Lecture 2: **Intermediate macroeconomics, spring 2016**

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Literature: ***Mankiw, chapters 3, 8 and 9.***

Topics

- **Production**
- **Labour productivity and economic growth**
- **The Solow model (neoclassical growth model)**
- **Endogenous growth**
- **Important determinants of growth**

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Basics about production functions

$Y = F\ (K, L)$

$MPL = F(K, L + 1) - F(K, L)$

$$MPL = \frac{dY}{dL} = \frac{dF(K,L)}{dL} = F_L$$

$MPK = F\ (K + 1, L) - F\ (K, L)$

$$MPK = \frac{dY}{dK} = \frac{dF(K,L)}{dK} = F_K$$

Figure 3-3: The production function

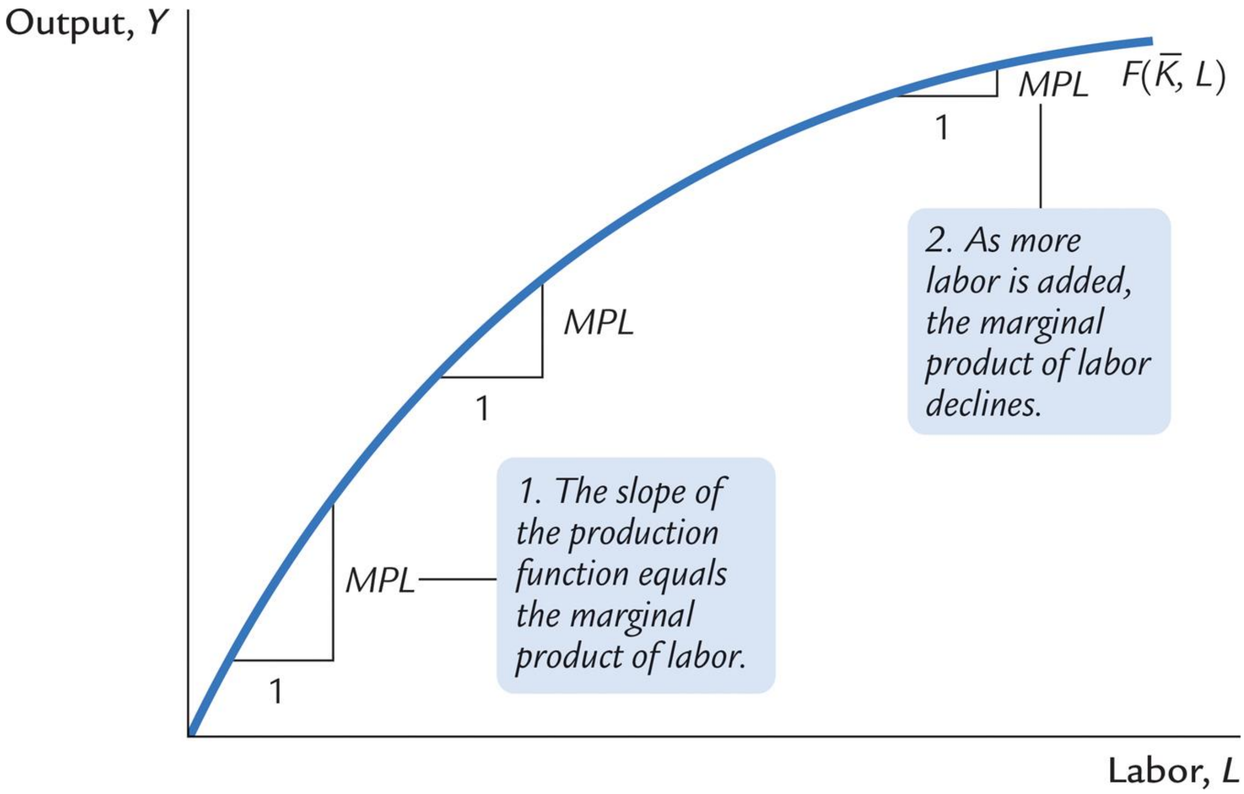
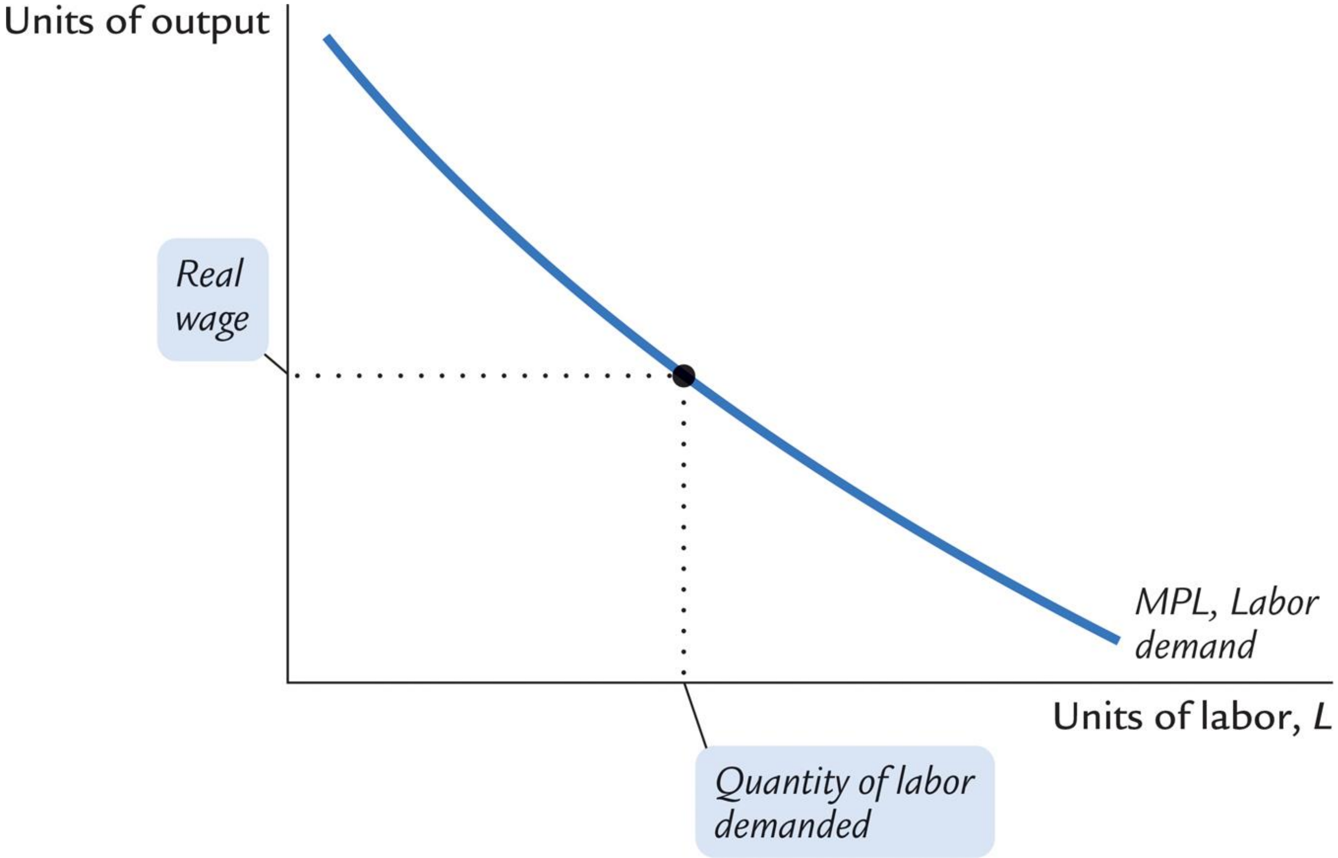


Figure 3-4: The marginal product of labour schedule



Profit maximisation

General: suppose $y = f(x,z)$. The first-order conditions (FOCs) for maximum of y are:

$$\frac{dy}{dx} = f_x = 0$$

$$\frac{dy}{dx} = f_z = 0$$

Profit maximization

$$\pi = PY - RK - WL = PF(K,L) - RK - WL$$

$$\frac{d\pi}{dL} = PF_L - W = 0 \leftrightarrow F_L = \frac{W}{P}$$

$$MPL = \frac{W}{P}$$

$$\frac{d\pi}{dK} = PF_K - R = 0 \leftrightarrow F_K = \frac{R}{P}$$

$$MPK = \frac{R}{P}$$

Production function

$Y = AF(K, L)$ — A = total factor productivity

$Y = AK^{\alpha}L^{1-\alpha}$ — Cobb-Douglas production function

It holds that:

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

$\alpha =$ **capital income share**
1- $\alpha =$**labour income share**

GDP growth = total factor productivity growth
+ contribution from growth of the capital stock
+ contribution from growth of the labour force

Growth accounting

The Solow-residual:

$$\frac{\Delta A}{A} \approx \frac{\Delta Y}{Y} - \alpha\frac{\Delta K}{K} - (1-\alpha)\frac{\Delta L}{L}$$

Figure 3-5: The ratio of labour income to total income

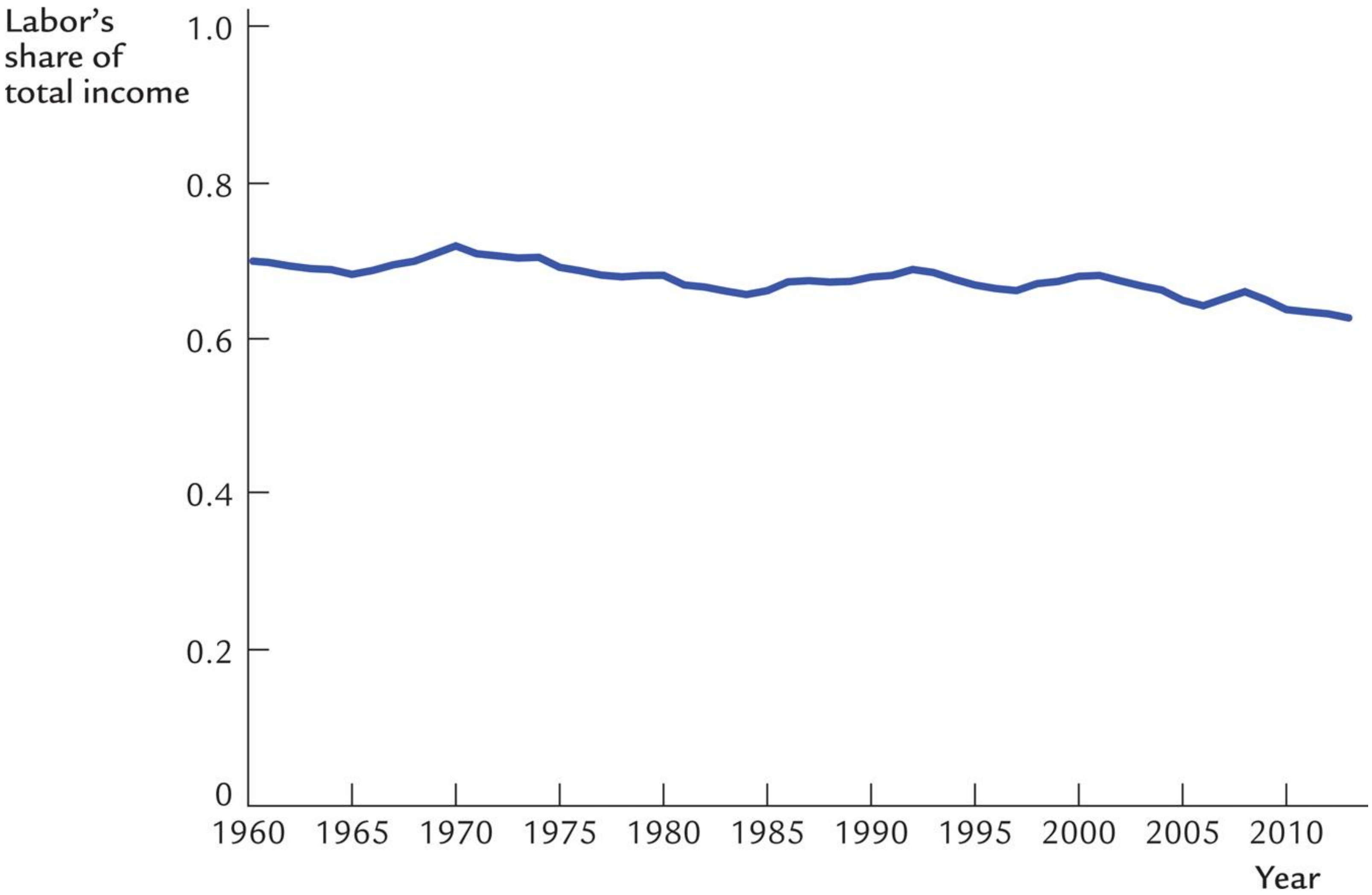
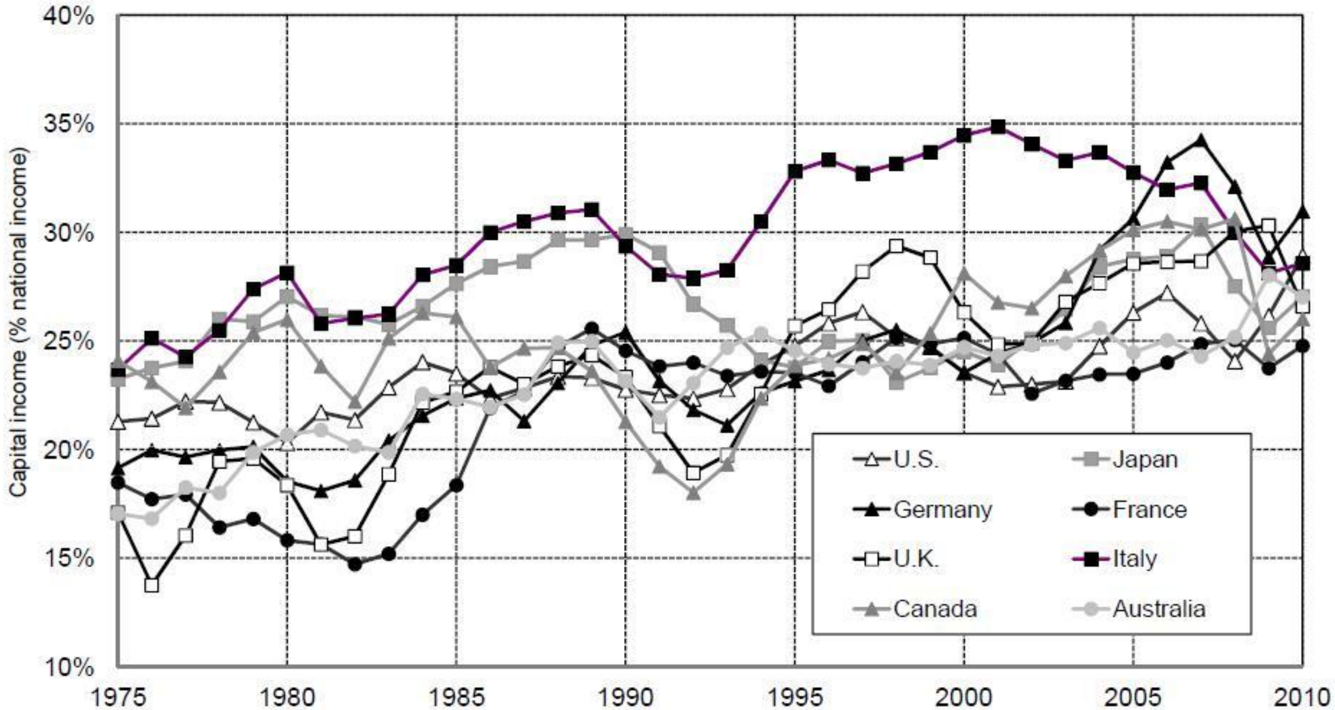
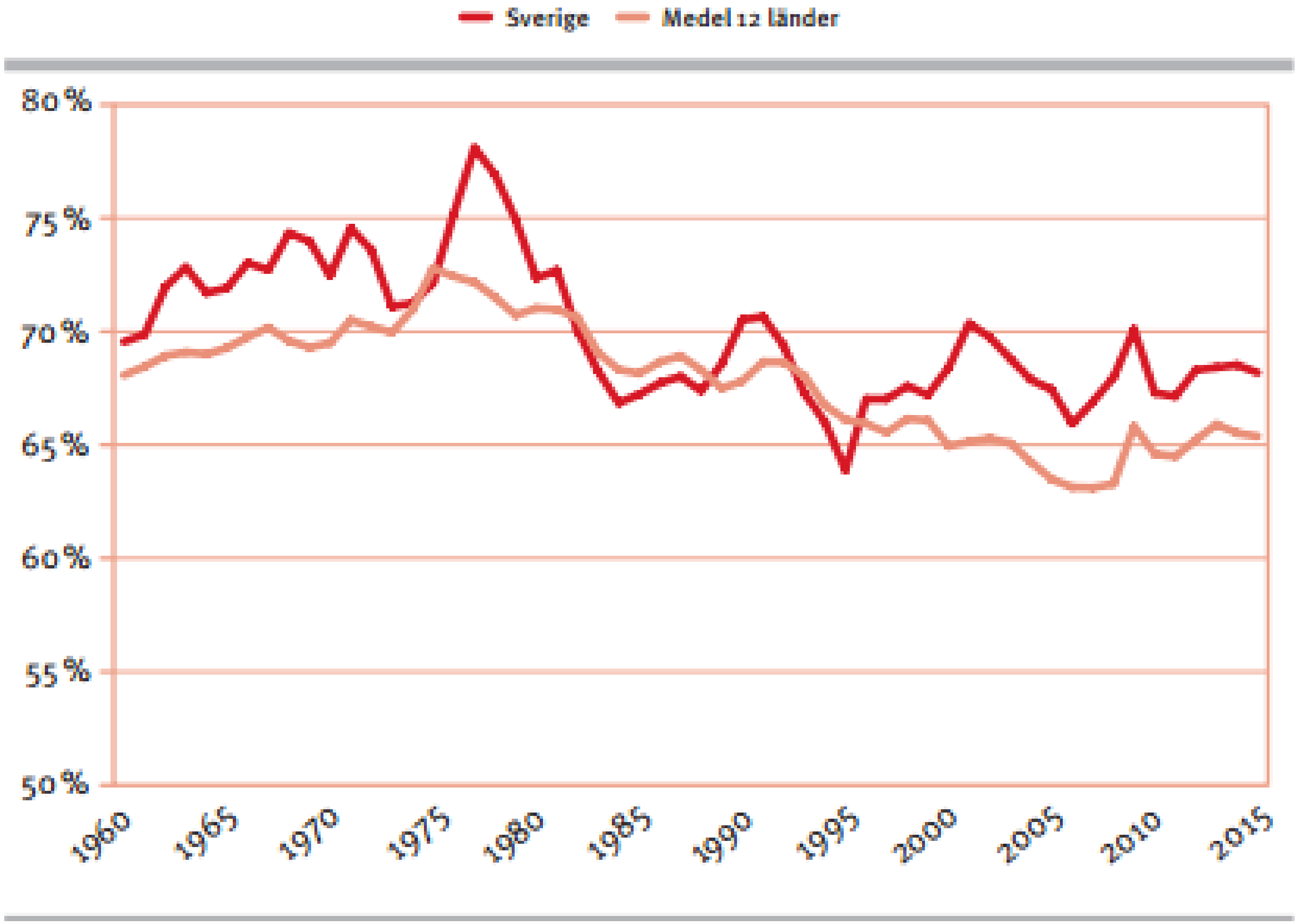


Figure 6.5. The capital share in rich countries, 1975-2010



Capital income absorbs between 15% and 25% of national income in rich countries in 1970, and between 25% and 30% in 2000-2010. Sources and series: see piketty.pse.ens.fr/capital21c

Labour income share in Sweden and twelve other countries



Rules of differentiation

If $y = f(g)$ and $g = g(x)$ so that

$$y = f(g(x))$$

then

$$\frac{dy}{dx} = \frac{\partial f}{\partial g}\frac{dg}{dx} = f_g\, g_x \qquad (1)$$

and for polynomials:

$$\frac{d(x^\gamma)}{dx} = \gamma x^{\gamma-1} \qquad (2)$$

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Table 9-3:Accounting for economic growth in the United States

			SOURCES OF GROWTH				
Years	Output Growth $\Delta Y/Y$	=	Capital $\alpha\Delta K/K$	+	Labor $(1-\alpha)\Delta L/L$	+	Total Factor Productivity $\Delta A/A$
			(average percentage increase per year)				
1948–2013	3.5		1.3		1.0		1.2
1948–1972	4.1		1.3		0.9		1.8
1972–1995	3.3		1.4		1.4		0.5
1995–2013	2.9		1.1		0.6		1.1

Data from: U.S. Department of Labor. Data are for the non-farm business sector. Parts may not add to total due to rounding.

Profit maximisation with Cobb-Douglas production function

$$\pi = PY - RK - WL = PAK^{\alpha}L^{1-\alpha} - RK - WL$$

$$\frac{d\pi}{dL} = (1-\alpha)PAK^{\alpha}L^{-\alpha} - W = 0$$

$$MPL = (1-\alpha)AK^{\alpha}L^{-\alpha} = \frac{W}{P}$$

$$MPL \ = \ \frac{(1-\alpha)AK^{\alpha}L^{-\alpha} \cdot L}{L} = \frac{W}{P}$$

$$MPL \ = \ \frac{(1-\alpha)AK^{\alpha}L^{1-\alpha}}{L} = \frac{W}{P}$$

$$MPL \ = (1-\alpha)\frac{Y}{L} = \frac{W}{P}$$

$$1-\alpha = \frac{WL}{PY} = \text{the labour share}$$

Table 3-1: Growth in labour productivity and real wages: The U.S. Experience

Time Period	Growth Rate of Labor Productivity	Growth Rate of Real Wages
1960–2013	2.1%	1.8%
1960–1973	2.9	2.7
1973–1995	1.5	1.2
1995–2013	2.3	2.0

Data from: U.S. Department of Labor. Growth in labor productivity is measured here as the annualized rate of change in output per hour in the nonfarm business sector. Growth in real wages is measured as the annualized change in compensation per hour in the nonfarm business sector divided by the implicit price deflator for that sector.

Decomposition of labour productivity growth

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

$$\frac{\Delta Y}{Y} - \frac{\Delta L}{L} \approx \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} - \frac{\Delta L}{L}$$

$$\frac{\Delta Y}{Y} - \frac{\Delta L}{L} \approx \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} - \alpha \frac{\Delta L}{L} - (1-\alpha)\frac{\Delta L}{L}$$

$$\frac{\Delta Y}{Y} - \frac{\Delta L}{L} \approx \frac{\Delta A}{A} + \alpha \left[\frac{\Delta K}{K} - \frac{\Delta L}{L}\right]$$

$$\frac{\Delta Y}{Y} - \frac{\Delta L}{L} \approx \frac{\Delta (Y/L)}{Y/L}$$

$$\frac{\Delta K}{K} - \frac{\Delta L}{L} \approx \frac{\Delta (K/L)}{K/L}$$

$$\frac{\Delta (Y/L)}{Y/L} \approx \frac{\Delta A}{A} + \frac{\Delta (K/L)}{K/L}$$

Labour productivity growth $\approx$ Total factor productivity growth + Growth of capital per worker

= Total factor productivity growth + Capital deepening

Dekomponering av produktivitetstillväxten i näringslivet

Bidrag till tillväxten i arbetsproduktiviteten i näringslivet

	1994–2006	2007–2014
Kapitalfördjupning	1,1	0,5
Arbetskraftens kompetens och förmåga	0,5	0,0
TFP	2,0	−0,4
Arbetsproduktivitet	**3,5**	**0,2**

Anm. Genomsnittlig årlig procentuell förändring för arbetsproduktiviteten.

Källa: SCB.

Table 8-1: International differences in the standard of living

Country	Income per person (2012)	Country	Income per person (2012)
United States	$51,749	Philippines	6,110
Japan	35,618	Nigeria	5,535
Russia	23,589	India	5,138
Mexico	16,426	Vietnam	4,998
Brazil	14,551	Pakistan	4,437
China	10,960	Bangladesh	2,405
Indonesia	9,011	Ethiopia	1,240

Data from: The World Bank. Data are PPP-adjusted—that is, the income figures account for differences in the cost of living among countries.

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Table 2.2. The law of cumulated growth

An annual growth rate equal to...	.. is equivalent to a generational growth rate (30 years) of...	...i.e. a multiplication by a coefficient equal to...	...and a multiplication after 100 years by a coefficient equal to...	...and a multiplication after 1000 years by a coefficient equal to...
0.1%	3%	1.03	1.11	2.72
0.2%	6%	1.06	1.22	7.37
0.5%	16%	1.16	1.65	147
1.0%	35%	1.35	2.70	20 959
1.5%	56%	1.56	4.43	2 924 437
2.0%	81%	1.81	7.24	398 264 652
2.5%	110%	2.10	11.8	52 949 930 179
3.5%	181%	2.81	31.2	...
5.0%	332%	4.32	131.5	...

An annual growth rate of 1% is equivalent to an annual growth rate of 35% per generation (30 years), a multiplication by 2.7 every 100 years, and by over 20 000 every 1000 years.

Constant returns to scale

$Y = F(K, L)$

$zY = zF(K, L) = F(zK, zL)$

10 % larger input of capital and labour raises output also by 10 %.

$$z = \frac{1}{L} \Rightarrow$$

$$\frac{Y}{L} = F(\frac{K}{L}, 1)$$

$\frac{Y}{L} = y =$ **output per capita**

$\frac{K}{L} = k =$ **capital intensity (capital stock per capita)**

$$y = F(k, 1) = f(k)$$

Output per capita is a function of capital intensity

The Cobb-Douglas case

Suppose that $Y = K^{\alpha}L^{1-\alpha}$:

$$y = \frac{Y}{L} = \frac{K^{\alpha}L^{1-\alpha}}{L} = K^{\alpha}L^{-\alpha} = \left(\frac{K}{L}\right)^{\alpha} = k^{\alpha}$$

Including total factor productivity (A) so that $Y = AK^{\alpha}L^{1-\alpha}$:

$$y = \frac{Y}{L} = \frac{AK^{\alpha}L^{1-\alpha}}{L} = AK^{\alpha}L^{-\alpha} = A\left(\frac{K}{L}\right)^{\alpha} = Ak^{\alpha}$$

<u>The Solow model</u>

(1)	$\boldsymbol{y = c + i}$	**Goods market equilibrium**
(2)	$\boldsymbol{c = (1 - s)\, y}$	**Consumption function, s is the savings rate**
(3)	$\boldsymbol{y = f(k)}$	**Production function**
(4)	$\boldsymbol{d = \delta k}$	**Capital depreciation, $\boldsymbol{\delta}$ is the rate of depreciation**
(5)	$\boldsymbol{\Delta k = i - \delta k}$	**Change in the capital stock**

Change in the capital stock = Gross investment – Depreciation

Substituting the consumption function (2) into the goods market equilibrium condition (1) gives:

$$\boldsymbol{y = (1 - s)y + i}$$

$$\boldsymbol{i = sy}$$

Investment = Saving

Substitution of the production function into the investment-savings equality gives:

$$\boldsymbol{i = sf(k)}$$

$$\therefore\ \boldsymbol{\Delta k = i - \delta k = sf(k) - \delta k}$$

In a steady state, the capital stock is unchanged from period to period, i.e. $\boldsymbol{\Delta k = 0}$, and thus:

$$\boldsymbol{sf(k) = \delta k}$$

<u>Figure 8-1: The production function</u>

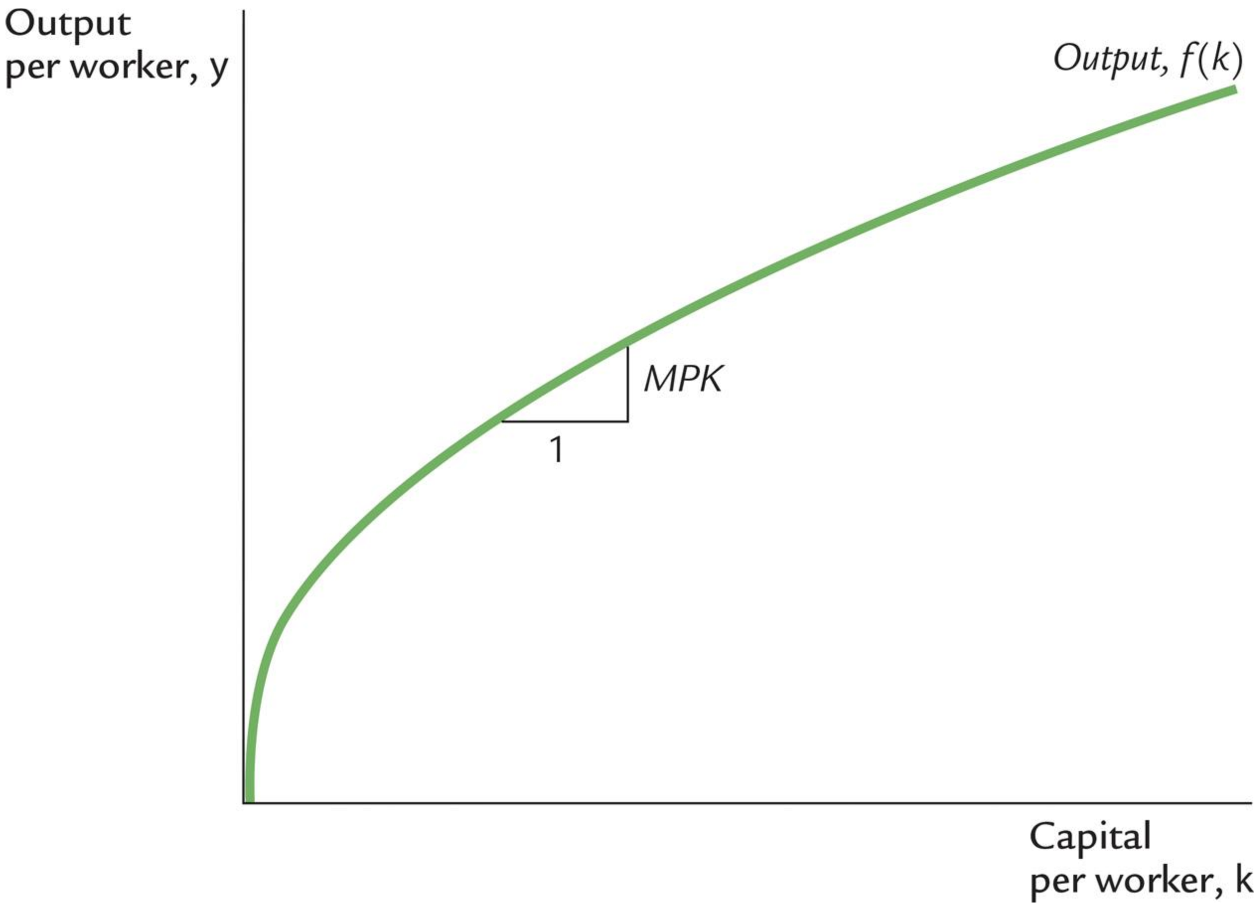


Figure 8-2: Output, consumption, and investment

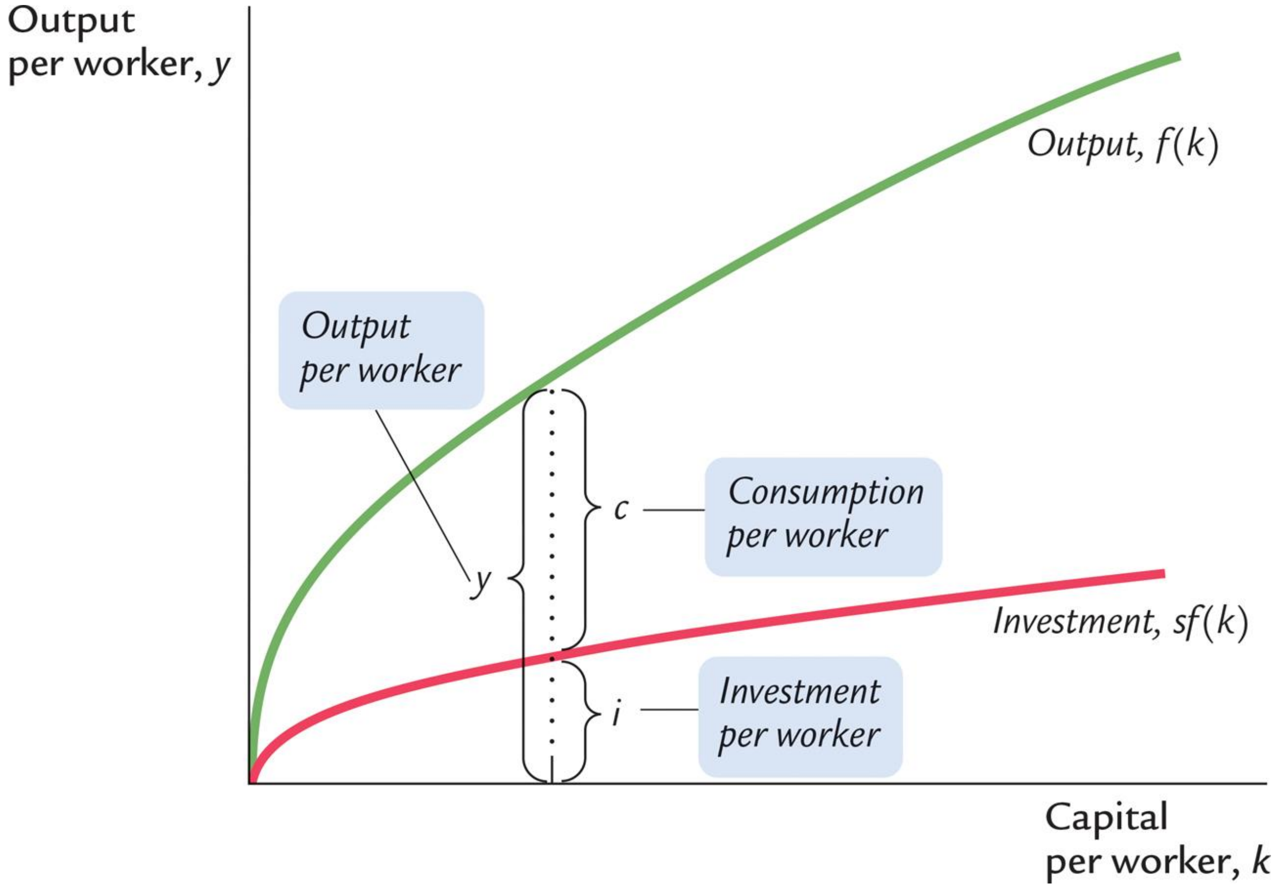


Figure 8-3: Depreciation

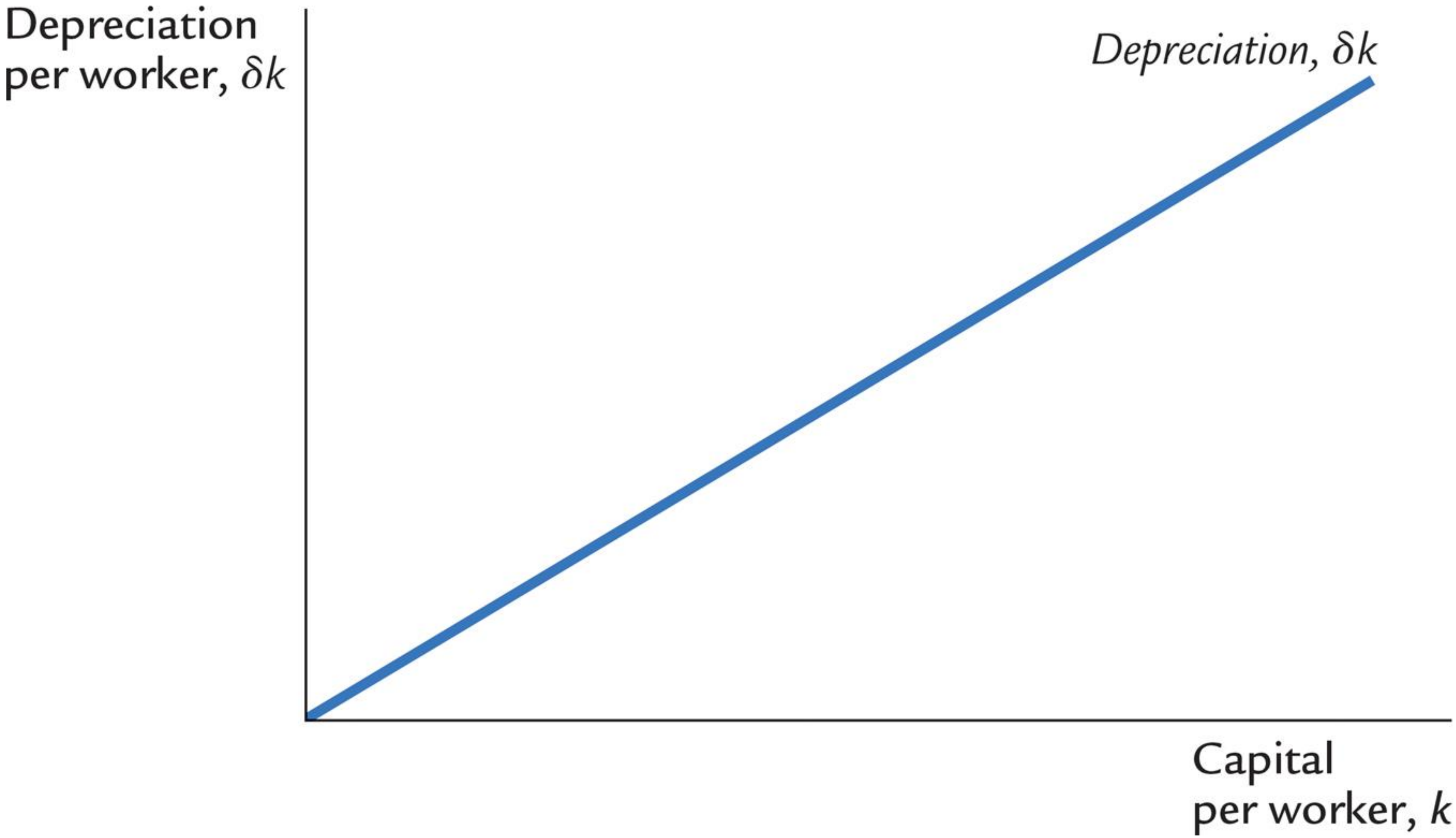
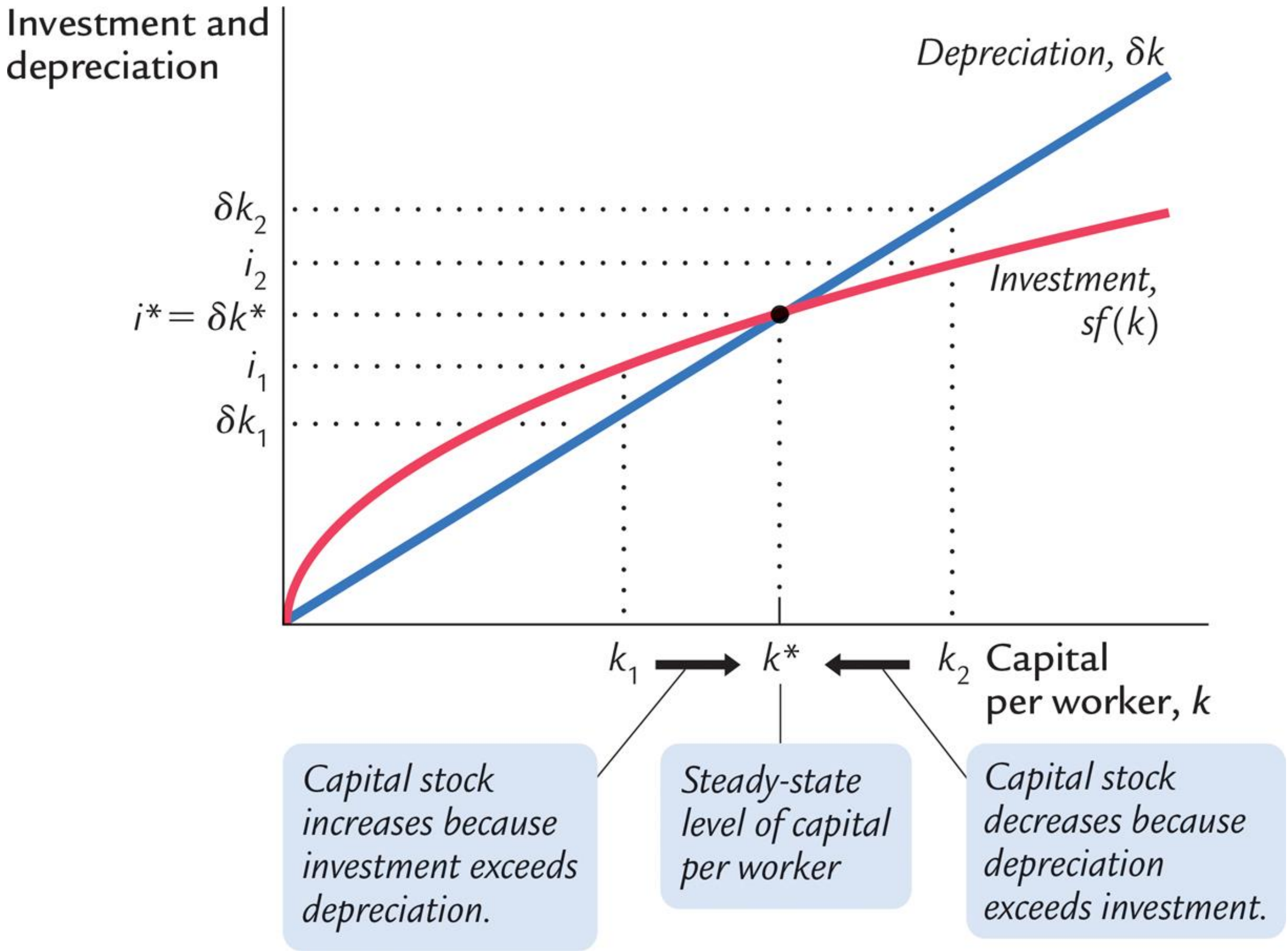


Figure 8-4: Investment, depreciation, and the steady state



Convergence of GDP per capita

- **Countries with different initial GDP per capita will converge (if they have the same production function, the same savings rate and the same depreciation rate) to the same GDP per capita**
- **The catch-up factor: Strong empirical support for the hypothesis that GDP growth is higher the lower is initial GDP per capita**

  - **when controlling for other factors**
  - **conditional convergence**
  - **convergence rate: 2 % per year**

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Figure 8-5: An increase in the saving rate

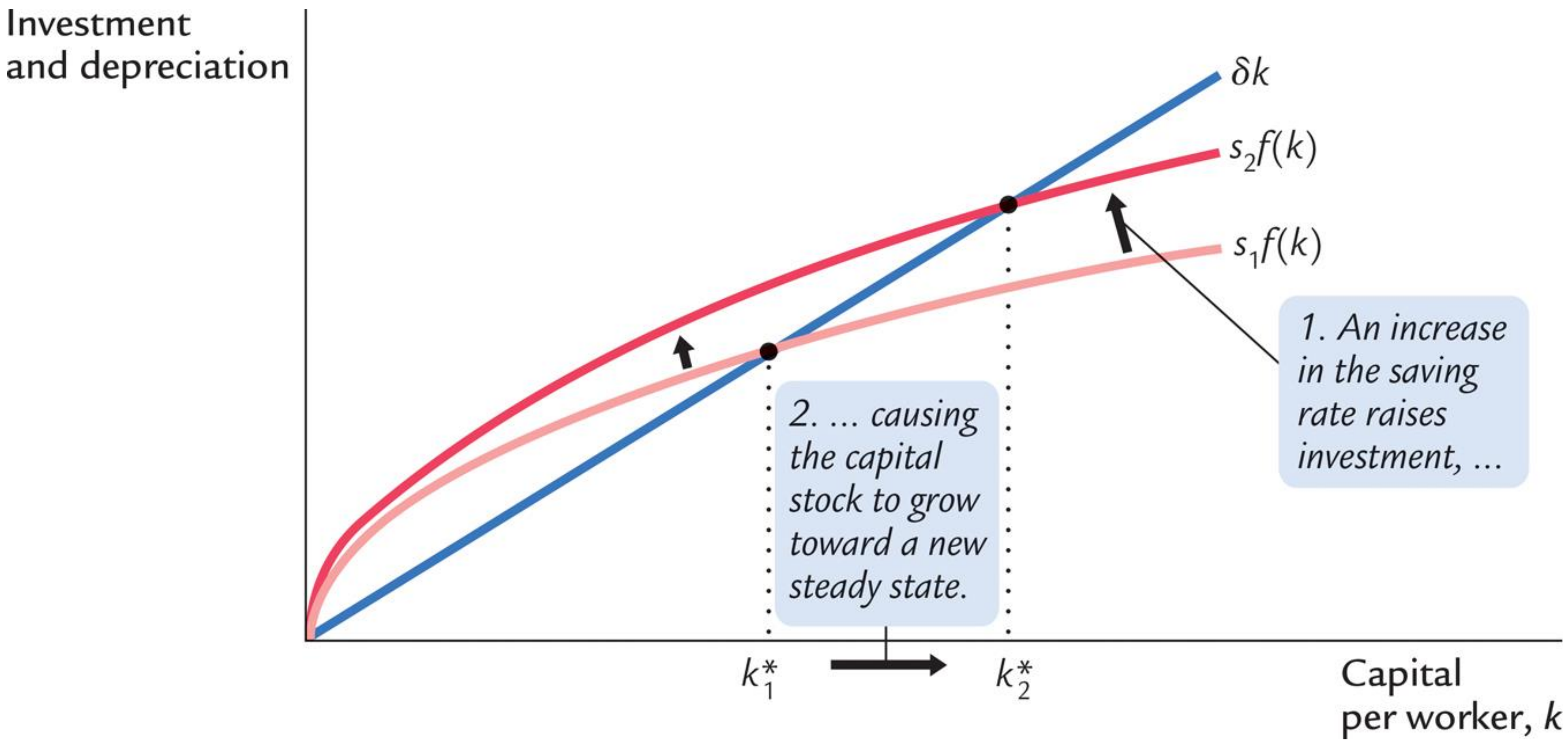
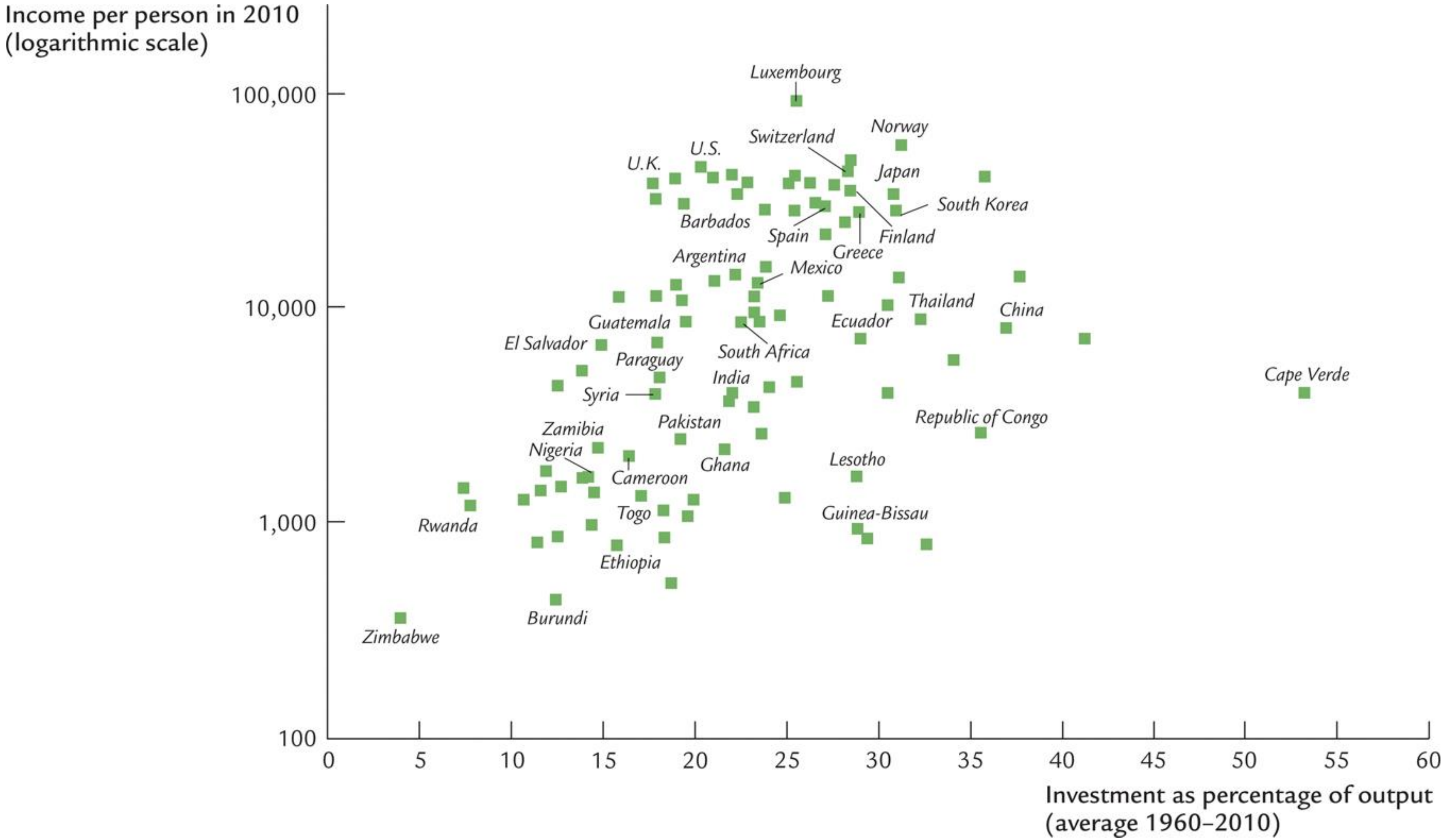


Figure 8-6: International evidence on investment rates and income per person



The golden rule level of capital

Which savings rate gives the highest per capita consumption in the steady state?

$y = c + i$

$c = y - i$

In a steady state, gross investment equals depreciation: $i = \delta k$

Hence:

$c = f(k) - \delta k$

Consumption is maximised when the marginal product of capital equals the rate of depreciation, i.e. $MPK = \delta$

Mathematical derivation

The first-order condition for maximisation of the consumption function:

$\partial c / \partial k = f_k - \delta = 0$

$f_k =$

Figure 8-7: Steady-state consumption

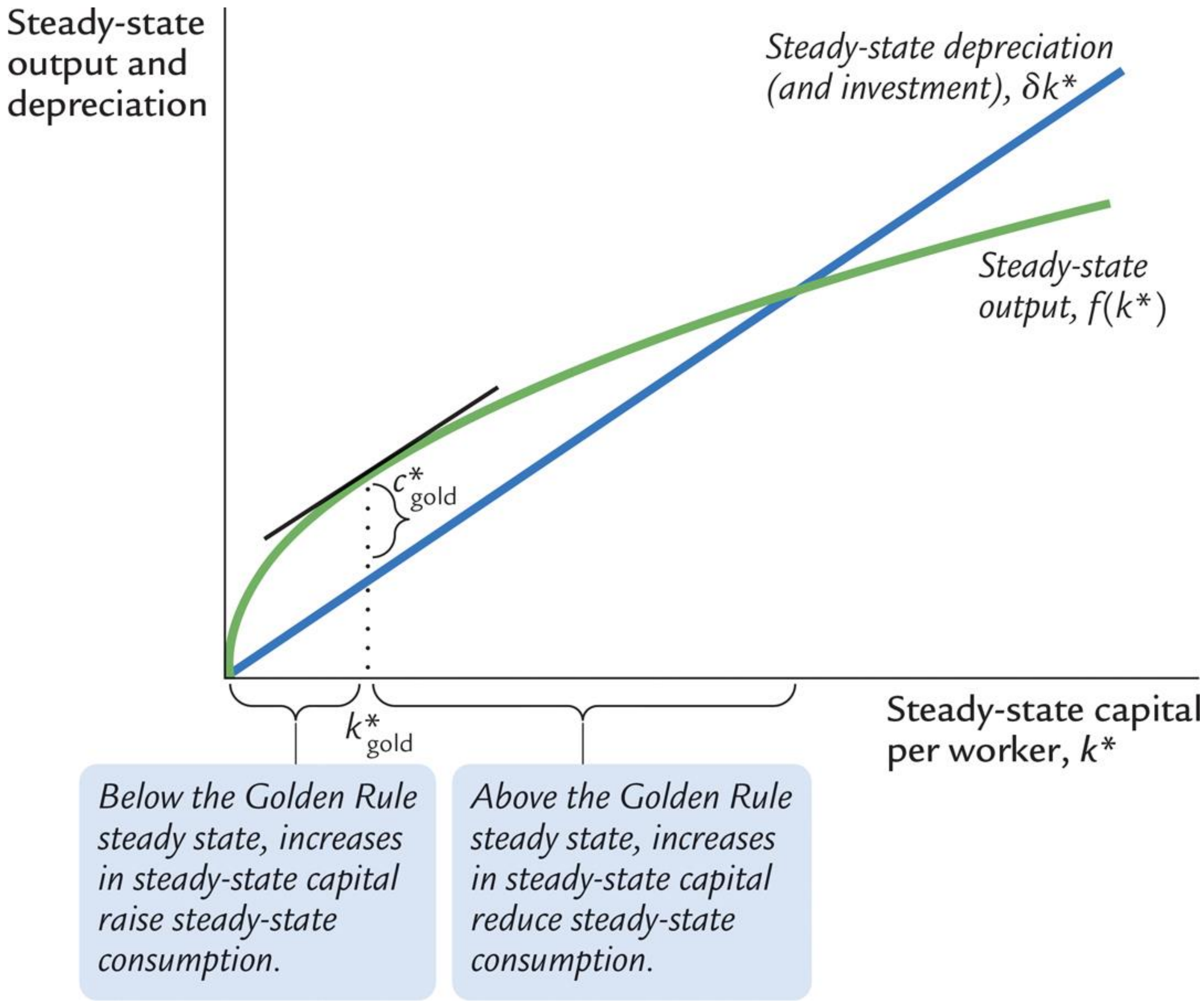


Figure 8-8: The saving rate and the golden rule

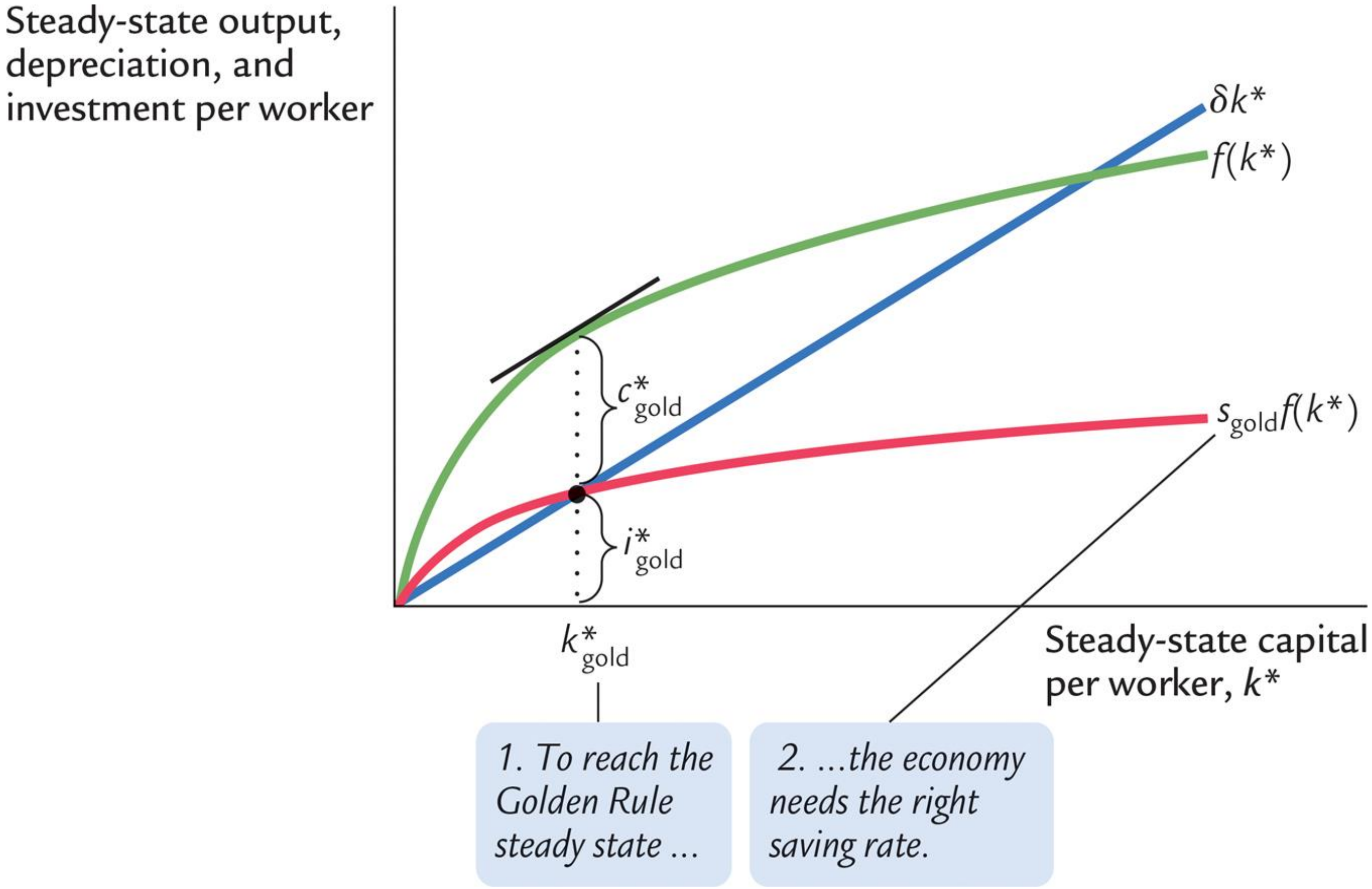


Figure 8-9: Reducing saving when starting with more capital than in the golden rule steady state

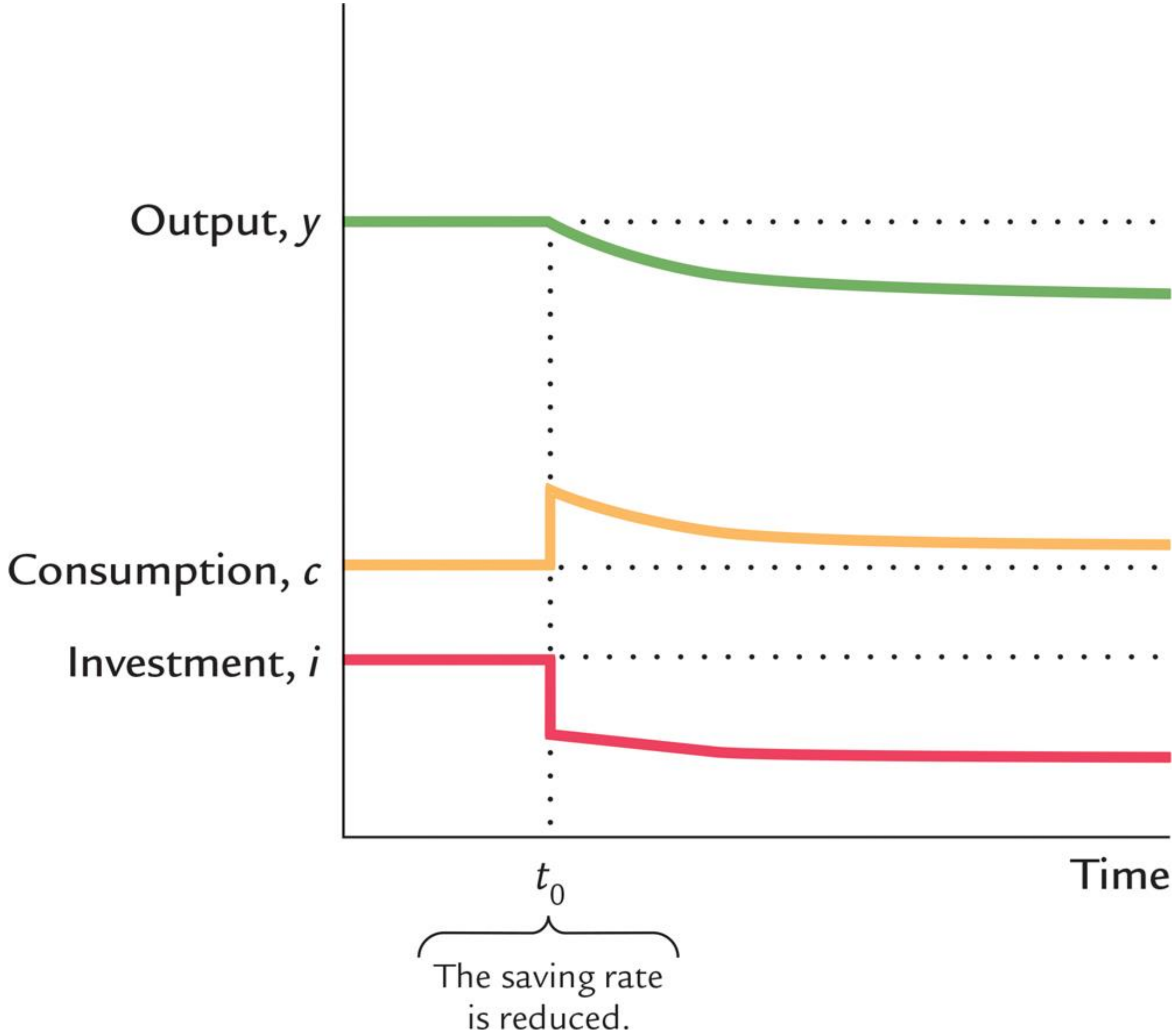
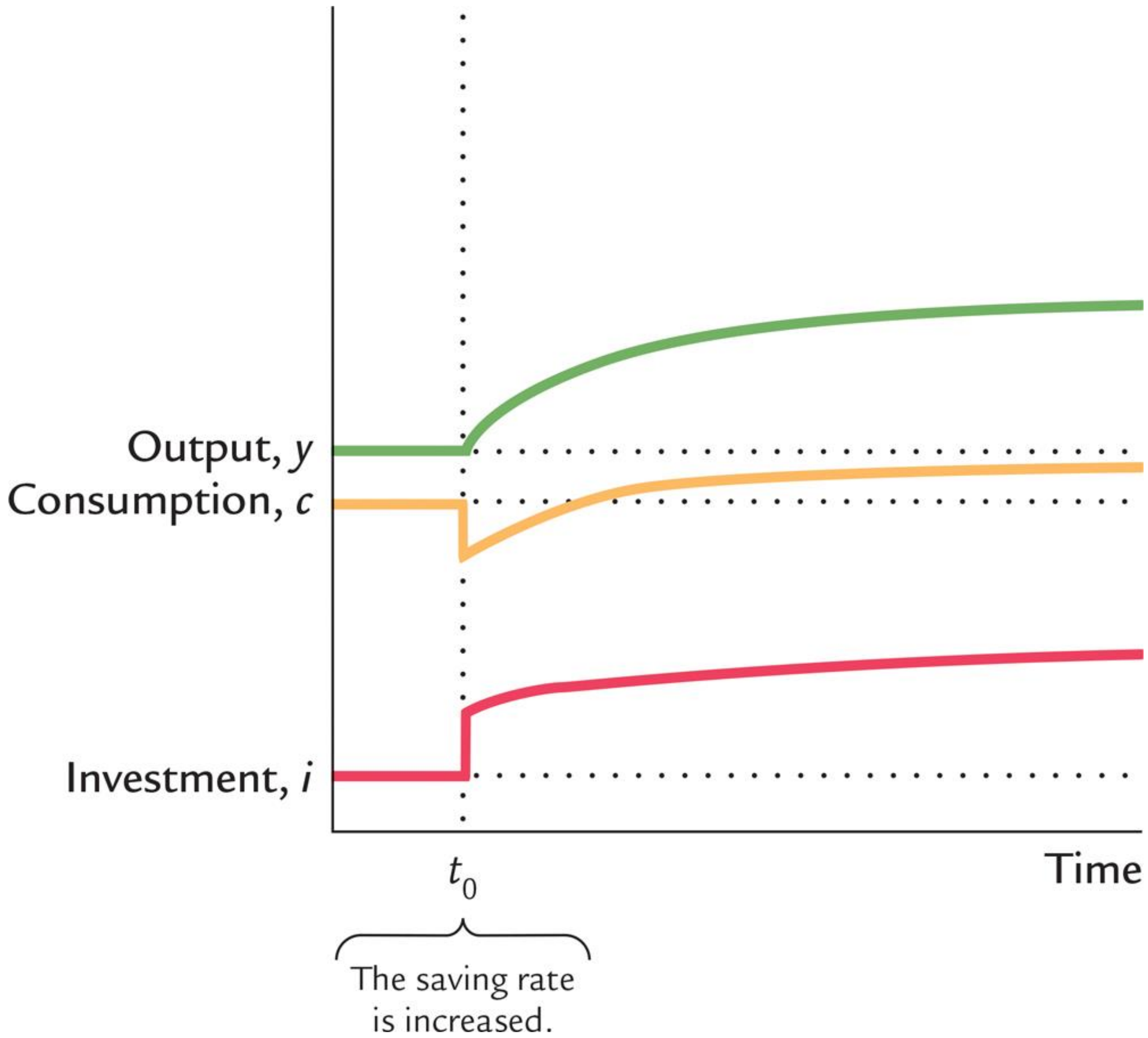


Figure 8-10: Increasing saving when starting with less capital than in the golden rule steady state



A steady state with population growth

$n = \frac{\Delta L}{L} =$ population growth

$\Delta k = i - \delta k - nk$

Change in capital intensity ($k = K/L$) = Gross investment – Depreciation – Reduction in capital intensity due to population growth

In a steady state:

$\Delta k = i - \delta k - nk = 0$

$i = (\delta + n)k$

Derivation of the capital growth equation

K = capital stock, I = gross investment, L = population
$k = K/L$ = capital stock per worker (capital intensity)
$i = I/L$ = gross investment per worker

$$\Delta K = I - \delta K$$

$$\frac{\Delta K}{K} = \frac{I}{K} - \delta$$

Use that:

$$\frac{\Delta k}{k} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} \text{ and } \frac{\Delta L}{L} = n$$

$$\frac{\Delta k}{k} \approx \frac{I}{K} - \delta - n$$

Hence:

$$\frac{\Delta k}{k} \approx \frac{I}{L} \bullet \frac{L}{K} - \delta - n$$

$$\frac{\Delta k}{k} \approx \frac{i}{k} - \delta - n$$

Multiplying by k gives:

$$\Delta k \approx i - \delta k - nk = i - (\delta + n)k$$

<u>Figure 8-11: Population growth in the Solow model</u>

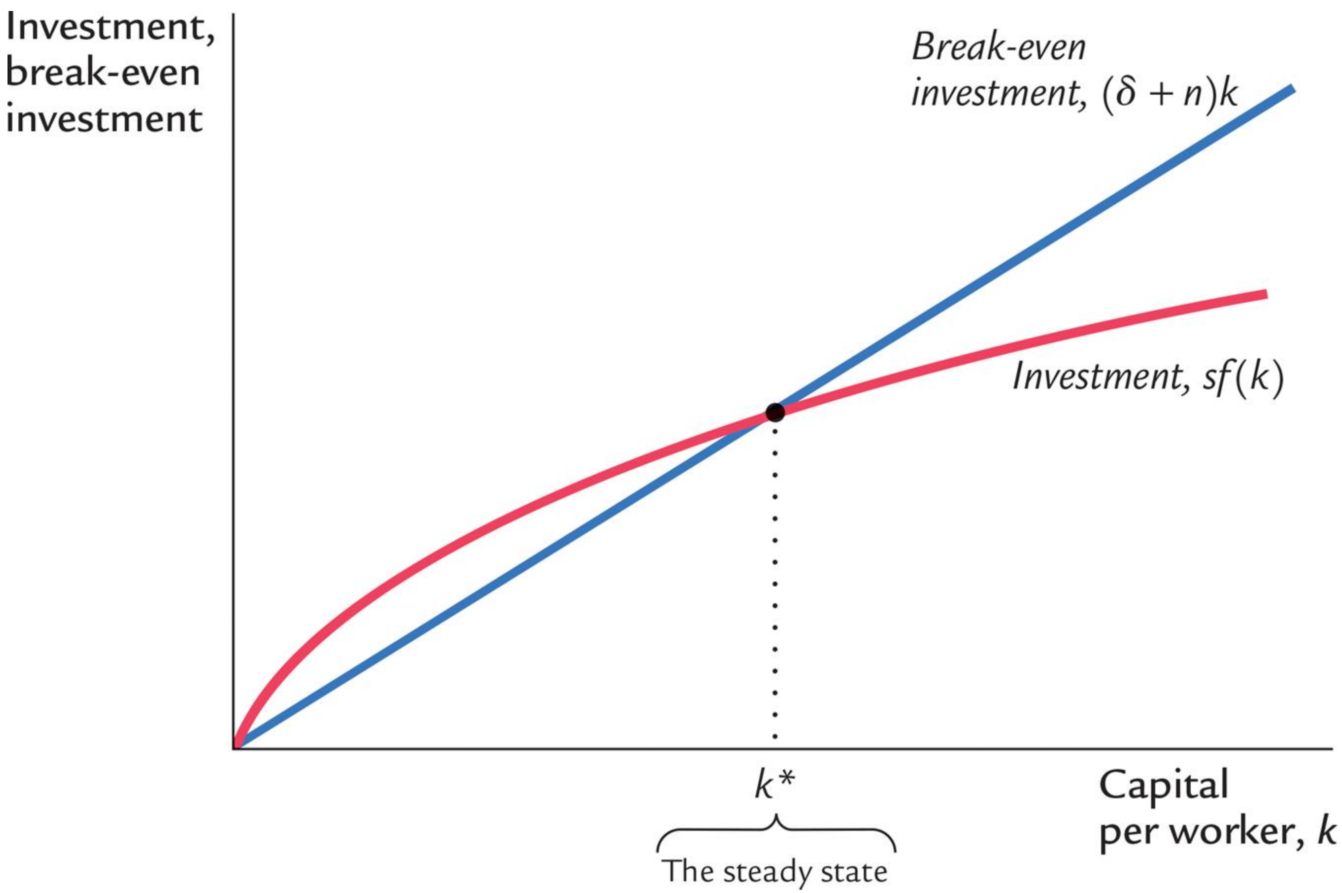


Figure 8-12: The impact of population growth

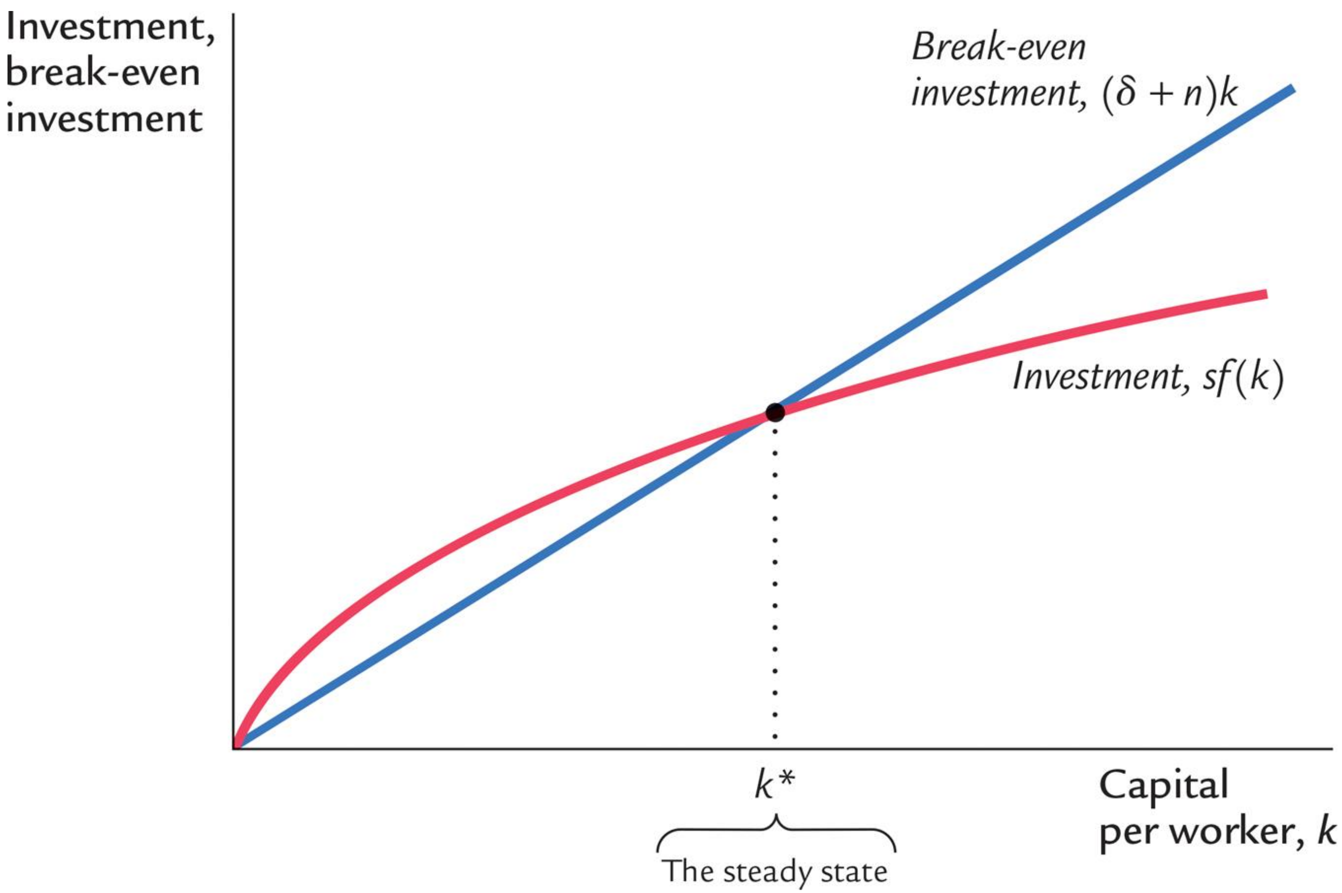
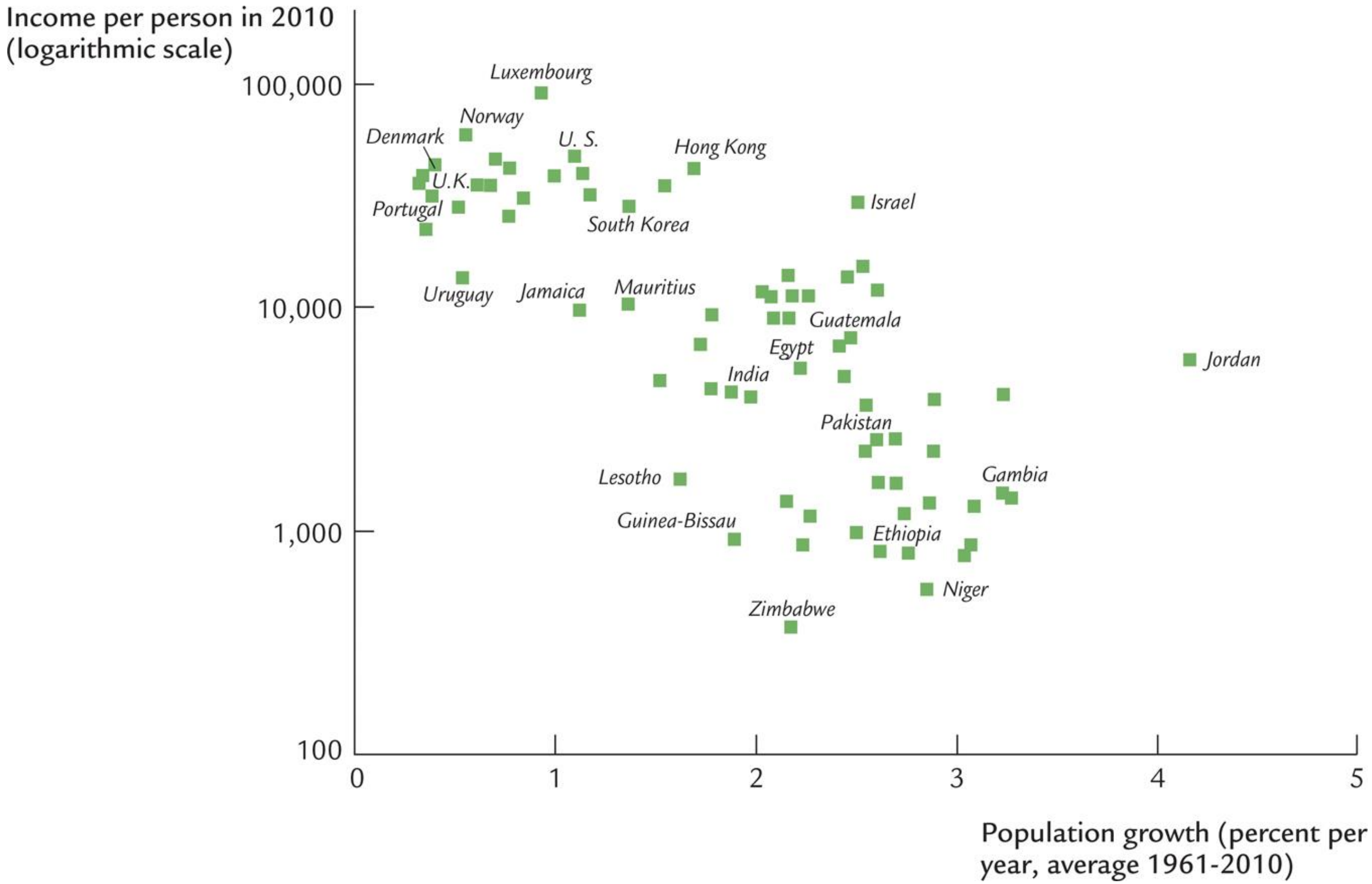


Figure 8-13: International evidence on population growth and income per person



A steady state with population growth

$$Y = F(K, L)$$

$$\frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

In a steady state, $k = K/L$ is constant. Because

$$\frac{\Delta k}{k} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} = 0,$$

We have

$$\frac{\Delta K}{K} = \frac{\Delta L}{L} = n$$

$$\because \text{ är } \frac{\Delta Y}{Y} \approx \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} = \alpha n + (1-\alpha)n = n$$

GDP growth = Population growth

Golden rule with population growth

$$c = y - i = f(k) - (\delta + n)k$$

Consumption per capita is maximised if *MPK* = $\delta + n$, i.e. if the marginal product of capital equals the sum of the depreciation rate and population growth

Alternative formulation: **The net marginal product of capital after depreciation (*MPK* – δ) should equal population growth (*n*)**

Mathematical derivation

Differentiation of c-function w.r.t *k* gives:

$$\partial c / \partial k = f_k - (\delta + n) = 0$$

$$f_k = \delta + n$$

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Alternative perspectives on population growth

1. **Malthus (1766-1834)**
   - **population will grow up to the point that there is just subsistence**
   - **man will always remain in poverty**
   - **futile to fight poverty**

2. **Kremer**
   - **population growth is a key driver of technological growth**
   - **faster growth in a more populated world**
   - **the most successful parts of the world around 1500 was the old world (followed by Aztec and Mayan civilisations in the Americas; hunter-gatherers of Australia)**

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<u>Labour-augmenting technical progress</u>

$$Y = F(K, L \bullet E)$$

$$E = \text{labour efficiency}$$

$$L \bullet E = \text{efficiency units of labour}$$

$$y = \frac{Y}{LE} = F(\frac{K}{LE}, 1) = F(k, 1) = f(k)$$

$$k = \frac{K}{LE}$$

<u>Steady state</u>

L grows by n % per year

E grows by g % per year

$\Delta k = sf(k) - (\delta + n + g)k = 0$

Gross investment = Depreciation + Reduction in capital intensity because of population growth + Reduction in capital intensity because of technological progress

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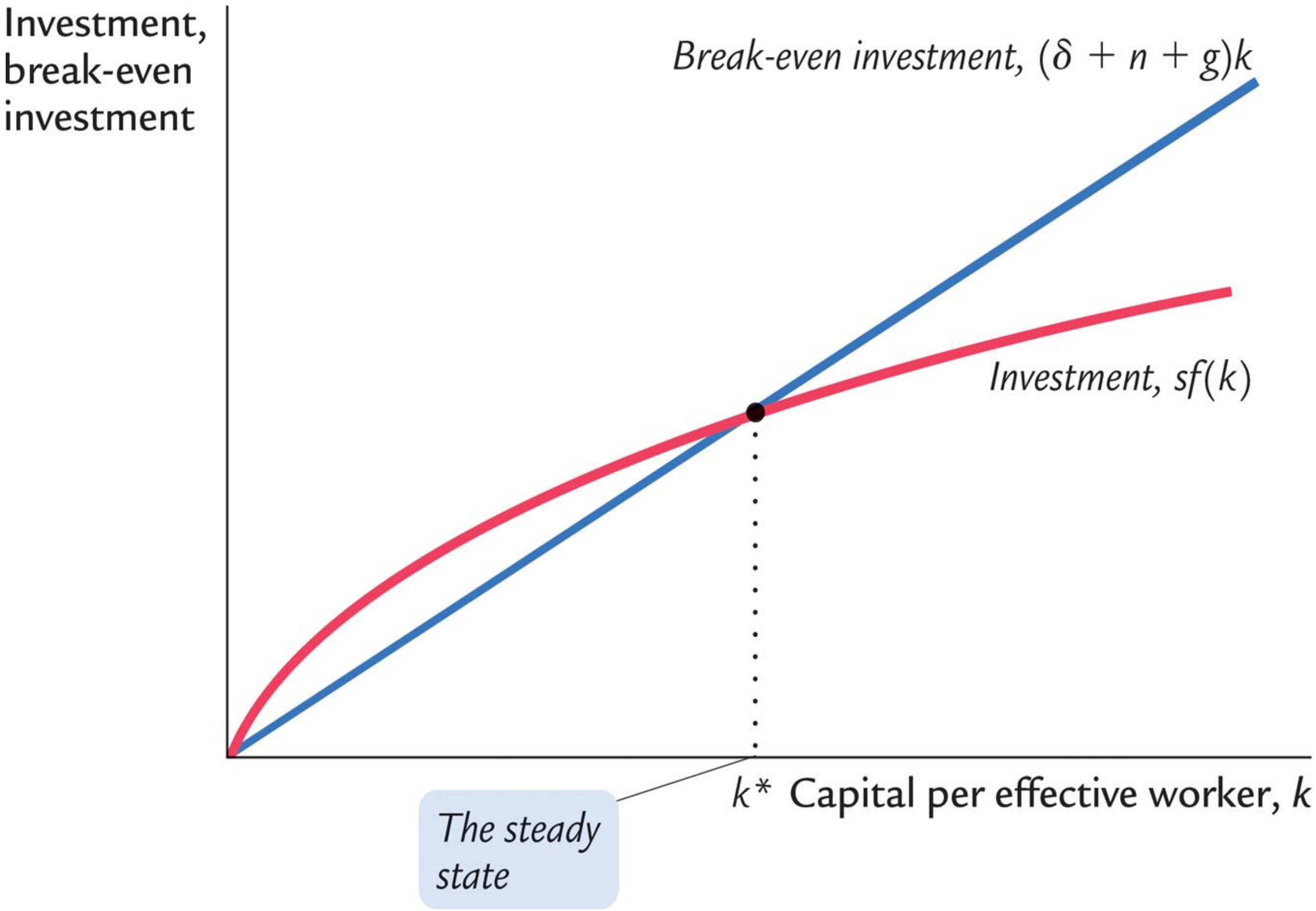
<u>Figure 9-1: Technological progress and the Solow growth model</u>



Growth and labour-augmenting technological progress

$$Y = K^{\alpha}(LE)^{1-\alpha}$$

$$\frac{\Delta Y}{Y} \approx \alpha\frac{\Delta K}{K} + (1-\alpha)(\frac{\Delta L}{L} + \frac{\Delta E}{E})$$

In a steady state *K/LE* is constant

$$(\Delta L/L + \Delta E/E) = n + g \Rightarrow \Delta K/K = n + g.$$

$$\frac{\Delta Y}{Y} \approx \alpha(n + g) + (1-\alpha)(n + g) = n + g$$

GDP growth = population growth+ technological progress

$$\frac{\Delta y}{y} \approx \frac{\Delta Y}{Y} - \frac{\Delta L}{L} = n + g - n = g$$

Growth in GDP per capita = rate of technological progress

Table 9-1: Steady-state growth rates in the Solow model with technological progress

Variable	Symbol	Steady-State Growth Rate
Capital per effective worker	$k = K/(E \times L)$	0
Output per effective worker	$y = Y/(E \times L) = f(k)$	0
Output per worker	$Y/L = y \times E$	g
Total output	$Y = y \times (E \times L)$	$n + g$

Golden rule with technological progress

$c = f(k) - (\delta + n + g)k$

Consumption per efficiency unit is maximised if

$MPK = \delta + n + g$

The marginal product of capital should equal the sum of depreciation, population growth and technological progress

Alternative formulation: **The net marginal product (*MPK* - δ) should equal GDP growth (*n* + *g*).**

Mathematical derivation

Differentiation w.r.t. *k*:

$$\partial c / \partial k = f_k - (\delta + n + g) = 0$$

Golden rule with technological progress, cont.

$$f_k = \delta + n + g$$

Real world capital stocks are smaller than according to the golden rule. The current generation attaches a larger weight to its own welfare than according to the golden rule.

Table 2.1: World growth since the industrial revolution			
Average annual growth rate	**World output**	World population	Per capita output
0-1700	**0.1%**	0.1%	0.0%
1700-2012	**1.6%**	0.8%	0.8%
incl.: 1700-1820	***0.5%***	*0.4%*	*0.1%*
1820-1913	***1.5%***	*0.6%*	*0.9%*
1913-2012	***3.0%***	*1.4%*	*1.6%*

Between 1913 and 2012, the growth rate of world GDP was 3.0% per year on average. This growth rate can be broken down between 1.4% for world population and 1.6% for per capita GDP.

Sources: see piketty.pse.ens.fr/capital21c.

Endogenous or exogenous growth

- **In the Solow model growth is exogenously determined by population growth and technological progress**
- **Recent research has focused on the role of human capital**
- **A higher savings rate or investment in human capital do not change the rate of growth in the steady state**
- **The explanation is decreasing marginal return of capital (*MPK* is decreasing in *K*)**

The AK-model

$Y = AK$

$\Delta K = sY - \delta K$

Assume *A* to be fixed!

$\Delta Y/Y = \Delta K/K$

$\Delta K/K = sAK/K - \delta K/K = sA - \delta$

$\Delta Y/Y = sA - \delta$

A higher savings rate *s* implies permanently higher growth

Explanation: constant returns to scale for capital

Complementarity between human and real capital

A two-sector growth model

- **Business sector**
- **Education sector**

$Y = F\,[K, (1-u)EL]$ **Production function in business sector**

$\Delta E = g(u)E$ **Production function in education sector**

$\Delta K = sY - \delta K$ **Capital accumulation**

u **= share of population in education**

$\Delta E/E = g(u)$

- **A higher share of population, *u*, in education raises the growth rate permanently (cf *AK*-model – here human capital)**
- **A higher savings rate, *s*, raises growth only temporarily as in the Solow model**

More about growth

- **Human capital is crucial for growth**

  - **strong empirical regularity**
  - **educational level**
  - **R & D expenditure**

- **Free trade appears to promote growth**

  - **comparison between open and closed economies**
  - **effects after trade liberalisations**
  - **other factors?**

- **Industrial policy**

  - **good if technological externalities and if the government can identify them**
  - **but can governments do this?**

- **Institutions**

  - **legal protection for shareholders and creditors leads to better functioning capital markets**
  - **quality of government: protection of property rights or "grabbing hand"**

The importance of geography

- **Direct negative effect of tropical conditions on productivity**
- **Impact of geography on institutions (Acemoglu, Johnson and Robinson)**
- **During colonial times Europeans settled in non-tropical areas - legal systems protecting individual**
- **Instead "extractive institutions" in tropical areas Strong path dependence of institutions**