EC402 - Problem Set 2

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4th of February 2009

Introduction

Last time we talked about the difficulties to satisfy any version of **A3** with non-experimental data.

Today we talk about how even with experimental data it is difficult to estimate some well-defined treatment effect. In particular we will (sensibly) assume that there are **heterogeneous treatment effects** and **no perfect compliance**.

We will talk about different quantities you might want to estimate (ATE, TOT), how they relate to what you actually can estimate (OLS, ITT, IV/LATE) and how this depends on the type of compliance.

1A. Using OLS

With **heterogeneous treatment** effects, what will the simple OLS estimate of a regression of the outcome variable on the received treatment status give?

First note that the OLS estimate with a constant gives the same coefficient as the OLS estimate when using the data in mean-deviations form and skipping the constant coefficient. Let us do the later, for notational simplicity.

$$
\begin{aligned}
y_i &= \alpha_i R_i + \epsilon_i = \bar{\alpha} R_i + \underbrace{\epsilon_i + (\alpha_i - \bar{\alpha_i}) R_i}_{\epsilon_i^*} \\
\hat{\alpha}_{\text{OLS}} &= \bar{\alpha} + (R'R)^{-1} R' \epsilon^* = \bar{\alpha} + \frac{\sum_{R_i=1} \epsilon_i}{T_R} + \frac{\sum_{R_i=1} (\alpha_i - \bar{\alpha})}{T_R} \\
E[\hat{\alpha}_{\text{OLS}}] &= \bar{\alpha} + E[\epsilon_i | R = 1] + E[\alpha_i - \bar{\alpha} | R = 1]
\end{aligned}
$$

Conclusion: So receiving treatment needs to be mean independent of ϵ_i **and** of the size of the treatment effect. With random assingment and perfect compliance this will be the case. However, generally there won't be perfect compliance and hence a simple OLS estimate is biased.

1B. Our Example: Compliance an issue for us?

```
. tab t_random t_final, row
```

Key
frequency
row percentage

Police Sheet Color	Final Disposition arrest	advise	suspect t	other	Total
pink	91 97.85	0 0.00	1 1.08	1 1.08	93 100.00
yellow	19 17.27	84 76.36	5 4.55	2 1.82	110 100.00
blue	26 20.47	5 3.94	83 65.35	13 10.24	127 100.00
Total	136 41.21	89 26.97	89 26.97	16 4.85	330 100.00

'Coddle' is defined as 'treatment' here (though that seems a little counter-intuitive). Hence we have few **'always takers'**, but quite some **'never takers'**.

1C. Our Example: Consequence of non-compliance for OLS?

```
. tab reason2
```

Reason for Not Complying with Random Assignment	Freq.	Percent	Cum.
blank	313	94.85	94.85
party assaults police officer	1	0.30	95.15
victim makes citizen's arrest	1	0.30	95.45
injury constitutes an aggravated assaul	5	1.52	96.97
victim has order of protection against/	1	0.30	97.27
other	4	1.21	98.48
unknown	5	1.52	100.00
Total	330	100.00	

We see that there are quite some 'never-takers'. Why do they not seem to get the coddle treatment? What does this mean for $E[\alpha_i - \bar{\alpha}|R = 1]$ and hence the bias of OLS as estimate of ATE?

2A. Understanding ITT vs ATE

$$
\begin{aligned}
ITT &= E[Y_i|T=1] - E[Y_i|T=0] \\
\\
&= E[Y_i|T=1\&R=1]\cdot(1-p_n) + E[Y_i|T=1\&R=0]\cdot(p_n) \\
&\quad -E[Y_i|T=0\&R=1]\cdot(p_a) + E[Y_i|T=0\&R=0]\cdot(1-p_a) \\
\\
&= E[Y_i|T=1\&R=1] - E[Y_i|T=0\&R=0] \\
&\quad -p_n(E[Y_i|T=1\&R=1] - E[Y_i|T=1\&R=0) \\
&\quad -p_a(E[Y_i|T=0\&R=1] - E[Y_i|T=0\&R=0) \\
\\
&= E[Y_{1i}|T=1\&R=1] - E[Y_{0i}|T=1\&R=1] \\
&\quad +E[Y_{0i}|T=1\&R=1] - E[Y_{0i}|T=0\&R=0] \\
&\quad -p_n(E[Y_i|T=1\&R=1] - E[Y_i|T=1\&R=0) \\
&\quad -p_a(E[Y_i|T=0\&R=1] - E[Y_i|T=0\&R=0)
\end{aligned}
$$

where $p_n \equiv P(R=0|T=1)$ and $p_a \equiv P(R=1|T=0)$.

2A. Understanding ITT vs ATE

Hence generally:

$$\begin{aligned} ITT &= \text{ATE} \mid \text{treatment assigned and perfect compliance} \\ &\quad + \text{selection bias} \mid \text{perfect compliance} \\ &\quad - \text{imperfect compliance bias} \end{aligned}$$

But...

a ... *with perfect compliance* $p_n = p_a = 0$ and we are left with

$$ITT = \text{ATE} \mid \text{treatment assigned} + \text{selection bias}$$

b ... and the 'selection bias' is zero *under randomization of the treatment*, since then $E[Y_{0i}|T = 1] - E[Y_{0i}|T = 0]$. This is the standard argument for randomization.

Note: We want to know ATE, but the only thing we can calculate here is ITT. In our example compliance is not perfect. What does this mean for the size of ITT relative to ATE?

2B. Understanding TOT vs LATE

Another thing we might be interested in is TOT. As opposed to ATE we ask:
'What is the treatment effect on those actually treated?'

$$
\begin{aligned}
TOT &= E[Y_{1i}|R=1] - E[Y_{0i}|R=1] \\
&= (p_r)(E[Y_{1i}|R=1\&T=1] - E[Y_{0i}|R=1\&T=1]) \\
&\quad -(1-p_r)(E[Y_{1i}|R=1\&T=0] - E[Y_{0i}|R=1\&T=0]) \\
&= (p_r)(E[Y_{1i}|R=1\&T=1] - E[Y_{0i}|R=0\&T=0]) \\
&\quad -(1-p_r)(E[Y_{1i}|R=1\&T=0] - E[Y_{0i}|R=1\&T=0]) \\
&= (p_r)(\text{complier treatment effect}) \\
&\quad -(1-p_r)(\text{always taker treatment effect})
\end{aligned}
$$

where $p_r \equiv P(T=1|R=1)$.

Note: None of this we can calculate. But we can use the IV method, which can be interpreted as LATE of the compliers (under the assumption that there are no defiers). How close this is to the TOT depends on whether the assumption is right, how many 'always takers' there are and how big their treatment effect is.

2C. Our example: Which type of compliance problem do we have?

We have very few ‘always takers’, hence we would expect the IV estimate to recover reasonably well the TOT.

3A. An example: ITT

```
. regress  reoffend1  coddle_assigned

      Source |       SS       df       MS              Number of obs =     330
-------------+------------------------------           F(  1,   328) =    4.85
       Model |  .714701115     1  .714701115           Prob > F      =  0.0284
    Residual |   48.376208   328  .147488439           R-squared     =  0.0146
-------------+------------------------------           Adj R-squared =  0.0116
       Total |  49.0909091   329   .14921249           Root MSE      =  .38404

------------------------------------------------------------------------------
   reoffend1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
coddle_ass~d |   .1034436   .0469916     2.20   0.028     .0110006    .1958865
       _cons |   .1075269   .0398233     2.70   0.007     .0291855    .1858682
------------------------------------------------------------------------------
```

3B. An example: ITT with controls

```
. regress reoffend1 coddle_assigned  y82 q1 q2 q3 nonwhite mixed anyweapon s_influence
```

Source	SS	df	MS
Model	1.7384865	9	.193165167
Residual	47.3524226	320	.147976321
Total	49.0909091	329	.14921249

Number of obs =	330
F(9, 320) =	1.31
Prob > F =	0.2330
R-squared =	0.0354
Adj R-squared =	0.0083
Root MSE =	.38468

reoffend1	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
coddle_ass~d	.09811	.0475365	2.06	0.040	.0045865	.1916336
y82	.0132675	.0529572	0.25	0.802	-.0909207	.1174558
q1	-.0368912	.0805851	-0.46	0.647	-.1954347	.1216524
q2	.0529932	.0651508	0.81	0.417	-.0751848	.1811712
q3	-.0292877	.0698676	-0.42	0.675	-.1667456	.1081701
nonwhite	.0276911	.0436444	0.63	0.526	-.058175	.1135572
mixed	-.0198068	.0496336	-0.40	0.690	-.1174563	.0778427
anyweapon	-.0770743	.0489911	-1.57	0.117	-.1734597	.019311
s_influence	.0169292	.0439631	0.39	0.700	-.069564	.1034224
_cons	.1049738	.0768676	1.37	0.173	-.046256	.2562035

3C. An example: OLS

```
. regress reoffend2  coddle_received

      Source |       SS       df       MS              Number of obs =     330
-------------+------------------------------           F(  1,   328) =    6.39
       Model |  .901740724     1  .901740724           Prob > F      =  0.0119
    Residual |  46.2528047   328  .141014649           R-squared     =  0.0191
-------------+------------------------------           Adj R-squared =  0.0161
       Total |  47.1545455   329  .143326886           Root MSE      =  .37552

------------------------------------------------------------------------------
   reoffend2 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
coddle_rec~d |   .1062007    .041997     2.53   0.012     .0235832    .1888183
       _cons |   .1102941   .0322005     3.43   0.001     .0469486    .1736397
------------------------------------------------------------------------------
```

3D. An example: IV/LATE

```
. ivreg  reoffend1  (coddle_received = coddle_assigned)

Instrumental variables (2SLS) regression

      Source |       SS       df       MS              Number of obs =     330
-------------+------------------------------           F(  1,   328) =    4.76
       Model | -.135460603     1 -.135460603           Prob > F      =  0.0298
    Residual |  49.2263697   328  .150080395           R-squared     =       .
-------------+------------------------------           Adj R-squared =       .
       Total |  49.0909091   329   .14921249           Root MSE      =   .3874

------------------------------------------------------------------------------
   reoffend1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
coddle_rec~d |   .1311702   .0601084     2.18   0.030     .0129236    .2494167
       _cons |    .104706   .0412729     2.54   0.012      .023513     .185899
------------------------------------------------------------------------------
Instrumented:  coddle_received
Instruments:   coddle_assigned
------------------------------------------------------------------------------
```

3E. An example: Interpreting the effects

```
. sum reoffend1
```

Variable	Obs	Mean	Std. Dev.	Min	Max
reoffend1	330	.1818182	.3862803	0	1

- **ITT** The effect of 0.103 means an increase in the reoffence rate of 57%.
- **OLS** The effect of 0.106 means an increase in the reoffence rate of 59%.
- **IV** The effect of 0.131 means an increase in the reoffence rate of 73%.

It is reasonable to think that ATE and TOT are of similar size, since the treatment was assigned randomly. Which of the above comes closest to this? What did we say about the biases in ITT, OLS and IV?

4. External validity

Suppose we manage to design the experiment carefully and tackle convincingly the issues in questions 1, 2 and 3 (**internal validity**).

Then the second question is whether the results from the experiment translate to a possible policy (**external validity**). Think about what is different when actually implementing the policy vs. running the experiment.