Correction - Problem Set #1, Q4.b)

We are asked to proof that mean independence of two random variables, $E(x|y) = E(x)$, implies that they are uncorrelated.

For the proof we first need to show that

$$cov(x, y) \equiv E(x - E(x)) \cdot (y - E(y)) = E(x \cdot y) - E(x) \cdot E(y) = 0$$

which then implies that $corr(x, y) = 0$.

To show the above it is sufficient to show that under mean independence $E(x \cdot y) = E(x) \cdot E(y)$. We start out by rewriting $E(x \cdot y)$

$$\begin{aligned} E(x \cdot y) &= \int \int x \cdot y \cdot f(x, y) dx dy \qquad (1) \\ &= \int \int x \cdot y \cdot f(x|y) \cdot f_Y(y) dx dy \\ &= \int y \cdot f_Y(y) \underbrace{\int x \cdot f(x|y) dx}_{E(x|y)} dy \end{aligned}$$

Now, under mean independence $E(x|y) = E(x)$ holds. But $E(x)$ is a constant is the integration over y and can hence be taken out of the integral.[1] Then

$$\begin{aligned} E(x \cdot y) &= E(x) \int y \cdot f_Y(y) dy \qquad (2) \\ &= E(x) \cdot E(y) \end{aligned}$$

which concludes the proof.

[1] Thanks to Sibylle for pointing this out.