Question 3.2

First of all: No doubt this is one of the hardest (if not the hardest) question you will encounter on monopolies in this course. But you'll see: *The basic logic is exactly the same as in any easier question on monopolists.*
The complication arises because the firm does not seem to be a monopolist in the sense you are used to. In fact, there are other firms here. However, importantly these firms are not producing the exact same product! So each firm is a monopolist in the market for it's specific product.[1] We are told that the firms are aware of this (so they are **not** price takers!) and take into account the effect their output quantity has on the price. If you look at the demand function given, you see that - as usual - a higher output is purchased only at a lower price. The only thing that makes things seem a little more complicated is that the other firms' outputs of their specific goods influence the demand for our firm's product as well.[2] But this does not really complicate our analysis: As our firm cannot do anything about the quantities produced by the other firms, we can think that it takes them as given, so they are just like a constant in the firm's decision problem.

Part (1) - Derive q_i^*

Now let us try to solve the question. What would the firm do? As always, we would think that it maximised profits

$$\Pi_i = p_i(q_i)\, q_i - C(q_i)$$

Note that as the firm is not a price taker, the price is not taken as constant. Rather the firm takes the effect of it's output on the price achievable in the market into account. We can just plug in the info from the question to find the profit function in the example to be

$$\Pi_i = \frac{A{q_i}^{\alpha}}{q_1^{\alpha} + \ldots + {q_i}^{\alpha} + \ldots + q_n^{\alpha}} - C_0 - c q_i$$

How would you find for which q_i profits are maximised? Exactly, just take the FOC. Two things that should not get you confused: (i) the firm can not choose over the other firm's quantities, so they are just constants here and (ii) q_i appears at three places in the profit function. The FOC is:

$$\frac{\partial \Pi_i}{\partial q_i} = \frac{A\alpha q_i^{\alpha-1}}{K} - \frac{A\alpha q_i^{2\alpha-1}}{K^2} - c = 0$$

where $K = \sum_{j=1}^{n} q_j^{\alpha}$.
What would you normally do now? 'Just' solve for q_i, right? You could try to do this here, but it would be very tedious, or actually impossible. This is where the hint "using the symmetry of the equilibrium" is useful. The idea is the following: Since all firms face the same cost function and same type of demand, why should they produce different quantities in the end?[3] The question tells you to believe that indeed the final output will be the same for all firms, say q^*. But this simplifies our algebra greatly! We have $K = nq^{*\alpha}$ and the FOC can be rewritten and solved for q^*.

$$q^* = \frac{A\alpha(n-1)}{n^2 c} \tag{1}$$

Part (1) - Elasticity of demand

Frank likes these elasticity questions, practise them! The point is most of the time to know the different ways of how to calculate the elasticity (here of demand). Here they are:

[1] Sometimes the different products produced by the different firms are referred to as 'varieties'.
[2] A way to motivate this demand function is to think that the consumers want to purchase a basket of different goods as diverse as possible, but they would purchase less of those goods which are more expensive.
[3] This is not a very rigourous idea, as there are many economic applications where despite the same initial conditions the final outcome is asymmetric.

$$\frac{d\log q}{d\log p} \approx \frac{1}{q}\frac{1}{\partial \log p/\partial q} = \frac{p}{q}\frac{1}{\partial p/\partial q} = \frac{\partial q/q}{\partial p/p} \equiv \eta$$

What is important for us is that $\frac{1}{\eta} = q\frac{\partial \log p}{\partial q}$ cause we can rewrite the demand function as

$$\log p_i = \log A + (\alpha - 1)\log q_i - \log \sum q_j^\alpha$$

and find

$$\frac{\partial \log p_i}{\partial q_i} = (\alpha - 1)\frac{1}{q_i} - \frac{1}{\sum q_j^\alpha} \cdot q_i^{\alpha-1} \cdot \alpha$$

Using the assumption that in equilibrium all q_j are the same and the formula for $1/\eta$ you can find the stated elasticity.

Part (2)

Let me be clear: I think this is a very difficult question. But interesting, too! It asks you to determine the number of firms in the industry. The economists answer to solve this is the following idea: As long as there are economic profits to be made, we would expect firms to enter the market. But if firms enter the market, competition will drive the profit per firm (and the total profit) down.[4] Once there are no more profits, no more firms will enter. So the question is: How many firms need to be in the market for economic profits to be zero?
Now the answer can be found easily: Just write down the profits a firm makes at the optimal choice of q_i (plug in the solution in (1)), set this to 0 and solve for n.

Motivation/Use

What you have just gone through, is part of the seminal model of Dixit and Stiglitz (1977)[5] of monopolistic competition. It is widely used in lots of economics today, in fact in most of the stuff that is called 'new', like 'new Keynesian macroeconomics' or 'new trade theory'. The basic reason is that it is an easy (compared to some other stuff) and tractable way to model what seems to characterise many economies: There are many firms, they produce a specific product, have some monopoly power in this market, but their monopoly power is limited by the consumers ability to buy more of some of the other monopolists when the price increases. Acknowledging this market structure allowed economist to explain many phenomena which could hardly or not at all be explained if one believes all markets were perfectly competitive.

[4] Note why this happens, at least in our case: As an additional firm enters the market, the consumers will want to buy some of its output variety - and this will be at the expense of how much they buy of the other goods. Mathematically you see that the demand curve shifts inwards as the sum in the denominator increases. This will shift the marginal revenue curve inwards. The cost structure however stays the same. Look at the graphs in the book on monopolistic competition to see who profits are driven to zero.

[5] Dixit, A. K., and J. E. Stiglitz (1977) "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67(3), 297-308.