Examples of Solution to 2nd graded exercise

Attached you find the solution of a student to Q1a) and Q2) c)ii). His answer to Q1)a) is not what we expected you to do. However some of you have tried to show the consistency of the given utility function with the axioms required for a vNM representation. While his solution is not perfect, I think that if you go this way you should try to argue in a similar way as shown here. In Q2)c)ii) he derives the offer curve of consumer C for all p and shows that there is a unique general equilibrium price p. Most of you did not do this rigorously.

I explicitely do not suggest to study from this and it is not necessarily spot-on in every part of it. I rather posted it since I believe that some of you have difficulties to answer the questions in an appropriate way and I think you might learn from it what we consider to be a very good way of answering questions in this course.

EC202: Microeconomics 2 Mr: Kondrad Buchardi
Class group 6:

(Anh Le)

EC202: Class hand-in exercise:

(1) An individual has preferences by the function:

$$-\alpha y_0^{-\gamma} - \beta y_1^{-\gamma}$$

or some affine transformation

i.e the expected utility function:

$$\mathcal{E}u(y) = \pi u_0(y) + (1-\pi) u_1(y) = -\alpha y_0^{-\gamma} - \beta y_1^{-\gamma} \quad (1)$$

where π is the probability of state 0 occuring.

(a) Von Neuman - Morgenstern utility function:

- For the utility function to be consistent with Independence Axiom, we have:

$$\begin{cases} \pi u_0(y) = -\alpha y_0^{-\gamma} \\ (1-\pi) u_1(y) = -\beta y_1^{-\gamma} \end{cases}$$

I am not sure I see your point here. Are you saying that u_i should only depend on y_i and not on y_j, $j \in \Omega$ and $j \neq i$?

i.e $$\begin{cases} u_0(y) = -\frac{\alpha}{\pi} y_0^{-\gamma} \\ u_1(y) = \frac{-\beta}{1-\pi} y_1^{-\gamma} \end{cases}$$

- State - Irrelevance Axiom satisfies when $-\frac{\alpha}{\pi} = \frac{-\beta}{1-\pi}$

i.e $$\pi = \frac{\alpha}{\alpha+\beta}$$

nice!

hence $u(y) = -(\alpha+\beta)y^{-\gamma}$ is the utility function

- Suppose that $u_0(y_0, y_1) > u_1(y_0, y_1)$ for some $y_0 > y_1$ then it is clear from the utility function $u(y) = -(\alpha+\beta)y^{-\gamma}$ that $u_0 > u_1$ for all values of y_0 &> y_1 given that α, β & $\gamma > 0$.

I am not sure again that I see your logic here. I know what you mean, but your presentation is a little fast for the reader.

i.e this is consistent with Revealed Likelihood Axiom

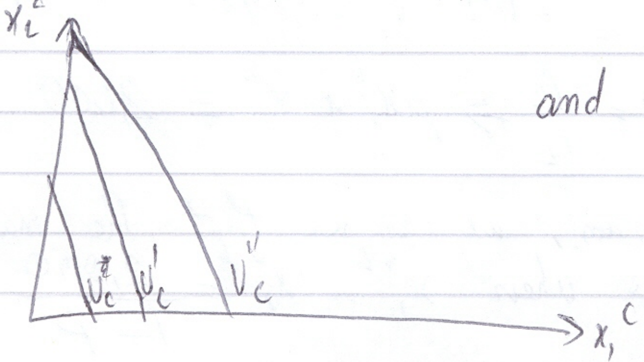
So the preferences are consistent with a Von-NeuMam Morgenstern utility function.

(9)

Ahn, this is clearly some levels higher than what was expected from you! But obviously very good!

(8) So the contract curve is $x_2^c = 3x_1^c - 1{,}000$. ✓

(ii) Person C's preferences : $U^c = 3x_1^c + x_2^c$
which has the indifference curve



and the budget constraint
$px_1^c + x_2^c \le 2{,}000$

(Anh Le)

So person c's demand will be:

$\circledast$ "$= \alpha x' + (1-\alpha)x''$, $\alpha \in [0,1]$ if $p = 3$"

$$\begin{cases} x^c = x', & \text{if } p > 3 \\ x^c \in \underbrace{[x', x'']}_{\circledast}, & \text{if } p = 3 \\ x^c = (2000/p, 0) & \text{if } p < 3 \end{cases}$$

Very good.

where $x' = (0; 2{,}000)$ and $x'' = (2000/3, 0)$

At equilibrium:
$$\begin{cases} x_1^{*a} + x_1^{*c} = 1{,}000 \\ x_2^{*a} + x_2^{*c} = 2{,}000 \end{cases} \quad (*)$$

. if $p > 3 \Rightarrow x_1^{*c} = 0,\ x_2^{*c} = 2{,}000$

$\Rightarrow$ $(*)$ cannot be satisfied

$\Rightarrow$ (rejected)

. if $p < 3 \Rightarrow x_1^{*c} = \frac{2000}{3},\ x_2^{*c} = 0$

$\Rightarrow x_1^{*a} + x_1^{*c} = 500 + \frac{2000}{3} > 1{,}000$

$\Rightarrow$ rejected

. if $p = 3$:
$$\begin{cases} 3x_1^{*c} + x_2^{*c} = 2{,}000 \\ 500 + x_1^{*c} = 1{,}000 \\ 500p + x_2^{*c} = 2{,}000 \end{cases}$$

$$\Rightarrow \begin{cases} x_1^{*c} = 500 \\ x_2^{*c} = 500 \end{cases} \text{ and } p = 3$$

So the equilibrium price $p = 3$ and the equilibrium allocation
$$\begin{cases} x_1^{*a} = 500 \\ x_2^{*a} = 1500 \end{cases} \quad \begin{cases} x_1^{*c} = 500 \\ x_2^{*c} = 500 \end{cases}$$

12

Excellent!