Ex 2.4(2) - Elasticity of substitution

I have been asked in some classes whether the answer to question 2.4 (1) changed if we change around the definition of the Elasticity of Substitution. This is just like redefining which input we call z_1 and z_2. Calculating

$$\sigma_{12} = -\frac{\partial log\,(z_2/z_1)}{\partial log\,(\phi_2(\mathbf{z})/\phi_1(\mathbf{z}))}$$

will give the same result as we found in class, $\sigma_{21} = 1$. Note that you obviously need to change around the indices both in the nominator and the denominator!

Ex 2.4(4) - Average cost curve

In some classes I did not have time to work through this. The average cost curve is $\frac{C(w,q)}{q}$. To find whether it is increasing, constant or decreasing, we have to differentiate the average-cost with respect to q and analyze when this expression is bigger, equal or smaller than 0, respectively. The average cost curve in this case and

$$\frac{C(w,q)}{q} = (\alpha_1+\alpha_2)\left[\left[\frac{w_1}{\alpha_1}\right]^{\alpha_1}\left[\frac{w_2}{\alpha_2}\right]^{\alpha_2}\right]^{\frac{1}{\alpha_1+\alpha_2}} \cdot q^{\frac{1}{\alpha_1+\alpha_2}-1}$$

taking the derivative gives

$$\frac{\partial(C(w,q)/q)}{\partial q} = \left(\frac{1-(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}\right)(\alpha_1+\alpha_2)\left[\left[\frac{w_1}{\alpha_1}\right]^{\alpha_1}\left[\frac{w_2}{\alpha_2}\right]^{\alpha_2}\right]^{\frac{1}{\alpha_1+\alpha_2}} \cdot q^{\frac{1}{\alpha_1+\alpha_2}-2}$$

The only potentially non-positive term is $(1-(\alpha_1+\alpha_2)/\alpha_1+\alpha_2)$. This will be positive if $\alpha_1+\alpha_2 < 1$, negative if $\alpha_1+\alpha_2 > 1$ and equal to 0 if $\alpha_1+\alpha_2 = 1$. So we find increasing average cost if the production function exhibits DRTS, we find decreasing average cost if the production function exhibits IRTS and constant average cost if the production function exhibits CRTS. This makes intuitive sense, you should make sure you understand this.

Note that this is a particular property of the production function we analyzed. But there is a general result that with IRTS the *marginal* cost is decreasing and equivalently for CRTS and DRTS.

Ex 2.5 - Particular Production Functions

In exercise 2.5 you are asked to deal with some particular production functions. I think there are two important points about them. First you should realize when not to use the Lagrangian method, i.e. when the function is not differentiable or the input requirement set is non-convex. And secondly you should practice how to solve the problem without the Lagrangian. The solutions will be posted on Friday, please go over them and in case you have any questions come to the office hour.

One note: The production function $q = \alpha_1 z_1 + \alpha_2 z_2$ actually has a convex input requirements set and is differentiable. So in fact it is possible to use the Lagrangian method. However, the problem here is that the isoquants touch the axes and you will need to allow for corner solutions, i.e. work with inequalities (Kuhn-Tucker-Method. If you feel comfortable about that, try it out. But it is as well possible to just draw the isoquants and think about which will be the cheapest input sets for different price ratios. This is how the cost function is found in the solutions. Now you might ask, how you can see in an exam which method to use?! As a rule of thumb I would suggest to first get an idea of what the isoquants look like. If they are such that they don't touch the axes, the input requirement set is strictly convex and the function is differentiable, use Lagrangian.

