Question 1

Intuition of Proof A

(i) Let us start with some q which can be produced at minimal costs with some input vector $z \in Z^0$. Another way of saying this is: $\phi(z) = q$ and $C(p,q) = \sum_i p_i \cdot z_i$. This is without any loss of generality.

(ii) We know that the average cost of producing q is $AC(q) = \frac{\sum_i p_i \cdot z_i}{\phi(z)} = \frac{C(p,q)}{q} = \frac{t \cdot C(p,q)}{t \cdot q}$. This holds **exactly**, cause that is how we defined z in (i).[1]

What we now need to do, is to take some bigger output than q, and see whether we can produce it at a lower AC.

(iii) Starting from z, let us multiply all inputs by a factor t, $t > 1$. We can call the output that we can produce with this new input vector $\hat{q} \equiv \phi(tz)$. This $\hat{q}$ is indeed bigger than q.

Remember, the question we try to answer is: What is the AC of producing $\hat{q}$.

(iv) We know we can produce $\hat{q}$ for sure with the inputs tz and hence at cost $\sum_i tp_i z_i = t \cdot C(p,q)$.

(v) But now there are **two reasons** which together allow us to believe that **with increasing returns to scale** the average costs of producing $\hat{q}$ are lower than those of producing q. Firstly, while zt allows us to produce $\hat{q}$, there might actually be a cheaper way of producing $\hat{q}$ with another input combination! So $C(p,\hat{q}) \leq t \cdot C(p,q)$.[2] And secondly, **by the increasing returns to scale property** the output that we produce, $\hat{q}$, is actually strictly bigger than tq.[3] Hence if you calculate the average cost of producing $\hat{q}$

$$AC(\hat{q}) = \frac{C(p,\hat{q})}{\hat{q}}$$

and compare it to the $AC(q)$ we see that the nominator is now at least not bigger (possibly smaller) and the denominator is certainly bigger. Hence $AC(q) > AC(\hat{q})$.

Why does this not work for Proof B?

Many of you tried to apply the exact same logic as above to the second part of the question. However, this does not work. The problem is in the last step, (v). Indeed the first statement still holds, the numerator might be smaller. But the second statement is now reversed: The denominator of $AC(\hat{q})$ is now certainly smaller. But then we cannot conclude on the overall effect for the average costs.[4]

And how does Proof B work?

Start the same way as above in (i)-(ii) and then say: *What we now need to do, is to take some* ***smaller*** *output than q, and see whether can produce it at a lower AC.* You can then choose some input bundle $\bar{z} = \frac{1}{t} z$ and define the output as $\bar{q}$. You can then argue that this input combination costs exactly one t^{th} of the minimum cost of producing q, but (a) you might produce $\bar{q}$ at lower costs then by using $\bar{z}$ and (b) the output $\bar{q}$ is strictly bigger than one t^{th} of q. Hence $AC(\bar{q}) < AC(q)$ and since $\bar{q} < q$ this means the average costs are strictly increasing.[5]

[1] A common mistake was to write the cost of producing q might be smaller than $C(p,q)$.

[2] Note that this might or might not be strictly true.

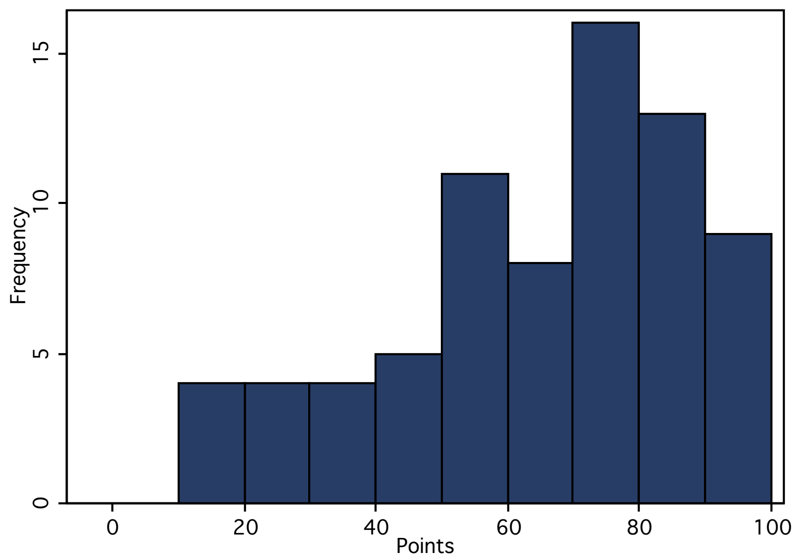
[3] Intuitively, it goes up by more than the cost of producing $\hat{q}$ compared to the cost of producing q for sure. Note that strictly speaking we require that both z and tz are in Z^0, cause otherwise the property might not hold given the information in the question.

[4] A common mistake was to believe that the first statement was reversed now.

[5] Note: Just like the first proof did not work for part B, this proof does not work for part A. Can you see why?

Point Distribution

If you are interested, here is the distribution of points. I have tried to mark as I would mark an exam and I guess my standards are (if anything) not particularly harsh. I do not say this to scare you; and neither do I post the graph to this end! The only purpose is for you to get a realistic idea of how the grading is done. If you find yourself below the median, do not get worried (at least not more than any student at LSE), there are tons of reasons for optimism: (a) you have a lot of time, (b) I think the first question is probably harder than what you see on an exam, (c) you might know a lot and only need to work a bit on how to present it[6] and (d) below median is not necessarily bad given your peer group![7]



[6] I have tried to give some hints in the homeworks, you should work on these points.
[7] But if you get less than 45, that should be a motivation to sit down and go through what we have been doing again.