Question 4.6(2)

The question translates into mathematics as: Find $\frac{\partial H^i}{\partial p_j}$.

Why do we identify the substitution effect with the change in the Hicksian/compensated demand? If the price of good j goes up, this will (possibly) change the amount the consumer purchases of good i and this change (the total effect of a change in p_j) is captured by $\frac{dD^i}{dp_j}$. In that way the **Marshallian** demands have an observable real-life analogue. But if you think of why the consumer changes the amount purchased, there are two things we can distinguish:

(i) Firstly, good j became more expense relative to the other goods and therefore we would by less of it and more of the other goods. This effect is called the substitution effect and it captures how the consumer changes his consumption choice *just because of the different price ratio*, but abstracting from the fact that he can afford less. The **Hicksian** demand provides this answer, as it does exactly this thought experiment: How does the consumer choice depend on prices, holding the utility achieved constant. It is because of this thought experiment that economists find the Hicksian demands interesting.

(ii) The second thing that happens is that the consumer can afford fewer consumption bundles, in a sense he gets less wealthy. Therefore he will buy less of all goods. This is the income effect, but this is not the point of the question.

How do we find $\frac{\partial H^i}{\partial p_j}$? The answer can be found from realising that

$$H^i(p,v) = D^i(p, C(p,v))$$

Why does this hold? The left-hand side answers: How do you achieve utility level v at the least cost. The answer is "Find the isocost line such that it is tangential to the indifference curve. Where they touch is the optimal consumption bundle (Hicksian demand)." The right-hand side says: Suppose we take the answer from the left-hand side and calculate the cost of achieving utility v. Then lets suppose we have exactly this as income and try to maximise utility. Where would we do so? The answer is: "Find the utility curve that is tangential to the isocost line. Where they touch is the optimal consumption bundle (Walrasian demand)." As the isocost line *is the same in both parts*, the answer is the same in both parts, or: The above identity holds.[1]

But from the above identity we can now find

$$\frac{dH^i(p,v)}{dp_j} = \frac{\partial D^i}{\partial p_j} + \frac{\partial D^i}{\partial y}\frac{\partial C(p,v)}{\partial p_j}$$

where we know that $\frac{\partial C(p,v)}{\partial p_j} = x_j^*$ by Shepard's Lemma and the missing information on the right-hand side can be plugged in from the results in part 1 of the question.

Question 4.6(3)

We know that **all** *compensate* and *uncompensated demand functions* derived from expenditure minimisation or utility maximisation exhibit certain properties. So in particular the demand function in the exercise will have these properties if they are derived from a utility maximising problem. Reversely, if they do not have these properties, we can conclude that they are not derived from a utility-maximisation or - stated otherwise - are not consistent with utility-maximisation. So which are the properties we might check?!

We know that:

[1] Note that this argument does not depends on the utility function being smooth or there being no corner solution. It is very general.

- $D(p, y)$ is homogeneous of degree 0 in (p, y). So if we calculate $D(tp, ty)$ this should turn out to be equal to $t^0 \cdot D(p, y) = D(p, y)$. Note that this makes intuitive sense if you think of for example a currency reform of 10:1.
- $\sum_{i=1}^{n} D^i(p, y) = y$, since otherwise you are either spending more than your budget (so you do not satisfy the budget constraint) or you do not spend all of your budget (so you do not maximise utility).
- $H_i^i(p, v) < 0$. This says that the own-price substitution e?ect is always negative. It can be proven from the concavity of the expenditure function (second derivative < 0) and ShepardÕs Lemma.
- $H_j^i(p, v) = H_i^j(p, v)$.

We know $D(p, y)$ from the question. $H_j^i(p, v)$ and $H_i^i(p, v)$ we have calculated in 4.6(2) using the Slutzky decomposition. Given those you can check whether all conditions are satisfied.

