ARTICLE IN PRESS
JID: DYNCON [m3Gsc;May 17, 2022;18:3]

Journal of Economic Dynamics & Control xxx (xxxx) xxx

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

On the possibility of Krusell-Smith Equilibria[☆]

Tobias Broer[a,*], Alexandre N. Kohlhas[b], Kurt Mitman[c], Kathrin Schlafmann[d]

[a] *Paris School of Economics, 48 boulevard Jourdan, Paris 75014, France*
[b] *Institute for International Economic Studies, Stockholm SE-106 91, Sweden*
[c] *Institute for International Economic Studies, Stockholm SE-106 91, Sweden*
[d] *Copenhagen Business School, Frederiksberg DK-2000, Denmark*

ARTICLE INFO

Article history:
Available online xxx

JEL classification:
C53
D83
E30

Keywords:
Expectations
Heterogeneity
Information

ABSTRACT

Solutions to macroeconomic models with wealth inequality and aggregate shocks often rely on the assumption of *limited* but *common* information among households. We show that this assumption is inconsistent with rational information choice for plausible information costs. To do so, we embed information choice into the workhorse heterogeneous-agent model with aggregate risk (Krusell and Smith, 1998). First, we demonstrate that the benefits of acquiring more precise information about the state of the economy depend crucially on household wealth. Second, we show that such heterogeneous incentives to acquire information combine with the strategic substitutability of savings choices to imply that equilibria in which households acquire the *same* information do not exist for plausible information costs. Finally, we document that a representative-agent equilibrium may not exist even in the absence of exogenous sources of wealth heterogeneity.

1. Introduction

Over the past two decades, macroeconomics has increasingly focused on the importance of heterogeneity among economic actors for understanding the causes and consequences of economic fluctuations (e.g., Krueger et al., 2016; Yellen, 2016). One of the challenges to further progress has been that workhorse models of household and firm heterogeneity in macroeconomics—combining aggregate risk and incomplete markets—are analytically and numerically intractable. This intractability stems from the entire wealth distribution, an infinite-dimensional object, being a state-variable (Den Haan et al., 2010). To make progress, macroeconomists commonly combine the standard model with a notion of *boundedly-rational equilibria*, following the pioneering work of Krusell and Smith (1998): Instead of possessing full information about the evolution of infinite-dimensional state variables, agents forecast future prices using a reduced system consisting only of a small set of state variables. Crucially, this limited information is assumed to be the *same across all agents* in the economy.[1]

☆ First draft: February 2021. For comments and useful suggestions, we thank Per Krusell, Mirko Wiederholt, our discussant, Sergei Glebkin, and seminar and conference participants at the IIES, SED Annual Meeting 2018, the Econometric Society European Meeting 2020, and the 2021 Conference on Markets & Economies with Information Frictions. This research received financial support from *Ragnar Soderberg Stiftelsen*. Mitman thanks the European Research Council (ERC Starting Grant 759482) for financial support.

* Corresponding author.
E-mail addresses: tobias.broer@psemail.eu (T. Broer), alexandre.kohlhas@iies.su.se (A.N. Kohlhas), kurt.mitman@iies.su.se (K. Mitman), ksc.fi@cbs.dk (K. Schlafmann).

[1] Recent methodological innovations have emphasized more sophisticated methods for reducing the dimensionality of the state space (e.g., Ahn et al., 2018; Fernández-Villaverde et al., 2019, among others), but still rely on the common information assumption. A complementary strand of the literature has

https://doi.org/10.1016/j.jedc.2022.104391
0165-1889/© 2022 Published by Elsevier B.V.

In this paper, we ask whether such equilibria—where households employ the same set of limited information—can arise as the endogenous outcome of a model where households optimally choose the information to use to forecast future prices. To answer this question, we embed household information choice into an otherwise standard infinite-horizon incomplete markets model with aggregate risk. In our model, households optimally choose their information about the state of the economy depending on individual wealth and income, in contrast to the previous literature that assumes common, limited information across all households. As a result, the extent to which households have limited information in our framework is a consequence of households' optimal choices, and not a restriction on the information sets imposed by the researcher. Through this lens, we propose a micro-foundation for equilibria in economies with distributions of heterogenous agents and aggregate risk. We find that the standard equilibria assumed by researchers do not exist for plausible information costs.

We start by showing that differences in wealth and employment status naturally imply heterogeneity in the incentives to acquire information. In our model, low-wealth households, who save little regardless of aggregate conditions, forgo acquiring information even if the costs are small. Similarly, an uninformed, constant savings rule may have low utility costs for households with substantial levels of financial wealth. By contrast, households in the middle of the wealth distribution, whose future consumption depends mostly on the value of future labor earnings, highly value information about the current state of the economy because it allows them to better predict future incomes, and thus to make better savings choices.

Next, we explore how such heterogenous incentives to acquire information at the micro-level interact with aggregate dynamics. We show that household information choices are strategic substitutes: An individual household's benefits of information about the state of the economy falls as the average degree of information in the economy rises. All else equal, a large number of informed households reduces the volatility of the aggregate capital stock. As result, higher levels of household informativeness strongly dampen economic fluctuations. This, in turn, lowers the individual benefits of acquiring information about the current state of the economy by decreasing the volatility of future prices and wages. We show that such strategic substitutability in information choice naturally implies that symmetric-information equilibria, where all households make the same once-and-for-all information choice, may not exist. Moreover, even without (exogenous) income and wealth heterogeneity, we show that a representative-agent equilibrium with information choice may not be present.

To illustrate our main results, we first focus on a simplified, two-period version of our standard neoclassical economy. With log-preferences, households naturally split into three groups, according to their first-period resources: The first group are poor households, for whom costs of information acquisition outweigh the limited benefits they can obtain from information. The second group consists of those households who are rich enough for consumption to be approximately unaffected by future wages. They will not pay large utility cost of information either, even if it perfectly reveals future wages and returns, and therefore also do not acquire information.[2] The final group consists of households with an intermediate level of current resources. They, in contrast, strictly benefit from information as it improves savings choices.

The three groups interact in general equilibrium. Because the savings of informed households are high when they expect low capital in the second period (and vice versa), informed savings reduce the dispersion of the future capital stock, and hence the dispersion of future wages and interest rates. This, in turn, lowers the cost of uninformed savings decisions today. The more households that are in the intermediate group, the more important this strategic substitutability becomes. And thus, the larger is the range of information cost parameters for which there is no symmetric equilibrium in pure strategies. By implication, we show that there may not exist a representative-agent equilibrium even in an economy that consists of ex-ante identical agents.

While our theoretical results in the simplified model show that there are regions where symmetric equilibria do not exist, the question remains what region is empirically relevant. To answer this question, we study once-and-for-all information choice in the full, infinite-horizon economy model with aggregate and idiosyncratic risk and incomplete markets. The model is calibrated to match the dynamics of the U.S. economy. We then quantify the relationship between the average degree of household informativeness and individual costs of uninformed decisions. In the standard Krusell-Smith economy, where all agents have the same (limited) information, uninformed savings decisions are not very costly; the capital stock is tightly distributed around its steady-state level. The average utility losses from uninformed decisions amount to 0.13 percent in permanent consumption equivalent terms. Further, the minimal ex-ante loss from deviating to an information set with less information about future prices is only on the order of 0.01%, or about \$4 per year at current prices. We conclude that, for any plausible costs of acquiring information, no Krusell-Smith type equilibria exist.

At the other extreme, when all individuals make uninformed savings choices the variance of the capital stock is 35 percent higher. This increases the average (maximum) utility loss from uninformed savings to 1 percent (3.7 percent), a more than fivefold increase. As a result, we show that no symmetric-information equilibrium exists when information acquisition costs fall in the range of $\$4 - \$1,566.00$ per year.

The rest of the paper is organized as follows: Section 2 presents the economic environment, while Section 3 derives analytical results in a simplified model. Section 4 quantitatively assesses households' incentive to acquire information. We conclude in Section 5. An appendix contains all proofs, in addition to further quantitative results.

focused on solving models in the sequence space (Boppart et al., 2018) taking time as the state variable, side-stepping the need for having the infinite-dimensional distribution as a state variable.

[2] This result depends, however, on the assumption of log-preferences: with higher risk-aversion, there is a wealth threshold beyond which households always acquire information (Section 3).

2. Economic environment

We consider a standard incomplete-markets economy with aggregate risk. The environment closely follows that of Krusell and Smith (1998) but with a modified information structure.

2.1. Technology and preferences

Firms:

The production sector consists of a representative competitive firm, which maximizes profits. Output Y_t is produced in accordance with a Cobb-Douglas production function that aggregates labor services and capital:

$$Y_t = z_t K_t^{\alpha} \left(L_t \bar{l} \right)^{1-\alpha}, \tag{2.1}$$

where K_t and L_t denote economy-wide capital and labor in period t, respectively, and $\alpha \in (0, 1)$. Each household in the economy is endowed with $\bar{l}$ units of time, which it supplies inelastically to the labor market. Total factor productivity z_t is stochastic and follows a first-order Markov chain that takes on two values $z_t \in \{z_l, z_h\}$ with $z_h > z_l$. We assume that markets for labor and capital are perfectly competitive, so that factor prices for labor w_t and capital r_t are given by their respective marginal products:

$$w_t = z_t (1-\alpha) K_t^{\alpha} \left(L_t \bar{l} \right)^{-\alpha}, \quad r_t = z_t \alpha K_t^{\alpha-1} \left(L_t \bar{l} \right)^{1-\alpha}. \tag{2.2}$$

Households:

The household sector comprises of a continuum of ex-ante identical households of unit mass, who have logarithmic preferences over non-durable consumption:

$$\mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log c_t, \tag{2.3}$$

where $\beta \in (0, 1)$ denotes the time discount factor and c_t the household's consumption at time t. Each household is endowed with $\bar{l}$ units of time, which it supplies inelastically to the labor market. Household labor productivity ϵ_t is stochastic and can take on two values $\epsilon_t \in \{0, 1\}$, which we interpret as unemployment and employment, respectively. We assume that ϵ_t follows a two-state, first-order Markov chain $\Pi_{z',\epsilon'|z,\epsilon}$, which depends on both ϵ_{t-1}, z_t, and z_{t-1}. A household earns wage w_t when employed and receives unemployment benefits μw_t when unemployed, where $\mu \in (0, 1)$ is the replacement rate. Households cannot borrow but can save in physical capital k_t, whose net return is $r_t - \delta$, where $\delta \in (0, 1)$ denotes the rate of depreciation on capital. In addition to the borrowing constraint and a non-negativity constraint on consumption, household consumption choices have to satisfy the per-period budget constraint:

$$c_t + k_{t+1} = (1-\tau_t)\epsilon_t w_t \bar{l} + \mu (1-\epsilon_t) w_t + (1 + r_t - \delta) k_t, \tag{2.4}$$

where τ_t denotes the tax rate on labor income. We denote the right-hand side of (2.4) by $m_t \equiv (1-\tau_t)\epsilon_t w_t \bar{l} + \mu(1-\epsilon_t) w_t + (1 + r_t - \delta) k_t$, and refer to m_t as household *cash-at-hand*.A household seeks to maximize its utility in (2.3) subject to the budget constraint in (2.4).

Finally, we assume that the share of households in a given idiosyncratic employment state only depends on current total factor productivity z_t. Hence, the unemployment rate is a function only of z_t, and thus only takes on two values u_h and u_l with $u_h < u_l$.

2.2. Government

The government runs a balanced-budget unemployment insurance scheme, such that $\tau_t = \frac{\mu u_t}{\bar{l} L_t}$, where L_t and $u_t = 1 - L_t$ are the employment and unemployment rates, respectively.

2.3. Timeline and information structure

The economy proceeds through two stages. In the first stage, at the start of period $t = 0$, households first draw an initial employment status ϵ_0 and asset state k_0 from Γ_0, the cross-sectional distribution of capital and employment. Then, households choose once-and-for-all which signals $\mathcal{I}_t$ for $t \geq 0$ they want to receive about the current state of the economy (described below) at utility cost $\kappa > 0$ per signal acquired in $\mathcal{I}_t$. The signals that households receive are restricted to be the same in each period, and we impose a maximum signal set $\mathcal{I}_t^{\max}$ that contains the signals that the household can choose between. We assume that $\mathcal{I}_t^{\max}$ can include current market signals such as aggregate capital. The household's information set Ω_t accumulates in future periods $t > 0$ according to $\Omega_t = (\mathcal{I}_s)_{s=0}^{t}$. Once households have committed to their information choices, the economy transitions to the second stage, which it stays in for all $t \geq 0$. Conditional on their information choices, households make consumption and savings decisions, firms produce, and goods and input prices adjust to clear markets. Nature determines the realization of the innovations ϵ_t and z_t at the start of each period.

2.4. Discussion

Notice that the state space of our economy is highly-dimensional. Even if households have full information about all shocks, because the saving choices that arise from (2.3) and (2.4) are non-linear functions of capital holdings, there is no law of motion for aggregate capital that households can use to accurately predict future wages and returns. The state of the economy comprises the entire joint distribution of capital holdings and employment status.

Motivated by this fact, standard solutions of incomplete market economies with aggregate risk (e.g., Krusell and Smith, 1998) assume that households employ a law of motion that uses information only about a limited set of moments of the cross-sectional distribution. Indeed, predictions of future prices are often extremely accurate in standard models when households' information comprises *only* the current level of productivity z_t and the current mean of the capital distribution $\overline{k}_{t+1}$ (equal to the aggregate capital stock K_t) (Den Haan et al., 2010). Consistent with this convention we take as a benchmark $\mathcal{I}_t^{\max} = \{z_t, K_t\}$. Consistent with rational expectations, we assume that agents use the equilibrium law of motion H to update their posterior distribution about future variables, conditional on their information.

Whenever $\mathcal{I}_t$ contains the same variables over time and identical for all households, and includes z_t and K_t, our approach is identical to that in Krusell and Smith (1998). However, in contrast to Krusell and Smith (1998), we also consider information sets that do not include aggregate productivity z_t, or the mean of the capital distribution K_t, and we allow information sets to differ across households. Furthermore, while we retain the requirement that, in equilibrium, households' perceived law of motion H captures the conditional distribution of elements in $\mathcal{I}_t$, we do not necessarily require H to describe their dynamics accurately in a statistical sense. Households use the (asymptotically) best linear unbiased estimator of the aggregates conditional on their information set. While this means that household forecasts are unbiased, they may make non-negligible expectational errors in equilibrium. By contrast, in models that follow the Krusell and Smith (1998) approach to computing equilibria, researchers typically increase the elements in $\mathcal{I}_t$ until conditional errors are negligible. Here, households instead choose their information sets optimally, to maximize their utility, internalizing that some information sets may result in non-negligible expectation errors.

Because households observe current factor prices w_t and r_t, one objection to our approach is that households could, in principle, always back out $\mathcal{I}_t^{\max}$ from the observation of current factor prices in (2.2). Consistent with the spirit of the rational inattention literature (e.g., Sims, 2003) and the literature on costly interpretation of asset and goods prices (e.g., Mondria et al., 2021), we view the utility cost κ that households incur from the observation of an additional variable as also describing the cognitive costs that such inversion of prices entail. We therefore invoke the previously-used dichotomy between the variables a household *observes* and those that it *uses* to forecast future prices (e.g., Gabaix, 2019).

Finally, we note that the presence of market-generated information further complicates the state space meaningfully. Since households can learn about the state of the economy from market outcomes, such as the cross-sectional average of capital holdings, households need to not only form beliefs about the cross-sectional distribution of capital and employment, but also about each household's beliefs about it, and so on ad infinitum (Townsend, 1983). Section 4 describes how we simplify this double-infinity state-space.

2.5. Recursive household problem and equilibrium

We now proceed to write the household problem recursively and define our notion of equilibrium:

The physical aggregate state $S = (\Gamma, z)$, consists of, Γ, the cross-sectional distribution of capital and employment at $t \geq 0$, and the current productivity realization, z. We denote household i's first-order belief about S by $\mathcal{P}_i(S)$, where $i \in [0,1]$. Household i's second-order belief about household j's belief is denoted as $\mathcal{P}_{ij}(S)$, and so on ad infinitum. Individual household beliefs are summarized by: $p^i = \left\{\mathcal{P}_i, \left(\mathcal{P}_{ij}\right)_{j\in[0,1]}, \ldots, \left(\mathcal{P}_{ij\ldots k}\right)_{j,\ldots,k\in[0,1]^{n-1}}, \ldots\right\}$. Let $\mathcal{P}$ denote the set of all such beliefs $\mathcal{P} = \left\{(\mathcal{P}_i)_{i\in[0,1]}, \left(\mathcal{P}_{ij}\right)_{i,j\in[0,1]^2}, \ldots, \left(\mathcal{P}_{ij\ldots k}\right)_{i,j,\ldots,k\in[0,1]^n}, \ldots\right\}$. The aggregate state of the economy can then be described by $\Sigma = (S, \mathcal{P})$, while the individual state variables are $\sigma^i = \left(m^i, \epsilon^i, \mathcal{I}^i, p^i\right)$. Households solve their dynamic problem in two stages.

At the start of $t = 0$, households choose what information to acquire each period $\mathcal{I} \in \mathcal{I}^{\max}$:[3]

$$V^0\left(m_0, \epsilon_0, \underline{p}, \underline{\Sigma}\right) = \max_{\mathcal{I}} \mathbb{E}[W(m_0, \epsilon_0, p_0, \Sigma_0) - \kappa(\mathcal{I}) \mid \underline{\Omega}], \tag{2.5}$$

where V^0 is the $t = 0$ value function before information choice, and W is the recursive value function for all periods after the once-in-for-all information choice. Information acquisition entails a utility cost κ per signal acquired in $\mathcal{I}$. We note that households' expectations in the first stage are computed using their ex-ante prior $\underline{p}$.[4] We assume that every period households rationally use their information, together with the equilibrium law of motion for the aggregate state, which we denote by H, i.e., $\Sigma' = H(\Sigma, z, \left(\mathcal{I}^i\right)_i)$, and the exogenous transition matrix Π^z, to form a prior about today's state variables from yesterday's posterior.

[3] We once again abstract from the superscript i, to ease notation.

[4] The ex ante expectation $\mathbb{E}[\cdot \mid \underline{\Omega}]$ that corresponds to $\underline{p}$ is the *unconditional* expectation. We note that $\underline{\Sigma} = (S_0, \underline{\mathcal{P}})$, where $\underline{\mathcal{P}}$ denotes the set of all initial (higher-order) beliefs, which we equate to the appropriate unconditional distributions.

After deciding on their information set $\mathcal{I}$, every period $t \geq 0$, households choose consumption c and savings k' given their updated information set $\Omega = \{\Omega_{-1} \cup \mathcal{I}\}$:

$$W(m, \epsilon, p, \Sigma) = \max_{c,k' \geq 0} \log c + \beta \mathbb{E}\left[W(m', \epsilon', p', \Sigma') \mid \Omega\right] \tag{2.6}$$

subj. to

$$c + k' = m$$

$$m' = (1 - \tau(\Sigma'))w(\Sigma')\bar{l}\epsilon' + \mu(1 - \epsilon')w(\Sigma') + (1 + r(\Sigma') - \delta)k' \tag{2.7}$$

We let g denote the function that characterizes a household's savings choice $k' = g(\sigma)$, and ι_0 the function that characterizes its information choice $\mathcal{I} = \iota_0(\underline{\sigma})$. Finally, today's posterior beliefs p' are linked to yesterday's p through Bayes' Rule and the information choice $\mathcal{I}$.

Given this formulation of a household's problem, our equilibrium is a law of motion $H : \Sigma' = H(\Sigma, z, (\mathcal{I}^i)_i)$, a pair of individual functions W and V, which describe the value functions at the two separate stages of a household's problem, a pair of policy functions (g, ι_0), and pricing functions (r, w) such that (i) (W, V^0, g, ι_0) solves a household's problem, (ii) r and w are competitive, (iii) H is generated by ι_0 and g and Bayes' Rule, using the information contained in $\mathcal{I}$ and current beliefs summarized in $\mathcal{P}$.

3. Analytical results in a simplified model

This section analyzes a simplified version of our baseline economy, whose analytical tractability allows us to highlight the forces that affect households' information choices. We collapse all future periods into a terminal period $t = 1$, and consider information choices in the initial period $t = 0$. We assume that all households are always employed $\epsilon_0 = \epsilon_1 = 1$, normalize labor supply $\bar{l} = 1$, and restrict the capital share $\alpha \in [\frac{1}{3}, 1]$. We proceed in two steps. First, because the relevant variables for consumption and savings choices are future prices, we initially let households acquire information directly about second-period wages and rates of return. This allows us to highlight how households' information choices differ across the wealth distribution. Second, we proceed to characterize equilibrium information choices, where period-two prices are determined by exogenous productivity shocks and the capital stock that results from period-one savings decisions. We show that a symmetric, pure strategy equilibrium may not exist for household information choices.

3.1. Heterogenous benefits of information

Consider a household with cash-at-hand m_0, who has the option to purchase *perfect information* about period-two wages w_1 and returns r_1, before choosing period-one consumption c_0 and savings k_1. The household enters the initial period with a non-degenerate prior distribution $\Phi(w_1, r_1)$ over w_1 and r_1 on the bounded support $\Psi = [\underline{w}, \overline{w}] \times [\underline{r}, \overline{r}]$ for $\underline{w} < \overline{w}$ and $\underline{r} < \overline{r}$. The household's Euler equations for k_1, with and without information choice are, respectively:

$$\frac{1}{m_0 - k_1} \geq \beta \frac{1}{k_1 + \frac{w_1}{r_1}}, \quad \frac{1}{m_0 - k_1} \geq \beta \mathbb{E}\left[\frac{1}{k_1 + \frac{w_1}{r_1}}\right], \tag{3.1}$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator with respect to Φ. Eq. (3.1) holds with equality whenever $k_1 > 0$. Proposition 1 characterizes a household's expected benefits from acquiring information about w_1 and r_1, using the household's utility function in (2.3).

Proposition 1. *Consider households' information choice problem in the simplified two-period version of the model with an exogenous prior about second-period prices w_1 and r_1:*

(i) *There exists a threshold $\underline{m} \equiv \underline{w}(\beta\overline{r})^{-1} > 0$ such that households, whose first-period cash-at-hand m_0 is less than $\underline{m}$, have zero benefit of acquiring information.*
(ii) *There exists another threshold $\bar{m} > \underline{m}$ such that households whose first-period cash-at-hand m_0 exceeds $\bar{m}$, have a strictly positive benefit of acquiring information, which strictly decreases towards zero in cash-at-hand m_0.*
(iii) *Finally, there exists cash-at-hand values $m_0 \in [\underline{m}, \bar{m}]$ for which the benefit of acquiring information is strictly positive and increases in cash-at-hand m_0.*

Proposition 1 shows that expected benefits of information follow an inverted u-shape in cash-at-hand. The reason for this shape arises from the effects that household wealth have on savings choices k_1, the sole intertemporal decision that households make.[5]

The first part of the proposition shows that households whose income is low enough to save zero $k_1 = 0$, irrespective of the present discounted value of future wages $\frac{w_1}{r_1}$, will never pay for information about future prices. Households for

[5] The latter assumption ensures that direct iteration on a household's best-response function for k' can be used to show a unique solution for k' (see Appendix A and the argument in Angeletos and Pavan, 2007).

which $m < \underline{m}$ are constrained by the no-borrowing limit for *any*value of future wages and interest rate (3.1 holds with strict inequality). As a result, these households choose to save zero irrespective of future wages or returns, and hence do not value information that helps better predict these prices.

By contrast, the second part of the proposition shows that households that save a positive amount for some value of future prices, those for which $m_0 > \bar{m}$, *do value* acquiring information. This is because additional information helps these household make better savings choices; savings choices that are more in line with future wages and returns. However, a key feature of this incentive to acquire information about future prices is that it decreases with household wealth. Increases in m_0 make a household's savings choices (with log-preferences) less responsive to future wages and returns, and hence decrease the expected benefit of information acquisition. Indeed, for households that are sufficiently rich $\left(\frac{\overline{w}}{m_0} \to 0\right)$, the expected benefit of information about future wages and rates of return convergences back to zero. We discuss below how these conclusions change for different household preferences.

Finally, the last part of the proposition follows from the continuity of households' utility function. It shows that there exists cash-at-hand values $m_0 \in [\underline{m}, \bar{m}]$ for which the benefit of information is strictly positive. However, unlike households for which $m_0 > \bar{m}$, the expected benefit here increases with cash-at-hand. This is because the increase in household savings k_1 arising from the relaxation of the no-borrowing limit, due to an increase in m_0, dominates the decreased sensitivity of household savings due to the income effect.

Fig. 1 illustrates Proposition 1 numerically, by depicting the expected utility loss from uninformed savings choices for different prior distributions. We transform the utility losses into percentage differences in permanent consumption as a function of first period cash-at-hand m_0. Losses follow an inverse u-shape pattern, and approach zero as first-period cash-at-hand m_0 rises. Expected losses are furthermore lower when wages and interest rates are perceived to be less volatile, or co-move more strongly. The latter arises because the present discounted value of future wage payments $\frac{w_1}{r_1}$ that determine informed savings in (3.1) is less variable when wages co-move more with interest rates. This will be important later.

Risk aversion and information.

The benefits of information at high-levels of cash-at-hand depend on households' relative risk aversion. With a higher level of risk aversion than in (2.3), expected losses are higher and no longer inverse u-shaped. Instead, losses have a local minimum at intermediate values for m_0, where future income is most diversified across wages and returns on savings. This is, moreover, a more powerful force when wages and interest rates correlate negatively. Appendix B demonstrates these results for the case in which households' felicity function is equal to $u(c) \equiv \frac{c^{1-\gamma}-1}{1-\gamma}$, where $\gamma > 1$. As cash-at-hand m_0 rises further, expected losses then gradually converge to a strictly positive limit. These differences in results arise because with increased risk aversion substitution effects of interest rate changes dominate income effects (while the two cancel with log-preferences). Households thus always want to align their savings choices with future interest rates, and hence value information more.

3.2. Information choice in general equilibrium

The utility benefits of information depend crucially on the joint distribution of wages and rates of return on capital (e.g., Fig. 1). In equilibrium, these in turn depend on aggregate productivity z_1, as well as households' savings and information choices that determine the aggregate capital stock K_1. In this subsection, we illustrate the consequences of this circular relationship for the existence of an equilibrium. We note that the unobserved fundamentals of the economy are productivity z_1 and the full distribution of cash-at-hand values $\left(m_0^i\right)_{i\in[0,1]}$.

Information about productivity.

We start by analyzing the incentives to acquire information about productivity z_1. Consider Eq. (2.2) at time $t = 1$. This equation shows that information about second-period factor prices is embedded in information about future productivity z_1 and current aggregate savings K_1. Knowledge of z_1 and K_1 suffice to predict the future value of wages w_1 and the rate of return on capital r_1 since in equilibrium labor supply $L_t = 1$. Consider now the Euler Eq. (3.1). This shows that (i) savings choices k_1 either depend only on the present discounted value of future wage payments $\frac{w_1}{r_1}$ (if the household is on its Euler equation); or (ii) are independent of future wages and returns altogether ($k_1 = 0$ if the household is off its Euler equation). As a result, aggregate savings K_1 is completely unaffected by movements in wages and returns that are caused by productivity shocks z_t. Indeed, directly substituting (2.2) into (3.1) shows that

$$\frac{1}{m_0 - k_1} \geq \beta \frac{1}{k_1 + \frac{\alpha}{1-\alpha}K_1}, \quad \frac{1}{m_0 - k_1} \geq \beta \mathbb{E}\left[\frac{1}{k_1 + \frac{\alpha}{1-\alpha}K_1}\right], \tag{3.2}$$

which is independent of productivity z_1. We conclude that households that know the current distribution of cash-at-hand $\left(m_0^i\right)_{i\in[0,1]}$, and thus correctly anticipate aggregate savings K_1, do not wish to acquire additional information about productivity.

Proposition 2. *When households know the distribution of cash-at-hand* $\left(m_0^i\right)_{i\in[0,1]}$, *a unique equilibrium exists; no household chooses to acquire information about current productivity* z_1.

Information about distribution of cash-at-hand.

By contrast, households do have an incentive to acquire information about the capital stock K_1, and hence about the distribution of cash-at-hand $(m_0^i)_i$. To illustrate this result, we proceed in two steps. First, we consider the special case in which all households have the same level of cash-at-hand $m_0 = M_0 \in \mathbb{R}_+$ but are unaware of this equality. This will allow us to show that a representative-household equilibrium does not necessarily exist for our economy. We then proceed with the case in which households are also heterogenous in their cash-at-hand values.

Representative household case. Suppose that each household has the prior about the aggregate level of cash-at-hand $M_0 = \mu_m + u$, where $u \sim \mathcal{N}\left(0, \sigma_u^2\right)$ and $\mu_m > 0$. If a household *does not* acquire information, it observes the noisy signal $s_i = u + \eta_i$, where $\eta_i \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$ and $\mathbb{E}\left[\eta_i \eta_j\right] = 0$ for $i \neq j$. In contrast, if a household *does* acquire information, it knows aggregate cash-at-hand M_0 with certainty.[6] Furthermore, we set $\beta = 1$ in the following, and also assume that borrowing constraints are absent.

The case in which all households have the same cash-at-hand $m_0 = M_0$, and are all on their Euler equation, closely resembles the classical case studied in Grossman and Stiglitz (1980). As in Grossman and Stiglitz' analysis, households' actions are strategic substitutes: the Euler equation in (3.2) directly shows that

$$\frac{\partial k_1}{\partial K_1} = -\frac{\alpha}{1-\alpha} \frac{\mathbb{E}\left[\frac{1}{\left(k_1 + \frac{1-\alpha}{\alpha} K_1\right)^2}\right]}{\frac{1}{(M_0 - k_1)^2} + \beta \mathbb{E}\left[\frac{1}{\left(k_1 + \frac{1-\alpha}{\alpha} K_1\right)^2}\right]} < 0. \tag{3.3}$$

This substitutability of savings choices, in turn, decreases the volatility of the capital stock in equilibrium, and hence decreases the value of additional information. In fact, as in Grossman and Stiglitz (1980) and Hellwig and Veldkamp (2009), strategic substitutability in actions leads to strategic substitutability in information choice. As a result, a range of values of the information cost parameter κ exists for which there is no pure strategy equilibrium.

Proposition 3. *Suppose $m_0^i = M_0$ for all i and σ_u is small.[7] Then, a range of values for $\kappa \in \mathbb{R}_+$ exists for which there is no equilibrium with a representative household with cash-at-hand $m_0^i = M_0$: If all (other) households purchase information about M_0, the change in expected utility from information $\mathbb{E}\Delta\mathcal{U} - \kappa$ is negative, and vice versa.*

The results in Proposition 3 connect the neoclassical environment in Section 2 with the class of quadratic games studied in Hellwig and Veldkamp (2009) and Colombo et al. (2014), in addition to the CARA-Gaussian asset pricing models analyzed in Grossman and Stiglitz (1980), Hellwig (1980), and Veldkamp (2011). But, although in each case, strategic substitutability of actions leads to strategic substitutability in information choice, the mechanism by which this occurs differs in our model from that in the previous literature.

The impact of (other) households' information choices does, namely, not arise through the observation of *endogenous information*, nor through a *direct payoff externality*. Instead, other households' information choices here matter because of a *pecuniary effect*. The more other households' purchase information, the less volatile the aggregate capital stock becomes, all else equal. The strength of the strategic substitutability in (3.3) is stronger when the average household is more informed, and does not dampen its responses due to imperfect information. This, in turn, decreases the incentive for an individual household to acquire information about capital, and hence the net-present value of future wages $\frac{1-\alpha}{\alpha}K$, in the first place.

Finally, because of the strategic substitutability of information choices, a representative-agent equilibrium, in which all households make the same information choice, may not exist for our economy. This is even though all households have the same cash-at-hand.

Heterogenous household case. The introduction of wealth heterogeneity into our simplified framework merges the insights of Proposition 1 with those of Proposition 3. To illustrate this result, we assume that m_0 takes on three values $m_0 \in \mathcal{M}_0 = \{m_0^L, m_0^M, m_0^H\}$, and denote the mass of households at each point as $\pi_x, x \in \{L, M, H\}$ with $\pi_L = \pi_H$, and $\pi_M = 1$. We choose the points such that households at $m = m_0^L$ are always constrained, and hence have no incentives to acquire information and save ($K_1^L = 0$). By contrast, households at $m = m_0^H$ face second-period wages that are a negligible fraction of m_0^H, and we assume that their saving choices are common knowledge and constant $\left(K_1^H \in \mathbb{R}_+\right)$.[8] Consequently, only middle-wealth households make non-trivial savings and information choices. We choose the same source of uncertainty about the cash-at-hand distribution of middle-wealth households and the same constraints as we did in the representative household case.

Proposition 4. *Consider the three-point distribution for cash-at-hand $\left(m_0^i\right)_i$, where $m_0^i \in \mathcal{M}_0$, and suppose σ_u is small. Then, there exists for each $\pi_L, \pi_H > 0$ a fixed cost $\kappa > 0$ such that no equilibrium with symmetric, pure-strategy information choices about m_0^M exists. The (absolute value norm of the) range of fixed cost parameters for which no symmetric, pure-strategy equilibrium exists further strictly decreases in the mass of low- and high-wealth households.*

[6] We further follow convention and assume that $\int_0^1 \eta_i di = 0$.

[7] We set σ_η to a small value to avoid Jensen's inequality terms in households' best response functions in (3.2) (see the proof of Proposition 3 in Appendix A and Section 4 for the full non-linear solution).

[8] Indeed, the latter follows from the assumption that second-period wages are a negligible fraction of m_0^H.

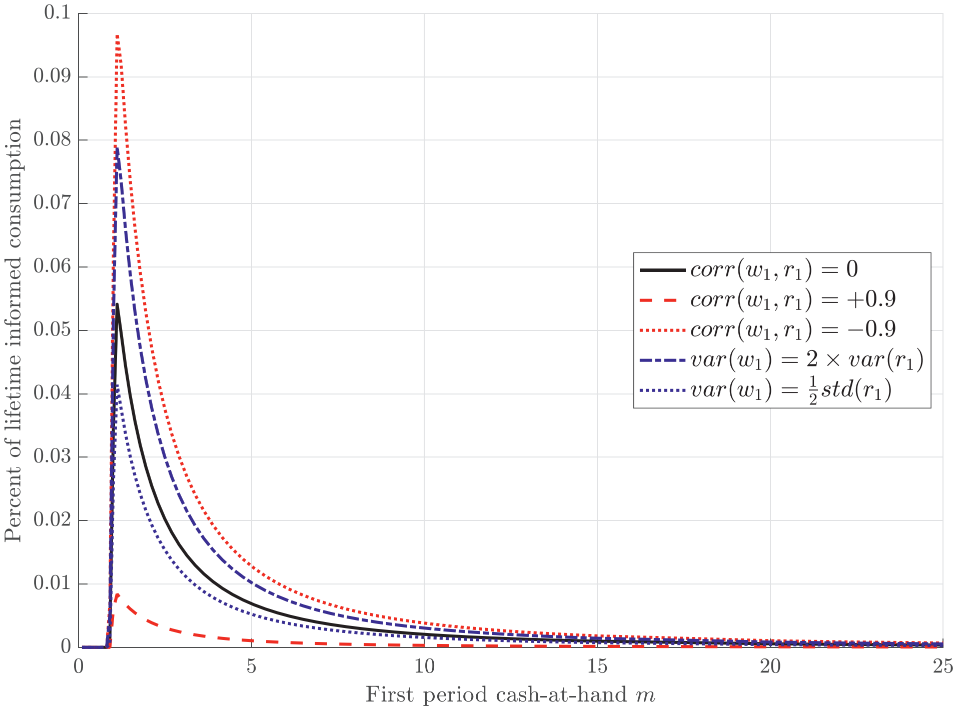

Fig. 1. Utility losses in the two-period model. The figure depicts the informed consumption-equivalent (CE) utility loss from not acquiring information in the simple two-period model with $\beta = 0.99$ and a joint normal distribution for w_1 and r_1 with means of 1 and standard deviations of 0.05, respectively. The correlation between w_1 and r_1 is $\sigma_{wr}/0.05^2$, where $\sigma_{wr} = \mathbb{C}ov(w_1, r_1)$. Appendix B shows the analogues figure for a household with risk-aversion that exceeds 1.

Proposition 4 shows that the non-existence of pure strategy equilibria, where all households make the same information choice, identified in Proposition 3, extends to the case with wealth heterogeneity. To see how wealth heterogeneity, nevertheless, modifies our previous insights, consider the addition of the two types of households to our setup.

Low-cash-at-hand households are constrained, and in all states of the economy choose to save zero. As a result, the savings choices of these households do not feature the strategic substitutability highlighted in the previous subsection. This, all else equal, decreases the range of fixed cost parameters for which the non-existence of equilibria is present. High-cash-at-hand households are through this lens similar. Proposition 1 shows that high cash-at-hand households also have a smaller incentive to acquire information than medium wealth households, all else equal, and their savings choices are similarly unaffected by realizations of future prices. Hence, the presence of a large mass of high wealth households also narrows the range of fixed cost parameters for which the non-existence of symmetric equilibria is present.

Fig. 2 illustrates the insights of Proposition 4. The figure shows that as we increase the mass of low - and high-wealth households in the population the smaller the range of fixed costs parameters becomes for which no-symmetric pure strategy equilibrium exists. All else equal, medium wealth households and their information choices are those that contribute to the volatility of the capital stock. As their share of the population decreases the uncertainty about the future capital stock falls. Because the expected benefit of information is higher the more uncertain the capital stock is, the range of fixed costs parameters that are too high for all middle-wealth households to buy information, but too low for no-one to buy it also decreases in the mass of high- and low- wealth households. This, in turn, delivers the results visible in Fig. 2. Combined, Proposition 4 and Fig. 2 demonstrate the crucial interaction that exists between wealth inequality and information choice.

3.3. Summary and discussion

The simple model analyzed in this section shows that the incentives to acquire information about the state of the economy are heterogeneous across the wealth distribution, and largest for middle-wealth households. Our model further shows that there is strategic substitutability in information choice at the heart of neoclassical environments. The benefits from informed savings choices are lower the more other (middle-wealth) households are informed. Combined, our results imply that symmetric-information equilibria, where all households make the same information choice, may not exist. The next section shows how our results extend to the general model, with an endogenous, continuous wealth distribution, idiosyncratic earnings risk, endogenous dynamics of the capital stock, and an infinite planning horizon (that makes information

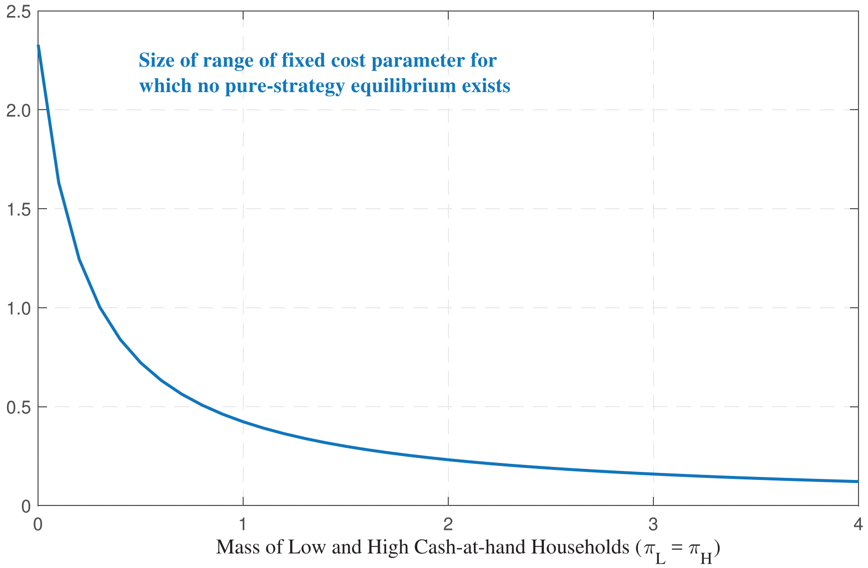

Fig. 2. Equilibrium information choice in the two-period model. The figure shows the norm of the range of fixed cost parameters consistent with no symmetric pure-strategy equilibrium. Let $\Delta(a, a)$ denote the expected net utility benefit to a household of acquiring information when every other household does so too, while $\Delta(a, na)$ denotes the expected net benefit when others does not acquire information. The figure depicts $\Delta(a, na) - \Delta(a, a)$ converted into consumption-equivalent losses for different values of $\pi_L = \pi_H$. The parameters used are: $\alpha = 0.40$, $K_H = 100$, $w = 1/2$, $m_0^M \sim \mathcal{N}(10, 1)$.

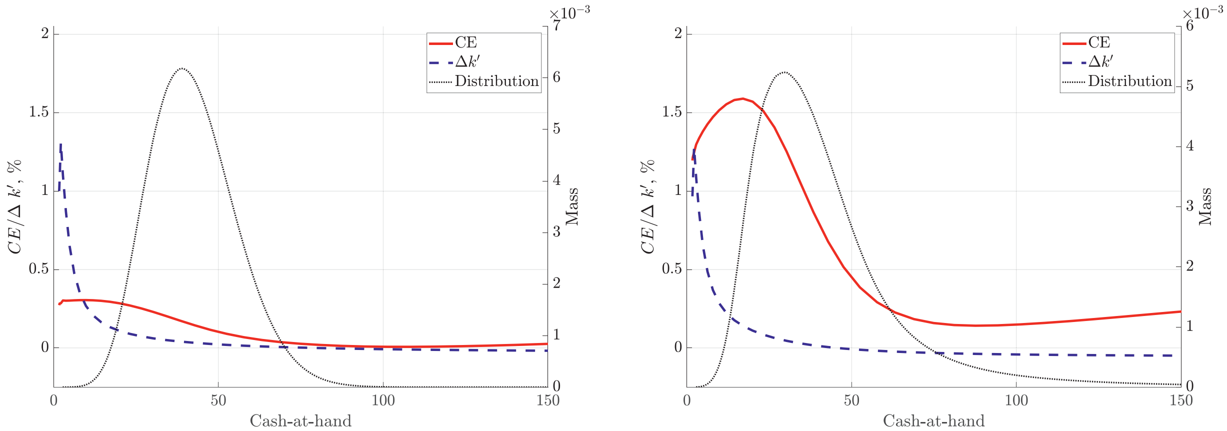

Fig. 3. Heterogenous benefits and losses from information. The figure depicts consumption-equivalence differences $CE_{\mathcal{I}}(k, \epsilon; \tilde{\mathcal{I}})$ of not acquiring information (solid lines), differences in policy functions k' (dashed lines), and the average cross-sectional distribution of households (dotted lines). It does for a *high-information economy* ($\tilde{\mathcal{I}} = \{z, K\}$, left-hand side panel) and a *low-information economy* ($\tilde{\mathcal{I}} = \{\emptyset\}$, right-hand side panel).The information choice considered is $\mathcal{I} = \{z, K\} = \mathcal{I}_{\max}$ versus $\mathcal{I} = \{\emptyset\}$. The figure focuses on an employed household ($\epsilon = 1$) with median capital in the low-productivity state $z = z_L$. Appendix B shows how the results extend to other households.

beneficial even when it does not affect savings choices in the current period). We then conclude by discussing how our results cast doubt on the use of common solution methods used to solve dynamic macroeconomic models with incomplete markets and aggregate risk.

4. Information choice in the general model

This section considers household information choice in the infinite-horizon version of our economy. We compute symmetric-information equilibria, where all households acquire the same information $\tilde{\mathcal{I}}$ in each period, and study individual households' incentives to deviate. As in the two-period model, the expected benefits from information vary strongly with household wealth, and information choices are strategic substitutes. Because of these dependencies, we show that symmetric-information equilibria do not exist for plausible information costs.

4.1. Quantitative strategy

To quantify the expected benefits or losses associated with different information choices, we first solve for an equilibrium where households use the same (pre-specified) information $\widetilde{\mathcal{I}}$ in each period. We compute this symmetric equilibrium using the iterative algorithm proposed in Krusell and Smith (1998):

1. Choose the information set $\widetilde{\mathcal{I}}$ and postulate a law of motion for the aggregate state H.[9]
2. Solve the household problem conditional on $\widetilde{\mathcal{I}}$ and H.
3. Using the resulting decision rules, simulate a large number of households for a large number of periods. From this simulation, calculate time series for the elements in $\widetilde{\mathcal{I}}$, and use these to estimate a new law of motion H'.
4. Compare H' to H used in (2). If different, update conjecture for H and return to (1).[10]

With such an symmetric-information equilibrium at hand, we then calculate optimal decision rules for consumption and savings when a household uses *a different information set* $\mathcal{I} \neq \tilde{\mathcal{I}}$ in each period. Consistent with its information choice, the household also considers an alternative law of motion $H(\mathcal{I})$ for the endogenous variables in $\mathcal{I}$. Finally, we then calculate the expected utility associated with both information choices, conditional on aggregate and individual states at time zero, and comment on any differences.[11]

As a baseline for our analysis, we consider the information set studied in Krusell and Smith (1998): Each household observes economy-wide productivity and the aggregate capital stock in each period ($\widetilde{\mathcal{I}} = \{z, K\} \equiv \mathcal{I}_{\max}$). Krusell and Smith (1998) find that this information set allows for extremely accurate predictions of future wages and interest rates. We then study households incentives to acquire strictly less information in each period; that is to acquire instead either (i) $\mathcal{I} = \{z\}$, (ii) $\mathcal{I} = \{K\}$, or (iii) $\mathcal{I} = \emptyset$.[12] At the end of this section, we briefly comment on the potential benefits from observing additional information. We then proceed to also study other symmetric information sets ($\widetilde{\mathcal{I}} = \{z\}$, $\widetilde{\mathcal{I}} = \{K\}$, or $\widetilde{\mathcal{I}} = \{\emptyset\}$), and conduct a similar analysis. We characterize the expected utility benefits or losses of using $\mathcal{I}$ rather than $\widetilde{\mathcal{I}}$, and discuss how it depends on individual and aggregate states, as well as the average level of information in the economy. This allows us to discuss the existence of symmetric-information equilibria for different values of the fixed information cost parameter κ.

4.2. Parameters and calibration

Table A.1 in the appendix summarizes the parameters we use in our quantitative analysis. We interpret a time period as a quarter, and choose a felicity function u with constant relative risk aversion equal to γ. Given evidence that prediction errors are declining in wealth for high-wealth quintiles in US micro data (Broer et al., 2021), we choose a value of γ equal to 5.[13] We choose standard parameters for the discount factor β (0.99), the capital share α (0.36), and the depreciation rate δ (0.025). We calibrate the structure of aggregate and idiosyncratic uncertainty to capture key features of the dynamics of unemployment and job-finding rates in the post-world war II US economy, in the spirit of Krueger et al. (2016). Specifically, we specify transitions in aggregate productivity to capture good and bad times, defined as periods when unemployment is below and above trend, respectively.[14] Productivity z_t then captures the difference in average US total factor productivity during the periods thus identified. The resulting persistence of good and bad times is 0.88 and 0.82, respectively. The resulting values for z_l, z_h are 0.98 and 1.01, respectively. The parameters governing individual transition probabilities are specified to be similar to those observed in the US labor market. In particular, we choose an unemployment rate in booms and recessions equal to 6 and 10 percent, respectively. Job-finding rates are set such that unemployment spells are relatively short, as in US data, equal to 55 and 45 percent in booms and recessions, respectively. The remaining transition probabilities are then pinned down by the requirement that the unemployment rate depends only on current productivity, and are reported in Table A.2. Finally, we normalize the time endowment $\bar{l} = 1/0.9$ such that aggregate labor services in the bad aggregate state are equal to 1, and set the replacement rate μ equal to 0.4.

4.3. Quantitative results

4.3.1. Benefits and losses of information acquisition

Fig. 3 depicts a household's incentives to be informed about the state of the economy. The figure displays the expected utility loss incurred by an *employed* household taking savings choices without any knowledge of the current state of the economy (so that $\mathcal{I} = \emptyset$). The figure evaluates these losses relative to those incurred under the Krusell-Smith information choice ($\mathcal{I} = \{z, K\} = \mathcal{I}_{\max}$). The left-hand panel considers a *high-information economy* where all other households use $\widetilde{\mathcal{I}} =$

[9] As in Krusell and Smith (1998), we assume a log-linear law of motion.

[10] Given the log-linear nature of H, we use a simple regression to update the parameters.

[11] We evaluate the expectations in both cases using the law of motion H associated with the more comprehensive information set $\mathcal{I}_{\max}$.

[12] When $\mathcal{I} = \emptyset$, we assume that households use only average transitions and the unconditional mean of capital in their forecasts of future wages and interest rates.

[13] Section 4.3 shows that our results are very similar to those in the case with log-preferences, where $\gamma \to 1$. Compare, for example, Fig. 3 with Fig. 1.

[14] We use an HP filter with smoothing parameter 14,400 to construct the trend in the unemployment rate.

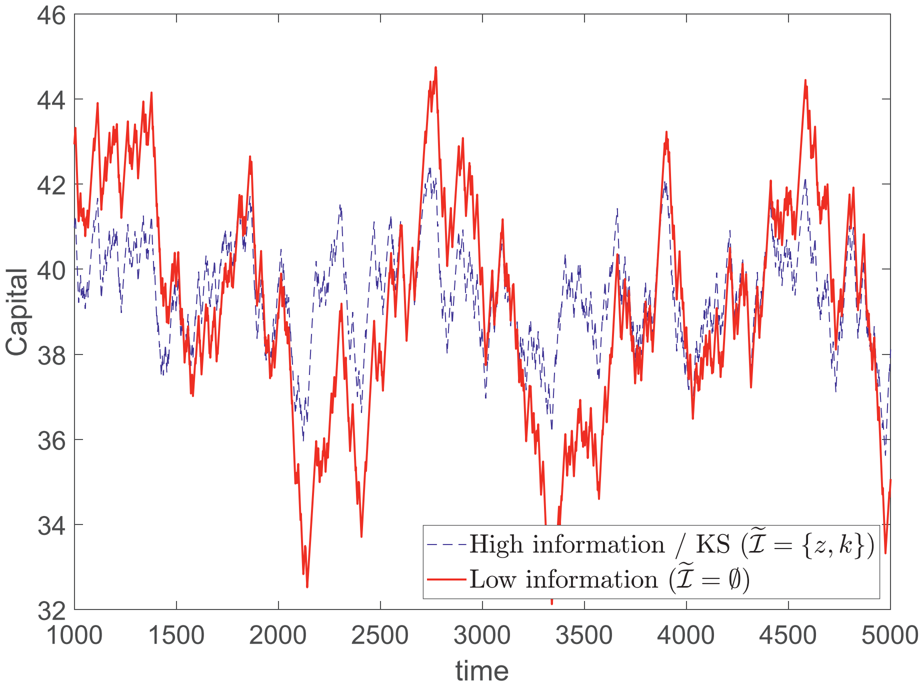

Fig. 4. Time series of the aggregate capital stock. The figure shows the time series of aggregate capital K_t from a simulation of the high-information (Krusell-Smith) economy ($\widetilde{\mathcal{I}} = \{z, K\}$, dashed blue line) and the low-information economy with uninformed savings ($\widetilde{\mathcal{I}} = \emptyset$, solid red line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$\{z, K\} = \widetilde{\mathcal{I}}_{\max}$, while the right-hand side panel considers a a *low-information economy*where all other households use $\widetilde{\mathcal{I}} = \{\emptyset\}$. For ease of interpretation, the figure depicts consumption-equivalent differences rather than raw utility differences.

We note that changes in welfare are markedly different at different levels of cash-at-hand, and that the overall pattern of the relationship between cash-at-hand and utility is similar to that observed in our simple two-period economy (Fig. 1). In both the right- and left-hand panel, expected utility changes are highest for low but positive values of cash-at-hand, where savings choices are unconstrained but dominated by the difference between current and future labor income. In this case, changes arise because predictions of future labor income are substantially improved by information about the current capital stock (which determines the level of future wages), as well as current aggregate productivity (whose persistence makes it a good predictor for future separation rates). At high productivity, for example, information raises the outlook for future employment, and lowers savings at low levels of cash-at-hand. As wealth rises further, the difference between informed and uninformed savings falls back, as do the associated losses from uninformed choices.[15]

One crucial difference between Figs. 1 and 3 lies in the losses from uninformed choices of the wealth-poorest, constrained households. In the two-period economy, the wealth poorest did not incur any utility losses, but in the more general version they now lose from uninformed savings choices in later periods. This shows the importance of the dynamic nature of households' savings for households' information acquisition strategies.

Finally, while the shape of losses from uninformed choices is similar in the left- and right-hand side panel of Fig. 3, their level is starkly different. For example, maximum utility losses are four times higher in the low-information economy. Fig. 4 sheds some light on the reason for why expected losses from uninformed savings choices are larger in the low-information economy. The reason is identical to that identified in Section 3: The capital stock is substantially more volatile when households do not condition their savings choices on current productivity and the current level of the capital stock. In equilibrium, uninformed savings choices thus strongly counteract the mean reversion inherent in neoclassical economies (whereby higher returns implied by a lower capital stock increase savings in bad times, and vice versa for good times). The implied widening of the capital distribution around its average level makes average, uninformed savings choices more costly. And the increase in persistence makes information about the current level of capital even more valuable to predict the future.

Table A.3 summarizes the expected and maximum gains or losses implied by deviations from the equilibrium information set $\widetilde{\mathcal{I}}$ in the high- and low-information economies, respectively. The table shows (i) *average* and *minimum ex-ante* expected losses from not acquiring any information $\mathcal{I} = \{\emptyset\}$ in the high-information economy ($\widetilde{\mathcal{I}} = \{z, K\}$) in columns one and two;

[15] Given the calibrated high job-finding rates (of 50 percent on average), the (forward-looking) expected losses for the *unemployed* are similar to those of the *employed* shown in Fig. 3. The main difference is that the expected losses are lower at low cash-at-hand levels, where the unemployed's current savings choice is constrained and expected losses only arise from uninformed choices in future unconstrained periods.

Table 1
Expected utility changes in percent CE.

	$\tilde{\mathcal{I}} = \{z, K\}$		$\tilde{\mathcal{I}} = \emptyset$	
	Average Loss	Minimum Loss	Average Gain	Maximum Gain
$\mathcal{I} = \emptyset$	−0.1297	−0.0152	·	·
$\mathcal{I} = \{z\}$	−0.1184	−0.0147	0.0619	0.2990
$\mathcal{I} = \{K\}$	−0.0705	−0.0705	0.9019	3.6495
$\mathcal{I} = \{z, K\}$	·	·	0.9415	3.6682

The table presents consumption-equivalent losses or gains from using different individual information sets $\mathcal{I}_t$ (indicated in the first column) rather than the information set used by all other households $\tilde{\mathcal{I}}$ (indicated in the top row). Expectations are taken across the 2.5–97.5 percentile range of the ergodic distribution of aggregate and individual states. Appendix B shows the equivalent table in raw utility differences as well the corresponding tables for the interim information choice cases ($\tilde{\mathcal{I}} = \{K\}$ and $\tilde{\mathcal{I}} = \{z\}$).

and (ii) *average and maximum* benefits from acquiring full information $\mathcal{I} = \{z, K\}$ in the low-information economy ($\tilde{\mathcal{I}} = \emptyset$) (columns three and and four). Benefits and losses are expressed in terms of permanent consumption differences,[16] and averages, maxima, and minima are calculated over the ergodic distribution of individual and aggregate state variables. Consistent with our earlier results, additional information affects ex-ante welfare substantially more in the low-information economy ($\tilde{\mathcal{I}} = \emptyset$), where average benefits from additional information are large. Consistent with Section 3, most of these benefits can be reaped by knowing the capital stock alone ($\mathcal{I} = \{K\}$), while the incremental benefit of information about productivity z_t is small. Average losses from foregoing information in the high-information economy ($\tilde{\mathcal{I}} = \{z, K\}$) are, by contrast, much smaller. Information about capital K_t is still more costly to forego than that about productivity z_t, but the difference is now smaller.

Combined, the results in this subsection confirm the main insight from Section 3. First, there is substantial heterogeneity in benefits of information acquisition along the wealth distribution. Second, the incentives to acquire information depend crucially on other households' information choices. All else equal, the more informed other households are, the less incentive there is for any given household to acquire information about future prices. The next subsection turns to the implications of the resulting strategic substitutability in information choice for the existence of symmetric-information equilibria.

4.3.2. Non-existence of symmetric-information equilibria

Because the benefits of information are low when average information is high (and conversely), our results in the previous subsection suggest that there exists a range of utility costs for which no symmetric equilibrium in information choice exists. To make this argument precise, we need to (i) consider expected utility differences (instead of the permanent-consumption differences depicted in Fig. 3 and Table 1); and to (ii) consider any possible deviation to subsets of $\mathcal{I}_{\max}$. (Recall that we take a pragmatic approach to the specification of information costs by assuming that acquiring any additional signal incurs the same fixed cost κ.) Tables A. 3 and A. 4 in Appendix B. 2 show expected average and extremum utility differences implied by deviations from $\tilde{\mathcal{I}}$ to any possible $\mathcal{I} \subseteq \mathcal{I}_{\max}$.

Two results stand out from this analysis. First, the Krusell-Smith outcome in which $\mathcal{I} = \{z, K\}$ for all households is not consistent with rational information choice for even minor costs of information. For fixed costs of information worth above 0.01 percent of permanent consumption (or around \$4 at current prices) at least one household would choose to acquire strictly less information if all other household choose to adopt the Krusell-Smith solution (Table 1 and Tables A.3).[17] Indeed, even the average household would want less information if it faces costs of information worth above 0.07 percent of permanent consumption (or \$30; Table 1 and Table A.3). Given the small costs involved for staying informed about relatively complex, economic objects, we conclude that the Krusell-Smith behavioral assumption does not appear robust to the introduction of costly rational information choice. Because of the relative stability of an economy under the Krusell-Smith solution, households would, on average, choose to be less informed if all other household adopt the Krusell-Smith outcome.

Second, a pure-strategy equilibrium, where all households acquire the same information, does not exist for our baseline calibration and moderate costs of information. Expected utility differences of deviating to high information ($\mathcal{I} = \{z, K\}$) in the no-information equilibrium are, for example, substantially larger than the utility loss of deviating to $\mathcal{I} = \emptyset$ in the Krusell-Smith economy. As a consequence, if utility costs are below around 3.7 percent of permanent consumption (or around \$1,566 per year), $\mathcal{I} = \emptyset = \tilde{\mathcal{I}}$ also does not constitute an equilibrium information choice (Table 1 and Tables A.3). Tables A.3 to A.5 in

[16] We transform raw relative utility differences from using $\mathcal{I}$ instead $\tilde{\mathcal{I}}$ into units of permanent consumption as follows:

$$CE_{\mathcal{I}}\left(k, \epsilon; \tilde{\mathcal{I}}\right) = \left[\frac{V_{\tilde{\mathcal{I}}} + \frac{1}{1-\beta}\frac{1}{1-\gamma}}{V_{\mathcal{I}} + \frac{1}{1-\beta}\frac{1}{1-\gamma}}\right]^{\frac{1}{1-\gamma}} - 1, \tag{4.1}$$

where $V_{\mathcal{I}}$ equals the discounted utility that a household with capital k and labor market status ϵ expects when they use the information set $\mathcal{I}$ in each period and the aggregate state of the economy is described by particular values of the elements in $\tilde{\mathcal{I}}$ (see also 2.5). Note that all terms in (4.1) also depend on the equilibrium law of motion for $\tilde{\mathcal{I}}$, suppressed for simplicity.

[17] In 2020, US per capita consumption was c. 42,500\$ (https://fred.stlouisfed.org/graph/?g=H0eE.).

Appendix B.2 furthermore show that neither of the intermediate information sets ($\mathcal{I} = \{z\}$ or $\mathcal{I} = \{K\}$) are rationalizable as symmetric equilibria. This is because deviation to the "other" intermediate information set incurs small but positive benefits (at no additional cost, because both choices have one signal). We conclude that a symmetric, pure strategy equilibrium information choice does not exist for moderate costs of information in the range between \$4 and \$1,566 per annum.

Finally, we have above considered welfare losses relative to the "comprehensive" Krusell and Smith (1998) information set $\mathcal{I}_{\max} = \{z, K\}$. This leaves the question of whether equilibria exists in which households acquire more information. However, Krusell and Smith (1998) show that, if all agents use $\mathcal{I}_{\max} = \{z, K\}$, then considering more information (in the form of additional moments of capital) has a "vanishingly small" (p. 878) effect on welfare. We confirmed this result in several exercises: The maximum welfare gains from increasing the information set to also contain the variance of individual capital, i.e. $\mathcal{I} = \{z, K, var(k)\}$, is less than one hundredth of a percent in consumption equivalent terms.[18] In other words, the relevant information choice in our standard neoclassical economy is about giving up information relative to the Krusell and Smith (1998) benchmark, not about adding more.

Overall, our results in this section cast doubt on the homogenous-information choice assumption $\widetilde{\mathcal{I}} = \mathcal{I}_{\max}$ used in much of modern macroeconomics to solve heterogenous-agents models with aggregate risk. Although such information choice allows for accurate predictions of future wages and rates of returns, the incentives for households to acquire such information conditional on others' behavior is small. In this sense, the classical dictum that "I am not worried if you are" seems appropriate for our workhorse neoclassical environment. Instead, our analysis suggests that if informational choice assumptions are to be invoked, to simplify household problems in heterogenous-agents models, one needs to consider either mixed-strategy equilibria, or state-dependent information choice rules. In ongoing work (Broer et al., 2021), we take the latter approach, and show how to compute an equilibria in the Krusell and Smith (1998) model that are consistent with households' incentive to acquire information. Importantly, such an equilibrium features different output and consumption dynamics, and substantially more wealth inequality. The introduction of state-dependent information choice further substantially modifies the economy's responses to shocks.

5. Conclusion

In this paper, we have shown that a standard behavioral assumption invoked to solve a broad class of heterogenous-agents models in macroeconomics appears inconsistent with rational information choice. We illustrated this result in the baseline framework of Krusell and Smith (1998), but our insights apply more broadly to settings in which agents' information choices interact with general-equilibrium dampening. While our findings may sound negative in nature, we see our contribution as opening up a fruitful and exciting avenue of research on household (and firm) information choice, and how it interacts with household (and firm) inequality. Indeed, recent novel empirical evidence suggests significant heterogeneity in expectations both at the household and firm level (e.g., Born et al., 2022; Coibion et al., 2018; D'Acunto et al., 2019; Vissing-Jorgensen, 2003, among others). Future research should consider more general, dynamic information-choice strategies that can simultaneously match the rich micro-heterogeneity in expectations and explore the subsequent macroeconomic implications. In Broer et al. (2021), we take an initial step in this direction. Finally, it would also be interesting to consider additional choices that information acquisition may improve, such as labour supply or portfolio choices, or consider the effects of more complex information cost functions (e.g., Hébert and La'O, 2020) for our results.

Appendix A. Analytical Results

Proof of Proposition 1. The proof proceeds in three steps: First, we show that households for which $m_0 \le \underline{m} \equiv \underline{w}(\beta\bar{r})^{-1} > 0$ have zero benefits of acquiring information. Second, we show that there exists another threshold $\bar{m} > \underline{m}$ such that households for which $m_0 > \bar{m}$ have a strictly positive benefit of acquiring information. Finally, we show that there exists cash-at-hand values $m_0 \in (\underline{m}, \bar{m})$ for which the benefit of acquiring information is strictly positive and increases in cash-at-hand m_0.

Step 1: Let $u(c) \equiv \log c$. Then, $m_0 \le \underline{m}$ implies that $u'(m_0) > \max_{r,w}\left[r_1 u'(w_1)\right]$. Hence, $u'(m_0) > \mathbb{E}[r_1 u'(w)]$. So the household would not choose a positive k_1 for any value of r_1 and w_1, and its choice would therefore be unchanged by information.

Step 2: Let k_1 and $k_1^\star$ denote the optimal savings choice without and with information, respectively. The expected utility differential of acquiring information is therefore

$$\mathbb{E}\Delta\mathcal{U} = \mathbb{E}[\log(m_0 - k_1) + \beta\log(r_1k_1 + w_1) - \log(m_0 - k_1^\star) - \beta\log(r_1k_1^\star + w_1)] \tag{A.1}$$

$$= \mathbb{E}\log\left(\frac{m_0 - k_1}{m_0 - k_1^\star}\right) + \beta\mathbb{E}\log\left(\frac{\alpha K_1^{\alpha-1}k_1 + (1-\alpha)K_1^\alpha}{\alpha K_1^{\alpha-1}k_1^\star + (1-\alpha)K_1^\alpha}\right)$$

$$= \mathbb{E}\log\left(\frac{m_0 - k_1}{m_0 - k_1^\star}\right) + \beta\mathbb{E}\log\left(\frac{\alpha K_1 k_1 + 1 - \alpha}{\alpha K_1 k_1^\star + 1 - \alpha}\right) < 0, \tag{A.2}$$

[18] We once more take the maximum over all individual and aggregate states.

where the last inequality follows from the concavity of the utility function, and the fact that $k_1^\star$ is the optimal choice under perfect foresight. The envelope theorem then shows that

$$\frac{\partial \mathbb{E}\Delta\mathcal{U}}{\partial m_0} = \mathbb{E}\left[\frac{1}{m_0 - k_1} - \frac{1}{m_0 - k_1^\star}\right] = \frac{1}{m_0 - k_1} - \mathbb{E}\left[\frac{1}{m_0 - k_1^\star}\right] > \frac{1}{m_0 - k_1} - \frac{1}{m_0 - \mathbb{E}k_1^\star} > 0,$$

where the last inequality follows from the Euler equation showing that $k_1 > \mathbb{E}k_1^\star$, and that $k_1 \leq m_0$.[19]

Step 3: Follows from the continuity of (A.2), and the results in *Step 1* and *2.* □

Proof of Proposition 2. There are two types of households: First, households that are *on their Euler equation*. They set k_1 in accordance with

$$\frac{1}{m_0 - k_1} = \beta\mathbb{E}\left[\frac{1}{k_1 + \frac{1-\alpha}{\alpha}K_1}\right],$$

which is independent of z_t. Second, households that are *off their Euler equation*. They set $k_1 = 0$, which is also independent of z_t. We conclude that households never have any incentive to acquire information about z_t, because their decisions are in all states of the world unaffected by realizations in z_t. The rest of the statement follows from information costs $\kappa > 0$ being strictly positive. □

Proof of Proposition 3. The proof is simplified by first defining some additional notation. Let $\mathbb{E}\mathcal{U}(a, a_{-1})$ denote the expected utility of a household, who either buys information ($a = b$) or not ($a = nb$), conditional on other households' information acquisition strategy. The condition for the non-existence of a pure strategy equilibria can then be stated as follows: there exists a $\kappa \in \mathbb{R}_+$ such that

$$\mathbb{E}\mathcal{U}(b, nb) - \mathbb{E}\mathcal{U}(nb, nb) > \kappa > \mathbb{E}\mathcal{U}(b, b) - \mathbb{E}\mathcal{U}(nb, b).$$

Let $\Delta(\cdot) \equiv \mathbb{E}\mathcal{U}(b, \cdot) - \mathbb{E}\mathcal{U}(nb, \cdot)$ denote the expected utility difference between buying information and not. A few simple derivations, using the household utility function, show that

$$\Delta(\cdot) = \mathbb{E}\left[\log\left(\frac{m - k_1^b}{m - k_1^{nb}}\right) + \log\left(\frac{\alpha k_1^b + (1-\alpha)K_1}{\alpha k_1^{nb} + (1-\alpha)K_1}\right)\right]. \tag{A.3}$$

Now, notice that *to a first order* it follows from a household's Euler equation:[20]

$$\frac{1}{m - k_1} = \frac{\alpha}{\alpha k_1 + (1-\alpha)\mathbb{E}[K_1]}. \tag{A.4}$$

Thus, using (A.4) with and without information choice (A.3) becomes

$$\Delta(\cdot) = \mathbb{E}\left[\log\left(\frac{\alpha k_1^b + (1-\alpha)K_1}{\alpha k_1^{nb} + (1-\alpha)\mathbb{E}[K_1]}\right)\left(\frac{\alpha k_1^b + (1-\alpha)K_1}{\alpha k_1^{nb} + (1-\alpha)K_1}\right)\right].$$

We can now once more use (A.4) to show that

$$k_{1i}^b = \frac{1}{2}m - \frac{1-\alpha}{\alpha}\frac{1}{2}K_1 \tag{A.5}$$

$$k_{1i}^{nb} = \frac{1}{2}m - \frac{1-\alpha}{\alpha}\frac{1}{2}\mathbb{E}_i[K_1]. \tag{A.6}$$

It follows from these two equations that

$$\Delta(\cdot) = \mathbb{E}\left[\log\left(\frac{\alpha m + (1-\alpha)K_1}{\alpha m + (1-\alpha)\mathbb{E}K_1}\right)\left(\frac{\alpha m + (1-\alpha)K_1}{\alpha m + (1-\alpha)\mathbb{E}K_1 + \frac{\beta(1-\alpha)}{1+\beta}(K_1 - \mathbb{E}K_1)}\right)\right]$$

$$= 2\mathbb{E}[\log(\alpha m + (1-\alpha)K_1)] - \mathbb{E}[\log(\alpha m + (1-\alpha)\mathbb{E}K_1)] \tag{A.7}$$

$$-\mathbb{E}\log\left(\alpha m + (1-\alpha)\mathbb{E}K_1 + \frac{1-\alpha}{2}(K_1 - \mathbb{E}K_1)\right).$$

Now, in a symmetric equilibrium, $K_1 = \int_0^1 k_{1i}di$ equals, using (A.6):

$$K_1 = \frac{1}{2}\sum_{j=0}^{\infty}\left(\frac{\alpha - 1}{2\alpha}\right)^j \bar{\mathbb{E}}^j[m], \tag{A.8}$$

[19] Notice that Jensen's inequality is here strict because w_1 and r_1 are defined over a bounded support.

[20] To a first order here implies that $\mathbb{E}[f(X)] = f(\mathbb{E}X)$ for some random variable X and some continuous function $f : \mathbb{R} \rightarrow \mathbb{R}$.

where $\bar{\mathbb{E}}[\cdot] \equiv \int_0^1 \mathbb{E}_i[\cdot]di$ and $\bar{\mathbb{E}}^j[\cdot] \equiv \int_0^1 \mathbb{E}_i\left[\bar{\mathbb{E}}^{j-1}[\cdot]\right]di$ with $\bar{\mathbb{E}}^0[m] = m$.

Solving (A.8) shows that

$$K_1 = \frac{\alpha}{1+\alpha}\mu_m + \frac{\alpha}{2\alpha + (1-\alpha)w}u, \tag{A.9}$$

where $w = 1$ if all households acquire information, or $w \in (0, 1)$ if all households do not.

Inserting (A.9) into (A.7) we thus arrive at:

$$\Delta(b) = \mathbb{E}\{2\log(2\alpha u + \alpha(1-\alpha)\mu_m) - \log[(1+\alpha+(1-\alpha)w)\alpha u + \alpha(1-\alpha)\mu_m] \tag{A.10}$$
$$-\log\left[(1+\alpha+(1-\alpha)w)\alpha u + \frac{(1-\alpha)\alpha(1-w)u}{2} + \alpha(1-\alpha)\mu_m\right]\Bigg\},$$

while

$$\Delta(nb) = \mathbb{E}\left\{2\log\left(\left[\alpha^2 + \alpha + \alpha(1-\alpha)w\right]u + \alpha\frac{(1-\alpha)(2\alpha+(1-\alpha)w)}{1+\alpha}\mu_m\right)\right. \tag{A.11}$$
$$-\log\left[(2\alpha(1-w)+2w)\alpha u + \alpha\frac{(1-\alpha)(2\alpha+(1-\alpha)w)}{1+\alpha}\mu_m\right]$$
$$\left.-\log\left[(2\alpha(1-w)+2w)\alpha u + \frac{(1-\alpha)\alpha(1-w)u}{2} + \alpha\frac{(1-\alpha)(2\alpha+(1-\alpha)w)}{1+\alpha}\mu_m\right]\right\},$$

Because $w \in (0, 1)$ and $\alpha < 1$, the formula for the mean of log-normal random variable, then shows that

$$\Delta(nb) > \Delta(b).$$

This completes the proof. □

Proof of Proposition 4. The proof has two steps. The first step shows that the value of acquiring information is increasing in the relative mass of informed middle-wealth households. The second step then shows that this implies that the range of fixed cost parameters for which no pure strategy equilibrium exists is increasing in the relative mass of informed middle-wealth households.

Step 1: Eq. (A.7) shows that $\Delta_M(\cdot)$ equals

$$\Delta_M(\cdot) = 2\mathbb{E}[\log(\alpha m_M + (1-\alpha)K_1)] - \mathbb{E}[\log(\alpha m_M + (1-\alpha)\mathbb{E}K_1)]$$
$$-\mathbb{E}\log\left(\alpha m_M + (1-\alpha)\mathbb{E}K_1 + \frac{1-\alpha}{2}(K_1 - \mathbb{E}K_1)\right),$$

where

$$K_1 = \pi_L \times 0 + 1 \times K_1^M + \pi_H \times K_1^H. \tag{A.12}$$

In a symmetric equilibrium, where all middle-wealth households make the same information choice, we therefore have that

$$K_1^M = \frac{1}{2}m_M - \frac{(1-\alpha)\pi_H}{2\alpha}K_1^H - \frac{(1-\alpha)}{2\alpha}\bar{\mathbb{E}}\left[K_1^M\right]$$
$$= \frac{\alpha}{2\alpha+(1-\alpha)w}u_M - \frac{1}{2\alpha+(1-\alpha)}\left[(1-\alpha)\pi_H K_1^H - \alpha\mu_m\right], \tag{A.13}$$

where $w = 1$ if all middle-wealth households acquire information, and $w \in (0, 1)$ if all middle-wealth households do not. Eq. (A.13) has been derived using (A.6) and (A.12). Thus,

$$K_1 = \frac{\alpha}{2\alpha+(1-\alpha)w}u_M + \frac{2\alpha\pi_H K_1^H + \alpha\mu_m}{2\alpha+(1-\alpha)}.$$

It now follows that

$$\Delta_M(b) = 2\mathbb{E}\left[\log\left(\alpha u_M + \frac{\alpha(1-\alpha)}{2\alpha+(1-\alpha)}u_M + \frac{(1-\alpha)\left(2\alpha\pi_H K_1^H + \alpha\mu_m\right)}{2\alpha+(1-\alpha)}\right)\right]$$
$$-\mathbb{E}\left[\log\left(\alpha u_M + \frac{\alpha(1-\alpha)w}{2\alpha+(1-\alpha)}u_M + \frac{(1-\alpha)\left(2\alpha\pi_H K_1^H + \alpha\mu_m\right)}{2\alpha+(1-\alpha)}\right)\right]$$
$$-\mathbb{E}\left[\log\left(\alpha u_M + \frac{\alpha(1-\alpha)w}{2\alpha+(1-\alpha)}u_M - \frac{(1-\alpha)\left(2\alpha\pi_H K_1^H + \alpha\mu_m\right)}{2\alpha+(1-\alpha)} + \frac{1-\alpha}{2}\frac{\alpha(1-w)}{2\alpha+(1-\alpha)}u_M\right)\right]$$

while

$$\begin{aligned}\Delta_M(nb) = 2\mathbb{E}\left[\log\left(\alpha u_M + \frac{\alpha(1-\alpha)}{2\alpha+(1-\alpha)w}u_M + \frac{(1-\alpha)\left(2\alpha\pi_H K_1^H + \alpha\mu_m\right)}{2\alpha+(1-\alpha)}\right)\right]\\ -\mathbb{E}\left[\log\left(\alpha u_M + \frac{\alpha(1-\alpha)w}{2\alpha+(1-\alpha)w}u_M + \frac{(1-\alpha)\left(2\alpha\pi_H K_1^H + \alpha\mu_m\right)}{2\alpha+(1-\alpha)}\right)\right]\\ -\mathbb{E}\left[\log\left(\alpha u_M + \frac{\alpha(1-\alpha)w}{2\alpha+(1-\alpha)w}u_M + \frac{(1-\alpha)\left(2\alpha\pi_H K_1^H + \alpha\mu_m\right)}{2\alpha+(1-\alpha)} + \frac{1-\alpha}{2}\frac{\alpha(1-w)}{2\alpha+(1-\alpha)w}u_M\right)\right].\end{aligned}$$

We note that $\Delta_M(nb) > \Delta_M(b) > 0$.

Let $x_0 \equiv \alpha$, $x_1 \equiv \alpha(1-\alpha)$, $x_2 \equiv \alpha(1-\alpha)w$, $x_3 \equiv x_0 + \frac{1-\alpha}{2}\frac{\alpha(1-w)}{2\alpha+(1-\alpha)}$, and $x_4 \equiv x_0 + \frac{1-\alpha}{2}\frac{\alpha(1-w)}{2\alpha+(1-\alpha)w}$. Further, let $q_0 \equiv 2\alpha + (1-\alpha)$, $q_1 \equiv 2\alpha + (1-\alpha)w$, and $d \equiv \frac{(1-\alpha)\left(2\alpha\pi_H K_1^H+\alpha\mu_m\right)}{2\alpha+(1-\alpha)}$. Then,

$$\begin{aligned}\frac{\partial[\Delta_M(nb)-\Delta_M(b)]}{\partial d} &= \mathbb{E}\left[\frac{2}{(x_0+\frac{x_1}{q_1})u+d} - \frac{2}{(x_0+\frac{x_1}{q_0})u+d}\right]\\ &-\mathbb{E}\left[\frac{1}{(x_0+\frac{x_2}{q_1})u+d} - \frac{1}{(x_0+\frac{x_2}{q_0})u+d}\right] - \mathbb{E}\left[\frac{1}{(x_4+\frac{x_2}{q_1})u+d} - \frac{1}{(x_3+\frac{x_2}{q_0})u+d}\right].\\ &= 2x_1\left(\frac{1}{q_0}-\frac{1}{q_1}\right)\mathbb{E}\left[\frac{u}{\left[(x_0+\frac{x_1}{q_1})u+d\right]\left[(x_0+\frac{x_1}{q_0})u+d\right]}\right]\\ &-x_2\left(\frac{1}{q_0}-\frac{1}{q_1}\right)\mathbb{E}\left[\frac{u}{\left[(x_0+\frac{x_2}{q_1})u+d\right]\left[(x_0+\frac{x_2}{q_0})u+d\right]}\right]\\ &-\mathbb{E}\left[\frac{\left(x_4-x_3+x_2\left(\frac{1}{q_0}-\frac{1}{q_1}\right)\right)u}{\left[(x_4+\frac{x_2}{q_1})u+d\right]\left[(x_3+\frac{x_2}{q_0})u+d\right]}\right].\end{aligned}$$

Now, notice that

$$\begin{aligned}\frac{\partial[\Delta_M(nb)-\Delta_M(b)]}{\partial d} &< (2x_1-x_2)\left(\frac{1}{q_0}-\frac{1}{q_1}\right)\mathbb{E}\left[\frac{u}{\left[(x_0+\frac{x_1}{q_1})u+d\right]\left[(x_0+\frac{x_1}{q_0})u+d\right]}\right]\\ &-\mathbb{E}\left[\frac{\left(x_4-x_3+x_2\left(\frac{1}{q_0}-\frac{1}{q_1}\right)\right)u}{\left[(x_0+\frac{x_1}{q_1})u+d\right]\left[(x_0+\frac{x_1}{q_0})u+d\right]}\right].\\ &= \left[x_4-x_3+(2x_1-2x_2)\left(\frac{1}{q_0}-\frac{1}{q_1}\right)\right]\mathbb{E}\left[\frac{u}{\left[(x_0+\frac{x_1}{q_1})u+d\right]\left[(x_0+\frac{x_1}{q_0})u+d\right]}\right]\\ &= -\frac{3a(1-a)^2(1-w)^2}{2(1+a)(2a+(1-a)w}\mathbb{E}\left[\frac{u}{\left[(x_0+\frac{x_1}{q_1})u+d\right]\left[(x_0+\frac{x_1}{q_0})u+d\right]}\right] < 0.\end{aligned}$$

We conclude that

$$\frac{\partial[\Delta_M(nb)-\Delta_M(b)]}{\partial \pi_H} = \frac{\partial[\Delta_M(nb)-\Delta_M(b)]}{\partial d} \times \frac{2(1-\alpha)\alpha}{2\alpha+(1-\alpha)}K_1^H < 0. \tag{A.14}$$

Step 2: The range of κ consistent with no-pure strategy equilibria is given by:

$$\kappa \in (\Delta_M(b), \Delta_M(nb)),$$

The rest of the statement then follows directly from (A.14). □

Appendix B. Quantitative Results

B1. Simple two-period model

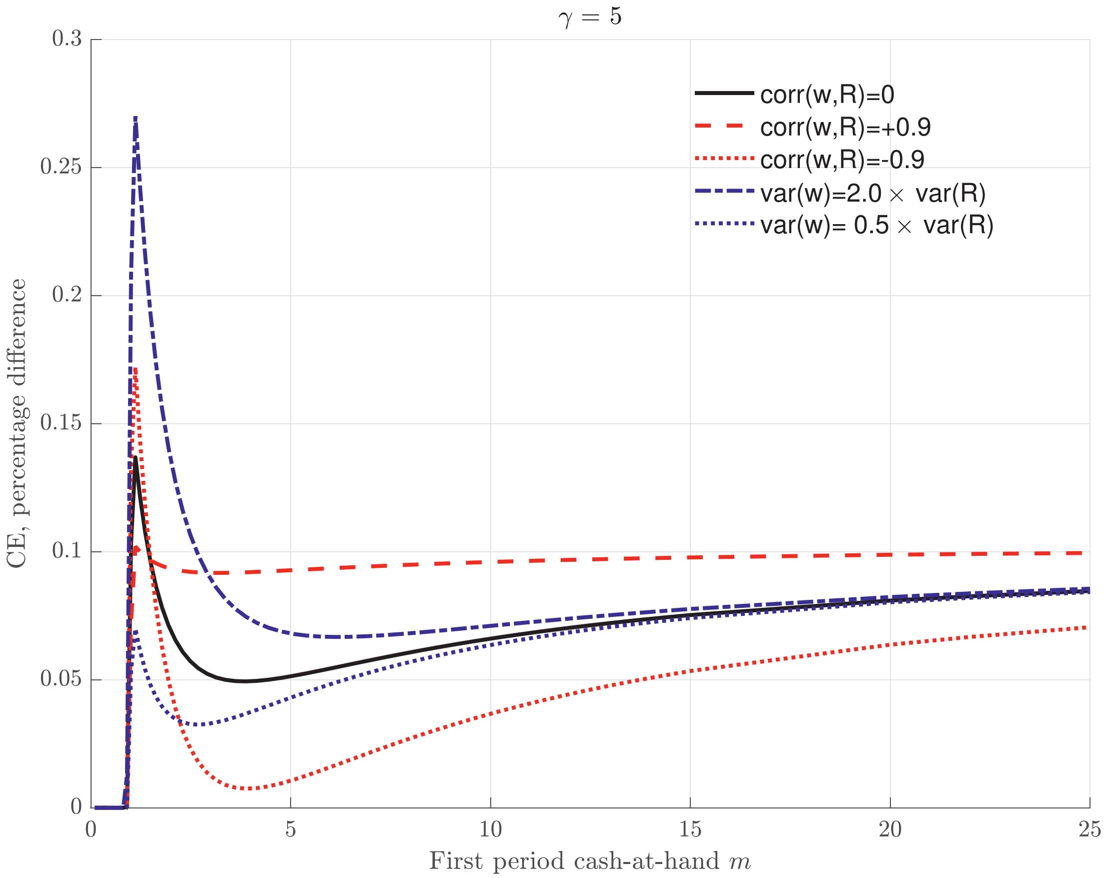

Fig. A.1. Utility losses in the two-period model ($\gamma > 1$). The figure depicts the expected certainty-equivalent (CE) utility loss from not acquiring information in the simple two-period model with $\beta = 0.99$ and a joint normal distribution for w_1 and r_1 with means of 1 percent and standard deviations of 5 percent, respectively. The figure assumes the felicity function $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$,where $\gamma > 1$. The figure uses $\gamma = 5$. We note that Fig. 1 in the paper corresponds to the case in which $\gamma \to 1$.

B2. Quantitative model

Table A.1
Benchmark parameters.

	β	γ	α	δ	$\bar{l}$	μ	Z_l	Z_h
Values	0.99	5.00	0.36	0.025	1/0.90	0.40	0.98	1.01

Table A.2
Transition probabilities.

	$0\|Z_l$	$1\|Z_l$	$0\|Z_h$	$1\|Z_h$
$0\|Z_l$	0.45	0.37	0.10	0.08
$1\|Z_l$	0.04	0.78	0.00	0.18
$0\|Z_h$	0.05	0.06	0.40	0.49
$1\|Z_h$	0.01	0.11	0.03	0.85

Table A.3
Expected Utility Changes: KS vs No-Information.

	$\widetilde{\mathcal{I}} = \{z, \bar{k}_t\}$		$\widetilde{\mathcal{I}} = \emptyset$	
	Average Loss	Minimum Loss	Average Gain	Maximum Gain
$\mathcal{I} = \emptyset$	−0.0022	-9.60e-05	·	·
$\mathcal{I} = \{z\}$	−0.0020	-9.41e-05	0.0013	0.0135
$\mathcal{I} = \{\bar{k}_t\}$	0.0011	-5.29e-05	0.0178	0.1161
$\mathcal{I} = \{z, \bar{k}_t\}$	·	·	0.0184	0.1169

The table presents raw expected losses or gains from using different individual information sets $\mathcal{I}_t$ (indicated in the first column) rather than the information set used by all other households $\widetilde{\mathcal{I}}_t$ (indicated in the top row). Expectations are taken across the 2.5–97.5 percentile range of the ergodic distribution of aggregate and individual states.

Table A.4
Average expected utility changes: interim cases.

	$\widetilde{\mathcal{I}} = \{z, \bar{k}_t\}$	$\widetilde{\mathcal{I}} = \{\bar{k}_t\}$	$\widetilde{\mathcal{I}} = \{z\}$	$\widetilde{\mathcal{I}} = \emptyset$
$\mathcal{I} = \emptyset$	−0.0022	−0.0001	−0.0017	·
$\mathcal{I} = \{z\}$	−0.0020	0.0001	·	0.0013
$\mathcal{I} = \{\bar{k}_t\}$	−0.0011	·	0.0181	0.0178
$\mathcal{I} = \{z, \bar{k}_t\}$	·	0.0011	0.0190	0.0184

The table presents raw expected losses or gains from using different individual information sets $\mathcal{I}_t$ (indicated in the first column) rather than the information set used by all other households $\widetilde{\mathcal{I}}_t$ (indicated in the top row). Expectations are taken across the 2.5–97.5 percentile range of the ergodic distribution of aggregate and individual states.

Table A.5
Average expected utility changes in percent CE (Interim Cases).

	$\widetilde{\mathcal{I}} = \{z, \bar{k}_t\}$	$\widetilde{\mathcal{I}} = \{\bar{k}_t\}$	$\widetilde{\mathcal{I}} = \{z\}$	$\widetilde{\mathcal{I}} = \emptyset$
$\mathcal{I} = \emptyset$	−0.1297	0.0095	−0.0748	·
$\mathcal{I} = \{z\}$	−.1184	0.0160	·	0.0619
$\mathcal{I} = \{\bar{k}_t\}$	−0.0705	·	0.9098	0.9019
$\mathcal{I} = \{z, \bar{k}_t\}$	·	0.0670	0.9712	0.9415

The table resents expected certainty-equivalence losses or gains from using different individual information sets $\mathcal{I}_t$ (indicated in the first column) rather than the information set used by all other households $\widetilde{\mathcal{I}}_t$ (indicated in the top row). Expectations are taken across the 2.5–97.5 percentile range of the ergodic distribution of aggregate and individual states.

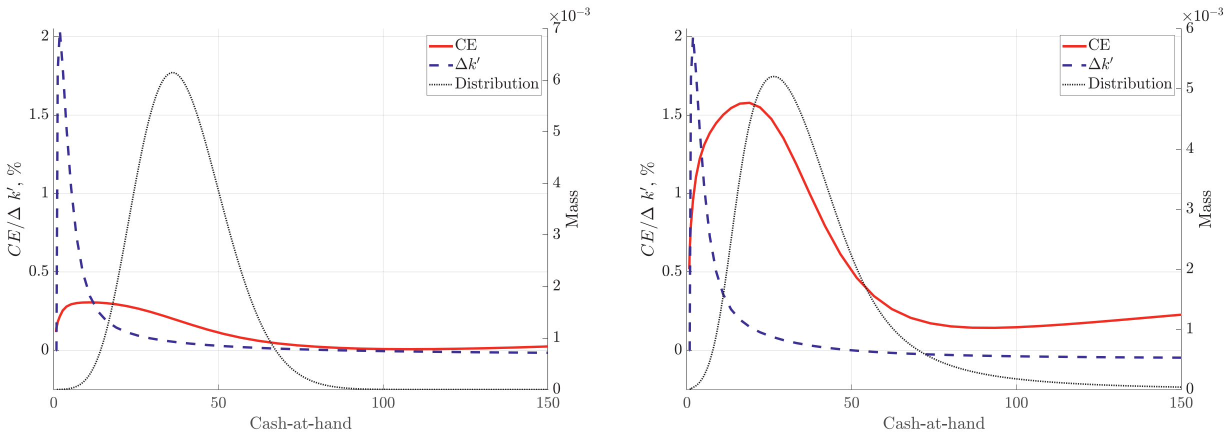

Fig. A.2. The figure depicts consumption-equivalence differences $CE_{\mathcal{I}}(k, \epsilon; \widetilde{\mathcal{I}})$ of not acquiring information (solid lines), differences in policy functions k' (dashed lines), and the average cross-sectional distribution of households (dotted lines). It does for a *high-information economy* ($\widetilde{\mathcal{I}} = \{z, K\}$, left-hand side panel) and a *low-information economy* ($\widetilde{\mathcal{I}} = \{\emptyset\}$, right-hand side panel). The information choice considered is $\mathcal{I} = \{z, K\} = \mathcal{I}_{\max}$ versus $\mathcal{I} = \{\emptyset\}$. The figure focuses on an unemployed household ($\epsilon = 0$) with median capital in the low-productivity state $z = z_L$.

References

Ahn, S., Kaplan, G., Moll, B., Winberry, T., Wolf, C., 2018. When inequality matters for macro and macro matters for inequality. NBER Macroecon Annu 32 (1), 1–75. doi:10.1086/696046.
Angeletos, G.-M., Pavan, A., 2007. Efficient use of information and social value of information. Econometrica 75 (4), 1103–1142.
Boppart, T., Krusell, P., Mitman, K., 2018. Exploiting mit shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. Journal of Economic Dynamics and Control 89, 68–92. doi:10.1016/j.jedc.2018.01.002. Fed St. Louis-JEDC-SCG-SNB-UniBern Conference, titled: “Fiscal and Monetary Policies”. https://www.sciencedirect.com/science/article/pii/S0165188918300022
Born, B., Enders, Z., Müller, G.J., Niemann, K., 2022. Firm expectations about production and prices: facts, determinants, and effects. Handbook of Economic Expectations.
Broer, T., Kohlhas, A., Mitman, K., Schlafmann, K., 2021. Information and wealth heterogeneity in the macroeconomy. IIES mimeo..
Coibion, O., Gorodnichenko, Y., Kumar, S., 2018. How do firms form their expectations? new survey evidence. American Economic Review 108 (9), 2671–2713. doi:10.1257/aer.20151299.
Colombo, L., Femminis, G., Pavan, A., 2014. Information acquisition and welfare. Rev Econ Stud 81 (4), 1438–1483.
D’Acunto, F., Hoang, D., Paloviita, M., Weber, M., 2019. IQ, Expectations, and Choice. Technical Report doi:10.3386/w25496.
Den Haan, W.J., Judd, K.L., Juillard, M., 2010. Computational suite of models with heterogeneous agents: incomplete markets and aggregate uncertainty. Journal of Economic Dynamics and Control 34 (1), 1–3. doi:10.1016/j.jedc.2009.07.001. Computational Suite of Models with Heterogeneous Agents: Incomplete Markets and Aggregate Uncertainty http://www.sciencedirect.com/science/article/pii/S0165188909001286.
Fernández-Villaverde, J., Hurtado, S., Nuño, G., 2019. Financial Frictions and the Wealth Distribution. Technical Report doi:10.3386/w26302.
Gabaix, X., 2019. Behavioral inattention. In: Handbook of Behavioral Economics: Applications and Foundations 1, Vol. 2. Elsevier, pp. 261–343.
Grossman, S.J., Stiglitz, J.E., 1980. On the impossibility of informationally efficient markets. Am Econ Rev 70 (3), 393–408.
Hébert, B.M., La’O, J., 2020. Information Acquisition, Efficiency, and Non-fundamental Volatility.
Hellwig, C., Veldkamp, L., 2009. Knowing what others know: coordination motives in information acquisition. Rev Econ Stud 76 (1), 223–251.
Hellwig, M.F., 1980. On the aggregation of information in competitive markets. J Econ Theory 22 (3), 477–498.
Krueger, D., Mitman, K., Perri, F., 2016. Macroeconomics and household heterogeneity. In: Handbook of Macroeconomics, Vol. 2. Elsevier, pp. 843–921.
Krusell, P., Smith, A., 1998. Income and wealth heterogeneity in the macroeconomy. Journal of Political Economy 106 (5).
Mondria, J., Vives, X., Yang, L., 2021. Costly interpretation of asset prices. Manage Sci.
Sims, C.A., 2003. Implications of rational inattention. J Monet Econ 50 (3), 665–690.
Townsend, R.M., 1983. Forecasting the forecasts of others. Journal of Political Economy 91 (4), 546–588.
Veldkamp, L.L., 2011. Information Choice in Macroeconomics and Finance. Princeton University Press.
Vissing-Jorgensen, A., 2003. Perspectives on behavioral finance: does ”irrationality” disappear with wealth? evidence from expectations and actions. NBER Macroecon Annu 18, 139–194. doi:10.1086/ma.18.3585252.
Yellen, J., 2016. Macroeconomic research after the crisis. Board of Governors of the Federal Reserve System.