objects, such as asset prices. We hence view our evidence as a useful anchor for these models.

2 Empirical Evidence

In this section, we document three features of US inflation forecasts. We show that professional forecasters' average inflation forecasts underreact to information received between two periods. We then show that, at the individual level, the same forecasters by contrast make forecast revisions that are systematically too large. Lastly, we document that the overrevisions at the individual level mask evidence of both over- and underreactions to salient public signals.

2.1 Data

We focus on US inflation forecasts from the *Survey of Professional Forecasters* (SPF).[8] At the start of each quarter, the SPF asks its respondents for their forecasts of a number of key macroeconomic and financial variables, and publishes them, in anonymous format but with personal identifiers, shortly thereafter. We study SPF forecasts of the year-on-year percentage change in the GNP/GDP deflator, for which the survey includes consistent forecasts for the six quarters following the survey quarter. We focus on inflation forecasts for three reasons. First,

[7] See also, for example, Graham (1999), Laster *et al.* (1999), and Ottaviani and Sørensen (2006).

[8] The SPF is the oldest quarterly survey of individual macroeconomic forecasts in the US, dating back to 1968. The SPF was initiated under the leadership of Arnold Zarnowitz at the American Statistical Association and the National Bureau of Economic Research, which is why it is also still occasionally referred to as the ASA-NBER Quarterly Economic Outlook Survey (Croushore, 1993).

because inflation expectations play a central role in the economy as determinants of wages, goods and asset prices. Second, to compare our estimates to those of previous studies, which have focused disproportionally on inflation. And third, because data on inflation forecasts are available for a substantially longer time-span than forecasts of other variables. Throughout, we consider first-release realizations of inflation to most accurately capture the precise definition of the variable being forecasted. Importantly for our purposes, although the precise schedule over the quarter has changed over time, the administrators of the SPF have consistently and publicly published the average of survey results well before sending out the next round of the questionnaire.[9] The information set of respondents therefore includes the consensus (or average) forecast from the previous quarter.

2.2 Average Forecasts

We first study the properties of average inflation forecasts. We denote respondent i's forecast made in period t of inflation π in period $t+h$ as $f_{it}\pi_{t+h}$. We then calculate the average forecast as $f_t\pi_{t+h} = \frac{1}{N_t}\sum_{i=1,..,N_t} f_{it}\pi_{t+h}$, where N_t denotes the number of forecasters in period t. A respondent's forecast error is $\pi_{t+h} - f_{it}\pi_{t+h}$, while the average forecast error is $\pi_{t+h} - f_t\pi_{t+h}$.

A well-known implication of full information and rational expectations (with mean-squared error preferences) is that forecast errors should be uncorrelated with known information. Our first test explores this prediction by estimating the Coibion-Gorodnichenko (2015) regression:

$$\pi_{t+h} - f_t\pi_{t+h} = a + b\left(f_t\pi_{t+h} - f_{t-1}\pi_{t+h}\right) + v_t. \tag{2.1}$$

where $f_t\pi_{t+h} - f_{t-1}\pi_{t+h}$ denotes the average forecast revision between period $t-1$ and t, and v_t an error term. We hence estimate the relationship between average errors and average revisions. Table I presents the results for one-year ahead inflation forecasts ($h = 4$).

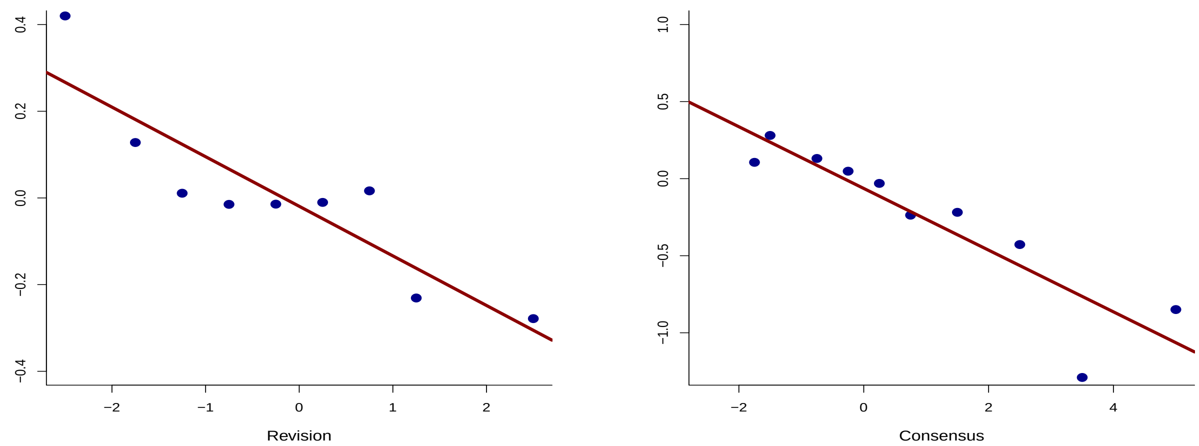
Average revisions are positively correlated with average errors ($b > 0$). This effect is strong and highly significant, in line with the results in Coibion and Gorodnichenko (2015), and others. On average, forecasters underrevise their expectations relative to the full information rational expectations benchmark, leading to a positive correlation between average errors, on the one hand, and average revisions, on the other hand.

Although inconsistent with full (common) information and rational expectations, $b > 0$ is in line with several popular models of rational expectations that allow for individual-specific noise in respondents' information (Coibion and Gorodnichenko, 2015).[10] In such noisy information

[9] See p.8 in the documentation: `https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf`.

[10] We note that with noisy *public information*, least-squares estimates of b in (2.1) are downwardly biased. As argued in Coibion and Gorodnichenko (2015) Online Appendix A, such downward bias, however, still entails

Figure 1: Forecast Errors from the Survey of Professional Forecasters



Note: The left-hand side panel depicts (on the vertical axis) the average of individual forecast errors taken within equally-sized bins of the distribution of individual forecast revisions (horizontal axis). The right-hand side panel shows (on the vertical axis) the average of individual forecast errors this time taken within bins of the distribution of consensus forecasts from the previous wave of the SPF (horizontal axis). All variables are demeaned by subtracting their (individual) averages during the SPF sample period (1970Q1-2020Q1).

Table I: Estimates from the Survey of Professional Forecasters

	Average Forecasts	*Individual Forecasts*		
	Forecast Error	*Forecast Error*	*Forecast Error*	*Forecast Error*
Forecast Revision	1.118***	-0.199***	–	-0.206***
	(0.287)	(0.067)		(0.067)
Previous Consensus	–	–	-0.192**	-0.200**
			(0.085)	(0.088)
Constant	-0.054	–	–	–
	(0.073)			
Observations	196	5,480	5,675	5,480
F Statistic	44.067	113.66	118.98	122.67
R^2	0.190	0.021	0.022	0.045

Note: Estimates of (2.1), (2.2), and (2.3) using SPF forecasts of one-year ahead inflation ($h = 4$). Column 1 presents estimates with a constant term. Column 2-4 include individual (respondent) fixed effects. Robust (double-clustered) standard errors in parentheses. Sample: 1970Q1-2020Q1. $*$ p<.1,$**$ p<.05,$***$ p<.01.

models, forecasters revise their expectations by less than a hypothetical agent would do if she could observe the average information in the population. This is because forecasters rationally downweigh their own information to account for its noisiness. However, because individual-specific noise terms cancel on average, this downweighing of new information leads to a positive correlation between average errors, on the one hand, and average revisions, on the other hand ($b > 0$). Relative to the full-information benchmark, noisy private information leads to an underrevision of average forecasts in response to average information.

We next turn to the statistical properties of individual inflation forecasts. We continue to explore the implication that rational errors should be orthogonal to known information.

2.3 Individual Forecasts

2.3.1 Overrevision of Individual Forecasts

Equation (2.1) studies the relationship between errors and *average revisions*.[11] However, basic introspection implies that *individual revisions* are always known to individual forecasters. This is even in the presence of noisy private information. This, in turn, implies that even with noisy information, rational (mean-squared optimal) errors should be uncorrelated with individual revisions. Estimates of the slope coefficient in (2.1) at the individual level should equal zero.

Figure 1 shows that this implication is *prima facie* not borne out by the data. The conditional means of individual errors are negatively associated with the means of individual revisions (left panel), suggesting a negative relationship. To test this implication more formally, we estimate a version of (2.1) at the individual level, using the benchmark specification:

$$\pi_{t+h} - f_{it}\pi_{t+h} = \alpha_i + \beta\left(f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h}\right) + \nu_{it}, \tag{2.2}$$

where α_i denotes a respondent fixed effect. Table I confirms our initial impressions.

The estimate of β is significantly negative and numerically large, inconsistent with the predictions of (noisy) rational expectations. This negative estimated value of β implies that positive individual revisions are associated with negative errors. Forecasters on average revise their forecasts by too much relative to the rational expectations benchmark, and hence on average overreact to the information received between subsequent survey rounds.

However, importantly, such overrevisions of individual forecasts do not inform us about the composition of responses that lead to a negative estimate for β. All we can conclude is that

that statistically significant findings of $b > 0$ imply average underreactions relative to full information. This is because least-squares estimates understate any positive association.

[11] We further note that equation (2.1) is equivalent to estimating the linear relationship between *individual* errors and *average* revisions. This is because $\mathbb{C}\text{ov}\left(\pi_{t+h} - f_t\pi_{t+h}, x_t\right) = \frac{1}{N}\sum_{i=1}^{N}\mathbb{C}\text{ov}\left(\pi_{t+h} - f_{it}\pi_{t+h}, x_t\right) di$ for some common variable x_t and assuming N ex-ante identical forecasters. In (2.1), $x_t \equiv f_t\pi_{t+h} - f_{t-1}\pi_{t+h}$.

forecasters overreact *on average*. In particular, estimates of (2.2) do not allow us to separate between (i) whether the overrevision of expectations is comprised exclusively of overreactions to new information, or (ii) whether the overall overrevision masks evidence of both over- and underreactions. As we argue in Section 3, this distinction is important for our analysis, as it will greatly constrain the set of models that are consistent with the data.

Finally, notice that a positive estimate of b in Table I corresponds to an average underrevision relative to the *full information and rational expectations* benchmark. By contrast, a negative estimate of β suggests an overrevision (or overreaction) relative to the *rational expectations* case, allowing for the presence of *noisy information*. We use both notions of over- and underrevisions interchangeably below when there is no cause for confusion.

2.3.2 Over- and Underreactions to Public Signals

In order to provide a first pass at a breakdown of the composition of responses that lead to $\beta < 0$ in Table I, our third test considers the relationship between errors and the public signals that forecast revisions are based on. (We focus on public signals because those are also observed by researchers.) In particular, we estimate the following regression equation:

$$\pi_{t+h} - f_{it}\pi_{t+h} = \alpha_i + \delta y_t + \nu_{it}, \tag{2.3}$$

where α_i denotes a respondent fixed effect, and y_t a public signal that is observed by forecasters. The third implication of rational expectations that we focus on is that δ should equal zero, as any non-zero coefficient would contradict the assumption that public information is used efficiently. The *Law of Iterated Expectations* implies that if the public signal is included in forecasters' information sets $y_t \in \Omega_{it}$ and forecasts are rational $f_{it}\pi_{t+h} = \mathbb{E}[\pi_{t+h} \mid \Omega_{it}]$, then there should always be zero correlation between errors and the public signal.[12]

The predicted zero correlation between rational errors and public information also allows for a clean interpretation of any non-zero $\delta-$estimates. Because $\pi_{t+k} - \mathbb{E}[\pi_{t+k} \mid \Omega_{it}]$ is uncorrelated with y_t, we can add and subtract $\mathbb{E}[\pi_{t+k} \mid \Omega_{it}]$ from the left-hand side of equation (2.3). This shows that δ is positive (negative) if and only if the rational expectations forecast $\mathbb{E}[\pi_{t+k} \mid \Omega_{it}]$ has a stronger (weaker) reaction to the public signal y_t than the actual forecast

[12]In particular, if respondents are rational:

$$\begin{aligned} \alpha_i + \delta \times y_t = \mathbb{E}[\pi_{t+h} - f_{it}\pi_{t+h} \mid y_t] &= \mathbb{E}[\pi_{t+h} \mid y_t] - \mathbb{E}\{\mathbb{E}[\pi_{t+h} \mid \Omega_{it}] \mid y_t\} \\ &= \mathbb{E}[\pi_{t+h} \mid y_t] - \mathbb{E}[\pi_{t+h} \mid y_t] = 0 + 0 \times y_t, \end{aligned} \tag{2.4}$$

where the second to last line follows from $y_t \in \Omega_{it}$ and the Law of Iterated Expectations. Hence, $\delta = 0$. Notice that one strength of the approach in (2.3) is that it allows forecasters to *rationally* choose to disregard other public signals $z_t \neq y_t$. The only requirement in (2.4) is that y_t is included in the information sets.

$f_{it}\pi_{t+h}$.[13] Consistent with earlier use of the terms, we say that forecasters *overreact* to y_t if $\delta < 0$. Conversely, we say that forecasters *underreact* if $\delta > 0$.

To estimate (2.3) requires a particular piece of public information that is at the same time publicly observed, relevant, and salient to professional forecasters. We first focus on a natural example of such public information within our context: that of the consensus forecast from the previous wave of the survey ($y_t = f_{t-1}\pi_{t+h}$). As argued in the introduction, and more forcefully in Ottaviani and Sørensen (2006), professional forecasters pay close attention to realizations of consensus. This is to assess how well they perform relative to their competitors. Consensus forecasts should therefore provide a conservative benchmark against which to test the orthogonality of individual forecast errors to public information.

Figure 1 (right panel) depicts the conditional means of individual forecast errors of one-year ahead inflation ($h = 4$), and shows that these decrease in previous period's consensus forecast. Table I confirms this impression. The estimate of δ in (2.3) is negative and statistically significant, inconsistent with rational expectations. Individual errors are, on average, more negative not only when individual revisions are more positive, but also when the previous consensus forecast is higher. We conclude that forecasters appear to overreact to the information contained in consensus forecasts. These overreactions are corroborated in the final column of Table I, where we report the coefficient estimates from a multivariate regression that includes both individual forecast revisions and consensus. These estimates suggest that even conditional on individual revisions forecasters overreact to consensus.

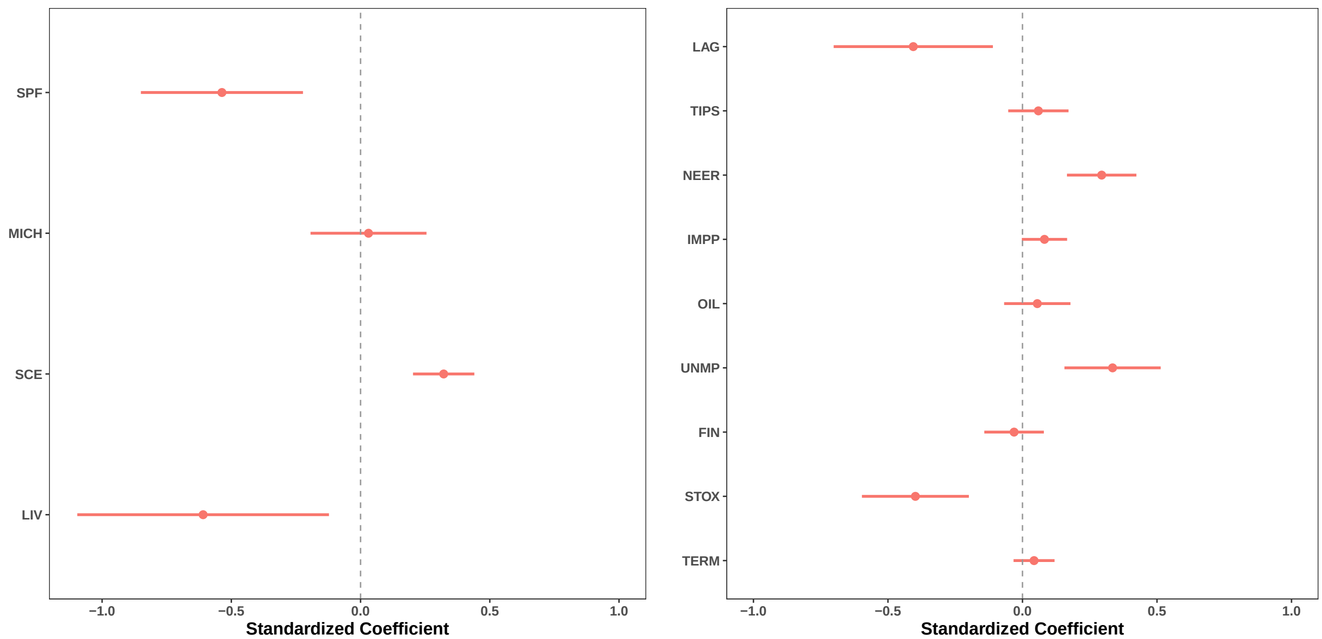
The negative estimate of δ in Table I may suggest that forecasters overreact to all information (implying $\delta < 0$ for all public signals). However, Figure 2 shows that such uniformity does not exist. The figure presents estimates of δ from (2.3) using a variety of public signals. We divide this evidence into two types: (i) alternative survey measures of future inflation, similar to consensus forecasts (left-hand side panel), and (ii) other publicly observable time series that are often used to predict inflation (right-hand side panel). We take the latter set of variables from the European Central Bank's published list of "important inflation indicators", to tie our hands with respect to variable selection.[14] A similar set of variables are used in Cecchetti (1995), Canova (2007), and Stock and Watson (2008), among many others. We note

[13]Specifically, we have that

$$\begin{aligned}\delta \times y_t = \mathbb{E}\left[\pi_{t+h} - f_{it}\pi_{t+h} \mid y_t\right] &= \mathbb{E}\left[\pi_{t+h} - \mathbb{E}\left[\pi_{t+k} \mid \Omega_{it}\right] + \mathbb{E}\left[\pi_{t+k} \mid \Omega_{it}\right] - f_{it}\pi_{t+h} \mid y_t\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[\pi_{t+k} \mid \Omega_{it}\right] - f_{it}\pi_{t+h} \mid y_t\right].\end{aligned}$$

[14]See, for example, https://www.ecb.europa.eu/pub/pdf/other/ebart201704_01.en.pdf. The main difference is that we avoid the inclusion of measures of the "output gap", since that would entail taking a stance on the structural determinants of deviations from the flex-price allocation.

Figure 2: Inflation Forecast Errors and Different Public Signals



Note: The figure depicts estimates of δ in (2.3) (horizontal axis) for various public signals (vertical axis). The left-hand side panel shows the coefficient estimates for previous period's consensus estimate of one-year ahead inflation ($h = 4$) from the Survey of Professional Forecasters (SPF), the Michigan Survey of Consumers (MICH), the Survey of Consumer Expectations (SCE), and the Livingston Survey (LIV). The right-hand side panel shows estimates of δ using one-period lagged inflation outcomes (LAG), 10-year inflation expectations from the TIPS market (TIPS), the year-over-year change in the nominal effective exchange rate (NEER), the year-over-year change in import prices (IMP), the year-over-year change in the WTI oil price (OIL), the unemployment rate (U), the Cleveland Fed's Financial Market-based measure of future inflation (FIN), the log-linear detrended level of the SP500 (STOX), and the 10-year-2-year term spread (TERM). All variables have been standardized, and have been signed such that an increase predicts higher inflation one year out. All variables and growth rates have also been derived using the latest available data at the time of the inflation forecast. Whisker-intervals correspond to 95 percent robust doubled-clustered confidence bounds. Online Appendix Table B.1 provides further details on the estimates.

that the alternative surveys of type (i) capture *different* public signal. This is because the alternative surveys measure the views of other respondents than those in the SPF. To make our estimates in Figure 2 comparable across series, all variables have been standardized, and have been signed such that an increase predicts higher inflation one year out.

On balance, we find that, although forecasters overreact to previous consensus forecasts from the SPF, the evidence for other public signals is more mixed. For example, the left-hand side panel in Figure 2 shows that forecasters *overreact* with similar strength to the observation of respondents consensus estimate from the *Livingston Survey* (Croushore, 1997). This is consistent with the *Livingston Survey* covering many of the same forecasters as the SPF. However, forecasters *underreact* to the information contained in the consensus outcome from the *Survey of Consumer Expectations* (Armantier *et al.*, 2017), in addition to estimates of consumer expectations from the *Michigan Survey of Consumers* (Dominitz and Manski, 2003), although the latter is not statistically significant.[15]

The right-hand side panel in Figure 2 confirms this picture of over- and underreactions in response to public signals other than measures of average expectations. When we estimate the relationship between individual inflation errors and nine common public signals of future inflation, we find significant overreactions to some (e.g., lagged outcomes, akin to extrapolation), but significant underreactions to others (e.g., changes to the exchange rate or the unemployment rate). A simple ANOVA exercise shows that the probability of all coefficients in Figure 2 occurring by chance in the absence of over- or underreactions is less than 0.0001.

Finally, two wider implications of our analysis are worth noting. First, the above analysis considers multiple public signals. However, our estimates do not attempt to directly estimate the relative weight placed on any specific signal compared to the rational expectations case. Such an exercise would require a full list of signals observed by forecasters, including those from private sources. Instead, our estimates explore the extent to which individual forecasts $f_{it}\pi_{t+h}$ are associated more or less with a public signal y_t than their rational counterpart $\mathbb{E}_{it}\pi_{t+h}$. Notwithstanding such concerns, Online Appendix Table B.2 shows that a multivariate version of (2.3) still confirms the above picture of over- and underreactions.

Second, our findings of overreactions to consensus estimates are robust to concerns of *limited attention*. Although professional forecasters track developments in the above public signals closely, if they instead of the consensus signal y_t were to observe $z_{it} = y_t + u_{it}$, with $u_{it} \sim \mathcal{N}(0, \sigma_u^2)$, due to limited attention, then estimates of δ would be upward biased. This is for the same reason that noisy information leads to a positive b in regression (2.1). We,

[15] The Livingston Survey is a bi-annual survey of forecaster expectations that covers many different types of forecasters. It is the oldest continuous survey of forecaster's expectations. The Federal Reserve Bank of Philadelphia took responsibility for the survey in 1990. The Michigan Consumer Survey and the Survey of Consumer Expectations are monthly surveys of a large number of U.S. households.

nevertheless, view inattention to salient public signals, such as consensus, to be unlikely for the professional forecasters that comprise our sample.

2.4 Alternative Estimates

We obtain similar estimates to those in Table I and Figure 2 beyond one-year ahead inflation forecasts from the US SPF. Table II and Figure 3 summarize alternative estimates of (2.1), (2.2), and (2.3) using different variables and other expectational surveys.

First, to complement our benchmark results using GNP/GDP inflation forecasts, we consider forecasts of CPI inflation and real output growth (Real GDP) from the *Survey of Professional Forecasters* (Table II). The estimated coefficients for b in (2.1), β in (2.2), and δ in (2.3) using past consensus outcomes ($y_t = f_{t-1}\pi_{t+h}$) all have the same sign as our benchmark results, and are all statistically significant, with the exception of the CPI estimate of b and the output estimate for δ. Similar results to those in Table I also hold when we restrict the sample to after 1992, when the Federal Reserve Bank of Philadelphia took over the administration of the SPF and substantially increased its coverage (and when inflation was also more stable).[16] We also document that similar patterns extend to a semi-annual forecast horizon ($h = 2$).

Second, we extend beyond the United States and consider professional forecasts for another geographic area, the Euro Area, as collected by the *ECB's Survey of Professional Forecasters* (Garcia, 2003). We once more find estimates of b, β, and δ using past consensus similar to those from the US SPF. While the point estimate of β for output is positive, the uncertainty around this estimate is large because of the short estimation sample that starts only in 2000. As we discuss below, our model in Section 4 can in any case also account for such observations.

Third, a large share of forecasters in the US and Euro Area SPF comes from financial sector institutions. We therefore also consider whether our results extend beyond financial sector forecasters. Table II shows that our results carry over with equal force to the non-financial sector forecasters in the US SPF, as well as to forecasters that are part of the semi-annual Livingston Survey, although the power of our estimates is here somewhat reduced. The non-financial sector forecasters in the US SPF mainly come from large private sector firms and consultancies, while the Livingston Survey covers a broader range of non-financial sector institutions (such as academic institutions and government entities, for example). Table B.3 in the Online Appendix shows that our results also extend to the five different classifications of forecasters in the Livingston Survey.

Fourth, to further complement our baseline results, Figure 3 summarizes estimates of the under-/overreaction coefficient δ in (2.3), using alternative forecaster surveys and other public

[16] The 1992Q1 observation corresponds to the first realization of five quarter-ahead inflation forecasts ($h+1$) from the SPF after the Federal Reserve Bank of Philadelphia took over the administration of the survey.

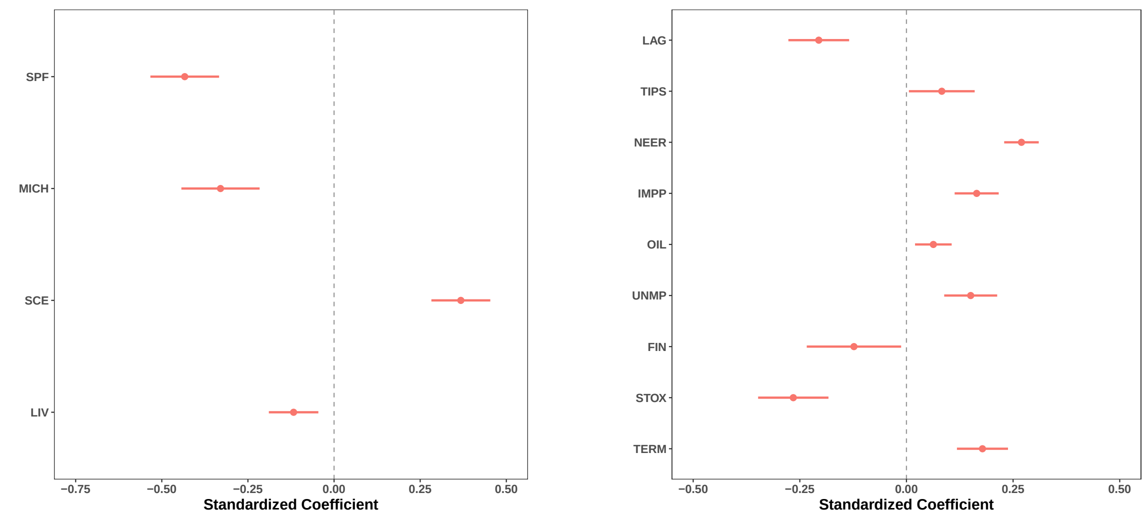
Table II: Robustness and Alternative Estimates

Description	*Avr. Forecast Error*		*Ind. Forecast Error*			
	b-coef	Std. error	β-coef	Std. error	δ-coef	Std. error
GDP Deflator (SPF)	1.118	(0.287)	-0.199	(0.067)	-0.192	(0.085)
CPI Inflation (SPF)	0.282	(0.230)	-0.293	(0.098)	-0.461	(0.079)
Real GDP (SPF)	0.783	(0.262)	-0.186	(0.061)	0.203	(0.161)
GDP Deflator (SPF, post '92)	0.572	(0.246)	-0.381	(0.048)	-0.391	(0.094)
CPI Inflation (SPF, post '92)	0.272	(0.414)	-0.279	(0.175)	-0.555	(0.172)
Real GDP (SPF, post '92)	0.601	(0.379)	-0.087	(0.135)	-0.584	(0.228)
GDP Deflator (SPF, h=2)	0.266	(0.146)	-0.287	(0.051)	-0.058	(0.080)
GDP Deflator (SPF, finan.)	0.609	(0.261)	-0.377	(0.058)	-0.361	(0.083)
GDP Deflator (SPF, non-finan.)	0.295	(0.212)	-0.379	(0.039)	-0.293	(0.118)
HICP Inflation (EASPF)	0.740	(0.406)	-0.169	(0.182)	**-0.669**	(0.665)
Real GDP (EASPF)	0.616	(0.226)	0.411	(0.170)	-0.905	(0.210)
CPI Inflation (LIV)	1.550	(0.733)	-0.270	(0.077)	-0.193	(0.206)
Real GDP (LIV)	0.430	(0.400)	-0.325	(0.130)	-0.709	(0.371)

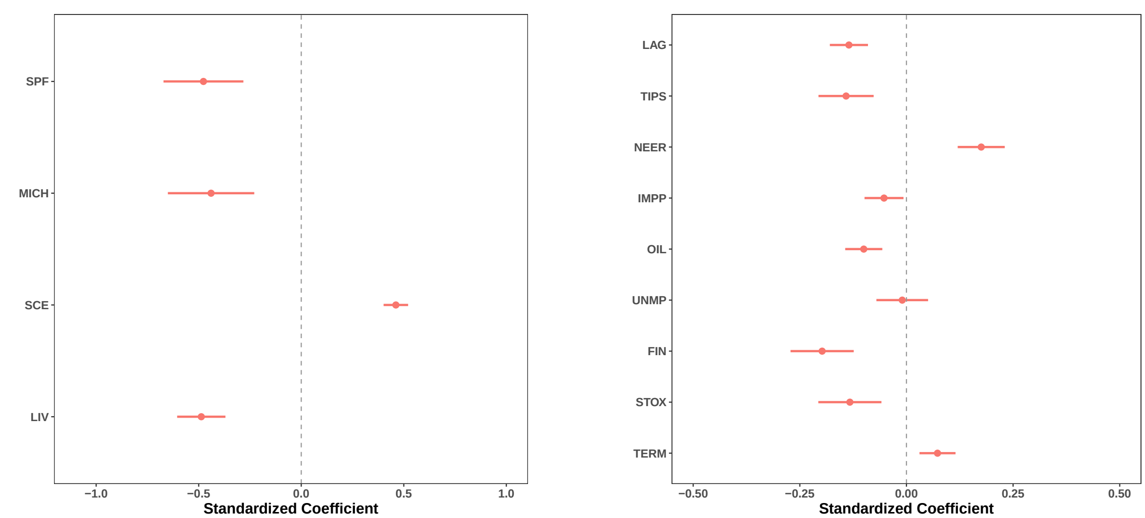
Note: Estimates of b in (2.3), β in (2.2), and δ in (2.3), where the estimates of δ use the previous period's consensus outcome from the survey in question. LIV denotes the Livingston Survey, while EASPF refers to the Euro Area Survey of Professional Forecasters. All estimates are computed using year-on-year growth rates that have been derived using the latest available data at the time of the forecast. Colored coefficients are significant at the five percent level. Robust (doubled-clustered) standard errors are used. Bold indicates a coefficient in which fewer than 50 time-clusters have been estimated, and that is significant using the adjustment in Cameron *et al.* (2010). Samples: SPF(1970Q1-2020Q1), LIV(1993Q1-2020Q1), EASPF(2000Q1-2020Q1).

Figure 3: Robustness and Different Public Signals

(a) Livingstone Survey: Inflation



(b) SPF: Alternative Inflation Measure (CPI)



Note: The figure shows estimates of δ in (2.3) (on the horizontal axis) for various public signals (on the vertical axis). The description of the different public signals is explained in the label for Figure 2. Figure B.1 in the Online Appendix shows similar estimates for the ECB's Survey of Professional Forecasters. All variables and growth rates have been derived using the latest available data at the time of the forecast. All variables have been standardized, and all variables have been signed in accordance with the estimates in Figure 2. Whisker intervals correspond to 95-percent robust clustered confidence bounds. Because of few time-series observations for the Livingstone Survey (LIV) when combined with several public signals, we follow the recommended adjustment in Cameron *et al.* (2010) and cluster at the individual level.

signals than previous consensus outcomes from the same survey. The estimates confirm our initial take-away from Figure 2. The overrevision of individual forecasts appear to be the product of both over- and underreactions to public signals ($\delta \lesseqgtr 0$). For example, similar to the baseline estimates from the US SPF, the broader set of forecasters from the Livingstone Survey (Panel a Figure 3) and the ECB's Survey of Professional Forecasters (Online Appendix Figure B.1) underestimate the inflationary effects of several publicly observable variables (such as movements in the exchange rate). However, forecasters equally overreact to the predictive power of other variables (such as financial prices). Panel (b) in Figure 3 further shows that this coincidence of over- and underreactions extends to US SPF forecasts of CPI inflation. Hence, more generally, the documented overrevision of individual forecasts $\beta < 0$ is comprised of both over- and underreactions to public information ($\delta \lesseqgtr 0$).

Finally, Tables B.4–6 in the Online Appendix contain further robustness checks. We document that the coincidence of over- and underreactions extend to cases where we consider median-individual estimates of b in (2.1), β in (2.2), and δ in (2.3), and that our results also carry over to the case where we winsorize outlier observation (Table B.5 and B.6). Further, Online Appendix Table B.4 shows that if we drop forecaster i from the SPF consensus in the individual-level regression (2.3) the overreaction to consensus documented above remains. This also holds when we exclude outlier observations from consensus. Lastly, consistent with the results in Clements (2018), Table B.4 documents a negative correlation between individual errors and past deviations of forecasts from consensus. This will important for later.

2.5 Summary and Discussion

In summary, our results suggest that *average* forecasts are consistent with models of noisy rational expectations with mean-squared error preferences ($b > 0$). This confirms the results of Coibion and Gorodnichenko (2015). However, *Individual* forecasts show patterns that strongly contradict such models. Specifically, forecasters systematically overreact, on average, to the news that they receive between subsequent survey rounds. This leads to too large forecast revisions relative to the noisy rational expectations benchmark ($\beta < 0$). Consistent with this pattern of overall overrevisions, we find strong evidence of overreactions to a particular public signal that is salient in the context of professional forecasts, namely the consensus forecast from the previous round of the survey ($\delta < 0$). We, nevertheless, also find evidence of sizable underreactions to other public signals ($\delta > 0$). As we have argued in the introduction, and will show formally below, several prominent models of forecaster behavior, both rational and behavioral, struggle to explain this coincidence of over- and underreactions. The next section makes this explicit using a workhorse noisy information framework.

However, before we turn to alternative models, we briefly discuss how our results relate to several contemporaneous papers. Complementary to our results, in independent work, Bordalo *et al.* (2020) demonstrate similar overrevisions of *individual-level* forecasts to those that we document above ($\beta < 0$). In contrast to their paper, we have shown that these overrevisions mask evidence of both over- and underreactions to salient public signals ($\delta \lesseqgtr 0$). We further argue below that this forcefully constrains the set of models that are consistent with the data. Kohlhas and Walther (2021) and Angeletos *et al.* (2020) also depart from Coibion and Gorodnichenko's (2015) findings, but focus on how forecasters extrapolate recent events. Such extrapolation can be viewed as an overreaction to a specific public signal: that of the past outcome of the forecasted variable. Through this lens, our results above have extended this observation by studying forecasters' responses to a wider set of salient public signals. Importantly, we have demonstrated that forecasters also *underreact* to publicly available information. We view this strand of research as presenting complementary steps towards a unified model of expectations that is consistent with the survey data.

3 Rational and Behavioral Models

A variety of popular models of forecaster behavior are consistent with the under- and overrevision of forecasts at the average ($b > 0$) and individual level ($\beta < 0$), respectively. In this section, we show that several of the most prominent of such explanations are nevertheless inconsistent with the documented over- and underreaction to public information ($\delta \lesseqgtr 0$).

3.1 Model Environment

We outline a model that captures several popular environments used to describe economic forecasts. The model is comprised of a continuum of measure one of forecasters, indexed by $i \in [0, 1]$. Forecasters minimize the mean-squared error of their forecasts f_{it} of the random variable π_{t+h} drawn from an uniform distribution over the real line. At time t, all forecasters have the prior belief that $\pi_{t+h} \sim \mathcal{N}(\mu_{it}, \tau_\pi^{-1})$ and observe two types of information.[17] Their own private information is summarized by the *private signal*

$$x_{it} = \pi_{t+h} + \epsilon_{it}^x, \quad \epsilon_{it}^x \sim \mathcal{N}(0, \tau_x^{-1}), \tag{3.1}$$

where the noise terms ϵ_{it}^x are independent across time and of π_{t+h}, and $\mathbb{E}[\epsilon_{it}^x \epsilon_{js}^x] = 0$ for all $j \neq i$ and $s \neq t$. The private signal of one forecaster is not observed by any other forecaster.

[17]We assume that all prior information forecasters observe is condensed into the initial signal $\mu_{it} = \pi_{t+h} + \nu_{ir}$, where $\nu_{ir} \sim \mathcal{N}\left(0, \tau_\pi^{-1}\right)$, observed in period $t-1$. Hence, before the observation of period-t information, forecasters' beliefs about $\pi_{t+h} \sim \mathcal{N}\left(\mu_{it}, \tau_\pi^{-1}\right)$. We further have that $\mu_{it} = f_{it-1}\pi_{t+h}$.

In addition to their private information, all forecasters observe the *public signal*

$$y_t = \pi_{t+h} + \epsilon_t^y, \quad \epsilon_t^y \sim \mathcal{N}(0, \tau_y^{-1}), \tag{3.2}$$

where ϵ_t^y is independent across time, and of π_{t+h} and ϵ_{it}^x for all t and $i \in [0,1]$. We note that this environment allows for rich heterogeneity in expectations, arising from both private information and heterogenous prior forecasts ($\mu_{it} \neq \mu_{jt}$ for $j \neq i$). Finally, we assume that individual forecasts follow the *Generalized Prediction Rule*:

$$f_{it}\pi_{t+h} = (1 - k_x - k_y)\,\mu_{it} + k_x x_{it} + k_y y_t, \tag{3.3}$$

where $k_x \neq k_x^\star$ and $k_y \neq k_y^\star$, and $k_x^\star \equiv \frac{\tau_x}{\tau_\pi+\tau_x+\tau_y}$ and $k_y^\star \equiv \frac{\tau_y}{\tau_\pi+\tau_x+\tau_y}$ denote the mean-squared optimal (rational) weights on private and public information, respectively. The rational weight on the prior μ_{it} is $\frac{\tau_\pi}{\tau_\pi+\tau_x+\tau_y}$. As with the conditional expectation $\mathbb{E}\left[\pi_{t+h} \mid \mu_{it}, x_{it}, y_t\right]$, an individual forecaster in (3.3) updates her prior expectation $\mu_{it} = f_{it-1}\pi_{t+h}$ in response to private and public information, x_{it} and y_t, respectively. But, importantly, relative to the conditional expectation, the forecaster can both over- or underreact to private and public information.

Depending on the precise use of information, the forecasts from (3.3) predict specific values for the regression coefficients b in (2.1), β in (2.2), and δ in (2.3). We note that if $k_x = k_x^\star$ and $k_y = k_y^\star$, the coefficients b, β, and δ are all equal to zero. Proposition 1 summarizes two other important cases, which combined capture a popular set of alternative models.[18]

Proposition 1. *Let individual forecasts $f_{it}\pi_{t+h}$ follow the Generalized Prediction Rule (3.3).*
(i) Then, if $k_x \in (k_x^\star, 1)$ and $k_y = (1-k_x)\frac{\tau_y}{\tau_\pi+\tau_y}$, so that

$$f_{it}\pi_{t+h} = (1-k_x)\,\mathbb{E}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] + k_x x_{it}, \quad k_x \in (k_x^\star, 1)\,, \tag{3.4}$$

$b > 0$ in (2.1), $\beta < 0$ in (2.2), but $\delta = 0$ in (2.3).
(ii) Then, if $k_x = k\frac{\tau_x}{\tau_x+\tau_y}$ and $k_y = k\frac{\tau_y}{\tau_x+\tau_y}$ with $k \in (k^\star, 1)$, where $k^\star \equiv \frac{\tau_x+\tau_y}{\tau_\pi+\tau_x+\tau_y}$, so that

$$f_{it}\pi_{t+h} = \mu_{it} + k\left(\mathbb{E}\left[\pi_{t+h} \mid x_{it}, y_t\right] - \mu_{it}\right), \quad k \in (k^\star, 1)\,, \tag{3.5}$$

$b > 0$ in (2.1), $\beta < 0$ in (2.2), but $\delta < 0$ in (2.3).

The first part of Proposition 1 characterizes individual responses when forecasters overemphasize private information $k_x \in (k_x^\star, 1)$. When forecasters attach more weight to private

[18] We note that we adjust for the bias caused by public information in our derivation of the regression coefficient b in Proposition 1. As mentioned in Section 2, because of the downward nature of this bias, our empirical findings of $b > 0$ are robust to the presence of public information (see also Online Appendix D).

information than optimal, forecasters will, on average, overreact to the information that they receive between two periods. This leads to a negative correlation between individual errors, on the one hand, and individual revisions, on the other hand. Furthermore, this negative correlation coincides with an underrevision of the average forecast ($b > 0$). This is because forecasters with $k_x < 1$ still respond less to private information than the optimal reaction to the *average private signal* ($\int_0^1 x_{it} di = \pi_{t+h}$), which in this case equals one.[19]

However, while an increased weight on private information is consistent with our first two stylized facts, it leads to neither an over- nor an underreaction to public information. In fact, when $k_x > k_x^\star$ (or $k_x \neq k_x^\star$), errors remain uncorrelated with the public signal ($\delta = 0$).

The reason is that a regression of individual errors onto any public signal only considers whether that source of information is used to minimize forecast errors. It does not consider more broadly whether all sources of information, in general, are accurately employed. Although forecasters in (3.4) do not optimally use private information to minimize errors, conditional on this misuse, they still use public information efficiently. The expression $\mathbb{E}[\pi_{t+h} \mid \mu_{it}, y_t]$ enters in (3.4). This, in turn, leads to a δ-coefficient that is equal to zero.[20]

The second part of Proposition 1 considers a natural extension that simultaneously skews forecasters use of private *and* public information away from their mean-squared optimal values. When k exceeds its optimal value ($k > k_\star$), forecasters in (3.5) over-emphasize new information contained in private and public signals relative to their prior expectation. In this sense, forecasters with $k > k_\star$ over-emphasize *all news* that is characteristic of updates relative to prior beliefs. When forecasters overreact to all information, the resulting forecasts from (3.5) can also be consistent with the documented behavior of forecast revisions ($b > 0$, $\beta < 0$). This occurs when $k \in (k_\star, 1)$. But, because forecasters overreact to all information such forecasts are also inconsistent with the documented underreaction to public signals ($\delta > 0$). Instead, such forecast always entail overreactions to public information ($\delta < 0$).

[19] We note that an increased weight on private information $k_x \in (k_x^\star, 1)$ is also consistent with our results in Online Appendix Table B.4, which documents a negative correlation between individual errors and past deviations of forecasts from consensus (Clements, 2018).

[20] Consider the forecast error that results from (3.4):

$$\pi_{t+h} - f_{it}\pi_{t+h} = \pi_{t+h} - k_x x_{it} - (1 - k_x)\,\mathbb{E}[\pi_{t+h} \mid \mu_{it}, y_t].$$

Taking conditional expectations based upon the public signal y then shows that

$$\begin{aligned} \mathbb{E}\left[\pi_{t+h} - f_{it}\pi_{t+h} \mid y_t\right] &= \delta \times y_t \\ &= (1 - k_x)\left(\mathbb{E}\left[\pi_{t+h} \mid y_t\right] - \mathbb{E}\left\{\mathbb{E}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] \mid y_t\right\}\right) = 0, \end{aligned}$$

where the last equality follows from the *Law of Iterated Expectations*. As a result, despite the erroneous use of private information, individual errors remain uncorrelated with the public signal ($\delta = 0$).

3.2 Applications and Extensions

A variety of popular models of forecaster behavior fall within the cases described in Proposition 1. These models are hence consistent with underrevisions of average forecasts ($b > 0$), overrevisions of individual forecasts ($\beta < 0$), but inconsistent with the co-existence of over- and underreactions to public information ($\delta \lesseqgtr 0$). Below, we outline several of these.

1. Strategic Diversification: Laster *et al.* (1999), Ottaviani and Sørensen (2006), and Marinovic *et al.* (2013) describe the market for professional forecasters as a winner-takes-all competition, where only the most accurate forecast is rewarded. As a consequence, in a symmetric equilibrium, all forecasters over-emphasize private information and follow (3.4) with $k_x > k_x^\star$.[21]

2. Reputational Considerations: In Ehrbeck and Waldmann (1996), forecasters are rewarded based on their perceived accuracy. One set of forecasters has access to more precise private information than another. As a result, the set of forecasters that receive less precise information overreact to their private information in an attempt to mimic their more informed competitors, and follow (3.4) with $k_x > k_x^\star$. Their more informed competitors set $k_x = k_x^\star$. The average individual forecast thus follows (3.4) with $k_x > k_x^\star$ (Online Appendix C.1).[22]

3. Behavioral Overconfidence: A considerable literature in psychology has documented that agents over-emphasize their own information (e.g., Moore and Healy, 2008). As discussed in, for example, Daniel *et al.* (1998), and more recently in Angeletos *et al.* (2020), such inherent overconfidence could provide a basis for overreactions to new information. Within our context, overconfident forecasters believe the precision of their private information to be higher than it actually is. Their forecasts thus follow (3.4) with $k_x \in (k_x^\star, 1)$. We return to how a suitably adjusted notion of behavioral overconfidence can capture our stylized facts in Section 4.

4. Models of Generalized Overreactions: A candidate explanation for the overreaction to individual information ($\beta < 0$) and consensus expectations ($\delta < 0$) that we have documented are models of *generalized overreactions*. This includes Bordalo *et al.* (2018b)'s theory of *diagnostic expectations* and Evans and Honkapohja (2012)'s theory of *excess Kalman Gain learning*. In the former case, forecasters overreact to all new information, because it is perceived to

[21] To see why, consider an individual forecaster who sets $k_x = k_x^\star$. Increasing the weight on private information ($k_x > k_x^\star$) leaves the probability of winning the contest approximately unchanged (as the posterior is flat at the conditional expectation). But more weight on private information also (in expectation) strictly reduces the mass of other forecasters that makes the same forecast. In equilibrium, all forecasters therefore choose to follow (3.4) and set k_x such that $k_x \in (k_x^\star, 1)$ (see, e.g., Proposition 4 in Ottaviani and Sørensen, 2006 and Proposition 1 and Corollary 1 in Marinovic *et al.*, 2013).

[22] See the results on p. 24 of Ehrbeck and Waldmann (1996). Online Appendix C.1 extends their model to explicitly account for public information. We assume that forecasters as well as clients observe the public signal y_t in (3.2). We summarize all initial information in the individual-specific prior μ_{it}. With the exception of these modifications all details are as in Ehrbeck and Waldmann (1996).

be diagnostic (or representative) of updates relative to prior information. In the latter case, forecasters instead overreact to increase their speed of learning. Within our framework, these models are captured by (3.5) with $k \in (k^\star, 1)$ (Online Appendix C.2).

5. Underreactions and Rational Inattention: We close this list by noting that several other, prominent models of forecaster behavior fall within the cases described in Proposition 1, but where $k_x \in (0, k_x^\star)$ or $k \in (0, k^\star)$.[23] As a result, these models cannot explain the documented overrevision of individual forecasts ($\beta < 0$). Finally, we note that models of rational inattention (e.g., Sims, 2003), or other rational models of limited attention (e.g., Gabaix, 2017), are likewise inconsistent with $\beta < 0$. This is because forecasts from these models equal conditional expectations, and hence satisfy the Law of Iterated Expectations.[24]

The above examples have shown that several prominent models of forecaster behavior are consistent with under- and overrevisions of expectations at the average ($b > 0$) and individual level ($\beta < 0$), respectively. However, none of these models have been simultaneously consistent with the documented over- and underreaction to public information ($\delta \lesseqgtr 0$). This insight extends beyond the specific applications considered above.

Online Appendix C.3 analyzes a more general model, where strategic incentives skew the optimal use of information away from its mean-squared optimal value. This appendix shows that, despite flexible strategic interactions, errors remain uncorrelated with public information. This result extends to cases with a common noise component in private information. Online Appendix C.4 shows that our results also extend to circumstances where trembling-hand noise drives a wedge between *reported* estimates and *actual* expectations.

Clearly, extensions or combinations of the above environments could potentially alter the prediction listed in Proposition 1. But, at this point, it is worth summarizing why these models fail to match the data. At its heart, the reason is that to explain the survey data forecasters have to flexibly misperceive public information. As Part (ii) of Proposition 1 shows, forecasters cannot, for example, always place an excessive weight on public information. Whatever misperception we consider has to result in both too much as well as too little weight placed on public signals. The next section shows that a natural candidate for such flexible misperceptions arises from forecasters' potentially incorrect views about other's information.

[23] For example, Graham (1999), Welch (2000), Lamont (2002), and Ottaviani and Sørensen (2006) describe models in which forecasters all have a rational incentive to herd, as in Scharfstein and Stein (1990). Hirshleifer *et al.* (2011) instead detail a model in which security analysts for behavioral reasons underreact to information. All of these explanations feature either $k_x \in (0, k_x^\star)$ in (3.4) or $k \in (0, k^\star)$ in (3.5).

[24] Let $x_{it}^\star$ denote the optimal signal observed by a capacity-constrained agent with entropy attention cost. Following Cover and Thomas (2012), $x_{it}^\star$ follows (3.1) but with a precision $\tau_x^\star \neq \tau_x$ that depends upon the capacity constraint. The agent's forecast equals $f_{it}\pi_{t+h} = \mathbb{E}\left[\pi_{t+h} \mid \mu_{it}, x_{it}^\star\right]$. But then the exact same steps as those taken in the proof of Proposition 1 show that $\beta = 0$, because of the Law of Iterated Expectations.

4 Absolute and Relative Overconfidence

In this section, we show that a simple model in which forecasters are overconfident in the precision of their own information (both relative to the truth and relative to their perception of others) can account for all three stylized facts. The next section then explores the potential of our model to also quantitatively match the magnitude of our empirical estimates.

4.1 Overconfidence and Public Information

We build our model of expectations from first principle starting with the well-documented overconfidence heuristic. In their overview of behavioral finance, De Bondt and Thaler (1985) state that "perhaps the most robust finding in the psychology of judgement is that people are overconfident" (p. 6). In particular, we call overconfident those individuals that are not only overconfident in the precision of their own information but also wrongly think that their information is better than others. We therefore merge the two related but distinct notions of overconfidence commonly used in the psychology literature (Moore and Healy, 2008). We refer to the first type as *absolute overconfidence* and the second type as *relative overconfidence* (Benoît *et al.*, 2015). Notice that it is the second, relative aspect of overconfidence that differentiates the notion of overconfidence studied here from that explored in Section 3. To motivate these assumptions, we briefly return to the survey data.

Panel a in Table III uses data on individual-level density forecasts of one-year ahead inflation from the US SPF. It shows that respondents' stated accuracy of their one-year ahead inflation forecasts exceeds their actual accuracy by a sizable amount. The estimated *coverage ratio* of respondents' 95 percent confidence interval, which describes the percentage of times when inflation outcomes fall inside an individual respondent's confidence interval, is only between 72 and 84 percent, depending on the estimation method. Closely related, Griffin and Tversky (1992) show that such absolute overconfidence tends to be more prevalent for forecasters that are faced with prediction tasks that are characterized by a large judgment component and delayed feedback, such as professional economic forecasters.[25]

Panel b in Table III uses the recent survey on firm managers' higher-order expectations of one-year ahead inflation undertaken by Coibion *et al.* (2021) in New Zealand.[26] Consistent with absolute overconfidence in individual-specific information, Coibion *et al.* (2021) document that the cross-sectional standard deviation of respondents' inflation forecasts is too large when

[25]Other prominent examples of overconfidence include the stated precision of forecasts produced by financial market traders, the certainty in the diagnosis of severe illnesses by physicians, and the probability of a positive verdict by procedural lawyers. See, for example, Oskamp (1965), Einhorn (1980), Froot and Frankel (1989), Baumann *et al.* (1991), Benoît *et al.*, 2015, and the summaries in Odean (1998), Thaler (2000), Moore and Healy (2008), and Moore and Dev (2017).

[26]We thank the editor, Olivier Coibion, and an unnamed referee for bringing this data set to our attention.

Table III: Overconfidence in Survey Data on Expectations

Panel a: Coverage Ratio of Forecasts

	Confidence Interval	
Estimation Method	95 percent	66 percent
Density Implied	0.84***	0.58**
Giordani and Soderlind (2003)	0.72**	0.48**

Note: The table uses SPF density forecasts for one-year ahead GDP deflator inflation. The table shows the coverage ratio (the fraction of cases when actual inflation is inside a forecaster's confidence band). If forecasters are rational a 95% confidence band will contain the true but unknown value 95% of the times. The confidence bands are derived assuming a normal distribution and are calculated as: mean of individual inflation densities $\pm$ critical value $\times$ standard deviation. Actual inflation is measured as the percentage change in the index (annual-average) in Q4 of each year. The significance of differences between the nominal confidence level and the actual are assessed using Christoffersen's (1998) test. * p<0.10, ** p<0.05, *** p<0.01. The sample period is 1981Q1 to 2018Q4. For reference, the table also includes estimates from Giordani and Söderlind (2003).

Panel b: Uncertainty and Dispersion of Higher-order Expectations

	Uncertainty	Std. dev.	Implied Higher-order Unc.	
	(1)	(2)	(1)	(2)
Inflation one-year ahead	1.11	3.06	1.76	1.80
Consensus inflation expectation	0.89	2.43	–	–

Note: The first column shows data on the average self-reported uncertainty of one-year ahead inflation expectations, as well as the average self-reported uncertainty about the consensus (average) expectation from the same survey. The second column shows the cross-sectional standard deviation (disagreement) of point forecasts of inflation and consensus. The data in the first two columns are taken from Table II [initial wave] in Coibion *et al.* (2021). The last two columns instead use the data in the first two columns to compute respondents' perception of other respondents' uncertainty of future inflation ("implied higher-order uncertainty"; Appendix A.6), measured as 1/(precision of prior + perceived precision of others' information). The third column uses the data in column one to compute this value, while the fourth column uses the data on disagreement in column two. Uncertainty is in all cases measured in terms of standard deviation. We throughout assume that the unconditional variance of inflation, the inverse of the prior precision, equals that realized post-1985 (Great Moderation) in New Zealand, the country to which the Coibion *et al.* (2021) survey pertains.

compared to the predictions from simple noisy rational expectation models. However, crucially, Panel b also shows that the elicited higher-order moments from the survey combine with their first-order counterpart to imply substantial amounts of relative overconfidence (Appendix A.6). In particular, we use the information encoded about individual views of other's information in the data on uncertainty and disagreement about consensus expectations. This suggest that respondent i's perception of respondent $j \neq i$'s uncertainty of future inflation is, on average, much above that of her own, consistent with the hallmark of relative overconfidence. Indeed, respondents' estimates suggest that individuals believe, on average, that their expectations are 45 percent more accurate than their competitors.

Finally, our model explicitly accounts for the fact that most public signals are *endogenous*. A central feature of the information landscape that people observe is that most of it reflects the realized choices of others. This is true whether one considers data releases on past inflation or output, the observation of asset or goods prices, or the observation of previous period's consensus estimate. Because of this endogeneity of public signals, any *equilibrium* model of expectation formation requires an assumption about individuals' views about the precision of others' information. Rational expectations commonly solves this issue by imposing the symmetry assumption that others' information is equal in quality to one's own. Relative overconfidence, by contrast, imposes the empirically motivated "better than others" perception. Individual views about others' information become especially salient when we consider public signals such as consensus, which reflect simple averages of individual expectations.

4.2 Environment with Overconfidence

We modify our previous environment from Section 3. We assume that inflation π_{t+h} is drawn from the normal distribution $\pi_{t+h} \sim \mathcal{N}(0, \underline{\tau}_{\pi}^{-1})$. At the start of period $t-1$ and t, each forecaster $i \in [0,1]$ receives the private signal $x_{i\tau}$ about the fundamental π_{t+h},

$$x_{i\tau} = \pi_{t+h} + \epsilon_{i\tau}^{x}, \quad \epsilon_{i\tau}^{x} \sim \mathcal{N}\left(0, \tau_x^{-1}\right), \tag{4.1}$$

where $\tau = \{t-1, t\}$ and $\epsilon_{i\tau}^{x}$ is independent of π_{t+h} with $\mathbb{E}\left[\epsilon_{i\tau}^{x}\epsilon_{js}^{x}\right] = 0$ for all $j \neq i$ and s.[27] We introduce the period $t-1$ signal to later allow the public signal that forecasters observe to depend on the previous period's consensus expectation. All forecasters exhibit absolute and relative overconfidence. They believe the precision of their private signals equals $\tau_x' > \tau_x$, and thus to be greater than the truth (absolute overconfidence). At the same time, forecasters also

[27]Hence, at time t, a forecaster receives two private signals: one for π_{t+h} and one for π_{t+h+1}. To avoid complicating the notation further, we do not add an additional subscript on $x_{i\tau}$ to keep a track of the distinct private signals observed at time t. We can do so because of the independence of π_{t+h} and π_{t+h+1}. Neither of our results, however, depend on this independence feature; it merely simplifies the exposition.

believe that other forecasters' private signals have a precision smaller than their own (relative overconfidence) equal to $\hat{\tau}_x < \tau'_x$. We make no assumptions about the relative size of $\hat{\tau}_x$ and τ_x. We note that the observation of x_{it-1} at time $t-1$ results in a period-t prior of π_{t+h} of the exact form used in Section 3 (see below).

At the start of period t, each forecaster, in addition, observes the *endogenous* public signal

$$y_t = \alpha_1 \pi_{t+h} + \alpha_2 f_t \pi_{t+h} + \epsilon_t^y, \quad \epsilon_t^y \sim \mathcal{N}\left(0, \tau_y^{-1}\right), \tag{4.2}$$

where $\alpha_j \geq 0$ for $j = \{1, 2\}$, and ϵ_t^y is independent of π_{t+h} and ϵ_{is}^x for all $i \in [0,1]$ and s.[28] The key difference between the public signal in (4.2) and that explored in (3.2) is the endogeneity of the signal to average individual expectations $f_t \pi_{t+h}$. For example, when $\alpha_1 = 0$ and $\alpha_2 = 1$, equation (4.2) directly becomes the consensus (average) forecast of inflation from the previous period. Vives (2010) and Veldkamp (2011) summarize the importance of public signals of the form (4.2) for the social value of public information, the benefits of social learning, and the volatility of asset prices and business cycles, among others.

We proceed in two steps. We first derive individual expectations of inflation π_{t+h} in period $t-1$ and t, and show how relative overconfidence causes forecasters to flexibly misperceive the public signal y_t. We then provide a set of sufficient conditions for individual expectations to be consistent with all three of our stylized facts.

Consider forecaster i's expectation of π_{t+h} in period $t-1$:

$$f_{it-1} \pi_{t+h} = v x_{it-1}, \tag{4.3}$$

where $v \equiv \frac{\tau'_x}{\tau'_x + \underline{\tau}_\pi}$ exceeds the mean-squared optimal weight on the private signal $v^\star \equiv \frac{\tau_x}{\tau_x + \underline{\tau}_\pi}$, because of forecaster i's (absolute) overconfidence in her private information. Importantly, the coefficient v also exceeds the weight that the forecaster believes others place on their private information (because of relative overconfidence), equal to $\hat{v} \equiv \frac{\hat{\tau}_x}{\underline{\tau}_\pi + \hat{\tau}_x}$.

Let $\mu_{it} \equiv f_{it-1} \pi_{t+h}$ denote forecaster i's prior expectation at the start of period t with perceived precision $\tau_\pi \equiv \underline{\tau}_\pi + \tau'_x$. To derive forecaster i's period-t expectation, we first need to differentiate between two different public signals: (i) the *realized public signal* y_t, and (ii) the *perceived public signal* $\hat{y}_t$. The former measures the actual signal in (4.2),

$$y_t = \alpha_1 \pi_{t+h} + \alpha_2 \int_0^1 f_{it-1} \pi_{t+h} di + \epsilon_t^y = \eta \pi_{t+h} + \epsilon_t^y, \tag{4.4}$$

where $\eta \equiv (\alpha_0 + \alpha_1 v) > 0$. The latter, by contrast, measures the public signal that forecasters

[28]We restrict the sign of α_1 and α_2 to avoid having to always separate between positive and negative signals of the fundamental in our discussions. Neither of our main results depend critically on this assumption.

believe they observe when confronted with observations of y_t,

$$\hat{y}_t = \hat{\eta}\pi_{t+h} + \epsilon_t^y, \tag{4.5}$$

where $\hat{\eta} \equiv (\alpha_0 + \alpha_1 \hat{v}) > 0$. Notice that the signals y_t and $\hat{y}_t$ differ only because of forecasters' misperception about the overconfidence of others ($\eta > \hat{\eta}$); that is, because all forecasters attach a weight of $v > \hat{v}$ to private information in (4.3). This shows how relative overconfidence boils down to a simple one-parameter deviation from rational expectations.

We are now ready to state forecasters' period-t expectation. Combining the public signal in (4.4) with forecasters' perception about it in (4.5), as well as with the period-t private signal in (4.2), shows that

$$\begin{aligned} f_{it}\pi_{t+h} &= (1 - k_x - k_y)\mu_{it} + k_x x_{it} + k_y \times \frac{1}{\hat{\eta}} y_t & (4.6) \\ &= (1 - k_x)\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] + k_x x_{it}, & (4.7) \end{aligned}$$

where $k_x \in (k_x^\star, 1)$, $k_y \lesseqgtr k_y^\star$, and $k_x^\star$ and $k_y^\star$ once more denote the mean-squared optimal weight on private and public information, respectively (Appendix A.2).

We conclude from (4.6) that forecasters' expectations are a special case of those from the generalized prediction rule in (3.3). Equation (4.7) shows that these expectations can also be recast in a form similar to that studied in case (i) in Proposition 1. The difference being that the conditional expectation $\mathbb{E}\left[\pi_{t+h} \mid \mu_{it}, y\right]$ in (4.7) is replaced with the overconfident forecast $\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y\right]$ that accounts for the misperception of the public signal; that is, the conditional expectation of π_{t+h} based on μ_{it} and y_t, but where a forecaster perceives y_t to be governed by equation (4.5) instead of equation (4.4).

4.3 Over- and Underreactions to Public Information

Because of the misperception of the public signal, a correlation naturally arises between individual errors, on the one hand, and the public signal, on the other hand. Taking conditional expectations of forecaster i's error based upon the *realized* public signal y_t shows that

$$\begin{aligned} \delta \times y &= \mathbb{E}\left[\pi_{t+h} - f_{it}\pi_{t+h} \mid y_t\right] \\ &= (1 - k_x)\left(\mathbb{E}\left[\pi_{t+h} \mid y_t\right] - \mathbb{E}\left[\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] \mid y_t\right]\right) \\ &= (1 - k_x)\,\mathbb{E}\left\{\mathbb{E}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] - \mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] \mid y_t\right\} \neq 0, & (4.8) \end{aligned}$$

where we have used the expectation in (4.7) and the Law of Iterated Expectation to arrive at the second and third condition, respectively. Unlike with case (i) in Proposition 1, the Law of Iterated Expectations in (4.8) does not imply orthogonality between individual errors and public information. This is because $\mathbb{E}\left[\pi_{t+h} \mid y_t\right] \neq \mathbb{E}\left\{\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] \mid y_t\right\}$. The misperception of the public signal breaks the implication of the Law of Iterated Expectations that forecast errors are orthogonal to public information. Proposition 2 computes an expression for the over- and underreaction coefficient δ in (4.8).

Proposition 2. *The over- and undereaction coefficient δ in (2.3) equals*

$$\delta = \Delta\left(\kappa^{\star} - \hat{\kappa}\right), \tag{4.9}$$

where $\Delta \in \mathbb{R}_+$, $\kappa^{\star} \equiv \frac{\eta^2\tau_y}{\tau_\pi + \eta^2\tau_y} \times \frac{1}{\eta}$ denotes the rational weight on the public signal y_t in $\mathbb{E}\left[\pi_{t+h} \mid y_t\right]$, while $\hat{\kappa} \equiv \frac{\hat{\eta}^2\tau_y}{\tau_\pi + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}}$ denotes the corresponding misperceived weight.

Intuitively, how forecasters respond to a public signal, such as past consensus outcomes, depends on their views about its precision (conditional variance) and its interpretation (conditional mean). Relative overconfidence causes forecasters to mistake both. On the one hand, it causes forecasters to underestimate the precision of public signals. The realized public signal y_t in (4.4) is more precise than the perceived public signal $\hat{y}_t$ in (4.5). The precision of the former is $\eta^2\tau_y$, while the precision of the latter is only $\hat{\eta}^2\tau_y$, where $\eta^2\tau_y > \hat{\eta}^2\tau_y$ since $v > \hat{v}$. This dismissal of other forecasters' information straightforwardly leads forecasters to *underreact* to the public signal ($\delta > 0$ as it causes $\kappa^{\star} > \hat{\kappa}$). On the other hand, relative overconfidence also causes forecasters to over-infer movements in fundamentals from public signals. The realized public signal y_t loads onto the fundamental π_{t+h} with η in (4.4), while the perceived public signal $\hat{y}_t$ only loads onto the fundamental with $\hat{\eta} < \eta$ in (4.5). Hence, a movement of $d\pi_{t+h} > 0$ in the fundamental causes forecasters to, all else equal, believe in a movement equal to $(\eta/\hat{\eta})d\pi_{t+h} > d\pi_{t+h}$, based on the observation of the public signal alone. This misinterpreation of the public signal, in turn, leads forecasters to *overreact* to its realizations. When forecasters over-infer values of the fundamental from observations of the public signal, they all else equal attach more weight to it than warranted ($\delta < 0$ as it causes $\kappa^{\star} < \hat{\kappa}$).

Depending on the relative strength of these effects, Proposition 2 shows that both under- and overreactions to a public signal can arise from individuals' dismissal of other's private information. Indeed, equation (4.9) provides the condition for $\delta \lesseqgtr 0$.

We close this subsection with two additional observations that follow from Proposition 2. First, we note that *overreactions (underreactions)* to public signals naturally arise when the public signal that forecasters observe is sufficiently precise (imprecise). Equation (4.9) shows that $\lim_{\tau_y \to 0} \delta > 0$ while $\lim_{\tau_y \to \infty} \delta < 0$. In Section 5, we relate this finding to our empirical estimates of δ for different public signals in Figure 2. Second, we note that when forecasters believe others' information is poor $\hat{\tau}_x \to 0$, equation (4.9) shows that underreactions always occur ($\delta > 0$). This can provide a lens through which to interpret some of our estimates using alternative consensus estimates in Section 2.

Corollary 1. *If the public signal y_t becomes sufficiently precise $\tau_y \to \infty$ (or imprecise $\tau_y \to 0$) overreactions (underreactions) always occur $\delta < 0$ ($\delta > 0$). Underreactions $\delta > 0$ also occur when forecasters almost fully disregard others' information $\hat{\tau}_x \to 0$.*

4.4 Data-consistent Expectations

Unlike the models in Section 3, the expectations in (4.7) can be consistent with all three stylized facts documented in Section 2. We show this concretely by focusing on our results in Table I ($b > 0$, $\beta < 0$, and $\delta < 0$), where we consider previous period's consensus estimate as the relevant public signal ($\alpha_1 = 0$ and $\alpha_2 = 1$). Section 5 explores the quantitative potential of our model to also match the magnitude of the empirical estimates.

Proposition 3. *Suppose $\alpha_1 = 0$ and $\alpha_2 = 1$, such that the public signal y_t corresponds to previous period's consensus estimate, and consider individual $i \in [0,1]$'s forecast*

$$f_{it}\pi_{t+h} = (1 - k_x)\,\mathbb{F}[\pi_{t+h} \mid \mu_{it}, y_t] + k_x x_{it}, \quad k_x \in (k_x^\star, 1)\,. \tag{4.10}$$

If $\eta^2\tau_y > \max(\chi, 1)\,\underline{\tau}_\pi$, where $\chi \equiv \frac{\underline{\tau}_\pi \hat{\eta}}{\tau_x'(1-\hat{\eta})}$, then there exists $c_0, c_1 \in \mathbb{R}_+$ such that, for $\epsilon > 0$ and $\tau_x' = \tau_x + c_0\epsilon$ and $\hat{\eta} = \eta - c_1\epsilon$, the coefficients satisfy $\beta < 0$, $\delta < 0$, and $b > 0$.

Proposition 3 combines the insights of Proposition 1 and 2. The first and second result in Proposition 3 ($b > 0$ and $\beta < 0$) resemble those in case (i) of Proposition 1. On the one hand, because of the dispersion in private signals, the average information across forecasters is more precise than any individual's. This, in turn, causes average forecasts to underreact to the average information observed ($b > 0$). On the other hand, despite these underreactions at the average level, at the individual level, forecasters overrevise their expectations ($\beta < 0$). This is once more in part due to forecasters' overconfidence in their own private information.

However, where Proposition 3 differs from case (i) of Proposition 1 is that forecasters also overreact to the past consensus outcomes ($\delta < 0$). These overreactions occur because forecasters' perceived and actual weight on private information are sufficient to ensure that the *perceived under-responsiveness of consensus* dominates its *perceived under-precision*. The condition $\eta^2\tau_y > \max(\chi, 1)\,\underline{\tau}_\pi$ ensures that relative overconfidence $\hat{\eta} < \eta$ delivers $\hat{\kappa} > \kappa^\star$ in (4.9) in Proposition 2. Combined with the dispersion and overconfidence in private information, this then ensures that the expectations from equation (4.7) are consistent with all three stylized facts documented in Table I.

5 Quantitative Implications

We have shown how our model of overconfidence can be qualitatively consistent with stylized facts about individual forecasts. Although our model is simple, in this subsection we explore the capacity of the model to also quantitatively match the survey data. We show that our model can account for the baseline estimates in Table I, and that our calibrated model entails degrees of absolute and relative overconfidence that are in line with auxiliary data. We also test several key implications of our model, and discuss its economic consequences.

5.1 Model Calibration

We use a simulated method of moments procedure to choose parameter values. Normalizing the variance of inflation to one and employing the restriction that $\hat{\tau}_x = \tau_x$, identification of the three parameters τ_x, τ_x', and τ_y requires at least three target moments. We choose the individual overrevision and overreaction coefficients β in (2.2) and δ in (2.3), respectively, documented in Table I. We choose the previous consensus expectation as the benchmark public signal because its structure is simple and known ($\alpha_1 = 0$, $\alpha_2 = 1$ in 4.2), and because its only relationship with future inflation is that of aggregating others' information. We then later show that the calibrated model also matches dimensions of the responses of individual errors to other public signals than consensus. Finally, we also include the estimate of information frictions b to our list of target moments. In particular, to account for the special feature that our baseline model only has one public signal, and not numerous as used by professional forecasters, we target the bias-adjusted measure of information frictions from Goldstein (2021). This estimate, in effect, bias-adjusts the $b-$coefficient in (2.1) for the presence of public information, and hence provides a more comparable estimate of the extent of information frictions to that of our model.[29] The criterion we choose to minimize is the sum of absolute deviations of target moments from model simulated moments.

[29] Online Appendix D provides details on the Goldstein (2021)-adjustment of the Coibion and Gorodnichenko (2015) estimate of information frictions in (2.1). In particular, the Goldstein (2021)-adjustment produces an unbiased estimate that is positively proportional to the bias-adjusted $b-$coefficient in (2.1).

Table IV: SMM Estimation: Inflation Forecasts

	β	δ	$b^\star$	Cons. dev.	$\sqrt{\tau_x}$	$\sqrt{\tau_y}$	$\sqrt{\underline{\tau}_\pi}$	$\sqrt{\tau'_x}$
Data	-0.19	-0.20	0.42	-0.59	–	–	–	–
Model	-0.18	-0.19	0.58	-0.79	0.41	4.74	1.00	0.95

Note: The table presents the values of the moments β in (2.2), δ in (2.3), and $b^\star$ from Goldstein (2021) (see also Online Appendix D). The table compares estimates using one-year ahead US SPF inflation forecasts (first row) and model estimates (second row). The table also reports the results for the (non-targeted) Clements (2018) regression (Cons. dev.), corresponding to column three in Online Appendix Table B.4, as well as the calibrated values of the model's precision parameters.

Table IV presents the results for one-year ahead inflation, where for ease of interpretation we report the square root of the precision, the inverse of the standard deviation. Our model is able to capture all three data moments well. We estimate private signals to be rather noisy ($\sqrt{\tau_x} = 0.41$) and the noise in consensus to be small ($\sqrt{\tau_y} = 4.74$). At a level of overconfidence that increases the square-root of the perceived precision of private signals by somewhat more than two, the model predicts accurately the overrevision of individual forecasts β and the overreaction to past consensus realizations δ. This is consistent with our previous discussion, which showed that the combination of a precise consensus and meaningful overconfidence, all else equal, makes overreactions more pervasive. The model also matches the level of information frictions well, although it entails somewhat too high information frictions. Finally, Table IV shows that our estimates also capture well the non-targeted Clements (2018) regression of individual errors onto consensus deviations.

5.2 Model Evaluation

5.2.1 Estimates of Overconfidence

The estimates in Table IV entail a noticeable degree of overconfidence. We next turn to how the implied estimates of absolute and relative overconfidence match those from survey data.

Estimates of Absolute Overconfidence: The individual density forecasts of one-year-ahead inflation, available in the US SPF, allow us to evaluate whether the implied degree of absolute overconfidence from our model is reasonable. Panel a in Table V presents this comparison in the form of *coverage ratios,* describing the percentage of times when actual inflation outcomes fall inside an individual forecaster's 95 (or 66) percent confidence band. Panel a contrasts the coverage ratios implied by our benchmark estimates in Table IV with those that are estimated from US SPF data in Table III. On balance, the implied degree of absolute overconfidence captures well that in the US SPF data. Forecasters' 95 percent confidence band has a coverage ratio of only 70 percent, consistent with a sizable amount of absolute overconfidence. This matches the magnitude of the US SPF estimate. In fact, Giordani and Söderlind (2003) find similar degrees of absolute overconfidence to those implied by our model estimates, using a somewhat more advanced estimation method to deduce individual confidence bands from reported forecast densities. We view the estimates in Panel a in Table V as important auxiliary evidence that corroborates our first main assumption of absolute overconfidence.

Estimates of Relative Overconfidence: Reported forecast densities can be used to assess the extent of *absolute overconfidence*, or the perceived precision of forecasters' information relative to the truth. In contrast, to assess the extent of *relative overconfidence* requires information about forecasters' perception of other forecasters' uncertainty (or expectations). This is typi-

Table V: Overconfidence in Survey and Model Data

Panel a: Coverage Ratio of Forecasts

Confidence Bands	*Confidence Level*	
	95 percent	66 percent
SPF Density Implied	0.82***	0.56**
Giordani and Soderlind (2003)	0.72**	0.48**
Model Implied	0.70	0.41

Note: The table shows the implied coverage ratio. The confidence bands from the SPF are derived assuming a normal distribution and are calculated as: mean of individual density forecast ± critical value × standard deviation. Actual inflation is measured as the percentage change in the GDP Deflator (annual-average). The significance of differences between the nominal confidence level and the actual are assessed using Christoffersen's (1998) test. * p<0.10, ** p<0.05, *** p<0.01. The sample is from 1981Q1 to 2018Q4.

Panel b: Ratio of Higher-order Uncertainty to First-order Uncertainty

	Level		*Share*	
	(1)	(2)	(1)	(2)
Survey Data	1.59	1.62	1.00	1.00
Model Implied	1.28	1.28	0.81	0.79

Note: The table estimates the ratio of higher-order uncertainty to first-order uncertainty in the survey data from Coibion *et al.* (2021), using the implied estimates in Table III. The table then compares these estimates to the model-implied counterparts. The model-implied results use the estimates of τ_x' and $\hat{\tau}_x = \tau_x$ from Table IV. An absence of relative overconfidence results in a ratio of one. Uncertainty is measured in units of standard deviation. We express the results in terms of share of the data in the final two columns to account for the different volatilities of inflation in New Zealand and the US, and to account for our model only being calibrated to US data. We note that there is only one model estimate, consistent with only one calibration of the model. Consistent with Table III, columns denoted with a (1) use survey data on uncertainty about consensus to estimate higher-order uncertainty. Columns denoted with a (2) instead use survey data on the cross-sectional dispersion of forecasts of consensus to estimate higher-order uncertainty.

cally not available in expectational survey data. The only exceptions that we are aware of are the survey of New Zealand firm managers, conducted by Coibion *et al.* (2021) and discussed in Table III, and the German ZEW survey (Köhler and Schmidt, 2021). The former asks its participants about their uncertainty about the consensus estimate of future inflation; the latter, by contrast, asks its respondents every month for their best forecast of the consensus estimate of an aggregate index of German economic activity.

Panel b in Table V shows the model-implied estimates of the ratio of higher-order uncertainty (i.e. a forecaster's estimate of another forecaster's uncertainty about future inflation) to first-order uncertainty of future inflation, using the parameters from Table IV. An absence of relative overconfidence results in a ratio of one. The table compares these estimates to those implied by the Coibion *et al.* (2021) survey, reported in Table III. Consistent with relative overconfidence, our model estimates show that forecasters perceive their own expectations to be around 30 percent more accurate than their competitors. As a result, our model accounts for around 80 percent of the relative overconfidence implied by the Coibion *et al.* (2021) data. That said, clearly, the implied estimates from our model, based on US SPF data, are not fully comparable to those from the Coibion *et al.* (2021) survey, because of differences in respondent types (professional forecasters vs. managers) and countries covered (US vs. New Zealand). Notwithstanding these discrepancies, the fact that the implied estimates in Panel b are of a similar magnitude is comforting, and the table does provide independent validation of our second main assumption of relative overconfidence.

Finally, Online Appendix F uses the time series data on higher-order expectations of economic activity, available from the ZEW survey, to directly estimate the actual and perceived weight on private information. Consistent with relative overconfidence, we estimate the actual weight on private information v to be around twice the perceived weight attached by others $\hat{v}$, although the difference is only borderline statistically significant when accounting for outlier observations. In the calibrated model, the weight on private information v is around three times the perceived weight.[30] We view these estimates, although pertaining to another variable and country, as lending further support to our assumption of relative overconfidence.

5.2.2 Heterogeneity in Responses to Public Information

We revisit the evidence in Figure 2, documenting heterogeneous responses to public information, ranging from over- to underreaction ($\delta \lesseqgtr 0$). In particular, we analyze how the over- and underreaction coefficient δ from our model changes with respect to the precision of public information. We then compare these predictions to estimates in the survey data.

[30] We note that the estimated weight on private information in the ZEW is smaller than the model-implied estimate. The estimate from the ZEW data is though roughly in line with that backed out by Coibion *et al.* (2021). This is consistent with the presence of several, additional public signals, beyond consensus estimates.

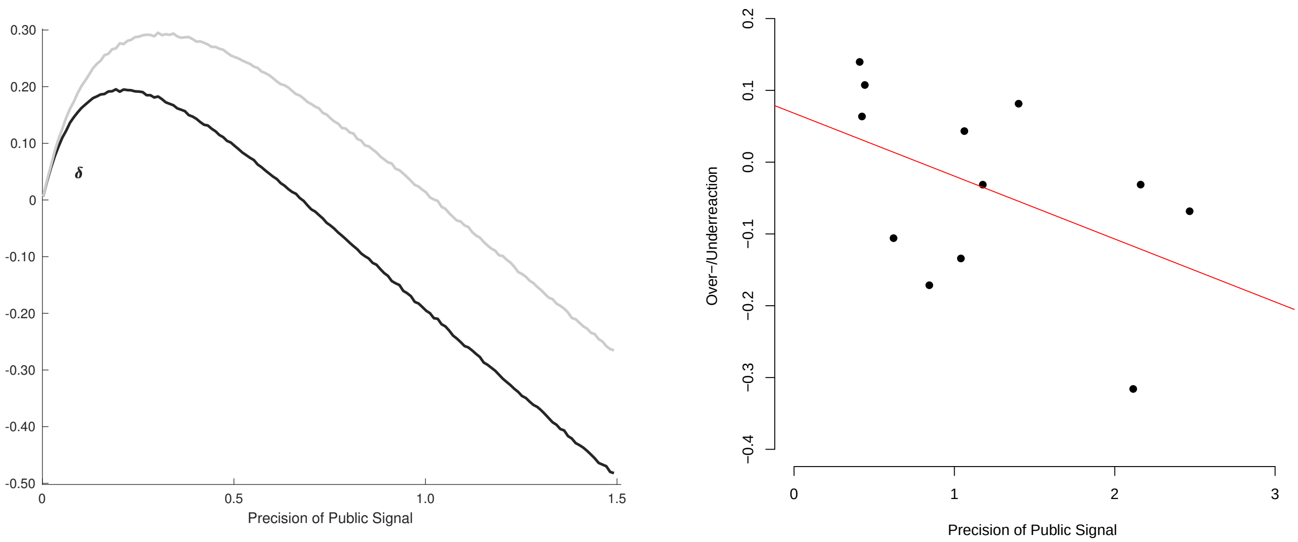
The left-hand panel in Figure 4 computes the model-implied estimates of δ as a function of the precision of the consensus signal in (4.4). The figure highlights two features of our model: First, all else equal, forecasters overreact more strongly to more precise signals (see also Corollary 1). Second, the precise parameters of the model determine the magnitude of over- and underreactions for any given precision of public information. The right-hand panel of Figure 2, by contrast, returns to the survey data. It illustrates the relationship between the precision of different public signals of one-year ahead inflation and the over- and underreaction coefficient δ in (2.3), using our estimates in Figure 2 but for a common sample. In line with the prediction of our model, we observe stronger overreactions to more precise signals. We note that the range of values in the left- and right-hand panel do not necessarily overlap, as the right-hand panel uses estimates employing other public signals than consensus. Overall, the results in Figure 4 lend credence to the notion that an important determinant of over- and underreactions to public signals is the noisiness of the signal in question.

5.3 Implications and Discussion

We conclude this section by discussing auxiliary implications of our calibrated model. The top left-hand panel in Online Appendix E shows the (demeaned) distribution of individual period-t forecasts implied by the model. Compared to rational, mean-squared optimal forecasts, the standard deviation of the overconfident forecast distribution is about three times larger. This is because overconfidence causes individuals to put additional weight on private information. Overconfidence in the precision of private information can thus help explain the *a priori* puzzling amount of dispersion in macroeconomic expectations (e.g., Mankiw *et al.*, 2003).

However, importantly, this increase in dispersion does not lead to substantially more imprecise expectations in equilibrium. Online Appendix E shows that the standard deviation of individual errors is only slightly larger in the overconfident case. As a result, forecasters in our model would face difficulty inferring from the accuracy of their own forecasts alone that they were indeed overconfident. The bottom panel in Online Appendix E illustrates the reason for this close equivalence: the endogenous public signal (consensus, in this case) is substantially more precise in the overconfident case. Because overconfident forecasters put more weight on private information, the endogenous consensus outcome embeds more of the sum of forecasters' private information, the only truly new information that forecasters can learn from each other. In effect, overconfidence in private information counteracts the standard learning externality that exists in markets with endogenous public information and which causes agents to attach too little weight to private information (e.g., Amador and Weill, 2010).

Figure 4: Overreaction and the Precision of Public Signals



Note: The left panel illustrates model-implied variations in δ as a function of the precision of consensus τ_y relative to its calibrated value from Table I (black line). A value of one on the horizontal axis, therefore, corresponds to a precision of public information equal to that in Table I. The gray line decreases the precision of private information τ_x and forecasters' beliefs about it τ_x' by 25 percent. The right hand panel shows the estimates of δ for different public signals (along the vertical axis), using the variables from Panel (a) and (b) in Figure 2, as a function of the signals' estimated precision (along the horizontal axis). Consistent with Section 4, we estimate the precision of public signals as the inverse of the variance of an error term. For consensus signals of the same forecast horizon (Panel a in Figure 2), the error term equals the difference between the realized value of one-year ahead inflation and its consensus forecasted value. For other public signals (Panel b in Figure 2), the error terms are instead constructed as the residuals from a linear regression of one-year-ahead inflation onto the public signal in question. To make the precision and δ-estimates comparable across series, we focus on the longest common sample available (1981Q1-2020Q1) and standardize the variables over this sample. Notice that this contrasts to Figure 2, where δ is estimated on the full-sample for each series. Finally, we drop the TIPS from the figure, as it is only available after 2015.

A core argument for rational, mean-squared optimal expectations is that such beliefs make agents as well-off as they can be. However, this rationale for rational expectations relies upon agents being strictly worse off with non-rational beliefs. As Online Appendix E shows, this is not necessarily the case in our model. This connects our results with those of Smith (1982) and others that attempt to find "group optimal explanations" for individual biases.

Finally, a substantial literature in macroeconomics has explored whether noise shocks to public information can explain business cycle fluctuations. Because agents in our model attach more weight to public information than optimal, any such shock also has a heightened effect on individual expectations. Compared to a rational model, our model therefore predicts larger responses to public noise shocks. This illustrates one potentially important implication of the combination of absolute and relative overconfidence. Other potential implications include: (i) increases in trade in financial assets, due to increases in the dispersion of (relative) beliefs; (ii) "over-shooting" of asset prices in response to public announcements; and (iii) increases in investments into new product lines. We leave these topics, and others, for future research.

6 Concluding Remarks

Expectations are central determinants of economic allocations. In part because of this central role, a considerable debate has arisen since Muth's (1961) seminal contribution about the best model of expectation formation. Recently, influential evidence has shown that *average* forecasts across a wide variety of surveys are consistent with models of noisy information and rational information use (Coibion and Gorodnichenko, 2015). By contrast, in this paper we have explored the implications of such models for *individual* professional forecasts.

We have demonstrated that the statistical properties of individual inflation forecasts contradict simple versions of noisy rational expectations. Specifically, we have documented two stylized facts: First, individual forecasters' overrevise their macroeconomic expectations. Second, such overrevisions mask evidence of both over- and underreactions to salient public signals. We have shown that such responses violate a basic tenet of noisy rational expectations, the Law of Iterated Expectations, and demonstrated that such violations also contradict several common agency-based and behavioral models of expectation formation.

In place, we have proposed a simple extension of noisy rational expectations, consistent with the stylized facts, building on two frictions: Forecasters believe that their own private information is not only better than it truthfully is (absolute overconfidence), but also better than that available to others (relative overconfidence). Combined, these biases entail that forecasters both overreact to private information and misperceive the informativeness of endogenous public signals that aggregate other agents' private information. We showed that the latter can cause forecasters to both under- and overreact to public signals in a manner that is consistent with the data. Finally, we have demonstrated that our model is not only *qualitatively* consistent with survey data but also captures key features *quantitatively*.

We hope that our paper may serve as a stepping stone for further empirical and theoretical research, along similar lines. Our model has illustrated how simple behavioral biases can combine with the endogeneity of public information to create rich patterns of predictability in individual forecast errors. This basic idea is more general than our particular forecaster application. For example, in future research, it would be valuable to consider asset price and business cycle implications of richer descriptions of absolute and relative overconfidence. This would also have the advantage of creating further testable predictions.

A Proofs and Derivations

A.1 Proof of Proposition 1

The proof studies each of the two cases in isolation.[31]

Case (i): Overreaction to private information. The prediction rule followed is

$$f_i\pi = (1-k_x)\,\mathbb{E}\left[\pi \mid \mu_i, y\right] + k_x x_i, \quad k_x \in (k_x^\star, 1), \tag{A1}$$

where the mean-squared optimal forecast can be written as

$$\mathbb{E}\left[\pi \mid x_i, \mu_i, y\right] = (1-k_x^\star)\,\mathbb{E}\left[\pi \mid \mu_i, y\right] + k_x^\star x_i, \quad k_x^\star = \frac{\tau_x}{\tau_\pi + \tau_x + \tau_y}. \tag{A2}$$

The coefficient β in (2.2) is, in this case,[32]

$$\begin{aligned}
\beta &\propto \mathbb{C}\mathrm{ov}\left[\pi - f_i\pi, f_i\pi - \mu_i\right] \\
&= \mathbb{C}\mathrm{ov}\left[\pi - \mathbb{E}\left[\pi \mid x_i, \mu_i, y\right] + \mathbb{E}\left[\pi \mid x_i, \mu_i, y\right] - f_i\pi, f_i\pi - \mu_i\right] \\
&= \mathbb{C}\mathrm{ov}\left[\mathbb{E}\left[\pi \mid x_i, \mu_i, y\right] - f_i\pi, f_i\pi - \mu_i\right] \\
&= (k_x^\star - k_x)\,\mathbb{C}\mathrm{ov}\left[x_i - \mathbb{E}\left[\pi \mid \mu_i, y\right], k_x x_i - \mu_i\right] = (k_x^\star - k_x)\,k_x \mathbb{V}\left[x_i - \mathbb{E}\left[\pi \mid \mu_i, y\right]\right],
\end{aligned}$$

where we have used (A1) and (A2). Thus, $\beta < 0$ since $k_x > k_x^\star$.

The coefficient δ in (2.3) is

$$\begin{aligned}
\delta &\propto \mathbb{C}\mathrm{ov}\left[\pi - f_i\pi, y\right] \\
&= \mathbb{C}\mathrm{ov}\left[\pi - k_x\pi - k_x\epsilon_i^x - (1-k_x)\mathbb{E}\left[\pi \mid \mu_i, y\right],\ y\right] \\
&= (1-k_x)\,\mathbb{C}\mathrm{ov}\left[\pi - \mathbb{E}\left[\pi \mid \mu, y\right], y\right] = 0.
\end{aligned}$$

Hence, $\delta = 0$.

Finally, notice that we can write $f_i\pi$ in (A1) as

$$\begin{aligned}
f_i\pi &= (1-k_x)\left[w\mu_i + (1-w)y\right] + k_x x_i, \quad w \equiv \frac{\tau_\pi}{\tau_\pi + \tau_y} \\
&= k_x x_i + w_y y + (1 - k_x - w_y)\,\mu_i, \quad w_y \equiv (1-k_x)(1-w).
\end{aligned}$$

Therefore,

$$f\pi = k\pi + (1-k)\mu + w_y\epsilon_y,$$

[31] In this section, to ease notation, we disregard time subscripts.

[32] We use the sign $\propto$ to denote "positively proportional to".

where $k \equiv k_x + w_y$ and $\mu \equiv \int_0^1 \mu_i di$. Using this expression, we arrive at

$$\begin{aligned}\pi - f\pi &= \frac{1-k}{k}(f\pi - \mu) - \frac{w_y}{k}\epsilon_y \\ &= \frac{(1-k_x)w}{k_x + (1-k_x)(1-w)}(f\pi - \mu) - \frac{(1-k_x)(1-w)}{k_x + (1-k_x)(1-w)}\epsilon_y.\end{aligned}$$

We conclude that, adjusting for the bias term $-\frac{(1-k_x)(1-w)}{k_x+(1-k_x)(1-w)} < 0$ caused by the presence of public information (Section 5; Online Appendix D), $b > 0$ in (2.3) since $k_x < 1$.[33]

Case (ii): Overreaction to all information. The prediction rule followed is

$$f_i\pi = \mu_i + k\left(\mathbb{E}\left[\pi \mid x_i, y\right] - \mu_i\right), \quad k \in (k^\star, 1), \tag{A3}$$

where the mean-squared optimal forecast can be written as

$$\mathbb{E}\left[\pi \mid x_i, \mu_i, y\right] = \mu_i + k^\star\left(\mathbb{E}\left[\pi \mid x_i, y\right] - \mu_i\right), \quad k^\star \equiv \frac{\tau_x + \tau_y}{\tau_\pi + \tau_x + \tau_y}. \tag{A4}$$

The coefficient β in (2.2) is now

$$\begin{aligned}\beta &\propto \mathbb{C}\text{ov}\left[\pi - f_i\pi, f_i\pi - \mu_i\right] \\ &= \mathbb{C}\text{ov}\left[\mathbb{E}\left[\pi \mid x_i, \mu_i, y\right] - f_i\pi, f_i\pi - \mu_i\right] \\ &= \left(k^\star - k\right)\mathbb{C}\text{ov}\left[\mathbb{E}\left[\pi \mid x_i, y\right] - \mu_i, f_i\pi - \mu_i\right] = \left(k^\star - k\right)k\mathbb{V}\left[\mathbb{E}\left[\pi \mid x_i, y\right] - \mu_i\right],\end{aligned}$$

where we have used (A3) and (A4). Hence, $\beta < 0$ since $k > k^\star$.

The coefficient δ in (2.3) is

$$\begin{aligned}\delta &\propto \mathbb{C}\text{ov}\left[\pi - f_i\pi, y\right] \\ &= \mathbb{C}\text{ov}\left[\mathbb{E}\left[\pi \mid x_i, \mu_i, y\right] - f_i\pi, y\right] \\ &= (k^\star - k)\mathbb{C}\text{ov}\left[\mathbb{E}\left[\pi \mid x_i, y\right] - \mu_i, y\right] = \left(k^\star - k\right)\left(1 - \frac{\tau_x}{\tau_x + \tau_y}\right)\frac{1}{\tau_y}.\end{aligned}$$

Thus, $\delta < 0$ since $k > k^\star$.

Lastly, notice that (A3) implies that

$$f_i\pi = \mu_i + k\left[\pi + w\epsilon_{xi} + (1-w)\epsilon_y - \mu_i\right], \quad w \equiv \frac{\tau_x}{\tau_x + \tau_y}.$$

[33] Recall that because presence of public information, the least-squares estimates for $b > 0$ that we document in Section 2 underestimate the true extent to which $b > 0$. The evidence for underrevisions of average forecasts is, in this sense, robust to the presence of public information.

Averaging this expression across i, and rearranging terms, then shows that

$$\pi - f\pi = \frac{1-k}{k}\left(f\pi - \mu\right) - \left(1-w\right)\epsilon_y. \tag{A5}$$

Thus, once more adjusting for the bias term $-\left(1-w\right) < 0$ caused by the presence of public information (Section 5; Online Appendix D), $b > 0$ in (2.3) because $k < 1$. □

A.2 Derivation of Equation (4.6) and (4.7)

We note that the perceived precision of the public signal y_t is $\hat{\eta}^2\tau_y$. The perceived responsiveness to the fundamental π_{t+h} is $\hat{\eta}$. The two private signals x_{it-1} and x_{it} about π_{t+h} have a perceived precision of τ_x' and a loading on the fundamental π_{t+h} of one.

Bayesian updating then provides us with

$$f_{it}\pi_{t+h} = k_x x_{it-1} + k_x x_{it} + k_y y_t, \tag{A6}$$

where

$$k_x = \frac{\tau_x'}{\underline{\tau}_\pi + 2\tau_x' + \hat{\eta}^2\tau_y}, \quad k_y = \frac{\hat{\eta}^2\tau_y}{\underline{\tau}_\pi + 2\tau_x' + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}}. \tag{A7}$$

Now, notice that $\mu_{it} = f_{it-1}\pi_{t+h} = v x_{it-1}$ with $v = \frac{\tau_x'}{\underline{\tau}_\pi + \tau_x'}$, and that $\tau_\pi = \underline{\tau}_\pi + \tau_x'$. Hence,

$$\begin{aligned} f_{it}\pi_{t+h} &= k_x \frac{1}{v}\mu_{it} + k_x x_{it} + k_y y_t \\ &= \frac{\tau_\pi}{\tau_\pi + \tau_x' + \hat{\eta}^2\tau_y}\mu_{it} + k_x x_{it} + k_y y_t \\ &\equiv k_\mu \mu_{it} + k_x x_{it} + k_y y_t \end{aligned} \tag{A8}$$

where

$$k_x = \frac{\tau_x'}{\tau_\pi + \tau_x' + \hat{\eta}^2\tau_y}, \quad k_y = \frac{\hat{\eta}^2\tau_y}{\tau_\pi + \tau_x' + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}}.$$

The mean-squared optimal weight on x_{it}, conditional on the prior μ_{it} and the misperception of the public signal y_t, is $k_x^\star = \frac{\tau_x}{\tau_\pi + \tau_x + \hat{\eta}^2\tau_y}$, so that $k_x \leq k_x^\star$.

Finally, let $\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] = \mathbb{F}\left[\pi_{t+h} \mid x_{it-1}, y_t\right]$ be the conditional expectation of π_{t+h} under the misperception that y_t is governed by (4.5). Hence,

$$\begin{aligned} \mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] &= \frac{\tau_x'}{\underline{\tau}_\pi + \tau_x' + \hat{\eta}^2\tau_y} x_{it-1} + \frac{\hat{\eta}^2\tau_y}{\tau_\pi + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}} y_t \\ &= \frac{\tau_\pi}{\tau_\pi + \hat{\eta}^2\tau_y}\mu_{it} + \frac{\hat{\eta}^2\tau_y}{\tau_\pi + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}} y_t. \end{aligned}$$

But then (A8) shows that

$$\begin{aligned} f_{it}\pi_{t+h} &= k_x x_{it} + (1-k_x)\left[\frac{\tau_\pi}{\tau_\pi + \hat{\eta}^2\tau_y}\mu_{it} + \frac{\hat{\eta}^2\tau_y}{\tau_\pi + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}} y_t\right] \\ &= k_x x_{it} + (1-k_x)\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right]. \end{aligned} \tag{A9}$$

A.3 Proof of Proposition 2

The forecast error at time t is

$$\pi_{t+h} - f_{it}\pi_{t+h} = (1-k_x)\left(\pi_{t+h} - \mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right]\right) - k_x \epsilon_{it}^x.$$

Thus,

$$\begin{aligned} \delta \times y = \mathbb{E}\left[\pi_{t+h} - f_{it}\pi_{t+h} \mid y_t\right] &= (1-k_x)\left\{\mathbb{E}\left[\pi_{t+h} \mid y_t\right] - \mathbb{E}\left[\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] \mid y_t\right]\right\} \\ &\propto \mathbb{E}\left[\pi_{t+h} \mid y_t\right] - \mathbb{E}\left[\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] \mid y_t\right], \end{aligned}$$

where

- $\mathbb{E}\left[\pi_{t+h} \mid y_t\right] = \frac{\eta^2\tau_y}{\mathcal{T}_\pi + \eta^2\tau_y} \times \frac{1}{\eta} y_t$
- $\mathbb{E}\left[\mathbb{F}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] \mid y_t\right] = \frac{\tau_x'}{\mathcal{T}_\pi + \tau_x' + \hat{\eta}^2\tau_y} \times \frac{\eta^2\tau_y}{\mathcal{T}_\pi + \eta^2\tau_y} \times \frac{1}{\eta} y_t + \frac{\hat{\eta}^2\tau_y}{\tau_\pi + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}} y_t.$

Hence,

$$\begin{aligned} \delta \times y &\propto \left[\frac{\eta^2\tau_y}{\mathcal{T}_\pi + \eta^2\tau_y}\frac{1}{\eta} - \frac{\tau_x'}{\mathcal{T}_\pi + \tau_x' + \hat{\eta}^2\tau_y}\frac{\eta^2\tau_y}{\mathcal{T}_\pi + \eta^2\tau_y}\frac{1}{\eta} - \frac{\hat{\eta}^2\tau_y}{\tau_\pi + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}}\right] y_t \\ &= \left[\frac{\mathcal{T}_\pi + \hat{\eta}^2\tau_y}{\mathcal{T}_\pi + \tau_x' + \hat{\eta}^2\tau_y}\frac{\eta^2\tau_y}{\mathcal{T}_\pi + \eta^2\tau_y}\frac{1}{\eta} - \frac{\hat{\eta}^2\tau_y}{\mathcal{T}_\pi + \tau_x' + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}}\right] y_t \propto \left[\frac{\mathcal{T}_\pi + \hat{\eta}^2\tau_y}{\mathcal{T}_\pi + \eta^2\tau_y}\eta\tau_y - \hat{\eta}\tau_y\right] y_t. \end{aligned}$$

Rearranging terms then shows that

$$\delta = \Delta\left(\frac{\eta^2\tau_y}{\mathcal{T}_\pi + \eta^2\tau_y} \times \frac{1}{\eta} - \frac{\hat{\eta}^2\tau_y}{\mathcal{T}_\pi + \hat{\eta}^2\tau_y} \times \frac{1}{\hat{\eta}}\right), \tag{A10}$$

where $\Delta > 0$. This completes the proof. □

A.4 Proof of Corollary 1

The proof proceeds in two steps.

First, let the coefficient γ be defined by

$$\gamma \equiv \frac{\mathcal{T}_\pi + \hat{\eta}^2 \tau_y}{\mathcal{T}_\pi + \eta^2 \tau_y} \times \eta - \hat{\eta}. \tag{A11}$$

Notice that γ determines the sign of δ in (A10). It follows that

$$\lim_{\tau_y \to 0} \gamma = \eta - \hat{\eta} > 0 : \quad \delta > 0,$$

and

$$\lim_{\tau_y \to \infty} \gamma = \frac{\hat{\eta}^2}{\eta^2} \times \eta - \hat{\eta} < 0 : \quad \delta < 0.$$

The first part of the statement then follows from $\Delta > 0$ for all τ_y.

Second, it follows from (A11) that

$$\lim_{\hat{\tau}_x \to 0} \gamma = \lim_{\hat{\eta} \to 0} \gamma = \frac{\mathcal{T}_\pi}{\mathcal{T}_\pi + \eta^2 \tau_y} \times \eta > 0.$$

Combined with $\Delta > 0$, this completes the proof of the statement. □

A.5 Proof of Proposition 3

The proof proceeds in two steps. We first derive expressions for the additional regression coefficients b and β. An expression for the coefficient δ is stated in (A10). We then consider a special case, which leads to the proposition.

Step 1: Additional Coefficients. We start with the coefficient β in (2.2).
The forecast error at time t is

$$\begin{aligned} \pi_{t+h} - f_{it}\pi_{t+h} &= \pi_{t+h} - k_x x_{it-1} - k_x x_{it} - k_y y. \\ &= (1 - 2k_x - \eta k_y)\pi_{t+h} - k_x \left(\epsilon^x_{it-1} + \epsilon^x_{it}\right) - k_y \epsilon^y_t, \end{aligned}$$

where k_x and k_y are given by the expression in (A7). The associated forecast revision is

$$\begin{aligned} f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h} &= (k_x - v)\, x_{it-1} + k_x x_{it} + k_y y_t \\ &= (2k_x + k_y \eta - v)\, \pi_{t+h} + (k_x - v)\, \epsilon^x_{it-1} + k_x \epsilon^x_{it} + k_y \epsilon^y_t. \end{aligned}$$

Hence,

$$\begin{aligned} \beta &\propto \mathbb{C}\text{ov}\left[\pi_{t+h} - f_{it}\pi_{t+h}, f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h}\right] \\ &= \left(1 - 2k_x - \eta k_y \left[2k_x + k_y \eta - \upsilon\right] \underline{\tau}_\pi^{-1} - \left(2k_x - \upsilon\right) k_x \tau_x^{-1} - k_y^2 \tau_y^{-1}. \end{aligned}$$

A few simple but tedious manipulations then show that this expression equals

$$\beta \propto \frac{\tau_x \tau_x' \hat{\eta}^2 \tau_y^2 \left(\eta - \hat{\eta}\right)\left(\hat{\eta} - 1\right) + \underline{\tau}_\pi \hat{\eta} \tau_y \left[\tau_x \tau_y \left(\eta - \hat{\eta}\right) + \hat{\eta} \tau_x' \left(\tau_x' - \tau_x\right)\right] + \underline{\tau}_\pi^2 \tau_x' \left(\tau_x - \tau_x'\right)}{\underline{\tau}_\pi \tau_x \left(\underline{\tau}_\pi + \tau_x'\right)\left(\underline{\tau}_\pi + 2\tau_x' + \hat{\eta}^2 \tau_y\right)}. \quad \text{(A12)}$$

We next turn to the $b-$coefficient in (2.1). Similar steps to those that lead to (A5) show that

$$\pi_{t+h} - f_{it}\pi_{t+h} = \frac{1-k}{k}\left(f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h}\right) - \frac{k_y}{k}\epsilon_t^y + \left(\hat{\eta} - \eta\right)\frac{k_y}{k}\upsilon \pi_{t+h},$$

where $k \equiv k_x + \eta k_y \in (0,1)$. We conclude that, adjusting for the usual bias caused by the presence of public information $-\frac{k_y}{k}\epsilon_t^y$, the coefficient b equals

$$b = \frac{1-k}{k} + \left(\hat{\eta} - \eta\right)\frac{k_y}{k}\upsilon \mathbb{C}\text{ov}\left(\pi_{t+h}, f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h}\right)\mathbb{V}\left[f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h}\right]^{-1}. \quad \text{(A13)}$$

Step 2: Over- and underrevisions and Overreactions. We first let $\tau_x' \to \tau_x$ from above. Equation (A12) shows that β is, in this case, positively proportional to

$$\beta \propto \left[\tau_x' \hat{\eta} \tau_y \left(\hat{\eta} - 1\right) + \underline{\tau}_\pi^2\right] \tau_x \tau_y \hat{\eta}\left(\eta - \hat{\eta}\right),$$

so that $\beta < 0$ if $\hat{\eta}^2 \tau_y > \chi \underline{\tau}_\pi$, where $\chi \equiv \frac{\underline{\tau}_\pi \hat{\eta}}{\tau_x'(1-\hat{\eta})}$, as $\hat{\eta} < \eta$.

We second let $\Omega \equiv \frac{\eta^2 \tau_y}{\underline{\tau}_\pi + \eta^2 \tau_y} \times \frac{1}{\eta} - \frac{\hat{\eta}^2 \tau_y}{\underline{\tau}_\pi + \hat{\eta}^2 \tau_y} \times \frac{1}{\hat{\eta}}$, so that $\delta = \Delta \Omega$ in (A10) with $\Delta > 0$. We note that $\frac{\partial \Omega}{\partial \hat{\eta}}_{\hat{\eta} = \eta} < 0$ if $\underline{\tau}_\pi < \eta^2 \tau_y$. Hence, $\delta < 0$ for $\hat{\eta} \to \eta$ from below.

We third notice that b in (A13) is positive for $\tau_x' \to \tau_x$ and $\hat{\eta} \to \eta$ as $k < 1$.

Combined, this shows that if $\eta^2 \tau_y > \max\left(\chi, 1\right) \underline{\tau}_\pi$, then there exists values $c_0, c_1 \in \mathbb{R}_+$ such that for $\epsilon > 0$ and $\tau_x' = \tau_x + c_0 \epsilon$ and $\hat{\eta} = \eta - c_1 \epsilon$ the coefficients $\beta < 0$, $\delta < 0$, and $b > 0$. The continuity of slope coefficients in the parameters ensures that this is the case. □

A.6 Relative Overconfidence and Survey data

We use the model framework from Section 4, although other model environments could also be employed. The key features necessary are (i) that forecasters can exhibit relative overconfidence, and (ii) that individual forecasts are in part based on individual-specific information.

Consider the (initial wave) period $t-1$ expectation of forecaster i:

$$f_{it-1}\pi_{t+h} = v x_{it-1}, \quad v = \frac{\tau_x'}{\underline{\tau}_\pi + \tau_x'}. \tag{A14}$$

The average forecast $f_{t-1}\pi_{t+h}$ and perceived average forecast $\hat{f}_{t-1}\pi_{t+h}$ across i is

$$f_{t-1}\pi_{t+h} = v\pi_{t+h}, \quad \hat{f}_{t-1}\pi_{t+h} = \hat{v}\pi_{t+h}, \tag{A15}$$

where $\hat{v} = \frac{\hat{\tau}_x}{\underline{\tau}_\pi + \hat{\tau}_x} < v$ because of relative overconfidence.

Let U_0 denote forecaster i's perceived mean-squared error of π_{t+h}. Wet let U_1 denote forecaster i's perceived mean-sqyared error about $f_{t-1}\pi_{t+h}$, accounting for the misperception embedded in the expression for $\hat{f}_{t-1}\pi_{t+h}$. Thus,

$$U_0 = \frac{1}{\underline{\tau}_\pi + \tau_x'}, \quad U_1 = \hat{v}^2 U_0. \tag{A16}$$

Let S_0 denote the realized cross-sectional variance of forecasts $\mathbb{E}\left[(f_{it-1}\pi_{t+h} - f_{t-1}\pi_{t+h})^2\right]$, and let S_1 denote its first higher-order expectation counterpart $\mathbb{E}\left[(f_{it-1}f_{t-1}\pi_{t+h} - f_{t-1}f_{t-1}\pi_{t+h})^2\right]$. Using equation (A14) and (A15), these are equal to

$$S_0 = v^2 \frac{1}{\tau_x}, \quad S_1 = \hat{v}^2 S_0. \tag{A17}$$

The moments in (A16) and (A17) are reported in Table II of Coibion *et al.* (2021). Assuming a value for $\underline{\tau}_\pi$ thus allows one to compute forecaster i's perception about forecaster $j \neq i$'s reported uncertainty of π_{t+h}, equal to $(\underline{\tau}_\pi + \hat{\tau}_x)^{-1}$, from either (A16) or (A17).

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Online Appendix to:

"Forecaster (Mis-)Behavior"

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Contents of Online Appendix:

This *Online Appendix* details additional extensions of the models analyzed in the paper.

B Additional Empirical Results

Table B.1: Regression of forecast errors on public signals

	Panel a: survey expectations		
	Estimate	Std. error	Observations
Survey of Prof. Forecasters	-0.536***	0.160	5,469
Michigan Survey of Consumers	0.031	0.115	4,499
Survey of Consumer Expectations	0.322***	0.061	775
Livingston Survey	-0.609**	0.248	2,762
	Panel b: other public signals		
	Estimate	Std. error	Observations
Lagged Outcomes	-0.406**	0.151	5,668
TIPS Spread	0.059	0.057	2,203
Nominal Eff. Exchange Rate	0.294***	0.066	5,668
Import Prices	0.082**	0.043	3,992
Oil Prices	0.055	0.063	5,668
Unemployment Rate	0.335***	0.091	5,668
Financial Inflation Index	-0.032	0.057	4,169
Stock Prices	-0.398***	0.101	5,668
Term Spread	0.043	0.039	4,169

Note: Estimates of (2.3) with respondent fixed effects using different public signals and one-year ahead inflation forecasts ($h = 4$) from the Survey of Professional Forecasters. Panel a shows the coefficient estimate on previous period's consensus forecast from the Survey of Professional Forecasters, the Michigan Survey of Consumers, the Survey of Consumer Expectations, and the Livingstone Survey. Panel b presents the coefficient estimate for one-period lagged inflation, 10-year inflation expectations from the TIPS market, the year-over-year change in the nominal effective exchange rate, the year-over-year change in import prices, the year-over-year change in the WTI oil price, the unemployment rate, the Cleveland Fed's Financial Market-based measure of future inflation, the log-linear detrended level of the SP500 stock prices, and the 10-year-2-year term spread. All variables have been standardized, and have been signed such that an increase predicts higher inflation one year out. All variables and growth rates have also been derived using the latest available data at the time of the inflation forecast. Double-clustered robust standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01.

Table B.2: Multivariate estimate of forecast errors on public signals

	Panel a: public signals		
	Estimate	Std. error	Observations
Survey of Prof. Forecasters	-0.364***	0.130	3,992
Lagged Outcomes	-0.173	0.208	3,992
TIPS Spread	–	–	–
Nominal Eff. Exchange Rate	0.176***	0.045	3,992
Import Prices	-0.150*	0.082	3,992
Oil Prices	0.060	0.067	3,992
Unemployment Rate	0.023	0.079	3,992
Financial Inflation Index	0.081	0.097	3,992
Stock Prices	0.039	0.113	3,992
Term Spread	0.045	0.057	3,992
	Panel b: survey expectations		
	Estimate	Std. error	Observations
Survey of Prof. Forecasters	0.212	0.326	400
Michigan Survey of Consumers	-0.325**	0.163	400
Survey of Consumer Expectations	1.378***	0.195	400
Livingston Survey	-0.217**	0.133	400

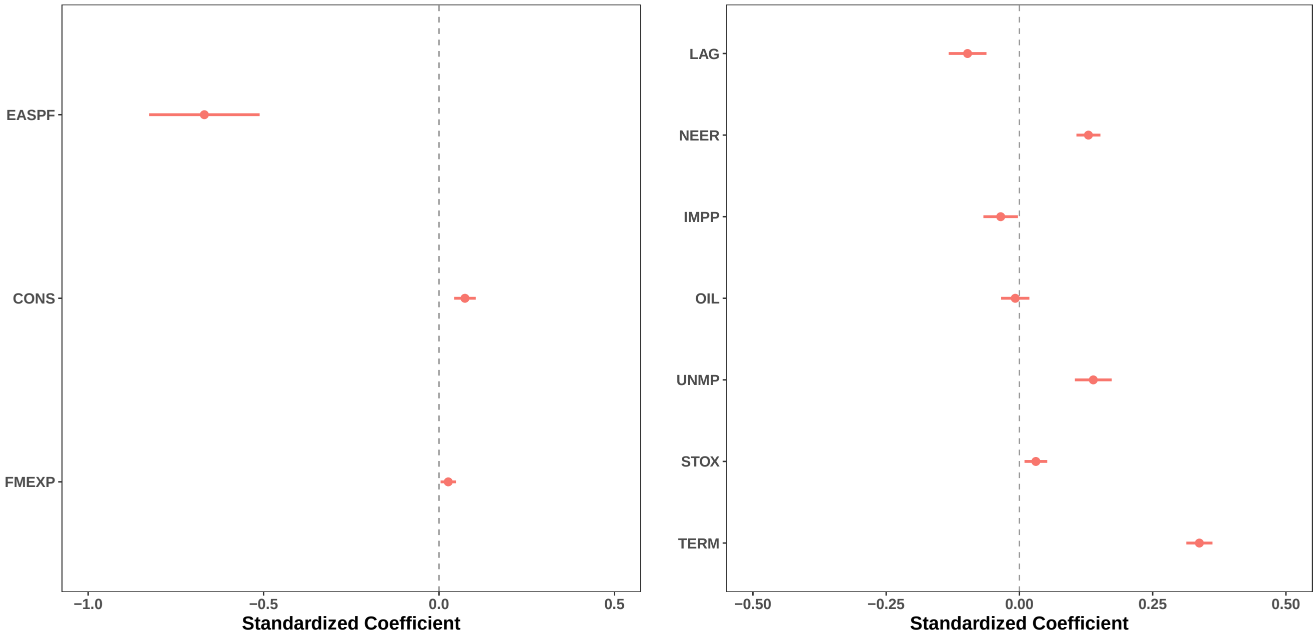
Note: Estimates of multivariate version of regression (2.3) with respondent (individual) fixed effects using public signals and one-year ahead inflation forecasts (h = 4) from the Survey of Professional Forecasters. Panel a shows the coefficient estimates on previous period's consensus forecast of inflation from the Survey of Professional Forecasters, one-period lagged inflation, the year-over-year change in the nominal effective exchange rate, the year-over-year change in import prices, the year-over-year change in the WTI oil price, the unemployment rate, the Cleveland Fed's Financial Market-based measure of future inflation, the log-linear detrended level of the SP500 stock prices, and the 10-year-2-year term spread. We exclude 10-year inflation expectations from the TIPS market from this panel due to sample limitations. Panel b shows that estimates on the consensus forecast of one-year ahead inflation from the Survey of Professional Forecasters, the Michigan Survey of Consumers, the Survey of Consumer Expectations, and the Livingstone Survey. All variables have been standardized, and have been signed such that an increase predicts higher inflation one year out. All variables and growth rates have also been derived using the latest available data at the time of the inflation forecast. Double-clustered robust standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01.

Table B.3: Livingstone Survey of Forecasters: CPI inflation, sub-groups

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Academic	Academic	Comm. Banking	Comm. Banking	Consulting	Consulting	Inv. Banking	Inv. Banking	Nonf. Business	Nonf. Business
Forecast revision	-0.792***		-0.236		-0.416**		-0.442***		-0.598***	
	(0.127)		(0.184)		(0.154)		(0.115)		(0.0755)	
Prev. consensus		-0.317		-0.417		-0.135		-0.642***		-0.362
		(0.321)		(0.342)		(0.372)		(0.214)		(0.369)
Constant	-0.0305	0.955	-0.190	1.057	-0.342*	0.112	-0.305**	1.777**	-0.128	0.908
	(0.176)	(0.900)	(0.169)	(0.965)	(0.195)	(0.987)	(0.125)	(0.655)	(0.177)	(1.015)
R^2	0.296	0.080	0.216	0.171	0.290	0.139	0.430	0.273	0.223	0.055
N	202	234	197	254	275	387	215	298	301	398

Note: Columns 1, 3, 5, 7, and 9 presents estimates of β, while columns 2, 4, 6, 8, and 10 of δ for one-year-ahead forecasts for CPI inflation from the Livingston Survey. The estimation sample comprises 1993 Q1 to 2016 Q4. Robust double-clustered standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01

Figure B.1: Euro Area Survey: inflation errors and public signals



Note: The figure shows estimates of δ in (2.3) (horizontal axis) for various public signals (vertical axis) using one-year ahead ($h = 4$) forecasts from the ECB's Survey of Professional Forecasters. EASPF denotes the previous period's consensus forecast from the ECB's Euro Area Survey, CONS consumers' one-year ahead inflation expectations from the European Commission's Consumer Survey, and lastly FM EXP financial market expectation of one-year ahead inflation as derived from Euro Area inflation swaps. The description of the other public signals used are in the label for Figure 2. We proxy developments in Euro Area stocks with the DAX index. All variables have been standardized and signed so that an increase predicts higher inflation one-year out. All variables and growth rates have also been derived using the latest data at the time of the forecast. Whiskers correspond to 95-percent robust clustered confidence bounds. Because of the presence of few time-clusters (often around 50), we follow the recommended adjustment in Cameron *et al.* (2010) and cluster at the individual level.

Table B.4: Regression of forecast errors on alternative consensus measures

	(1)	(2)	(3)	(4)
Previous Consensus	-0.192**	–	–	–
	(0.085)			
Previous Consensus $-i$	–	-0.182**	–	–
		(0.083)		
Previous Conensus Dev.	–	–	-0.591***	–
			(0.096)	
Previous Conensus Wins.	–	–	–	-0.187**
				(0.085)
Observations	5,675	5,675	5,675	5,675
F Statistic	118.98	116.83	1,025	136.85
R^2	0.022	0.021	0.160	0.025

Note: Estimates of (2.3) with respondent fixed effects. "Previous Consensus $-i$" denotes the previous consensus estimate that arises if, for all i, we drop respondent i from the consensus average. "Previous Consensus Deviation" denotes the difference between respondent i's forecast of inflation at time $t + h$ and the previous consensus estimate. Finally, "Previous Consensus Winsorized" winsorizes the top- and bottom one percent of forecast errors pre-estimation. Double-clustered robust standard errors in parentheses. $*$ p<.1,$**$ p<.05,$***$ p<.01.

Table B.5: Median individual estimates for inflation

	Forecast Error	Forecast Error	Forecast Error
	Panel a: individual forecasts PGDP inflation		
Avr. Forecast Revision	0.714	–	–
Ind. Forecast Revision	–	-0.100	–
Previous Consensus	–	–	-0.212
	Panel b: individual forecasts CPI inflation		
	Forecast Error	*Forecast Error*	*Forecast Error*
Avr. Forecast Revision	0.184	–	–
Ind. Forecast Revision	–	-0.224	–
Previous Consensus	–	–	-0.538

Note: Column 1: estimates of the cross-sectional median of b_i in $\pi_{t+h} - f_{it}\pi_{t+h} = \alpha_i + b_i\left(f_t\pi_{t+h} - f_{t-1}\pi_{t+h}\right) + v_{it}$. Column 2: estimates of the cross-sectional median of β_i in $\pi_{t+h} - f_{it}\pi_{t+h} = \alpha_i + \beta_i\left(f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h}\right) + v_{it}$. Column 3: estimates of the cross-sectional median of δ_i in $\pi_{t+h} - f_{it}\pi_{t+h} = \alpha_i + \delta_i\left(f_{it}\pi_{t+h} - f_{it-1}\pi_{t+h}\right) + v_{it}$. We consider forecasters with more than 20 individual forecasts. Estimates use one-year ahead SPF inflation forecasts ($h = 4$) and include respondent fixed effects. Sample: 1970Q1-2020Q1.

Table B.6: Estimates without outliers

		Individual Forecasts	
	Forecast Error	*Forecast Error*	*Forecast Error*
Ind. Forecast Revision	-0.132**	–	–
	(0.058)		
Avr. Forecast Revision	–	1.174***	–
		(0.273)	
Previous Consensus	–	–	-0.195**
			(0.084)
Observations	5,480	5,675	5,675
F Statistic	55.26	831.42	137.99
R^2	0.011	0.134	0.025

Note: Estimates of (2.1), (2.2), and (2.3), using individual forecast errors on the left-hand side of (2.1) and SPF forecasts of one-year ahead inflation ($h = 4$). All columns include individual (respondent) fixed effects. The top- and bottom one percent of forecast errors (in percent) have been winsorized. Robust (double-clustered) standard errors in parentheses. Sample: 1970Q1-2020Q1. $*$ p<.1,** p<.05,*** p<.01.

C Analysis of Alternative Models[1]

C.1 Forecasts with Reputational Considerations

We follow Ehrbeck and Waldmann (1996), but extend their setup to allow for public information. There is a continuum of measure one of forecasters $i \in [0,1]$ with prior beliefs $\pi \sim \mathcal{N}\left(\mu_i, \tau_\pi^{-1}\right)$. Each forecaster i observes a private signal

$$x_i^j = \pi + \epsilon^j, \quad \epsilon^j \sim \mathcal{N}\left[0, \left(\tau_x^j\right)^{-1}\right],$$

where $j = \{1,2\}$ and $\tau_x^1 > \tau_x^2$. In line with Ehrbeck and Waldmann (1996), we assume that $i \in [0, 1/2[$ observe x_i^1, while $i \in [1/2, 1]$ observe x_i^2. In addition, forecasters observes y in (3.2).

We consider linear equilibria, in which forecaster i's prediction rule is characterized by

$$f_i\pi = \left(1 - w^j\right) \mathbb{E}[\pi \mid \mu_i, y] + w^j x_i^j,$$

where w^j (potentially) differs from its mean-squared optimal value $w^\star$. Following the same steps as in Ehrbeck and Waldmann (1996) shows that if we only consider Nash equilibria in which able forecasters are frank (and forecasters care only about the posterior odds of being viewed as able by their clients), then $w^2 > w^\star$ for $i \in [1/2, 1]$. It now follows from Proposition 1 that, across all forecasters $i \in [0,1]$, $b > 0$, $\beta < 0$, but $\delta = 0$. We summarize these results in Proposition C.1

Proposition C.1. *In the extended Ehrbeck and Waldmann (1996) model with public information, the regression coefficients $b > 0$ in (2.1), $\beta < 0$ in (2.2) , but $\delta = 0$ in (2.3).*

C.2 Forecasts with Generalized Overreactions

The information contained in x_i and y can be summarized by $z_i \equiv \mathbb{E}[\pi \mid x_i, y]$. The expected value of π conditional on an information set Ω as well as x_i and y, $\mathbb{E}[\pi \mid x_i, y, \Omega]$, is equal to the expected value of π conditional on Ω and z, $\mathbb{E}[\pi \mid x_i, y, \Omega] = \mathbb{E}[\pi \mid z_i, \Omega]$. In this sense, z captures all of the new information observed by forecaster i. In Bordalo *et al.* (2018)'s theory of *diagnostic expectations,* forecaster i overreacts to z ($k > k^\star$), because it is perceived to be be diagnostic of updates relative to prior information. In Evans and Honkapohja (2012)'s theory of *excess Kalman Gain learning* forecasters instead overreact to increase their speed of learning.

Proposition C.2. *Let $k = (1+\chi)k^\star$, where $\chi > 0$. Then, forecasts that follow (3.5) in Proposition 1 exhibit a δ in (2.3) equal to $\delta = -\chi k_\star \left(\frac{\tau_x}{\tau_x+\tau_y} + 1\right) \frac{\tau_y}{\tau_x+\tau_y}$.*

Proof. The forecasting rule in (3.5) can be re-stated as:

$$f_i\pi = \mu_i + (1+\chi)\, k^\star \left(w_x x_i + w_y y - \mu_i\right) \tag{OA1}$$

[1]In this appendix, we abstract from time subscripts, to simply our notation.

where $w_x = \frac{\tau_x}{\tau_x + \tau_y}$ and $w_x + w_y = 1$. Now, notice that

$$\begin{aligned} \delta \times y &= \mathbb{E}\left[\pi - f_i\pi \mid y\right], \\ &= \mathbb{E}\left[\pi - \mathbb{E}\left[\pi \mid \mu_i, x_i, y\right] + \mathbb{E}\left[\pi \mid \mu_i, x_i, y\right] - f_i\pi \mid y\right] = \mathbb{E}\left[\mathbb{E}\left[\pi \mid \mu_i, x_i, y\right] - f_i\pi \mid y\right] \end{aligned}$$

where $\mathbb{E}\left[\pi \mid \mu_i, x_i, y\right]$ equals the forecast in (OA1) when $\chi = 0$. Thus,

$$\delta \times y = -\chi k^{\star}\mathbb{E}\left[w_x x_i + w_y y \mid y\right] = -\chi k_{\star}\left(w_x w_y + w_y\right) y,$$

and we conclude that $\delta = -\chi k_{\star}\left(w_x + 1\right) w_y < 0$. □

C.3 Forecasts with Strategic complementarity and Error Correlation

The paper discusses several models in which strategic incentives skew the optimal use of information away from its mean-squared optimal value. In place of these more specific models, a more general way to capture the basic idea that strategic incentives skew individuals' use of private and public information is to extend the baseline framework to allow for arbitrary strategic complementarity between individual forecasts. Suppose forecaster i's estimate of π follows

$$f_i\pi = (1-r)\mathbb{E}\left[\pi \mid \mu_i, x_i, y\right] + r\mathbb{E}\left[f\pi \mid \mu_i, x_i, y\right], \tag{OA2}$$

where $f\pi = \int_0^1 f_i\pi di$ and $r \in (-1, 1)$ is the amount of strategic complementarity (substitutability) between forecasters. The case where $r \to 1$ corresponds to the case where forecasters care only about aligning their forecasts to the average estimate. By contrast, $r = 0$ corresponds to the benchmark, mean-squared optimal case from Section 3. In addition, following Myatt and Wallace (2011), we allow for arbitrary correlation between the errors in public and private information. In particular, we allow for a common noise component: forecasters' private information takes the form $x_i = \theta + \epsilon_i + cu$, where $c \in \mathbb{R}$ and $u \sim \mathcal{N}\left(0, \tau_u^{-1}\right)$, while the public signal y is $y = \theta + u + \epsilon_y$. The coefficient c controls the correlation between the error terms.

As shown by Angeletos and Pavan (2007), the coefficient r maps directly into the weight on private information k_x. Specifically, whenever there is strategic complementarity ($r > 0$), the weight on private information falls below its mean-squared optimal value ($k_x < k_x^{\star}$), and conversely when there is strategic substitutability ($r < 0$). Proposition C.3 shows that, despite the flexible amount of strategic complementarity and the presence of error correlation, individual errors remain uncorrelated with public information ($\delta = 0$).

Proposition C.3. *If individual forecasts follow (OA2), then δ in (2.3) is equal to zero.*

Proof. The orthogonality of errors to public information follows from a similar argument to that which establishes Proposition 1. Since $f\pi = \int_0^1 f_i\pi di$, we can re-write (OA2) as

$$f_i\pi = \mathbb{E}\left\{ r\sum_{j=0}^{\infty}(1-r)^j\bar{\mathbb{E}}^j\left[\pi \mid \mu_i, x_i, y\right] \right\}, \tag{OA3}$$

where $\bar{\mathbb{E}}\left[\pi \mid \mu_i, x_i, y\right] = \int_0^1 \mathbb{E}\left[\pi \mid \mu_i, x_i, y\right] di$ and $\bar{\mathbb{E}}^j\left[\pi \mid \mu_i, x_i, y\right] = \int_0^1 \mathbb{E}\left\{\bar{\mathbb{E}}^{j-1}\left[\pi \mid \mu_i, x_i, y\right]\right\} di$ for $j \geq 1$. But now notice that from the Law of Iterated Expectations:

$$\mathbb{E}\left[f_i\pi \mid y\right] = r\sum_{i=0}^{\infty}(1-r)^i\mathbb{E}\left[\pi \mid y\right] = r\frac{1}{1-(1-r)}\mathbb{E}\left[\pi \mid y\right] = \mathbb{E}\left[\pi \mid y\right].$$

Hence,

$$\delta \times y = \mathbb{E}\left[\pi - f_i\pi \mid y\right] = 0.$$

□

C.4 Forecasts with Trembling-hand Noise

Let $\tilde{f}_i\pi \equiv f_i\pi + e_i$ denote forecaster i's *stated* forecast, while $f_i\pi$ denotes her *actual* forecast. We further assume that $e_i \sim \mathcal{N}\left(0, \tau_e^{-1}\right)$. In this case, forecasters stated predictions are subject to "trembling-hand" noise. The results in Proposition 1 to a large extent carry over to this case. In fact, as the below proposition shows, the only difference between such trembling-hand forecasts and those analyzed in Section 3 is that the coefficient on individual revisions becomes more negative. Let the slope coefficient from the associated regression using forecasters' stated predictions be denoted by $\tilde{\beta}$. Then, $\tilde{\beta} = \chi\left(\beta - \tau_e^{-1}\mathbb{V}\left[f_i\pi - \mu_i\right]^{-1}\right)$, where $\chi \equiv \frac{\tau_e}{\tau_e + \mathbb{V}[f_i\pi - \mu_i]^{-1}}$. As a consequence, even when forecasters are rational, and their actual forecasts correspond to their conditional expectation, forecasters still appear to overrevise their expectations ($\tilde{\beta} = -\frac{\mathbb{V}[f_i\pi - \mu_i]^{-1}}{\tau_e + \mathbb{V}[f_i\pi - \mu_i]^{-1}} < 0$). However, importantly, our results about the correlation between individual errors and public information remain as before. The coefficient δ equals that in Proposition 1 ($\tilde{\delta} = \delta$). In particular, it is still the case that conditional expectation forecasts remain uncorrelated with public information ($\tilde{\delta} = 0$). Finally, we note that for conditional expectation forecasts to be consistent with the estimate of β in, for example, Table 1, the standard deviation of the noise should be around 40 percent of the standard deviation of the forecast revision. Although we doubt that forecasters' stated predictions are subject to this much noise (see Juodis and Kucinskas, 2019), the result suggests that our third implication of noisy rational expectations ($\delta = 0$) provides a more robust test than the second ($\beta = 0$).

Proposition C.4. *Suppose forecasts are subject to noise $\tilde{f}_i\pi \equiv f_i\pi + e_i$, where $e_i \sim \mathcal{N}\left(0, \tau_e^{-1}\right)$, and let $\tilde{d} \in \left\{\tilde{\beta}, \tilde{\delta}\right\}$ denote the regression coefficient from (2.2) or (2.3) using the noisy forecast. The coefficient from this regressions using the underlying forecasts is denoted without a tilde. Then, $\tilde{\beta} = \chi\left(\beta - \tau_e^{-1}\mathbb{V}\left[f_i\pi - \mu_i\right]^{-1}\right)$, where $\chi \equiv \frac{\tau_e}{\tau_e + \mathbb{V}[f_i\pi - \mu_i]^{-1}}$, and $\tilde{\delta} = \delta$.*

Proof. We have that

$$\tilde{\beta} = \mathbb{C}ov\left(\pi - \tilde{f}_i\pi, \tilde{f}_i\pi - \mu_i\right)\mathbb{V}\left[\tilde{f}_i\pi - \mu_i\right]^{-1} = \frac{\mathbb{C}ov\left(\pi - f_i\pi, f_i\pi - \mu_i\right) - \tau_e^{-1}}{\mathbb{V}\left[f_i\pi - \mu_i\right] + \tau_e^{-1}}.$$

Thus,

$$\tilde{\beta} = \beta\frac{\tau_e}{\tau_e + \mathbb{V}\left[f_i\pi - \mu_i\right]^{-1}} - \frac{\mathbb{V}\left[f_i\pi - \mu_i\right]^{-1}}{\tau_e + \mathbb{V}\left[f_i\pi - \mu_i\right]^{-1}}.$$

However,

$$\tilde{\delta} = \mathbb{C}ov\left(\pi - \tilde{f}_i\pi, y\right)\mathbb{V}\left[y\right]^{-1} = \mathbb{C}ov\left(\pi - f_i\pi - e_i, y\right)\mathbb{V}\left[y\right]^{-1} = \delta.$$

□

D Accounting for Bias in Estimates of (2.1)

The estimate of the extent of information frictions b in regression (2.1) is biased by the presence of public information (e.g., Coibion and Gorodnichenko, 2015). Goldstein (2021) considers a simple solution to this problem. Consider the prediction rule from Section 3 and suppose forecasters report mean-squared optimal expectations. Their forecasts equal (Appendix A.1)

$$f_{it}\pi_{t+h} = (1 - k_x^\star)\,\mathbb{E}\left[\pi_{t+h} \mid \mu_{it}, y_t\right] + k_x^\star x_i, \quad k_x^\star = \frac{\tau_x}{\tau_\pi + \tau_x + \tau_y}. \tag{OA4}$$

Let $w \equiv \frac{\tau_\pi}{\tau_\pi + \tau_y}$ denote the optimal weight on the prior μ_{it} in $\mathbb{E}\left[\pi_{t+h} \mid \mu_{it}, y_t\right]$. Then, taking averages of (OA4) and manipulating the resulting expression shows that

$$\pi_{t+h} - f_t\pi_{t+h} = \frac{(1-k_x^\star)w}{k_x^\star + (1-k_x^\star)(1-w)}\left(f_t\pi_{t+h} - \mu_{it}\right) - \frac{(1-k_x^\star)(1-w)}{k_x^\star + (1-k_x^\star)(1-w)}\epsilon_t^y. \tag{OA5}$$

This shows that estimates of b in (2.1) are biased by the presence of public information, because of the correlation that arises between the noise term $-\frac{(1-k_x^\star)(1-w)}{k_x^\star+(1-k_x^\star)(1-w)}\epsilon_t^y$ and the forecast revision $(f_t\pi_{t+h} - \mu_{it})$ in (OA5). However, notice that because this bias is negative, any estimate of $b > 0$ from (2.1) still shows that an unbiased estimate of the coefficient is positive.

Goldstein (2021) considers a simple solution to the problem of a biased estimate of information frictions. Instead of regressing average errors onto average revisions, regress deviations from consensus onto its previous value; that is, consider the regression equation

$$f_{it}\pi_{t+h} - f_t\pi_{t+h} = \alpha_{it} + b^\star\left(f_{it-1}\pi_{t+h} - f_{t-1}\pi_{t+h}\right) + u_{it}. \tag{OA6}$$

Taking averages of (OA4) and manipulating the resulting equation shows that

$$f_{it}\pi_{t+h} - f_t\pi_{t+h} = (1-k_x^\star)w\left(\mu_{it} - \mu_t\right) + \epsilon_{it}^x, \tag{OA7}$$

so that $b^\star = (1-k_x^\star)\,w$, $\alpha = 0$, and $u_{it} = k_x^\star\epsilon_{it}^x$ in (OA6). Notice that $b^\star > 0$ if and only if the bias-adjusted value of b, equal to $\frac{(1-k_x)w}{k_x+(1-k_x)(1-w)}$, in (OA5) is positive. Both are positive if and only if $k_x^\star < 1$; that is, if and only if there are information frictions.

Table D.1 shows the estimates of regression (OA6) alongside those of regression (2.1).

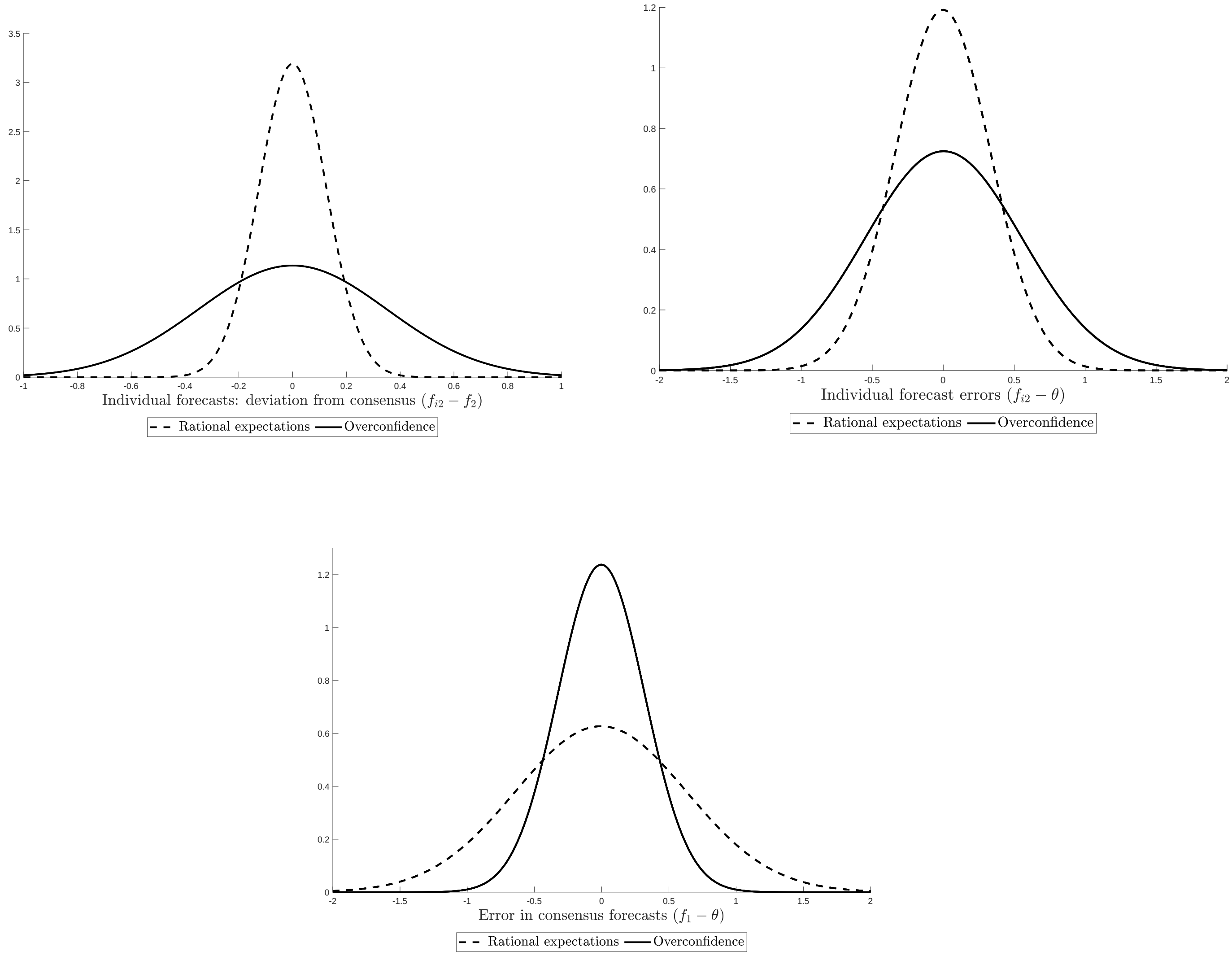
Table D.1: Goldstein (2021) Estimates of Information Frictions

	Average Forecasts *Forecast Error*	*Individual Forecasts* *Consensus Deviation*
Average Forecast Revision	1.118***	–
	(0.287)	
Previous Consensus Deviation	–	0.411***
		(0.057)
Constant	-0.054	–
	(0.073)	
Observations	196	5,480
F Statistic	44.067	1,151.4
R^2	0.190	0.181

Note: Column 1: estimates of the Coibion and Gorodnichenko (2015) regression in (2.1).
Column 2: estimates of the Goldstein (2021) regression in (OA6) with individual fixed-effects.
Robust (double-clustered) standard errors in parentheses. Sample: 1970Q1-2020Q1. $*$ p<.1,$**$ p<.05,$***$ p<.01.

E Distribution of Forecasts and Errors

Figure E.1: The Behavior of Calibrated Individual Forecasts



Note: The top left-hand panel depicts the distribution of the difference between individual period t forecasts (f_{i2}) and consensus (f_2). It does so for both the overconfidence model and the corresponding mean-squared optimal rational expectations model. In both cases, we use the parameters listed in Table IV. The top right-hand panel, by contrast, shows the corresponding distribution of individual forecast errors in the two cases, where $\theta = \pi_{t+h}$. The bottom panel depicts the distribution of the errors in the period $t-1$ consensus forecast (f_1).

F Relative Overconfidence and the ZEW Survey

Every month, the Zentrum für Europäische Wirtschaftsforschung (ZEW) asks its survey respondents not only for their own expectation of (an index of) aggregate German economic activity six months from now, but also for their forecast of the average (or consensus) estimate. Such information can, in turn, be used to construct a test for relative overconfidence.

Consider the consensus expectation of the fundamental $\theta = \pi_{t+h}$ from Section 4 ($\alpha_1 = 0$, $\alpha_2 = 1$ in 4.2):

$$f_{t-1}\theta = y_t = v\theta + \epsilon_t^y, \tag{OA8}$$

and compare it to the consensus estimate that forecasters perceive

$$\hat{f}_{t-1}\theta = \hat{y}_t = \hat{v}\theta + \epsilon_t^y, \tag{OA9}$$

where $v > \hat{v}$ due to relative overconfidence. Because of the misperception inherent to relative overconfidence, if forecasters are asked to provide an expectation of consensus, they will report a forecast of (OA9) instead of (OA8). As a result, a relationship arises between the average forecast error of consensus, on the one hand, and consensus and the fundamental itself, on the other hand. Specifically, (OA8) and (OA9) imply the linear relationship:

$$f_{t-1}\theta - f_{t-1}\left[f_{t-1}\theta)\right] = v\theta - \hat{v}f_{t-1}\theta + u, \tag{OA10}$$

where $u = \epsilon_t^y$ denotes an error term respectively.

Table F.1 provides the estimate of v and $\hat{v}$ in (OA10). Consistent with relative overconfidence, Table F.1 shows an estimate of v that exceeds that of $\hat{v}$. Section 5.2 contains further discussion of these results and their interpretation.

Table F.1: Relative Overconfidence and ZEW Forecasts

	Consensus Forecast Error
Fundamental (v)	0.02**
	(0.008)
−Consensus ($\hat{v}$)	0.01
	(0.013)
Obs.	209
F Statistic.	3.089**
R^2	0.029

Notes: The table shows estimates of v and $\hat{v}$ in (OA10). A constant term is included in the regression. The top and bottom 5 percent of forecast errors and consensus realizations have been winsorized. Robust standard errors in parentheses: * p<0.10, ** p<0.05, *** p<0.01. Sample: 2003:3 – 2019:10.

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