Lecture 1: Labour economics

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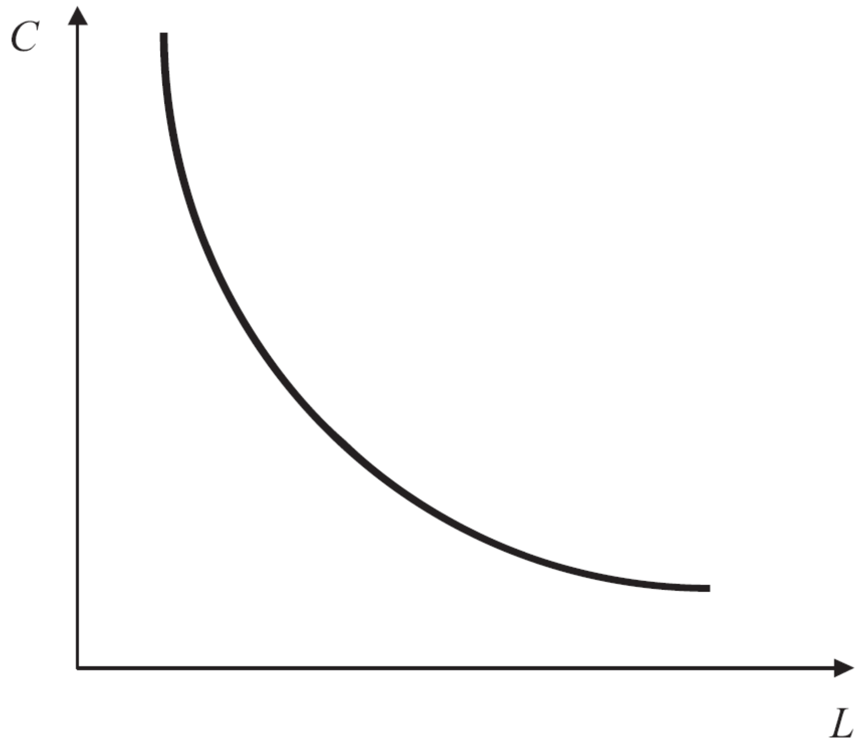
The choice between consumption and leisure

$U = U(C,L)$
C = consumption of goods
L = consumption of leisure
L_0 = total amount of time
$h = L_0 - L$ = working time

$U(C,L) = \overline{U}$ defines an indifference curve

Figure 1.1



$U(C,L) = U(C,L) = \overline{U}$ defines a function $C(L)$, which satisfies

$U[C(L),L] = \overline{U}$

Differentiation w.r.t *L* gives:

$$U_C C' + U_L = 0$$

$$C'(L) = -\frac{U_L(C,L)}{U_C(C,L)}$$

$$\left|C'(L)\right| = \frac{U_L(C,L)}{U_C(C,L)} = MRS_{C,L}$$

Indifference curves are negatively sloped.

Indifference curves are convex (absolute value of slope falling with *L*) if $C''(L) > 0$.

$C''(L)$ is obtained by differentiating $C'(L) = -U_L(C,L)/U_C(C,L)$ w.r.t *L* and substituting $-U_L/U_C$ for C' after differentiation.

We get:

$$C''(L) = \frac{U_L \left[2U_{CL} - U_{LL} \frac{U_C}{U_L} - U_{CC} \frac{U_L}{U_C} \right]}{(U_C)^2}$$

$C''(L) > 0$ if $2U_{CL} - U_{LL} \frac{U_C}{U_L} - U_{CC} \frac{U_L}{U_C} > 0$

This is certainly the case if $U_{CL} = 0$ since $U_{LL} < 0$ and $U_{CC} < 0$.

<u>The choice problem of the individual</u>

w = real hourly wage
wh = real wage income
R = other income

The individual's budget constraint: $C \leq wh + R$

Alternative formulation of budget constraint:
$C \leq w(L_0 - L) + R$
$C + wL \leq wL_0 + R \equiv R_0$

Interpretation:

- **The individual disposes of a potential income R_0 obtained by devoting all of his time to working and using other resources R. Leisure or consumer goods can be bought with this income.**
- **The wage is the <u>price</u> as well as the <u>opportunity cost</u> of leisure.**

The decision problem of the individual:

$$\max_{\{C,L\}} \; U(C,L) \quad \text{s.t.} \quad C + wL \leq R_0$$

Interior solution, such that $0 < L < L_0$ and $C > 0$.

$\mu > 0$ is the Lagrange multiplier.

The Lagrangian is:

$$£(C,L,\mu) = U(C,L) + \mu(R_0 - C - wL)$$

The FOCs are:

$$U_c(C,L) - \mu = 0$$

$$U_L(C,L) - \mu w = 0$$

The complementary slackness condition:

$$\mu(R_0 - C - wL) = 0 \text{ with } \mu \geq 0$$

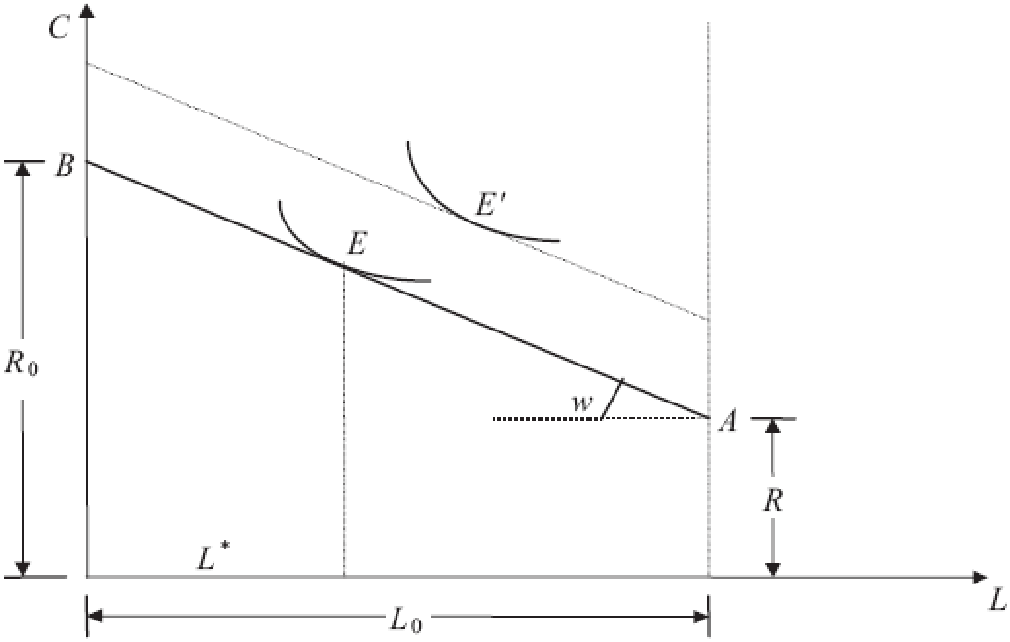
Since $\mu = U_c(C,L) > 0$ with an interior solution, it follows that the budget constraint is then binding, i.e. $C + wL = R_0$

The optimal solution is then:

$$\frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w^*$$

$$C^* + wL^* = R_0$$

Figure 1.2



Equation of budget line:

$$C + wL = R + wL_0 = R_0$$

$$C = R + w(L_0 - L)$$

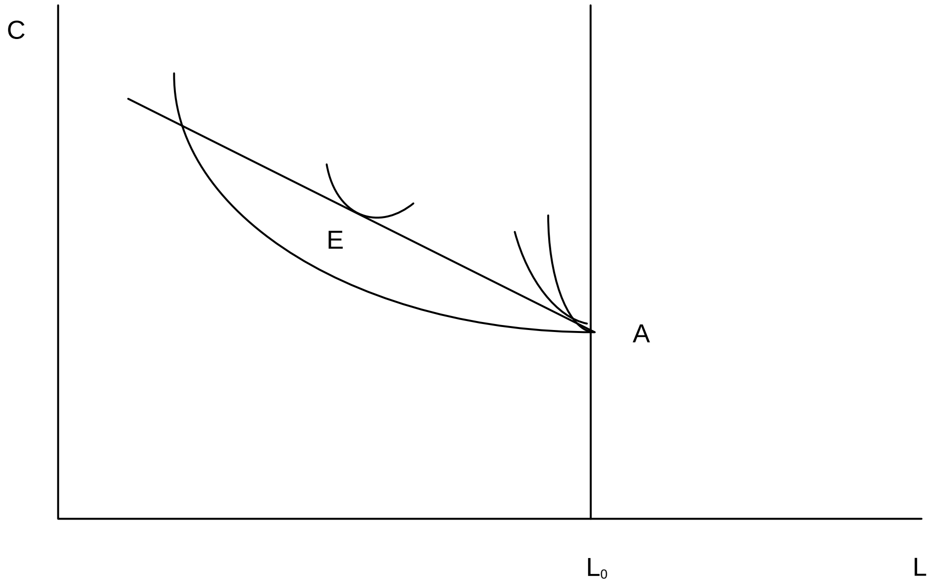
$$L = L_0 \Rightarrow C = R$$

$$L = 0 \Rightarrow C = R + wL_0 = R_0$$

- **Change in *w* rotates budget line around *A***
- **Change in *R* gives rise to a parallel shift of the budget line**

The reservation wage

- ***E* must lie to the left of *A* for there to be a positive labour supply ($L<L_0$)**



1. **Tangency point at *A*: $L = L_0$ and $h = L_0 - L = 0$ is interior solution**

2. **Indifference curve is more sloped than budget line at *A*: $L = L_0$ and $h = L_0 - L = 0$ is a corner solution**

3. **Indifference curve is less sloped than budget line at *A*: $L < L_0$ and $h = L_0 - L > 0$ is an interior solution**

***MRS* at point *A* is called the reservation wage, w_A**

$$w_A = \frac{U_L(R, L_0)}{U_C(R, L_0)}$$

- **An individual participates in the labour force only if $w > w_A$.**
- **The reservation wage depends on non-wage income.**
- **If leisure is a normal good (i.e. increases with income), then a higher non-wage income creates a disincentive for work.**

Properties of labour supply

$$\frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w \quad \textbf{and} \quad C^* + wL^* = R_0 \qquad \textbf{(2)}$$

Equation (2) implicitly defines labour supply.

$L^* = \wedge(w, R_0)$

$h^* = L_0 - L^*$ **is the Marshallian or uncompensated labour supply.**

The impact of R_0 on leisure:

From (2) we have:

$$wU_C\left(R_0 - wL^*, L^*\right) - U_L\left(R_0 - wL^*, L^*\right) = 0$$

Differentiate w.r.t L^*, w **and** R_0 **and use:**

$w = U_L / U_c$ **after the differentiation to get rid of** w**.**

We then obtain:

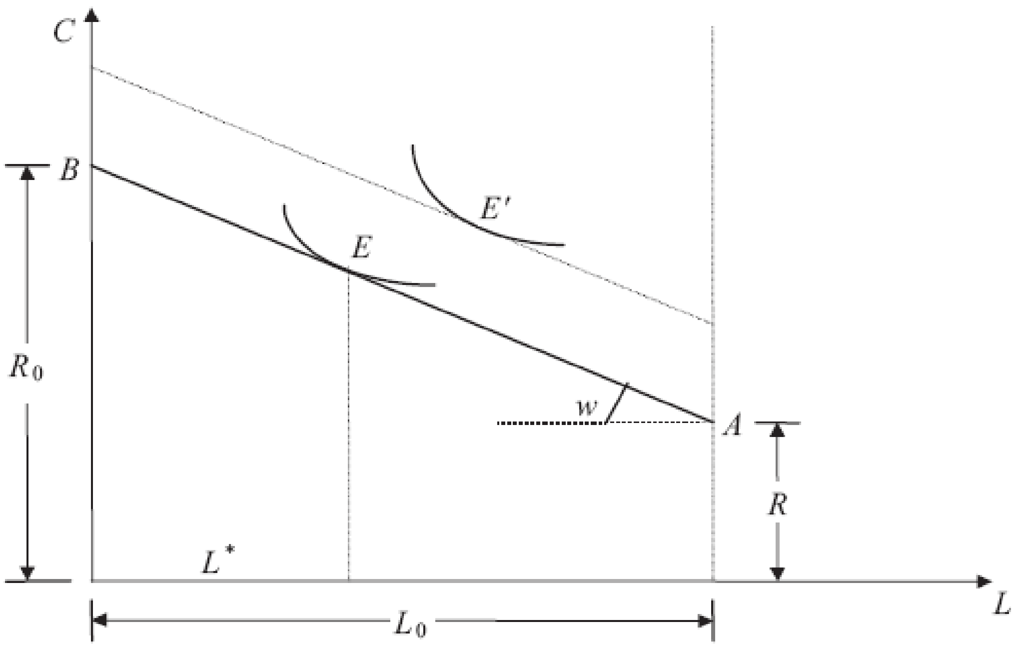
$$\Lambda_1 = \frac{\partial L^*}{\partial w} = \frac{-L\left(\frac{U_{CL}U_C - U_{CC}U_L}{U_L}\right) - U_C\left(\frac{U_C}{U_L}\right)}{\left[2U_{CL} - U_{LL}\left(\frac{U_C}{U_L}\right) - U_{CC}\frac{U_C}{U_L}\right]}$$

$$\Lambda_2 = \frac{\partial L^*}{\partial R_0} = \frac{\frac{U_{CL}U_C - U_{CC}U_L}{U_L}}{\left[2U_{CL} - U_{LL}\left(\frac{U_C}{U_L}\right) - U_{CC}\left(\frac{U_L}{U_C}\right)\right]}$$

- **From quasi-concavity (convex indifference curves) we have that the denominators of $\wedge_1$ and $\wedge_2$ are positive.**
- **Hence signs of $\wedge_1$ and $\wedge_2$ are determined by the numerators.**
- **$\wedge_2 > 0$ if $U_{CL}U_C - U_{CC}U_L > 0$. This is the condition for leisure to be a <u>normal good</u>, i.e. for leisure to increase if income increases.**
- **$\wedge_1 < 0$, i.e. leisure falls and labour supply increases if the wage increases, unambiguously only if leisure is a normal good.**
- **There is both an (indirect) income effect and a <u>substitution</u> effect. Both are negative if leisure is a normal good.**

The effect of an increase in non-wage income *R*:

Figure 1.2

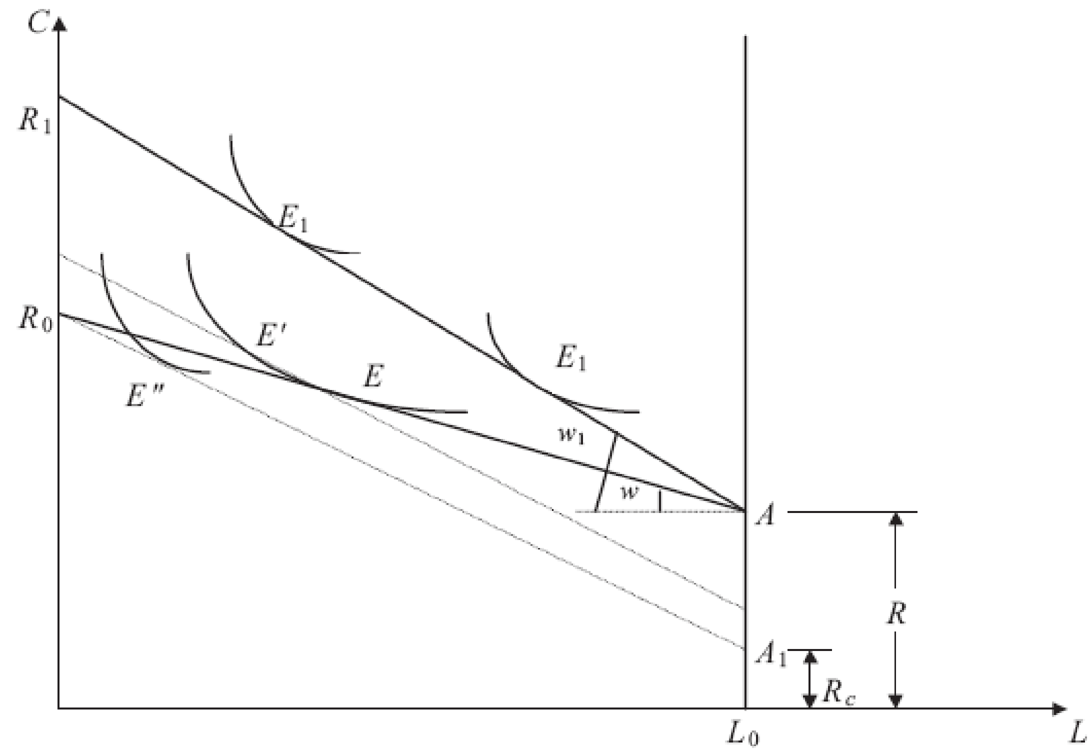


The effect of a wage increase

$$L* \;=\; \wedge(w, R_0) \qquad R_0 \;=\; wL_0 \;+\; R$$

$$\frac{dL^*}{dw} \;=\; \wedge_1 \;+\; \wedge_2 \frac{\partial R_0}{\partial w} \;=\; \overset{(-)}{\wedge_1} \;+\; \overset{(+)}{\wedge_2}\, L_0$$

Figure 1.3



- ***w*** **increases from *w* to w_1**

Keep R_0 unchanged. New budget line A_1R_0. As if decline from R to $R_c = R - (w_1 - w)L_0$.

R_c = compensated income. A_1R_0 is the compensated budget constraint.

1. **$E \rightarrow E'$ is substitution effect reducing leisure. (Outlays of the consumer are minimised under the constraint of reaching a given level of utility.)**

2. **$E' \rightarrow E''$ is (indirect) income effect reducing leisure farther if leisure is normal good.**

3. **$E'' \rightarrow E_1$ is (direct) income effect increasing leisure if leisure is a normal good. It represents the increase in potential income from the wage increase.**

<u>Conclusion</u>: Net effect of a wage increase on leisure/hours worked is ambiguous.

<u>Simpler analysis</u>:

1. **$E \rightarrow E^1$ is substitution effect**

2. **$E' \rightarrow E_1$ is global income effect (the indirect and direct income effects are aggregated)**

Compensated and uncompensated elasticity of labour supply

$L^* = \wedge(w, R_0)$ is the Marshallian (uncompensated) labour supply.

The Hicksian (compensated) labour supply is obtained as the solution to the problem:

$$\min_{L,C} \; C + wL \quad \text{s.t.} \quad U(C,L) \geq \bar{U}$$

One then obtains $\hat{L} = \hat{L}(w, \bar{U})$

The Slutsky equation:

$$\eta_w^{h^*} = \eta_w^{\hat{h}} + \frac{wh^*}{R_0}\eta_{R_0}^{h^*}$$

$\eta_w^{h^*}$ = the uncompensated labour supply elasticity w.r.t the wage

$\eta_w^{\hat{h}}$ = the compensated labour supply elasticity w.r.t the wage

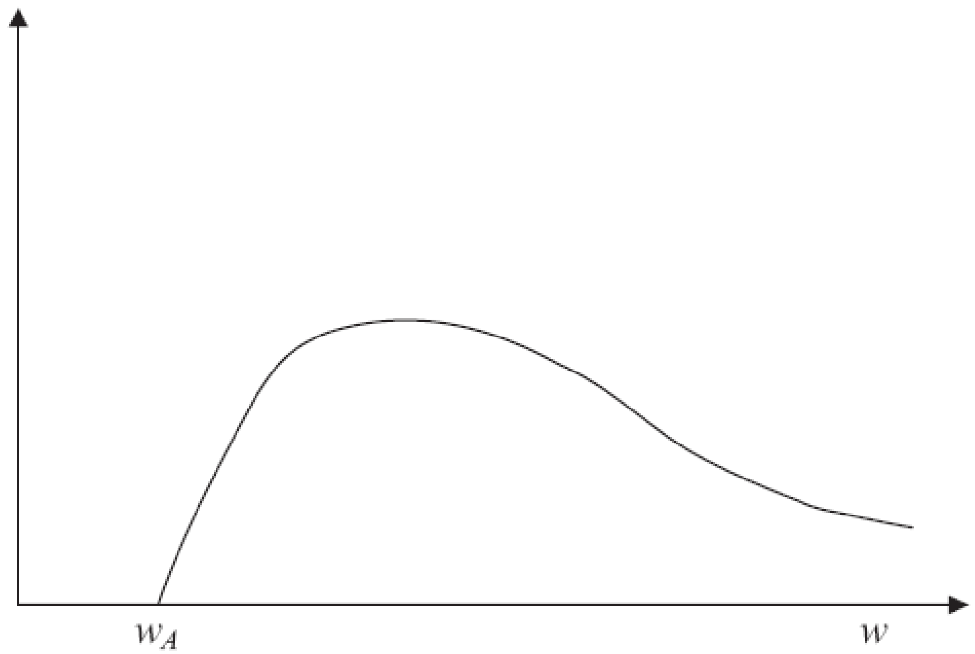
$\eta_{R_0}^{h^*}$ = the income elasticity of labour supply

$$R_0 = wL_0 + R$$

- **With constant elasticities,** $\frac{wh^*}{R_0}\eta_{R_0}^{h^*}$ **may increase relative to the substitution elasticity when the wage increases.**
- **The income effect may finally overtake the substitution effect.**

Figure 1.4

$L_0 - L = h$

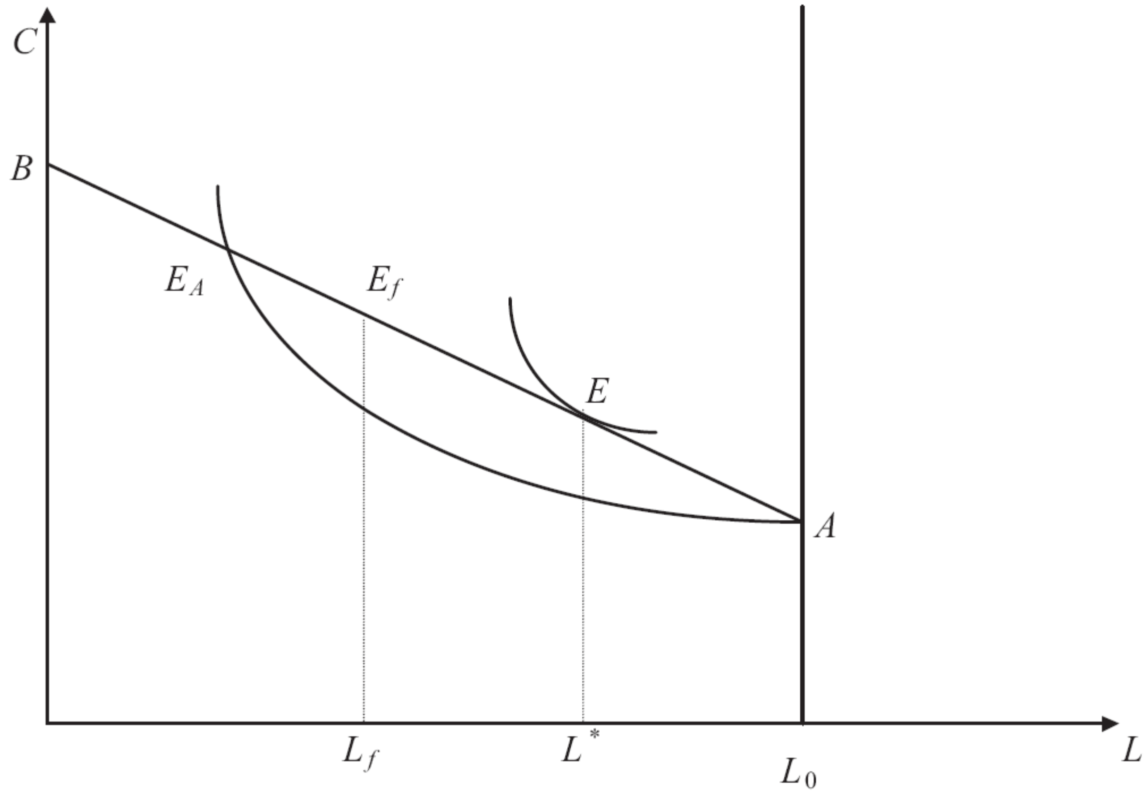


<u>Complications</u>

- **Higher overtime pay**
- **Progressive taxes**
- **Fixed cost to enter the labour market**
- **Only jobs with fixed number of hours**

$L_0 - L_f = h_0$ is the fixed number of hours demanded.

<u>Figure 1.5</u>



- ***E* is the unconstrained optimum.**
- **If *E* is to the left of E_f, the individual would have liked to supply more hours.**
- **If *E* is to the right of E_f, the individual takes the job only if E_f is to the right of E_A (i.e. offering higher utility). The individual is forced to work more than he would want.**
- **If E_f is to the left of E_A, the individual chooses not to work. <u>Involuntary</u> non-participation.**

The condition for taking a job is:

$$U\left[R + w(L_0 - L_f, L_f), L_f\right] \geq U(R, L_0)$$

$U\left[R + w_A(L_0 - L_f, L_f), L_f\right] = U(R, L_0)$ defines the reservation wage w_A.

<u>Aggregate labour supply and labour force participation</u>

- **Aggregate labour supply is obtained by adding up the total number of hours supplied by each individual.**

- **The existence of indivisibilities in working hours offered to agents implies that the elasticity of aggregate supply differs from that of the individual supply.**

- **Reservation wages differ among individuals**
  - **differences in preferences**
  - **differences in non-wage income**

- **The diversity of reservation wages $w_A \in [0, +\infty]$ is represented by the cumulative distribution function $\phi(w)$.**

- **$\phi(w)$ represents the participation rate, i.e. the proportion of the population with a reservation wage below *w*.**

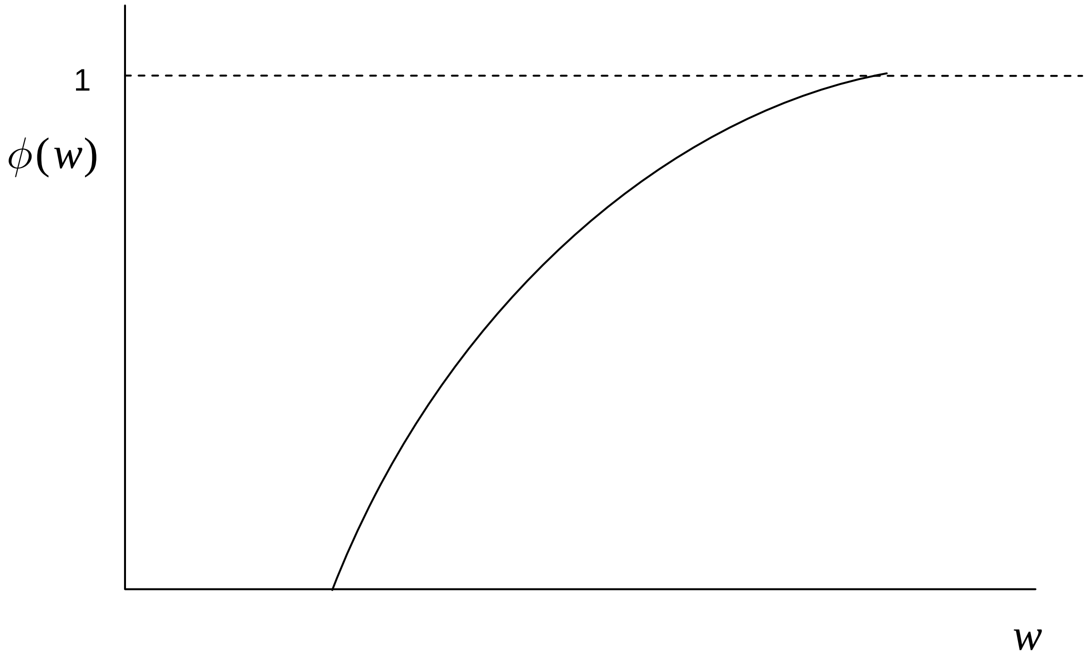
- **If the population size is *N*, $\phi(w)N$ is the labour force.**

- **Given *N*, the wage elasticity of the aggregate supply of labour is equal to that of the participation rate.**

- **The elasticity is positive, since a higher wage draws workers into the labour market.**

- **<u>Key empirical result</u>: the wage elasticity of the participation rate is much larger than the wage elasticity of individual labour supply.**

Cumulative distribution function



Labour supply with household production

$U = U(C,L)$

$C = C_D + C_M$

C_M = quantity of consumption goods bought in the market

C_D = home production of consumption goods

L_0 = total endowment of time

h_M = working hours in the market

h_D = working hours in the household production

L = leisure

$L_0 = h_M + h_D + L$

Home production function: $C_D = f(h_D)$

$f' > 0, \; f'' < 0$

wh_M = wage earnings

R = non-wage income

Choose C_M, C_D, h_D, h_M and L such that utility is maximised s. t. $C_M \leq wh_M + R$

$C_M \leq wh_M + R$

$h_M = L_0 - h_D - L \;\Rightarrow\; C_M \leq w(L_0 - h_D - L) + R$

$C_M + wL \leq wL_0 - wh_D + R$

$$wL_0 + R = R_0 \Rightarrow C_M + wL \leq R_0 - wh_D$$

$$\overbrace{C_M + C_D}^{C} + wL \leq R_0 + C_D - wh_D$$

$$C + wL \leq R_0 + [f(h_D) - wh_D]$$

<u>The consumer's programme</u>

$$\max_{C,L,h_D} U(C,L) \quad \text{s.t.} \quad C + wL \leq [f(h_D) - wh_D] + R_0$$

According to the budget constraint, the total income of the consumer is equal to the sum of potential income R_0 and "profit" from household production, $f(h_D) - wh_D$.

<u>Two-step solution</u>

Step 1: Choose h_D so as to maximise profit from household production and thus also total income:

$$f'(h_D^*) = w$$

<u>Step 2</u>: Given h_D, equivalent problem to that of the basic consumption/leisure model

- **Replace**

$$R_0 = wL_0 + R \text{ by } \overline{R}_0 = R_0 + f(h_D^*) - wh_D^* =$$

$$= wL_0 + R + f(h_D^*) - wh_D^*$$

The optimal solution is then defined by:

$$\frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w = f'(h_D^*) \text{ and } C^* + wL^* = \overline{R}_0 \qquad \textbf{(5)}$$

<u>Interpretation</u>:

- **Marginal rate of substitution between consumption and leisure is equal to the wage.**
- **Use time for household production up to the point when the marginal productivity of household production = the wage.**
- **The wage elasticity of labour supply is affected by the possibility to make trade-offs between household and market activities.**

(5) gives: $L^* = \wedge(w, \overline{R}_0)$

Differentiation w.r.t *w*:

$$\frac{dL^*}{dw} = \wedge_1 + \wedge_2 \frac{d\overline{R}_0}{dw} \qquad \textbf{with}$$

$$\frac{d\overline{R}_0}{dw} = L_0 - h_D^*$$

Since $h_M^* = L_0 - h_D^* - L^*$ **we have:**

$$\frac{dh_M^*}{dw} = -\frac{dh_D^*}{dw} - \frac{dL^*}{dw}$$

Since $w = f'(h_D^*)$ we have $\frac{dh_D^*}{dw} = \frac{1}{f''(h_D^*)} < 0$

Using that, we obtain

$$\frac{dh_M^*}{dw} = -\frac{1}{f''(h_D^*)} - \wedge_1 - \wedge_2 (L_0 - h_D^*) =$$

$$= -(\wedge_1 + \wedge_2 L_0) + \left[\wedge_2 h_D^* - \frac{1}{f''(h_D^*)}\right]$$

$-(\wedge_1 + \wedge_2 L_0)$ **is the impact on labour supply given household production: ambiguous sign.**

$\wedge_2 h_D^* - \frac{1}{f''(h_D^*)}$ **is unambiguously positive if leisure is a normal good** $(\wedge_2 > 0)$**.**

The possibility to make trade-offs between household production and market work increases the wage elasticity of labour supply.

- **Possible explanation of why female labour supply is more elastic than male labour supply: clearly the case if men are in a corner solution with** $h_D^* = 0$ **because** $w > f'(0)$.

- **Weakness: Disutility of household and market work is assumed to be the same.**

Intrafamily decisions

Interdependent decisions within a family

The unitary model

- **Extension of the basic model**
- **Utility of the family is $U = U(C, L_1, L_2)$**
  C = total consumption of goods of the family
  L_i (i = 1,2) = leisure of individual i
  Utility from consumption does not depend on distribution of consumption.

Programme of the household:

$$\max_{C, L_1, L_2} \; U(C, L_1, L_2)$$

$$\text{s.t.} \quad C + w_1 L_1 + w_2 L_2 \leq R_1 + R_2 + (w_1 + w_2) L_0$$

- **Distribution of non-wage incomes does not matter, only their sum $R_1 + R_2$ (income pooling).**
- **Empirically questionable**
  - Fortin and Lacroix find support only for couples with pre-school-age children.

The collective model

- **Household choices must arise out of individual preferences**
- **But Pareto-efficient decisions**

Programme:

$$\max_{C_1, C_2, L_1, L_2} U_1(C_1, L_1)$$

s.t. $$U_2(C_2, L_2) \geq \overline{U}_2$$

$$C_1 + C_2 + w_1 L_1 + w_2 L_2 \leq R_1 + R_2 + (w_1 + w_2) L_0$$

$\overline{U}_2$ **likely to depend on w_i and R_i.**

Chiappori (1992):

$$\max_{C_i, L_i} U_i(C_i, L_i) \quad \text{s.t.} \quad C_i + w_i L_i \leq w_i L_0 + \Phi_i$$

- **Φ_i is a sharing rule such that $\Phi_1 + \Phi_2 = R_1 + R_2$**

  Φ_i depends on w_i and R_i

- **Efficient allocations are solutions to individual programmes where each individual is endowed with a specific non-wage income which depends on the overall income of the household.**
- **Also extensions of basic model with specification of the individual's non-wage income.**

<u>Models of intrafamily decisions</u>

- **Explanation of specialization in either household or market work**
- **Interdependence of decisions**

  - w ↓ ⇒ reduction in household income ⇒ increased participation (from earlier non-participants)

  - but this <u>additional worker effect</u> does not seem empirically important

  - not negative but positive relationship between participation and average wage