Lecture 2: Intermediate macroeconomics, autumn 2009

Lars Calmfors

Topics

- Production
- Labour productivity and economic growth
- The Solow Model
- Endogenous growth
- Long-run effects of the current recession

Literature: Mankiw and Taylor, Chapters 3, 7 and 8; OECD Economic Outlook, Chapter 4, pp. 231-245.

GDP per capita, percent of OECD average, PPP-adjusted

| ition 1970 | Index |
|---------------|---|
| Switzerland | 154 |
| USA | 147 |
| Luxembourg | 119 |
| Sweden | 113 (105*) |
| Canada | 111 |
| Denmark | 109 |
| France | 105 |
| Australia | 103 |
| Netherlands | 102 |
| New Zeeland | 100 |
| Great Britain | 96 |
| Belgium | 95 |
| Germany | 93 |
| Italy | 89 |
| Austria | 89 |
| Norway | 88 |
| Japan | 86 |
| Finland | 85 |
| Iceland | 83 |
| Spain | 66 |
| Ireland | 55 |
| Greece | 53 |
| Portugal | 46 |
| Mexico | 40 |
| Turkey | 28 |
| | Switzerland USA Luxembourg Sweden Canada Denmark France Australia Netherlands New Zeeland Great Britain Belgium Germany Italy Austria Norway Japan Finland Iceland Spain Ireland Greece Portugal Mexico |

| Position 1980 | | Index |
|---------------|---|-------------------|
| 1 | USA | 140 |
| 2 | Switzerland Canada Luxembourg Iceland France Norway | 137 |
| 3 | Canada | 118 |
| 4 | Luxembourg | 118 115 110 |
| 5 | Iceland | 110 |
| 6 | France | 109 107 |
| 7 | Norway | 107 |
| | Sweden | 107 (98*) |
| 9 | Denmark | 105 |
| 10 | Belgium | 104 |
| 11 | Australia | 101 |
| 11 11 | Netherlands | 101 |
| 11 | Austria | 101 |
| 14 | Italy | 97 |
| 14 | Germany | 97 |
| 16 | Japan | 95 |
| 17 | Great Britain | 93 92 89 |
| 18 | Finland | 92 |
| 19 | New Zeeland | 89 |
| 20 | Spain | 68 |
| 21 | Greece | 61 |
| 21 | Ireland | 61 61 |
| 23 | Greece Ireland Portugal | 53 |
| 24 | Mexico | 45 |
| 25 | Turkey | 27 |

^{*} If Mexico and Turkey are excluded.

GDP per capita, percent of OECD average, PPP-adjusted

| n 1990 | Index | Pos | sition 1998 | Index |
|---------------|---|---|--|---|
| Luxembourg | 141 | 1 | Luxembourg | 156 |
| USA | 137 | 2 | USA | 138 |
| Switzerland | 131 | 3 | Norway | 124 |
| Canada | 114 | 4 | Switzerland | 120 |
| Japan | 110 | 5 | Denmark | 119 |
| Norway | 108 | 5 | Iceland | 119 |
| France | 107 | 7 | Canada | 111 |
| Iceland | 107 | 8 | Belgium | 109 |
| Denmark | 105 | 8 | Japan | 109 |
| Sweden | 105 (94*) | 10 | Austria | 108 |
| Belgium | 103 | 11 | Netherlands | 104 |
| Austria | 103 | 12 | Australia | 103 |
| Finland | 100 | 12 | Germany | 103 |
| Italy | 100 | 14 | Ireland | 102 |
| Australia | 99 | 15 | France | 100 |
| Germany | 99 | 16 | Finland | 98 |
| Netherlands | 98 | 16 | Italy | 98 |
| Great Britain | 98 | 18 | Great Britain | 96 |
| New Zeeland | 82 | 18 | Sweden | 96 (85*) |
| Spain | 73 | 20 | New Zeeland | 80 |
| Ireland | 70 | 21 | Spain | 76 |
| Portugal | 59 | 22 | Portugal | 69 |
| Greece | 57 | 23 | Greece | 65 |
| Mexico | 36 | 24 | Mexico | 36 |
| Turkey | 29 | 25 | Turkey | 30 |
| | Luxembourg USA Switzerland Canada Japan Norway France Iceland Denmark Sweden Belgium Austria Finland Italy Australia Germany Netherlands Great Britain New Zeeland Spain Ireland Portugal Greece Mexico | Luxembourg 141 USA 137 Switzerland 131 Canada 114 Japan 110 Norway 108 France 107 Iceland 107 Denmark 105 Sweden 105 (94*) Belgium 103 Austria 103 Finland 100 Italy 100 Australia 99 Germany 99 Netherlands 98 Great Britain 98 New Zeeland 82 Spain 73 Ireland 70 Portugal 59 Greece 57 Mexico 36 | Luxembourg 141 1 USA 137 2 Switzerland 131 3 Canada 114 4 Japan 110 5 Norway 108 5 France 107 7 Iceland 107 8 Denmark 105 8 Sweden 105 (94*) 10 Belgium 103 11 Austria 103 12 Finland 100 12 Italy 100 14 Australia 99 15 Germany 99 16 Netherlands 98 16 Great Britain 98 18 New Zeeland 82 18 Spain 73 20 Ireland 70 21 Portugal 59 22 Greece 57 23 Mexico 36 24 | Luxembourg 141 1 Luxembourg USA 137 2 USA Switzerland 131 3 Norway Canada 114 4 Switzerland Japan 110 5 Denmark Norway 108 5 Iceland France 107 7 Canada Iceland 107 8 Belgium Denmark 105 8 Japan Sweden 105 94*) 10 Austria Belgium 103 11 Netherlands Austria 103 12 Australia Finland 100 12 Germany Italy 10 14 Ireland Australia 99 15 France Germany 99 16 Finland Netherlands 98 16 Italy Great Britain 98 18 Great Britain New Zeeland |

^{*} If Mexico and Turkey are excluded.

GDP per capita, US dollars, PPP-adjusted, percent of OECD average, ranking by country

| | | 2007 | |
|--------|-----------------|------|--|
| 1 | Luxembourg | 245 | |
| 2 | Norway | 163 | |
| 3 | United States | 139 | |
| 4 | Ireland | 137 | |
| 5 | Switzerland | 126 | |
| 6 | Netherlands | 120 | |
| 7 | Canada | 118 | |
| 8 | Australia | 115 | |
| 9 | Austria | 114 | |
| 10 | Sweden | 112 | |
| 11 | Iceland | 111 | |
| 12 | Denmark | 110 | |
| 13 | United Kingdom | 109 | |
| 14 | Belgium | 108 | |
| 15 | Germany | 105 | |
| 16 | Finland | 106 | |
| 17 | Japan | 103 | |
| 18 | Euro area | 100 | |
| 19 | France | 100 | |
| 20 | Spain | 96 | |
| 21 | Italy | 93 | |
| 22 | Greece | 87 | |
| 23 | New Zealand | 82 | |
| 24 | Korea | 82 | |
| 25 | Czech Republic | 73 | |
| 26 | Portugal | 70 | |
| 27 | Hungary | 57 | |
| 28 | Slovak Republic | 61 | |
| 29 | Poland | 49 | |
| 30 | Mexico | 43 | |
| 31 | Turkey | 40 | |
| Source | OECD | | |

$$Y = F(K, L)$$

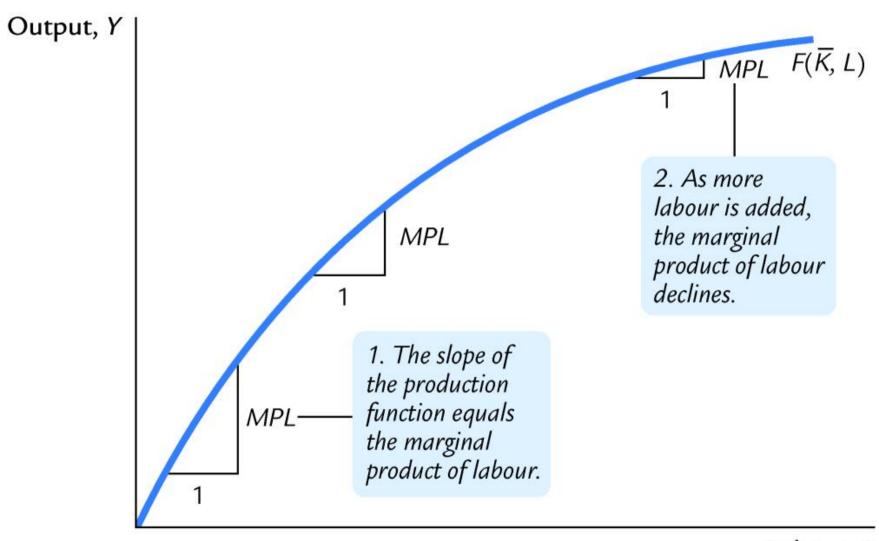
$$MPL = F(K, L + 1) - F(K, L)$$

$$MPL = \frac{dY}{dL} = \frac{dF(K, L)}{dL} = F_L$$

$$MPK = F(K + 1, L) - F(K, L)$$

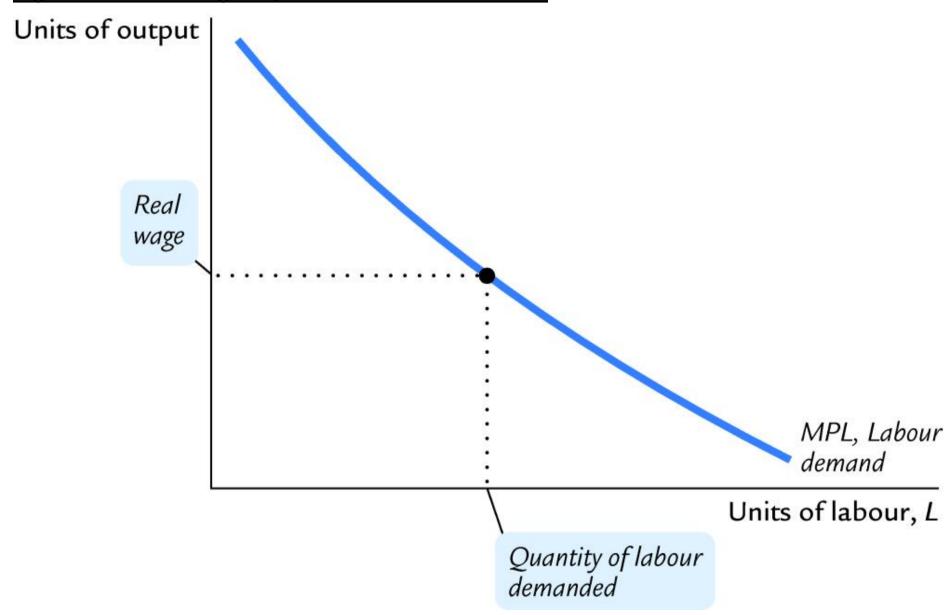
$$MPK = \frac{dY}{dK} = \frac{dF(K, L)}{dK} = F_K$$

Figure 3-3: The production function



Labour, L

Figure 3-4: The marginal product of labour schedule



Profit maximisation

General: suppose y = f(x, z). The first-order conditions (FOCs) for maximum of y are:

$$\frac{dy}{dx} = f_x = 0$$

$$\frac{dy}{dz} = f_z = 0$$

Profit maximisation

$$\pi = PY - RK - WL = PF(K, L) - RK - WL$$

$$\frac{d\pi}{dL} = PF_L - W = 0 \quad \Leftrightarrow \quad F_L = \frac{W}{P}$$

$$\frac{d\pi}{dK} = PF_K - R = 0 \quad \Leftrightarrow \quad F_K = \frac{R}{P}$$

Production function

$$Y = AF(K, L)$$
 $A = \text{total factor productivity}$

It holds that:

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha) \frac{\Delta L}{L}$$

 α = capital income share

 $1-\alpha$ = labour income share

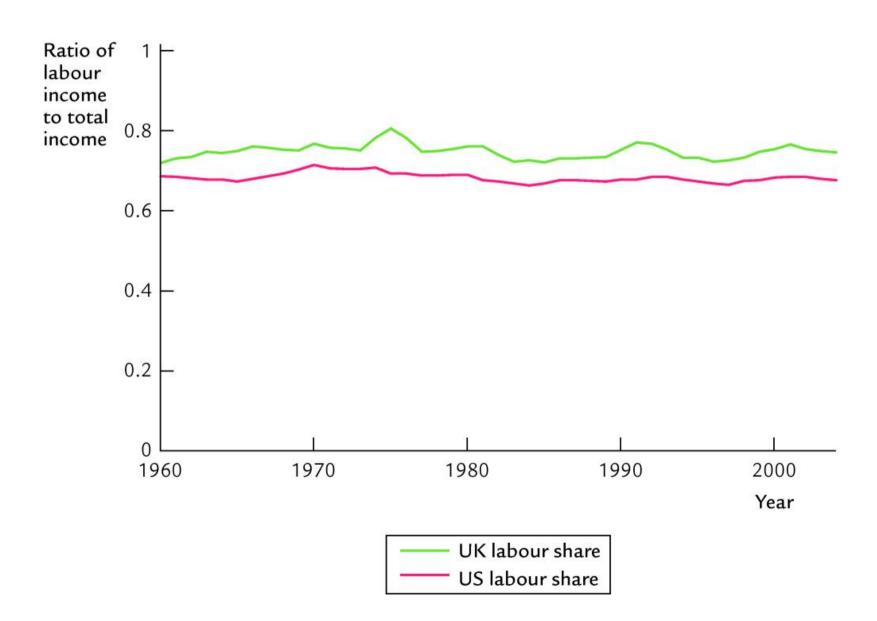
GDP growth = total factor productivity growth + contribution from growth of the capital stock + contribution from growth of the labour force

Growth accounting

The Solow-residual:

$$\frac{\Delta A}{A} \approx \frac{\Delta Y}{Y} - \alpha \frac{\Delta K}{K} - (1-\alpha) \frac{\Delta L}{L}$$

Figure 3-5: The ratio of labour income to total income in the US and the UK



Mathematical preliminaries: the natural logarithm

Recall that ℓn x is the natural logarithm of x. By definition:

$$x = e^a \iff a = \ell n x$$

Properties:

$$\ell n (xy) = \ell n x + \ell n y$$

$$\ell n \left(\frac{x}{y}\right) = \ell n x - \ell n y$$

$$\ell n x^{\beta} = \beta \ell n x$$

Rules of differentiation

If y = f(g) and g = g(x) so that

$$y = f(g(x))$$

then

$$\frac{dy}{dx} = \frac{\partial f}{\partial g} \frac{dg}{dx} = f_g g_x \tag{1}$$

Moreover, the derivative of the ℓn -function is given by:

$$\frac{d(\ln x)}{dx} = \frac{1}{x} \tag{2}$$

and for polynomials:

$$\frac{d(x^{\gamma})}{dx} = \gamma x^{\gamma-1}$$

Cobb-Douglas production function

$$Y = AF(K, L) = AK^{\alpha}L^{1-\alpha}$$

K, *L* and *A* and thus also *Y* are functions of time (continuous-time formulation).

$$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}$$

Taking logarithms:

$$\ell n \ Y(t) = \ell n \ A(t) + \ell n \ K(t)^{\alpha} + \ell n \ L(t)^{1-\alpha}$$

$$\ell n \ Y(t) = \ell n \ A(t) + \alpha \ \ell n \ K(t) + (1-\alpha) \ \ell n \ L(t)$$

Differentiation w.r.t. time gives:

$$\frac{d \ln Y(t)}{dt} = \frac{d \ln A(t)}{dt} + \alpha \frac{d \ln K(t)}{dt} + (1-\alpha) \frac{d \ln L(t)}{dt}$$

$$\frac{dY}{dt} \cdot \frac{1}{Y} = \frac{dA}{dt} \cdot \frac{1}{A} + \alpha \frac{dK}{dt} \cdot \frac{1}{K} + (1-\alpha)\frac{dL}{dt} \cdot \frac{1}{L}$$

Call
$$\frac{dY}{dt} = \dot{Y}$$
, $\frac{dA}{dt} = \dot{A}$, $\frac{dK}{dt} = \dot{K}$ och $\frac{dL}{dt} = \dot{L}$

$$\therefore \quad \frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha \frac{\dot{K}}{K} + (1 - \alpha) \frac{\dot{L}}{L}$$

 α = profit share

$$1-\alpha$$
 = wage share

The discrete-time equivalent is:

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha) \frac{\Delta L}{L}$$

$$\Delta Y = Y_t - Y_{t-1}$$
 etc.

Profit maximisation with Cobb-Douglas production function

$$\pi = PY - RK - WL = PAK^{\alpha}L^{1-\alpha} - RK - WL$$

$$\frac{d\pi}{dK} = \alpha PAK^{\alpha-1}L^{1-\alpha} - R = 0$$

$$\frac{d\pi}{dL} = (1-\alpha)PAK^{\alpha}L^{-\alpha} - W = 0$$

Re-arranging these equations implies:

$$\alpha = \frac{R}{PAK^{\alpha-1}L^{1-\alpha}} = \frac{RK}{PAK^{\alpha}L^{1-\alpha}} = \frac{RK}{PY}$$

$$1-\alpha = \frac{W}{PAK^{\alpha}L^{-\alpha}} = \frac{WL}{PAK^{\alpha}L^{1-\alpha}} = \frac{WL}{PY}$$

Growth in labour productivity

$$\frac{\Delta Y}{Y} = \alpha \, \frac{\Delta K}{K} + (1 - \alpha) \, \frac{\Delta L}{L} + \frac{\Delta A}{A} \tag{A}$$

GDP growth = contribution from growth of capital stock + contribution from growth of labour + total factor productivity growth

Labour productivity: Y/L

$$\frac{\Delta \left(\frac{Y}{L}\right)}{\left(\frac{Y}{L}\right)} \approx \frac{\Delta Y}{Y} - \frac{\Delta L}{L}$$

Subtracting $\frac{\Delta L}{L}$ from both sides of equation (A) gives:

$$\frac{\Delta Y}{Y} - \frac{\Delta L}{L} = \alpha \frac{\Delta K}{K} + (1-\alpha) \frac{\Delta L}{L} + \frac{\Delta A}{A} - \frac{\Delta L}{L}$$

$$\frac{\Delta Y}{Y} - \frac{\Delta L}{L} = \alpha \left(\frac{\Delta K}{K} - \frac{\Delta L}{L} \right) + \frac{\Delta A}{A}$$

Growth in labour productivity = contribution from capital deepening + total factor productivity growth

Capital deepening: Increase in capital intensity (capital relative to labour)

Capital deepening can be decomposed into ICT capital deepening and non-ICT capital deepening

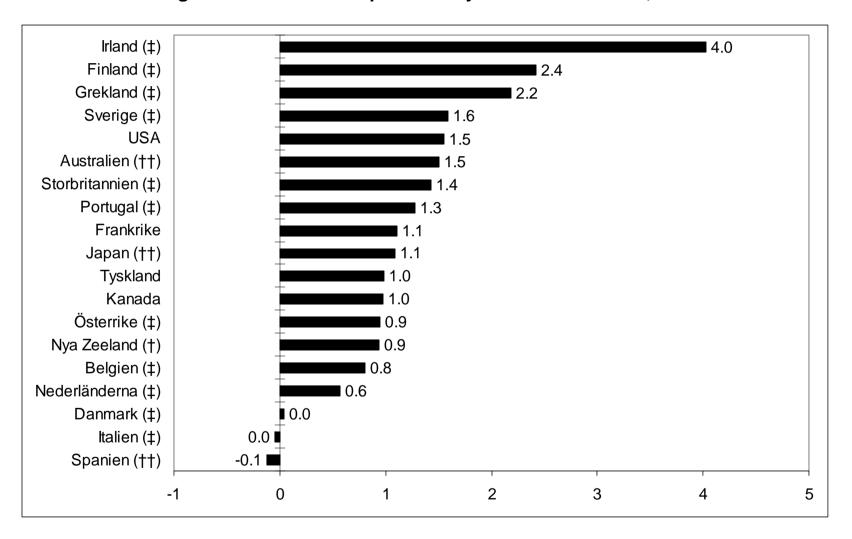
ICT = Information and Communications Technology

Table 4.5

Contributions to average annual growth in GDP per hour, percentage points, 1990–2004

| | Growth in GDP per hour | Contribution from ICT capital deepening | Contribution from non-ICT capital deepening | Total factor productivity growth |
|----------------------|---------------------------|--|---|-------------------------------------|
| Denmark | | | | |
| 1990-94 | 2.4 | 0.6 | 0.5 | 1.3 |
| 1995-99 | 1.8 | 1.0 | 0.5 | 0.3 |
| 2000-04 | 1.4 | 0.5 | 1.0 | - 0.1 |
| Finland | 2000 | ****** | | 0.000 |
| 1990-94 | 2.1 | 0.5 | 1.1 | 0.5 |
| 1995-99 | 2.7 | 0.5 | -0.7 | 2.8 |
| 2000-04 | 2.8 | 0.6 | 0.2 | 2.0 |
| Sweden | ***** | -57-00 | | 797000 |
| 1990-94 | 2.0 | 0.5 | 0.7 | 0.7 |
| 1995-99 | 2.4 | 1.0 | 0.2 | 1.2 |
| 2000-04 | 2.6 | 0.4 | 0.3 | 1.9 |
| Average Scandinavian | | | | |
| 1990-94 | 2.2 | 0.5 | 0.8 | 0.9 |
| 1995-99 | 2.3 | 0.9 | 0.0 | 1.4 |
| 2000-04 | 2.3 | 0.5 | 0.5 | 1.3 |
| Austria | | | | |
| 1990–94 | 0.9 | 0.3 | 0.6 | 0.0 |
| 1995–99 | 3.2 | 0.6 | 0.8 | 1.8 |
| 2000-04 | 1.4 | 0.4 | 0.8 | 0.2 |
| Belgium | 2000 | 591000 | | Page 200 |
| 1990-94 | 2.9 | 0.5 | 0.9 | 1.6 |
| 1995–99 | 2.7 | 0.9 | 0.2 | 1.5 |
| 2000-04 | 0.6 | 0.4 | - 0.1 | 0.3 |
| France | Danish. | 2000 | | 166,6000 |
| 1990–94 | 1.5 | 0.2 | 1.3 | 0.0 |
| 1995–99 | 2.1 | 0.4 | 0.6 | 1.1 |
| 2000-04 | 1.5 | 0.2 | 0.9 | 0.5 |
| Germany | | 979 | 119797 | |
| 1990–94 | 3.0 | 0.4 | 0.9 | 1.8 |
| 1995–99 | 1.9 | 0.5 | 0.4 | 1.0 |
| 2000-04 | 1.2 | 0.3 | 0.3 | 0.6 |

Annual growth of total factor productivity in OECD countries, 1995-2005



Explanations of high productivity growth in Sweden

- Large contributions from both ICT-producing and ICT-using sectors
- Encompassing deregulations of product and service markets
 - low level of regulation
 - early deregulations
- High educational level (complementarity between ICT technology and high-skilled labour)
- High R&D expenditures (Research and development)
- Creative destruction in the 1990s

Constant returns to scale

$$Y = F(K, L)$$

$$zY = zF(K, L) = F(zK, zL)$$

 $10\ \%$ larger input of capital and labour raises output also by $10\ \%.$

$$z = \frac{1}{L} \Rightarrow$$

$$\frac{Y}{L} = F(\frac{K}{L}, 1)$$

$$\frac{Y}{L} = y =$$
output per capita

$$\frac{K}{L} = k =$$
 capital intensity (capital stock per capita)

$$y = F(k, 1) = f(k)$$

Output per capita is a function of capital intensity

The Cobb-Douglas case

Suppose that $Y = K^{\alpha} L^{1-\alpha}$:

$$y = \frac{Y}{L} = \frac{K^{\alpha}L^{1-\alpha}}{L} = K^{\alpha}L^{-\alpha} = \left(\frac{K}{L}\right)^{\alpha} = k^{\alpha}$$

Including total factor productivity (A) so that $Y = AK^{\alpha}L^{1-\alpha}$:

$$y = \frac{Y}{L} = \frac{AK^{\alpha}L^{1-\alpha}}{L} = AK^{\alpha}L^{-\alpha} = A\left(\frac{K}{L}\right)^{\alpha} = Ak^{\alpha}$$

The Solow model

(1) y = c + i Goods market equilibrium

(2) c = (1-s)y Consumption function, s is the savings rate

(3) y = f(k) Production function

(4) $d = \delta k$ Capital depreciation, δ is the rate of depreciation

(5) $\Delta k = i - \delta k$ Change in the capital stock

Change in the capital stock = Gross investment – Depreciation

The Solow model (cont.)

Substituting the consumption function (2) into the goods market equilibrium condition (1) gives:

$$y = (1-s)y + i$$

$$i = sy$$

Investment = Saving

Substitution of the production function into the investmentsavings equality gives:

$$i = sf(k)$$

$$\therefore$$
 $\Delta \mathbf{k} = i - \delta k = sf(k) - \delta k$

In a steady state, the capital stock is unchanged from period to period, i.e. $\Delta k = 0$ and thus:

$$sf(k) = \delta k$$

Convergence of GDP per capita

- Countries with different initial GDP per capita will converge (if they have the same production function, the same savings rate and the same depreciation rate).
- The catch-up factor
- Strong empirical support for the hypothesis that GDP growth is higher the lower is initial GDP per capita

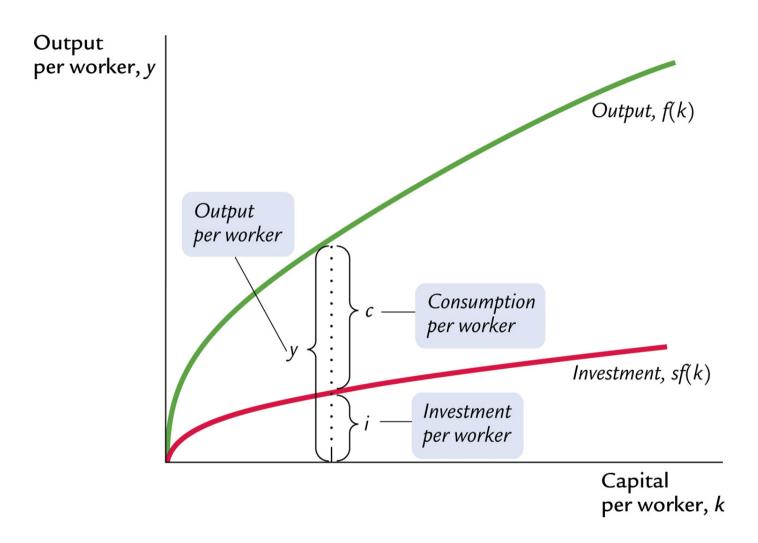


Figure 7-2: Output, consumption and investment

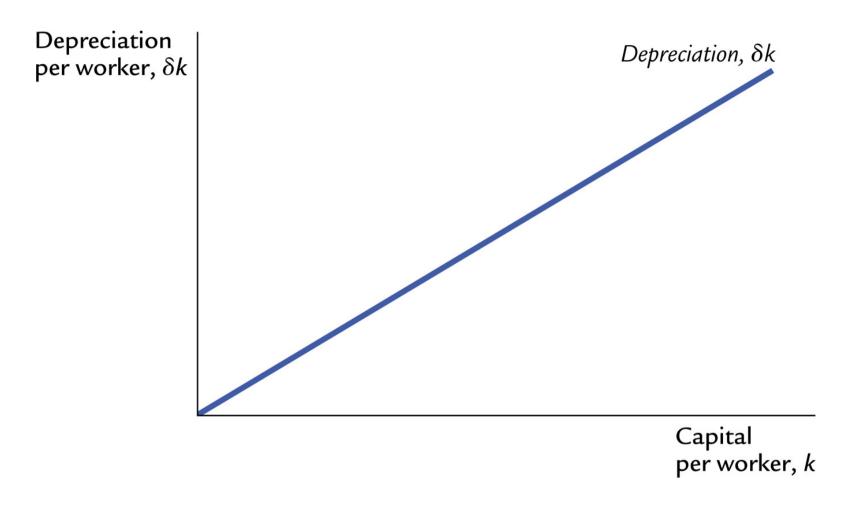


Figure 7-3: Depreciation

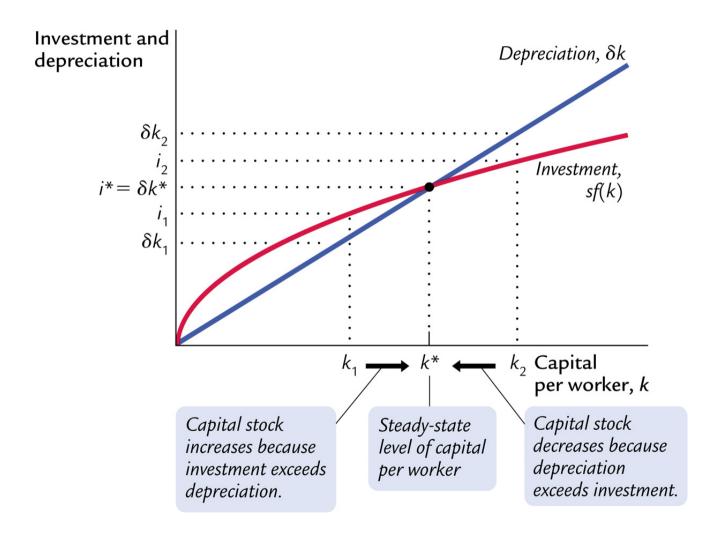


Figure 7-4: Investment, depreciation and the steady state

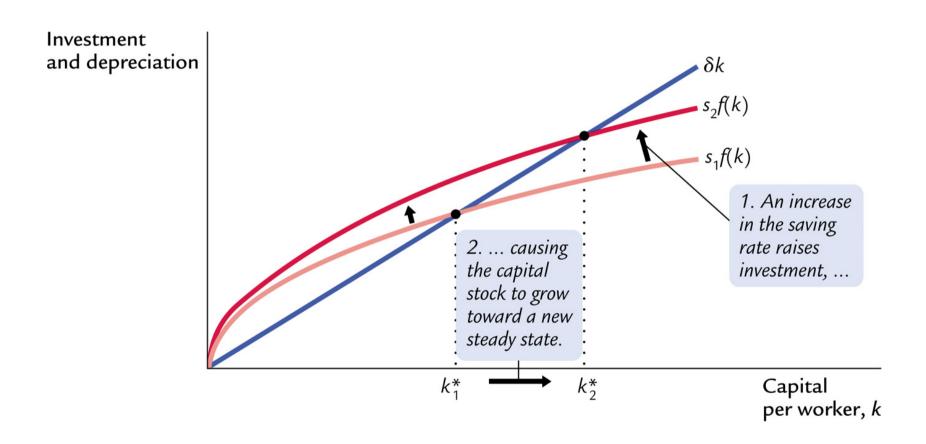


Figure 7-5: An increase in the saving rate

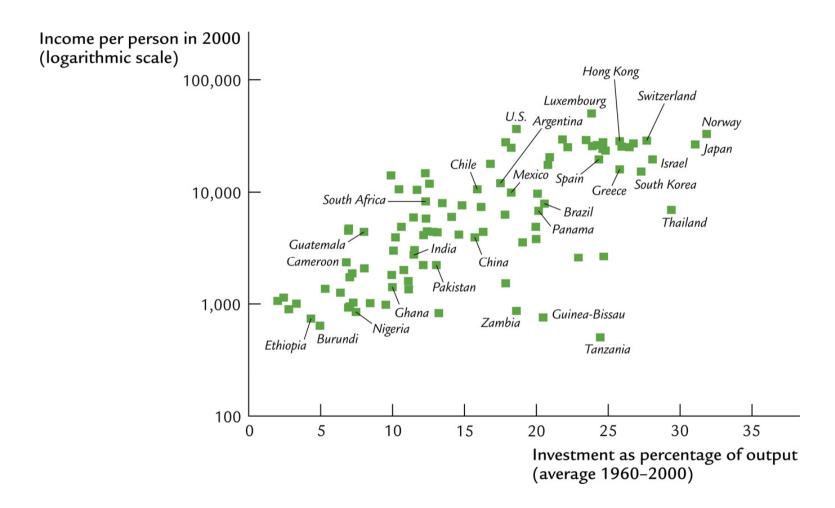


Figure 7-6: International evidence on investment rates and income per person

Golden rule of capital accumulation

Which savings rate gives the highest per capita consumption in the steady state?

$$y = c + i$$

$$c = y - i$$

In a steady state, gross investment equals depreciation:

$$i = \delta k$$

Hence:

$$c = f(k) - \delta k$$

Consumption is maximised when the marginal product of capital equals the rate of depreciation, i.e. $MPC = \delta$

Mathematical derivation

The first-order condition for maximisation of the consumption function:

$$\partial c/\partial k = f_k - \delta = 0$$

$$f_k = \delta$$

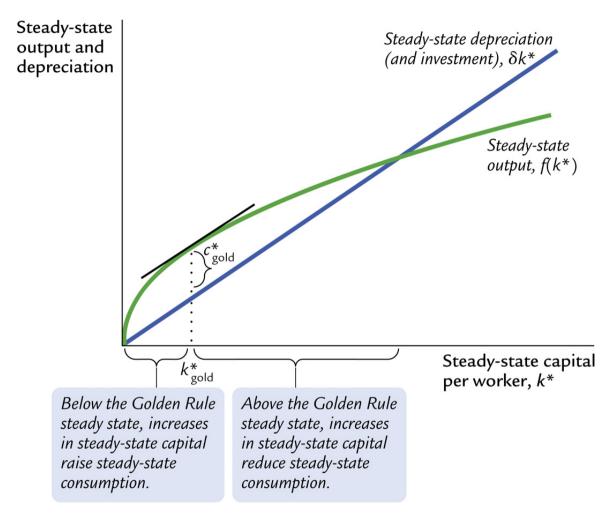


Figure 7-7: Steady-state consumption

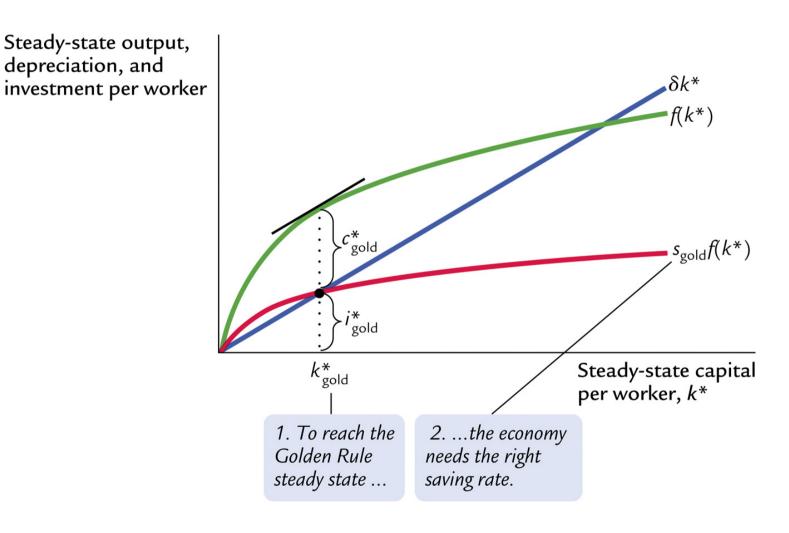


Figure 7-8: The saving rate and the golden rule

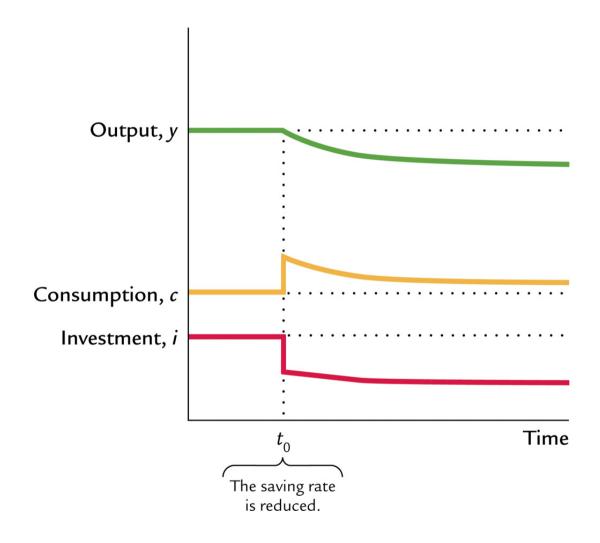


Figure 7-9: Reducing saving when starting with more capital than in the golden rule steady state

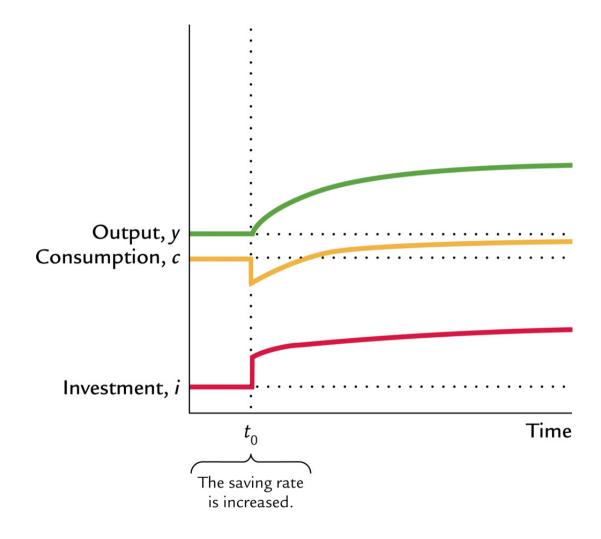


Figure 7-10: Increasing saving when starting with less capital than in the golden rule steady state

A steady state with population growth

$$n = \frac{\Delta L}{L}$$
 = population growth

$$\Delta k = i - \delta k - nk$$

Change in capital intensity (k = K/L) = Gross investment – Depreciation – Reduction in capital intensity due to population growth

In a steady state:

$$\Delta k = i - \delta k - nk = 0$$
, i.e. $i = (\delta + n)k = 0$

Derivation of the capital growth equation

 $K = \text{capital stock}, \quad I = \text{gross investment}, \quad L = \text{population}$ k = K/L = capital stock per worker (capital intensity)i = I/L = gross investment per worker

$$\Delta K = I - \delta K$$

$$\frac{\Delta K}{K} = \frac{I}{K} - \delta$$

Use that:

$$\frac{\Delta k}{k} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} \text{ and } \frac{\Delta L}{L} = n$$

$$\frac{\Delta k}{k} \approx \frac{I}{K} - \delta - n$$

Hence:

$$\frac{\Delta k}{k} \approx \frac{I}{L} \cdot \frac{L}{K} - \delta - n$$

$$\frac{\Delta k}{k} \approx \frac{i}{k} - \delta - n$$

Multiplying by *k* gives:

$$\Delta k \approx i - \delta k - nk = i - (\delta + n)k$$

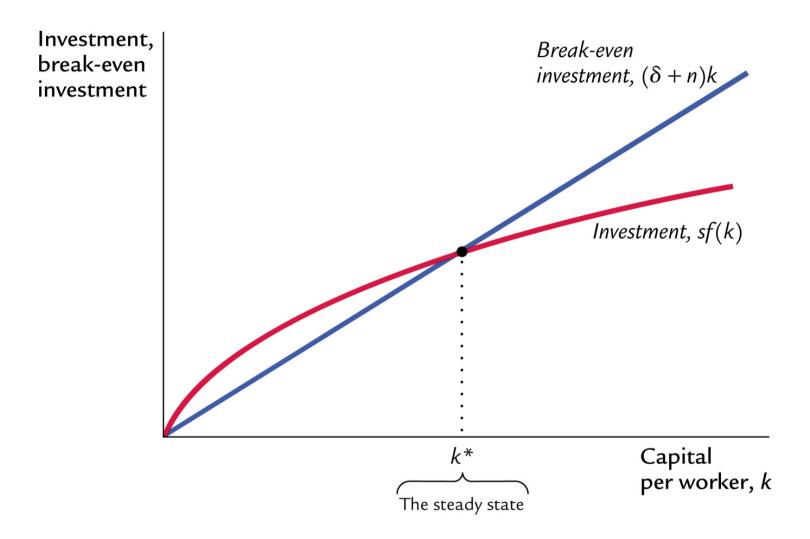


Figure 7-11: Population growth in the Solow model

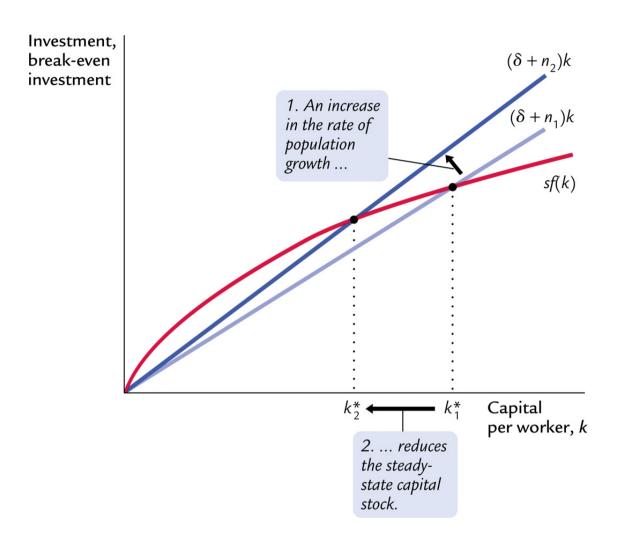


Figure 7-12: The impact of population growth

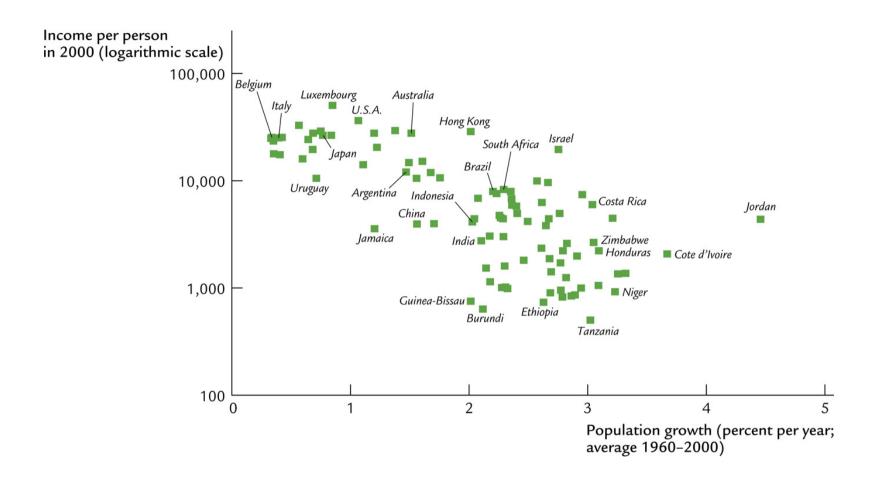


Figure 7-13: International evidence on population growth and income per person

A steady state with population growth

$$Y = F(K, L)$$

$$\frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha) \frac{\Delta L}{L}$$

In a steady state, k = K/L is constant. Because

$$\frac{\Delta k}{k} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} = 0,$$

We have

$$\frac{\Delta K}{K} = \frac{\Delta L}{L} = n$$

$$\because \operatorname{ar} \frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1 - \alpha) \frac{\Delta L}{L} = \alpha n + (1 - \alpha) n = n$$

GDP growth = Population growth

Golden rule with population growth

$$c = y - i = f(k) - (\delta + n)k$$

Consumption per capita is maximised if $MPC = \delta + n$, i.e. if the marginal product of capital equals the sume of the depreciation rate and population growth

Alternative formulation: The net marginal product of capital after depreciation $(MPK - \delta)$ should equal population growth (n)

Mathematical derivation

Differentiation of c-function w.r.t k gives:

$$\partial c/\partial k = f_k - (\delta + n) = 0$$

$$f_k = \delta + n$$

Labour-augmenting technical progress

$$Y = F(K, L \cdot E)$$

E = labour efficiency

 $L \cdot E = \text{efficiency units of labour}$

$$y = \frac{Y}{LE} = F(\frac{K}{LE}, 1) = F(k, 1) = f(k)$$

$$k = \frac{K}{LE}$$

Steady state

L grows by n % per year

E grows by g % per year

$$\Delta k = sf(k) - (\delta + n + g)k = 0$$

Gross investment = Depreciation + Reduction in capital intensity because of population growth + Reduction in capital intensity because of technological progress

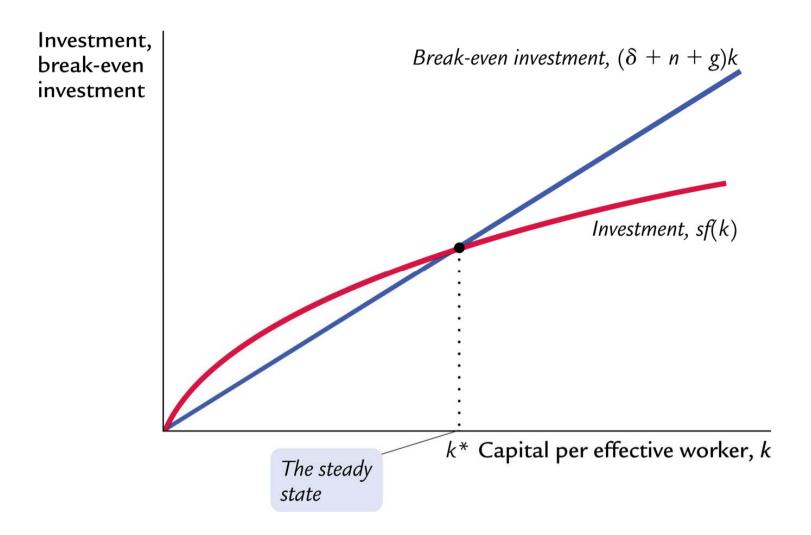


Figure 8-1: Technological progress and the Solow growth model

Growth and labour-augmenting technological progress

$$Y = K^{\alpha}(LE)^{1-\alpha}$$

$$\frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha)(\frac{\Delta L}{L} + \frac{\Delta E}{E})$$

In a steady state *K/LE* is constant

$$(\Delta L/L + \Delta E/E) = n + g \Rightarrow \Delta K/K = n + g.$$

$$\frac{\Delta Y}{Y} \approx \alpha(n+g) + (1-\alpha)(n+g) = n+g$$

GDP growth = population growth+ technological progress

$$\frac{\Delta y}{y} \approx \frac{\Delta Y}{Y} - \frac{\Delta L}{L} = n + g - n = g$$

Growth in GDP per capita = rate of technological progress

<u>Table 8-1</u>: Steady-State growth rates in the Solow model with technological progress

| Variable | Symbol | Steady-state growth rate |
|------------------------------|---------------------------|--------------------------|
| Capital per effective worker | $k = K/(L \times E)$ | 0 |
| Output per effective worker | $y = Y/(L \times E)$ | О |
| Output per worker | $(Y/L) = y \times E$ | g |
| Total output | $Y = y \times E \times L$ | n + g |

Golden rule with technological progress

$$c = f(k) - (\delta + n + g)k$$

Consumption per efficiency unit is maximised if $MPK = \delta + n + g$

The marginal product of capital should equal the sum of depreciation, population growth and technological progress

Alternative formulation: The net marginal product (MPK - δ) should equal GDP growth (n + g).

Mathematical derivation

Differentiation w.r.t. k:

$$\partial c/\partial k = f_k - (\delta + n + g) = 0$$

 $f_k = \delta + n + g$

Real world capital stocks are smaller than according to the golden rule. The current generation attaches a larger weight to its own welfare than according to the golden rule.

Endogenous or exogenous growth

- In the Solow model growth is exogenously determined by population growth and technological progress
- Recent research has focused on the role of human capital
- A higher savings rate or investment in human capital do not change the rate of growth in the steady state
- The explanation is decreasing marginal return of capital (MPK is decreasing in K)

The AK-model

$$Y = AK$$
$$\Delta K = sY - \delta K$$

Assume A to be fixed!

$$\Delta Y/Y = \Delta K/K$$

$$\Delta K/K = sAK/K - \delta K/K = sA - \delta$$

$$\Delta Y/Y = sA - \delta$$

- A higher savings rate *s* implies permanently higher growth
- Explanation: constant returns to scale for capital
- Complementarity between human and real capital

A two-sector growth model

- Business sector
- Education sector

Y = F[K, (1-u)EL] Production function in business sector

 $\Delta E = g(u)E$ Production function in education sector

 $\Delta K = sY - \delta K$ Capital accumulation

u =share of population in education

$$\Delta E/E = g(u)$$

- A higher share of population, *u*, in education raises the growth rate permanently (cf *AK*-model here human capital)
- A higher savings rate, s, raises growth only temporarily as in the Solow model

Human capital in growth models

- 1. Broad-based accumulation of knowledge in the system of education
- 2. Generation of ideas and innovations in research-intensive R&D sector
- 3. Learning by doing at the work place

Policy conclusions

- 1. Basic education incentives for efficiency in the education system incentives to choose and complete education
- 2. Put resources in top-quality R&D
- 3. Life-long learning in working life

Technological externalities / knowledge spillovers

Role of institutions

- Quality of institutions determine the allocation of scarce resources
- Legal systems secure property rights
 - "helping hand" from government (Europe)
 - "grabbing hand" from government
- Acemoglu / Johnson /Robinson
 - European settlers in colonies preferred moderate climates (US, Canada, NZ)
 - European-style institutions
 - Earlier institutions strongly correlated with today's institutions

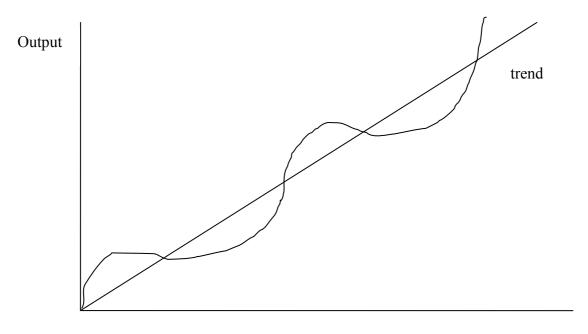
Will the current recession have long-run growth effects?

- Traditional view: a recession only represents a temporary reduction in resource utilisation
- Modern view a recession can have "permanent" effects on potential output growth

Effects on potential growth

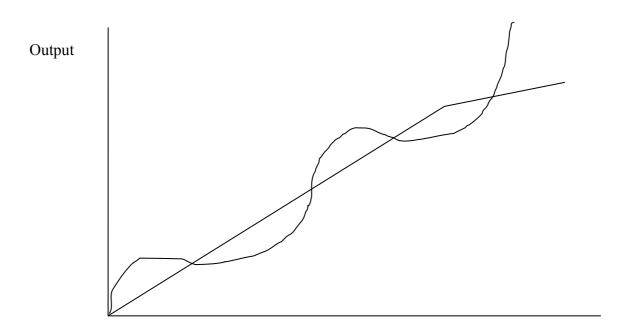
- Slower growth of capital input
 - lower investment because of lower output and credit crunch in the short run and because of higher risk premia (higher interest rates and thus higher capital costs) in the medium run
 - capital becomes obstacle
- Higher structural unemployment
- Slower growth in total factor productivity
 - lower R&D expenditure
 - but also closing down of least efficient firms

Temporary effects of a recession



Time

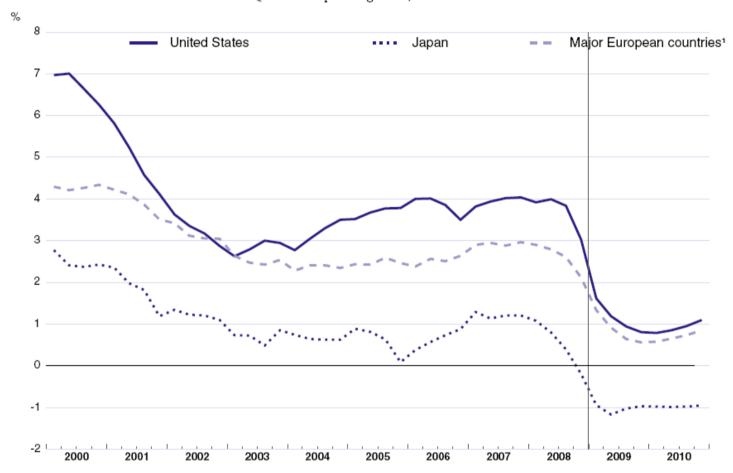
Permanent effects of a recession



Time

Figure 4.1. Growth in capital services, 2000-10

Quarter-on-quarter growth, annualised rate



1. Weighted average of Germany, France, Italy and the United Kingdom.

Source: OECD Economic Outlook 85 database.

Contributions to changes in potential output growth, 2009-10

Percentage point pa differences in the potential growth rate

| | | 2009 | | | 2010 | | | | |
|-----------------------|------------------------------|-----------------------------------|-----------------|-------|------------------------------|-----------------------------------|-----------------|-------|---------------------------------------|
| | from Potential Employmnet | from Total Factor Productivity | from Capital | Total | from Potential Employmnet | from Total Factor Productivity | from Capital | Total | Cumulative Contribution 2009-10 |
| Ireland | -1.5 | -1.1 | -1.7 | -4.3 | -2.1 | -1.1 | -2.8 | -6.1 | -10.4 |
| Spain | -1.4 | 0 | 0.1 | -1.3 | -1.3 | 0.1 | -0.2 | -1.4 | -2.7 |
| Sweden | -0.1 | 0 | -0.3 | -0.3 | -0.3 | 0 | -0.8 | -1.1 | -1.4 |
| US | -0.1 | 0 | -0.5 | -0.6 | -0.1 | 0 | -0.8 | -0.9 | -1.5 |
| Simple OECD average | -0.2 | -0.1 | -0.4 | -0.7 | -0.4 | -0.1 | -0.8 | -1.3 | -2.0 |
| Weighted OECD average | -0.2 | 0 | -0.4 | -0.6 | -0.3 | 0 | -0.7 | -1.0 | -1.5 |

Source: OECD Economic Outlook 85