Comments on Homework No. 4

Below you find some comments about common mistakes in the 4th hand-in exercise and recommendations on how you could better solve the problems in an exam situation. While they are intended to give you a hint on how to structure the answer and what are important points in answering the question, please note that I don't provide any solution. Neither have these notes been checked by Abhinay or Frank.

1 Exercise 1 / PBE games

In all "job market signaling" questions we have treated in this course, you were always ask to find conditions for the existence of either a pooling or a separating equilibrium. I think all of you understand what this vocabulary means. Now, the problem with those questions is, that while solving them can get a little confusing. But if you stick to the structure we developed in class, I think you can avoid getting confused. Just go through it step-by-step.

1.1 Structure of Signaling Games

So what is the basic structure of those games? There is one player who moves first (player 1) by choosing some strategy, and this strategy might depend on his type/quality [1]. Then there is a second player (player 2), who moves *after* player 1. He knows what strategy a player 1 of a certain type plays[1] but can only observe the action of player 1 and **not** his type. However, the only thing that interests him is player 1's type (and not player 1's action).[2] Therefore player 2 tries to infer from the observed action of player 1 on the type of player 1 (i.e. "forms believes" [2]). Given what he believes player 1's type to be, player 2 then chooses his optimal action [3] (e.g. offer a wage).

So far, sort of ok. Now what confused so of you, is to understand what an equilibrium is in such a game. As usual, an equilibrium will be a set of strategies of all players such that, given the others strategies, nobody can profitably deviate. All player are, player 1 of some type (say "type A"), player 1 of the other type (type B) and player 2. Note that to find players 2's optimal response given player 1's strategy is usually easy. We just need player 2's believes about player 1's type and then it is usually easy to calculate e.g. the wage offer (pay expected marginal product). The difficult part is the optimal strategy of player 1! Cause if he changes his strategy, he will need to take into account how this will effect the believe player 2 will form and consequently the wage player 2 will offer to him.

1.2 4 Steps to find the equilibria

Now in principle we could try to figure out all equilibria of such a game, as we do e.g. in a simple 2-by-2 normal form game. However, this is sometimes not easy and not what we did when we had exercises on this type of games. Instead *we focus on certain types of equilibria*, where we classify the equilibria by how the player 1 s' strategies related to each other. In particular we searched for equilibria where both types of player 1 choose the same strategy ("Pooling Equilibrium") or where they choose distinct strategies that will never lead to the same action ("Separating Equilibrium").

The way to find the above equilibria is fairly straight forward.

[1] Firstly we know something about the strategy of player 1s of type A and of type B. E.g. if we search for the strategies which sustain a pooling equilibrium, we know for sure that the strategies of

[1] So if player 1's strategy is pure, he knows the action player 1 will take, given player 1's type.

[2] So in the job-market example, the only thing that affects the employers pay-off is workers type/talent. His education is only interesting in so far as it signals something about his type. Obviously you might find it more realistic to say that both the talent and the education matter to the employer, and we could think of some game where both the type and the action matter for the pay-off of the employer, but this would not be a *pure* signaling game.

player 1 of type A and of type B need to be the same (otherwise it's not a pooling equilibrium).

[2] Given [1] allows us to look for the optimal response of player 2. Since this will depends on what he believes the type of player 1 to be, he will first need to form believes. For all equilibrium actions, he forms his believes using Bayes Rule[3]. The out-of-equilibrium believes you - as the modeler - can choose as you. You might not find this sensible, and others don't either, but it's the concept of a PBE. However, while you are free to pick the believe, **it is very important to specify *some* believe for *all* out-of-equilibrium actions!**[4] Many of you did not specify the out-of-equilibrium believes or they were not complete.

For example, if you want to find a separating equilibrium in the job market signaling game and e.g. high types pick $e^H = 4$ and low types pick $e^L = 2$,[5] forming the equilibrium believes is easy: If you see somebody with 4 years of education he is a high type and if you see somebody with 2 years of education, he is a low type. But now it is important that you specify as well what player 2 will believe about the type of player 1 if he sees $e < 2$ **and** when he sees $2 < e < 4$ **and** when he sees any $e > 4$. Sure, if $e^H = 4$ and $e^L = 2$ turn out to be a separating equilibrium you will never see the other e's as employer. Still it's important to think about what you would think if you saw them. You'll see in [4] why! Again, you are free to say anything here (but say it!). If you feel like (and just to clarify the point, thought obviously it would be a bad idea to complicate things in the exam), you might assume if player 2 sees $e < 4$ he believes it's a low type, if he sees $4 < e < 4.7$ he believes it's a high type, when he sees $e = 4.7$ he believes it's a low type and when he sees $e > 4.7$ he believes it's a high type. Important is only that you really specify believes for **all** out-of-equilibrium actions.[6]

[3] Now we can find the optimal response of player 2. Here again, a full strategy profile needs to specify some action not only for equilibrium actions of player 1, but as well for **all** out-of-equilibrium actions. In the example, you will need to find the optimal wage for all possible levels of education you might see, so not only 2 or 4, but as well for all $e \neq 2, 4$. Many did not do this.

To summarize till here, now we know player 2's optimal response, given that both player 1's play a pooling/separating strategy. The last thing we will make sure is that given player 2's strategy (which depends on his believes), no player 1 will want to deviate.

[4] So what we need to check is, given player 2's strategy and given player 1/type B's strategy, when will player 1/type A not want to deviate? And given player 2's strategy and given player 1/type A's strategy, when will player 1/type B not want to deviate? This will give us conditions under which both don't want to deviate and hence under those conditions nobody will want to deviate, so it's an equilibrium.

Note how the out-of-equilibrium believes play an important role here: In our example, to check that e.g. the high type does not want to deviate, we need to check he is not better of playing **any** $e \neq 4$. So not 2 and no other either! But to check this we need to know which wage he would get for any other e, which we found in [3]. And to find the out-of-equilibrium wage offers in [3] we first needed the out-of-equilibrium believes in [2].

[3](Don't worry about it, we only use a simple version of it.)

[4]Note, sometimes, in order to find an equilibrium, it is necessary to pick the "right" out-of-equilibrium believes. Roughly, in order to find e.g. all pooling equilibria, you should pick the out-of-equilibrium believes which make a deviation from the equilibrium strategy least profitable.

[5]Note that I only picked $e^L = 2$ to make the point about out-of-equilibrium believes. In the question we saw, in a separating equilibrium the low type would in fact never pick $e^L = 2$ cause he would rather prefer $e^L = 0$.

[6]And for the equilibrium actions obviously as well.

2 Exercise 2 / Moral Hazard

This question asks which wage scheme a risk-neutral principle would offer to a risk-neutral agent under different informational settings.

2.1 Exercise 2, b)

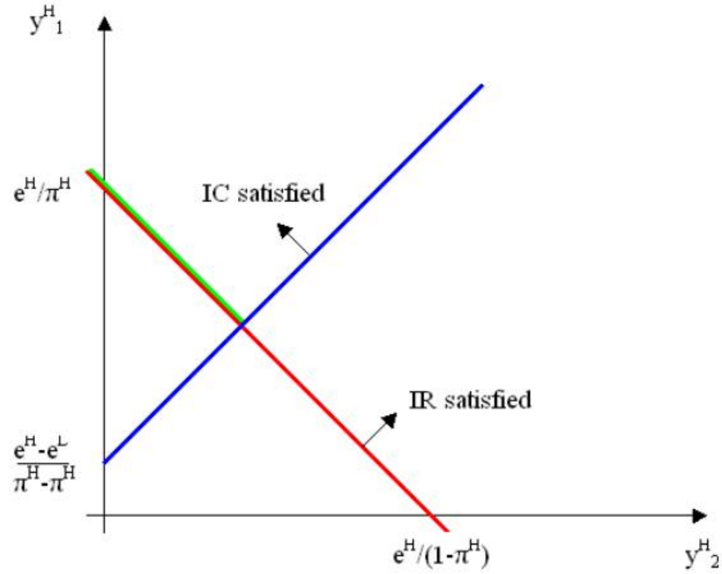
In b) most of you got right that under full information there is no IC, so only the IR needs to hold. Most of you as well argued, that to minimize the wage payment, it needs to hold with equality. However, some of you did assume the wage payment to be fixed. That's not right. Cause it exactly assumes the answer that we are interested in! If you recall what we were doing before, we had a risk neutral principle and a risk-averse agent and found the wage scheme to implement high effort to have some incentive pay and some insurance. The optimal wage scheme to implement low effort we found to be flat. Equivalently we found some optimal wage scheme if the agent is risk neutral and the principle is risk averse. Similarly in this question we are interested in the optimal wage schemes! And by assuming it to be flat one assumes the answer! (Moreover an incomplete one.)

Instead you should start from the principle offering some y_1^H and y_2^H if he contracts high effort and some y_1^L and y_2^L if he contracts low effort. Since he is risk-neutral he will then minimize the expected wage payments subject to the agent participating - which he does exactly when the IR is binding. This gives you a continuum of (y_1^{H*}, y_2^{H*}) and (y_1^{L*}, y_2^{L*}) which optimally implement high or low effort, respectively.

2.2 Exercise 2, c)

In c), with asymmetric information about the effort, you now need the IC constraint to ensure high effort. The principle will obviously still wants to minimize his expected wage payments - just now he does so subject to the IC constraint. From b) we already found a continuum of (y_1^{H*}, y_2^{H*}) which minimize his expected wage payments (and make the agent participate). But will the IC be satisfied for them? Well, for some of them: yes. We just need to pick a y_1^{H*} high enough (and y_2^{H*} then low enough) and at some point the IC will be satisfied. All (y_1^{H*}, y_2^{H*}) from b) with y_1^{H*} bigger than this will will minimize the expected wage payments for the principle and will satisfy the IR (with equality, actually) and the IC.

The below graph (adapted from Neel) visualizes this. All contracts right to the red line satisfy the IR, all on the red line satisfy the IR with equality. Those on the red line are the solutions to implement e^H in part b). The blue line is the IC from part c). All contract above and on the blue line satisfy the IC. The green line then shows the solution to c). Note that all lines extend to the area where $y_1^H < 0$ or $y_2^H < 0$.



y^H_1
e^H/π^H
IC satisfied
IR satisfied
$\frac{e^H-e^L}{\pi^H-\pi^H}$
$e^H/(1-\pi^H)$
y^H_2