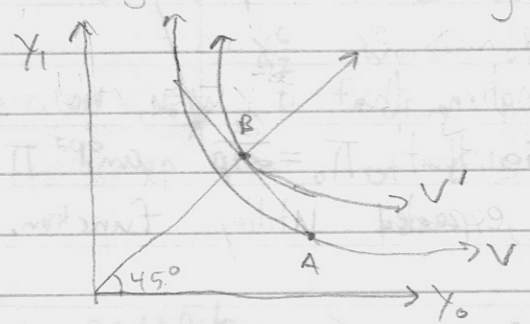
Examples of Solution to 2nd graded exercise

Attached you find the solution of a student to Q1 b)-c) and Q2) a)-c)i) that I found particularily good. He combines a rigorous mathematical argument with a clear economic intuition. **I explicitely do not suggest to study from this and it is not necessarily spot-on in every part of it.** I rather posted it since I believe that some of you have difficulties to answer the questions in an appropriate way and I think you might learn from it what we consider to be a very good way of answering questions in this course.

Part b:

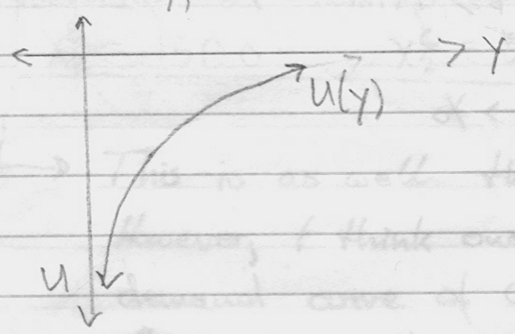
The person is risk averse. We can tell this from the shape of his indifference curves, in that they are quite clearly quasi-concave (it is clear from looking at the indifference curves that the "at-least-as-good-as" set is convex). ⟶ You should show this mathematically OR analyse concavity of the felicity funct. mathematically.

Looking at the following figure:



We can see that a certain prospect B, or any point between A and B, is prefered to A.

Additionally, if we look at the felicity function:



We can clearly see that it is concave.

Question 1, part b (continued)

More formally, we can look at the derivatives of the felicity function:

$$\frac{\partial U(y)}{\partial y} = \gamma y^{-(\gamma+1)} > 0$$ "Since we are told $\gamma > 0$, which also means that $(\gamma+1) > 1$.

and

$$\frac{\partial^2 U(y)}{\partial y^2} = -(\gamma+1)\gamma y^{-(\gamma+2)} < 0$$ for the same reasons

This shows that the felicity function is concave, and so the individual is risk averse. (enough)

Finally, we could also find the Hessian of the utility function:

$$|U''(y_0,y_1)| = \begin{vmatrix} U_{y_0y_0} & U_{y_0y_1} \\ U_{y_1y_0} & U_{y_1y_1} \end{vmatrix} = \begin{vmatrix} -\alpha(\gamma+1)\gamma y_0^{-(\gamma+2)} & 0 \\ 0 & -\beta(\gamma+1)\gamma y_1^{-(\gamma+2)} \end{vmatrix} > 0$$

and since $U_{y_0y_0} < 0$, we know that $U''(y_0,y_1)$ is negative definite and so $U(y_0,y_1)$ is quasi-concave. ✓

So, both graphically and algebraically, we have shown twice that the person is risk averse.

Excellent ! 10
(sorry for the comment on the other page)

Question 1, Part C

$\alpha = \beta = \gamma = 1$

$y_0 = 1$ million
$y_1 = 3$ million

for ease of notation, I will leave off the six zeros in everything but my final answer

We can find the certainty equivalent by normalising the subjective probabilities so they sum to 1: (Not really necessary.)

$$\frac{\alpha}{\alpha+\beta} = \pi_0 = \frac{1}{2}$$
$$\frac{\beta}{\alpha+\beta} = \pi_1 = \frac{1}{2}$$

And calculating the certainty equivalent $\mathcal{E}_x$

$$\mathcal{E}U(y) = U(\mathcal{E}_x)$$
$$\frac{1}{2}(-1^{-1}) + \frac{1}{2}(-3^{-1}) = -\mathcal{E}_x^{-1}$$
$$-\frac{1}{2} - \frac{1}{2}\left(\frac{1}{3}\right) = \frac{-1}{\mathcal{E}_x}$$
$$-\frac{2}{3} = \frac{-1}{\mathcal{E}_x} \quad \text{so} \quad \boxed{\mathcal{E}_x = \frac{3}{2}}$$

Very good.

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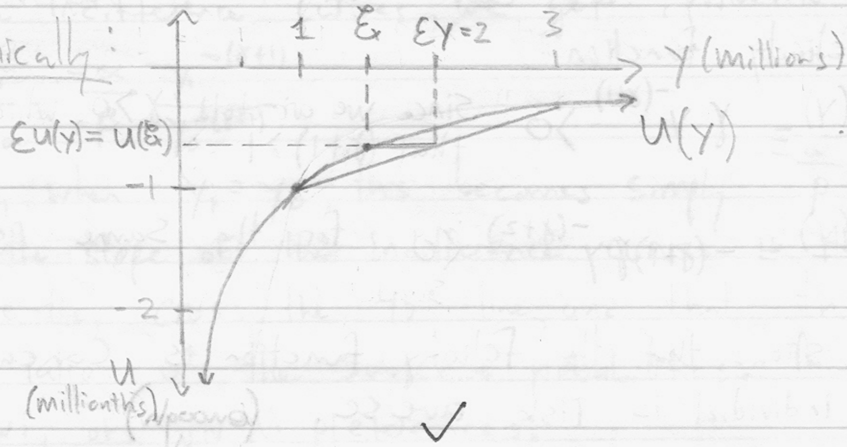
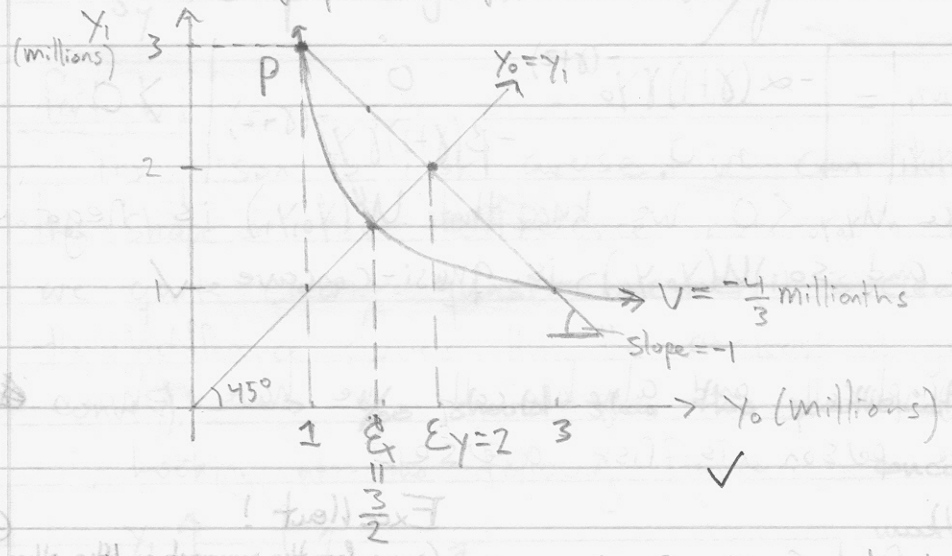
the certainty equivalent is \$1,500,000 ✓

Graphically:



✓

Alternate graphical Representation:



✓

the person faces prospect P, the certainty equivalent of which is \$1,500,000. ✓ we could add that the risk premium is $\mathcal{E}y - \mathcal{E}_x = \$500{,}000$

Excellent again!

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Question 2

Part a

(i) to determine the contract curve, the only additional information we would need is the preferences of the two traders. If their preferences are "well-behaved" (can be represented by indifference curves that are continuous, differentiable, and quasi-concave) then the contract curve will be defined by the points where their indifference curves are tangent. Even if preferences are not well-behaved, if we know what the preferences are, we can find the contract curve by looking for all points at which neither trader can be made better off without making the other worse off.

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(ii) To find the Competative Equilibrium (or equilibria), we need the initial allocation of the goods between the traders, in addition to the traders' preferences. This allows us to see which of the allocations on the Contract Curve can be supported

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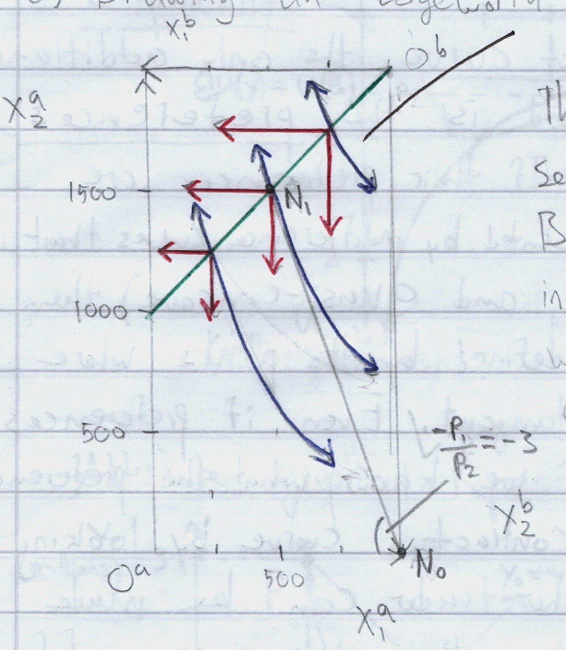
Question 2, Continued

Part b

Why do you draw the blue indifference curves assymetric? It's as well in the solutions, but I don't see why that should be.

(i) Drawing an Edgeworth box diagram:



The Contract curve (shown in green) is the set of points defined by the "corners" of B's Leontief indifference "curves" (shown in red). Taking O_a as the origin, we can write the equation of the contract curve as $x_2^a = 1000 + x_1^a$ ✓

This is the contract curve because at point not on the contract curve, A could be made better off without making B any worse off by a move to the "corner" of whichever of B's indifference curves the point happens to be on. Although these are not strictly speaking points of tangency, the idea is similar, in that the contract curve is the set of points where B's indifference "curves" touch A's indifference curves at a single point. ✓

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Very good.

(ii)

Finding A's demand functions:

$\max_{x_1, x_2} x_1^a x_2^a$ s.t. $p_1 x_1 + p_2 x_2 \leq 1000 p_1$

The Lagrangian is:

$\mathcal{L}(x_1, x_2; p_1, p_2) = x_1^a x_2^a + \lambda^a (1000 p_1 - p_1 x_1 - p_2 x_2)$

FOC: $\frac{\partial \mathcal{L}}{\partial x_1} = x_2^{a*} - \lambda^a p_1 = 0$

$\frac{\partial \mathcal{L}}{\partial x_2} = x_1^{a*} - \lambda^a p_2 = 0$

$\frac{\partial \mathcal{L}}{\partial \lambda^a} = 1000 p_1 - p_1 x_1 - p_2 x_2 = 0$

and so $x_1^{a*} = \lambda^a p_2$

$x_2^{a*} = \lambda^a p_1$

Plugging into the constraint & re-arranging: $\lambda^a = \frac{500}{p_2}$

Question 2, part b(ii) (continued):

So A's demand functions are:

$$x_1^{a*} = 500$$

$$x_2^{a*} = 500 \frac{p_1}{p_2}$$

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B Maximizes $U(x_1^b, x_2^b) = \min\{x_1^b, x_2^b\}$ Subject to $p_1 x_1^b + p_2 x_2^b \leq 2000 p_2$ which means that $x_1^{b*} = x_2^{b*}$ (otherwise, B could enjoy the same level of utility at a lower cost by consuming less of whichever good he had more of). So substituting $x_1^{b*} = x_2^{b*}$ into the budget constraint and re-arranging, we get:

$$x_1^{b*} = 2000 \frac{p_2}{p_1 + p_2}$$

$$x_2^{b*} = 2000 \frac{p_2}{p_1 + p_2} \quad ✓$$

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Setting excess demand for each good equal to zero will give us the price ratio:

$$E_1(p_1, p_2) = 500 + 2000 \frac{p_2}{p_1 + p_2} - 1000 = 0$$

$$2000 \frac{p_2}{p_1 + p_2} = 500$$

$$\frac{p_2}{p_1 + p_2} = \frac{1}{4}$$

$$\boxed{\frac{p_1}{p_2} = 3}$$

To confirm this, we can do the same for good 2, although Walras Law tells us that we do not need to do this.

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So the Equilibrium allocation is

$$x_1^a = 500 \qquad x_1^b = 500$$

$$x_2^a = 1500 \qquad x_2^b = 500$$

or point N_1 in the diagram (on p. 6).

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This answer is in my opinion just perfect.

⑧

Question 2, part C

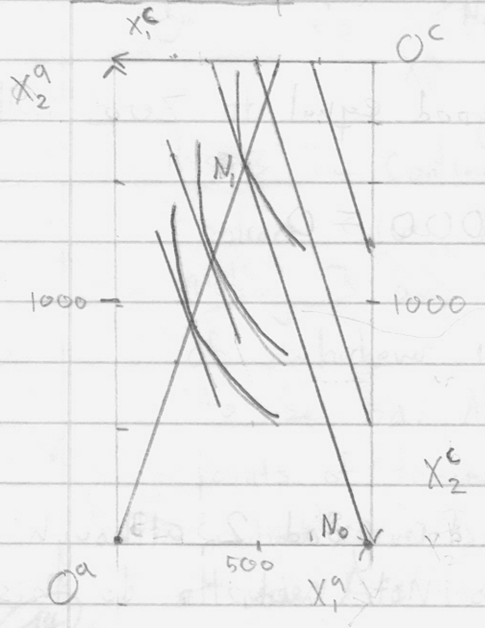
(i) In this case, the Contract Curve is the set of points where A's indifference curves are tangent to C's indifference lines. We find the Contract Curve by setting the Marginal Rates of Substitution Equal:

$$MRS^a_{21} = MRS^c_{21}$$ ~~[illegible]~~

$$\frac{U^a_{x^a_1}}{U^a_{x^a_2}} = \frac{U^c_{x^c_1}}{U^c_{x^c_2}}$$

$$\frac{X^a_2}{X^a_1} = \frac{3}{1}$$ so $\boxed{\text{the Contract Curve is } X^a_2 = 3X^a_1}$

Graphically:



C's indifference lines are shown in red, while A's indifference curves are shown in blue. The Contract Curve is shown in green. We can see that at points not on the contract curve, A could be made better off without making C worse off by moving up to the indifference curve that is tangent to whichever one of C's indifference lines the initial point was on.

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