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Aggregate implications of indivisible labor, incomplete markets, and labor market frictions ☆

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ABSTRACT

We study the impact of tax and transfer programs on steady-state allocations in a model with search frictions, an operative labor supply margin, and incomplete markets. In a benchmark model that has indivisible labor and incomplete markets but no trading frictions we show that the aggregate effects of taxes are identical to those in the economy with employment lotteries, though individual employment and asset dynamics can be different. The effect of frictions on the response of aggregate hours to a permanent tax change is highly nonlinear. There is considerable scope for substitution between "voluntary" and "frictional" nonemployment in some situations.

1. Introduction

Although labor market outcomes have always figured prominently in macroeconomic analyses, the way in which macroeconomists model the labor market has changed dramatically over the last 40 years. In particular, two underlying premises that bear on how to model the labor market have become commonplace during this period: the first is that labor supply matters for aggregate labor market outcomes, and the second is that trading frictions matter for aggregate labor market outcomes. Interestingly, both of these views can be traced to contributions that appeared in the Phelps (1970) volume, and each represented a radical departure from the canonical macroeconomic model of that time period. From the household perspective, the canonical model prevailing at the time assumed that desired hours of work were independent of any features of the economic environment, including such factors as wages, taxes, and transfer programs, and from the firm perspective this model assumed that employment could be costlessly and immediately increased in line with any increase in the demand for labor.

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Although these two premises are not in any sense in conflict, almost all work on aggregate labor market outcomes adopts one or the other but not both. This is reflected in the fact that the two standard frameworks for addressing issues related to the aggregate labor market are either the one-sector growth model (extended to include an endogenous labor supply decision as in Kydland and Prescott, 1982), or a version of the Diamond–Mortensen–Pissarides matching model. The former abstracts from any trading frictions in the labor market, while the latter abstracts from any labor supply considerations. One interpretation of this state of affairs is that each feature is important for its own particular subset of issues; in fact, however, both frameworks are routinely used to address the same set of issues, ranging from the nature of business cycle fluctuations to the effect of permanent policy changes. Moreover, in some cases the two models deliver results that are sharply different.

In view of this situation, we believe that it is important to develop a better understanding of the relative importance of these two features for specific issues of interest, and to assess whether there are important interactions among them. The goal of this paper is to take a first step in this agenda. Specifically, the contribution of the paper is twofold. First, we develop a general equilibrium model that incorporates both labor market frictions and a standard labor supply problem. Second, we use our model to address one important issue in aggregate labor market analysis: the effect of tax and transfer programs on steady-state hours of work. Following the work of Prescott (2004), this issue has attracted considerable attention and serves as a useful starting point for thinking about the relative importance of labor supply considerations and frictions.[1]

The model that we develop possesses three key features: indivisible labor, frictions, and incomplete markets. If one wants to capture trading frictions in the labor market, then indivisible labor is a natural assumption. While one can certainly formulate models of indivisible labor and trading frictions with complete markets, we believe that a market structure that does not include either markets for employment lotteries or insurance markets for the idiosyncratic income shocks that frictions generate is of particular interest.

Our analysis provides several interesting results. First, we provide analytic solutions to a benchmark model that includes indivisible labor and incomplete markets in the absence of trading frictions. We show that steady-state equilibrium allocations are identical to those that obtain in the economy when one permits trade of employment lotteries. Our result extends the similar finding of Prescott et al. (2007) that considered continuous time, finite horizons, and no discounting.[2] We also provide a complete characterization of the individual decision rules that obtain in this equilibrium. Two interesting properties emerge. One is that wealth effects are nonlinear in wealth. For either low or high wealth, increases in wealth lead to equal increases in consumption, but for intermediate levels of wealth the effect on consumption is zero. In contrast, for low and high levels of wealth the effect of wealth on labor supply is zero, but for intermediate levels the effect is positive, but only in a lifetime sense. This last statement follows from another interesting property: current labor supply is indeterminate for intermediate levels of wealth. Specifically, equilibrium imposes structure on the amount of labor supplied over one's lifetime but imposes very little structure on the timing of labor supply. This indeterminacy has important implications for how individuals respond to the presence of frictions.

Second, we find that the extent to which labor market frictions affect the response of aggregate steady-state hours to permanent changes in tax and transfer programs is also highly nonlinear. Specifically, in some regions of the parameter space the presence of frictions has effectively no effect on the response, while in other regions of the parameter space the presence of frictions leads to a dramatic reduction in the response of hours of work. But importantly, this effect is not linear. For example, in the case of tax reductions, the effect of frictions may only manifest itself for reductions beyond some threshold. Moreover, the magnitude of this threshold depends very much on the initial equilibrium: starting from some equilibria, frictions manifest themselves even for small changes. An important message is that one cannot generally conclude that frictions do or do not matter for a specific issue. Whether they matter depends very much on what region of the parameter space one is in and on the nature of the policy change being considered.

Third, we find that there is considerable scope for substitution between "voluntary" and "frictional" nonemployment in some regions. Specifically, an increase in frictions need not have any effect on steady-state equilibrium employment. Moreover, this can be true even though the length of nonemployment spells is completely determined by the extent of frictions. That is, individual level data on employment spells is not necessarily informative about the response of aggregates to changes in policy.

An outline of the paper follows. In the next section we provide some background information that helps to describe the context of the more general research issue concerning the interaction of labor supply and frictions. This section also summarizes some related literature. Section 3 describes and analyzes the benchmark frictionless model that features indivisible labor and incomplete markets. Section 4 introduces frictions into the model, and Section 5 presents quantitative results for the effect of permanent tax changes and how the effects depend upon frictions. Section 6 concludes and discusses directions for future research.

2. Motivation

In this section we report the results of a simple policy exercise in the context of two prototype models that are used for thinking about aggregate employment. The first is the indivisible labor model of Rogerson (1988), embedded into the

[1] Ljungqvist and Sargent (2008) also consider taxes in a model with incomplete markets, frictions and indivisible labor. But whereas we focus on how the presence of frictions matters for the effects of taxes, they focus on how the presence of human capital matters.

[2] Also see Mulligan (2001), Ljungqvist and Sargent (2006, 2008) and Nosal and Rupert (2007) for related analysis.

growth model and calibrated as in Hansen (1985). The second is the matching model of Pissarides (1985), calibrated as in Shimer (2005). Understanding the differences between the two exercises serves to highlight the need for the framework that we develop in this paper. Related to this, we argue that the results in the existing literature are easily misinterpreted regarding the effect of adding trading frictions to standard aggregate models.

At a superficial level, the steady-state allocations in the calibrated models of Hansen and Shimer seem very similar. Both models feature indivisible labor and generate steady states in which the employment rate is interior and workers move stochastically between employment and nonemployment.[3] However, despite this superficial similarity, the two models are fundamentally different in terms of economic mechanisms. Central to understanding this is to note the differences in the nature of the calibrated steady states. In the Hansen calibration exercise, preferences are written as

$$\sum_{t=0}^{\infty} \beta^t [\log c_t + \alpha \log(1 - h_t)],$$

where c_t is consumption in period t, and $h_t \in \{0, \bar{h}\}$ is time devoted to market work. The parameter α dictates what fraction of the population will be employed in the steady state. If this parameter is set sufficiently low, implying that individuals do not value leisure very much, then the steady-state equilibrium will entail everyone employed. Given that in the data the employment to population ratio is around 0.60, Hansen chose α so that the steady-state equilibrium matches this observation. Equivalently, in his steady-state equilibrium, workers only want to spend roughly 60% of their lifetime in employment.

Next consider this same issue in the context of the Shimer calibration. Preferences are now given by

$$\sum_{t=0}^{\infty} \beta^t [c_t - b h_t]$$

and the output of a worker-firm match is y. A key property of the model is that it is linear. If one removes the frictions from this model, then the steady-state employment rate is dictated by the relationship between the leisure parameter b and the productivity parameter y. Specifically, if $b < y$ and there are no frictions, then everyone will work in every period. Conversely, if $b > y$ then no one will ever work (with or without frictions). In the knife-edge case of $b = y$ then everyone is always indifferent between working and nonworking, so that if there are no frictions the equilibrium is indeterminate. In order for his analysis to be of any interest, Shimer necessarily calibrated his model so that $b < y$—in fact, he chose $b = 0.4$ and $y = 1$—so that the difference is quite large. Given this choice, the frictionless version of his model implies that everyone works all the time. In other words, he calibrates the model so that the labor supply decision is degenerate.

This has important implications for how one interprets results from the two models. For example, consider the case of asking what the impact would be on aggregate employment if we instituted a permanent change in the tax on labor earnings that is used to finance a lump-sum transfer. Both models predict that this will lead to a decrease in steady-state employment, though in general they will give different answers. Specifically, for the calibrations employed by Hansen and Shimer one finds that the decrease in employment is dramatically greater in the Hansen model. In particular, whereas a 10 percentage point increase in taxes leads to roughly a 10% decrease in employment in Hansen's model, the decrease is only around 1% in Shimer's model.

Because the Shimer model looks like an indivisible labor model with frictions, one might be tempted to conclude that adding frictions reduces the impact of tax changes on employment. But in fact, such a conclusion would be completely unwarranted, and the reason is precisely because of the calibration issue noted above. To see why, note that if Hansen had calibrated the value of α so that steady-state employment is at a corner solution, equal to 1 (and is a long way from being interior), he would find that an increase in taxes has no impact on employment. This is because the sole channel through which employment decreases in his model is through labor supply, and by calibrating it to effectively remove the labor supply choice, the effect of taxes on employment disappears. In Shimer's calibrated model, in contrast, the only mechanism through which employment changes in response to a permanent change in taxes is via changes in the level of frictions (which in his model is captured by the ratio of vacancies to unemployment).

To summarize, the two exercises tell us about two different channels through which taxes can affect employment. As a result, neither exercise can tell us how the presence of frictions affects the importance of the labor supply channel, since one exercise has no frictions and the other has no labor supply.

This discussion suggests what we find to be an open and interesting question: What are the properties of models which feature both nontrivial labor supply decisions and search frictions? This basically calls for a merging of the two frameworks discussed earlier, which one can view as either adding labor supply to the matching model or adding frictions to the indivisible labor model. One may reasonably ask whether it is important to merge these two frameworks; one view is that

[3] Although these features are similar, the models might be viewed as connecting to the data in slightly different ways—the indivisible labor model is probably best viewed as distinguishing between employment and nonemployment, without any implications for different categories of nonemployment, i.e., unemployed versus nonparticipating. The matching model, on the other hand, also has employed and nonemployed individuals, but the nonemployed individuals in this model seem to reflect what the data captures as unemployed workers rather than nonparticipating workers. The matching model also has predictions for one variable that the indivisible labor model does not—the number of vacancies that are posted.

the two frameworks are relevant for different issues (one for unemployment and one for participation) and that merging them is a mechanical exercise with little expected payoff in terms of understanding substantive issues. To this we counter that it seems very intuitive that trading frictions would necessarily interact with the participation decision. Moreover, our analysis shows that there are important interactions between the two and that the nature of the interaction is very dependent upon where one is in parameter space.

It is important to note that the issue of merging the two frameworks is not simply to add some dimension of labor supply to a model with frictions but rather to add it in a manner that would allow the model to be consistent with applied work on labor supply. In particular, specifications that feature utility that is linear in consumption are ultimately going to be of little interest. Furthermore, one should not think that adding labor supply to matching models is synonymous with adding curvature. As our discussion of the Hansen model indicates, the mere presence of curvature in preferences does not imply that an indivisible labor model will feature a nondegenerate labor supply decision.

If one seeks to merge these two frameworks there is another issue that must be dealt with. In particular, another difference of interest between the Shimer and Hansen analyses has to do with the existence of markets for insurance. Hansen implicitly assumes that individuals can perfectly insure against idiosyncratic income shocks associated with employment lotteries. Shimer assumes no insurance markets, though of course this has no real consequences in his framework given that preferences are linear. If one studies a model with indivisible labor, a nondegenerate labor supply problem and trading frictions, then the market structure does matter. In what follows we will study a market structure that does not include employment lotteries or insurance markets for idiosyncratic income shocks. Given that trading frictions necessarily lead to idiosyncratic fluctuations in income, it seems natural to consider a structure in which the presence of these income fluctuations is not effectively assumed away. If one considers the Social Planner's problem for a model with complete insurance and trading frictions (e.g., Merz, 1995 or Andolfatto, 1996), one can see that these models are isomorphic to models that exhibit aggregate adjustment costs on labor in the aggregate production function but no trading frictions.[4] In this sense, adding complete insurance effectively severs the connection between the idiosyncratic uncertainty associated with trading frictions and labor supply. Given that we are precisely interested in the implications of trading frictions for labor supply, it makes sense to not assume complete insurance. Moreover, there is a growing literature that has already uncovered many interesting aspects of how labor supply interacts with this form of market incompleteness in the absence of frictions.[5]

For thinking about many policy issues, this market structure is natural. For example, if one wants to think about the effects of unemployment insurance programs on aggregate labor market outcomes, then it is natural to start with a market structure that does not already provide a full set of insurance markets. Moreover, to the extent that one thinks that the incomplete-markets structure is a better description of reality than the complete-markets model, and that market structure influences the nature of individual employment and consumption paths, then a strategy of using micro data to help calibrate the key features of the model will be more reliable in the incomplete-market setting.

3. A frictionless benchmark economy

As stated in the previous section, our objective is to study a class of models that features indivisible labor, incomplete markets, and frictions. The perspective that we adopt in this work is to ask how the addition of frictions to an indivisible labor model with incomplete markets affects the implications of the model in the context of a specific issue, which we take to be the effect of tax and transfer schemes on steady-state allocations. The benchmark model for this analysis is a version of the growth model that features indivisible labor, incomplete markets, and no trading frictions. This section is devoted to an analysis of this benchmark. In addition to being of interest as the natural benchmark for the question that we ask, this analysis is also of interest for two additional reasons. First, we can obtain analytic results for individual decision rules in the steady-state equilibrium, and the features of these decision rules will turn out to be very important in understanding how our findings are affected by the addition of frictions in the next section. Second, this benchmark model allows one to assess the extent to which the implications of an indivisible labor model with trade in employment lotteries are affected by the consideration of alternative market structures.

3.1. Environment

The environment is basically the same as the model in Hansen (1985) without shocks, except that we consider a market structure that rules out all insurance markets and trade in employment lotteries. Instead, following Krusell and Smith (1998), we consider a market structure in which individuals can only hold capital as an asset. The specifics of the model follow.

[4] We note that although Merz and Andolfatto both have curvature in preferences over consumption, both models imply that the employment rate would be one if trading frictions were removed.

[5] Recent papers that examine labor supply in models with incomplete markets but no trading frictions include Chang and Kim (2006, 2007), Pijoan-Mas (2006) and Domeij and Floden (2006).

There is a continuum of measure one of identical households, each with preferences given by

$$\mathrm{E}\left[\sum_{t=0}^{\infty}\beta^t[\log(c_t)-d(h_t)]\right],$$

where $c_t \geqslant 0$ is consumption in period t and $h_t \in \{0,1\}$ is hours devoted to market work in period t. Given that labor is assumed to be indivisible we need only assume that d is increasing. The discount factor β satisfies $0<\beta<1$.

There is an aggregate production function that uses capital (K_t) and labor (H_t) to produce output (Y_t) according to

$$Y_t = K_t^{\alpha}H_t^{1-\alpha}.$$

Aggregate labor input is simply the integral of individual labor supply across households. Output can be used either as consumption or investment, investment is reversible, and capital depreciates at rate δ, with $0<\delta<1$.

We assume that the government levies a constant proportional tax τ on labor earnings and that the tax revenues are used to finance an equal lump-sum transfer payment T_t to all individuals subject to a period-by-period balanced budget condition:

$$T_t = \tau w_t H_t,$$

where w_t is the wage rate at time t.

We consider a recursive representation of the competitive equilibrium. In a steady-state equilibrium, prices for both capital and labor services will be constant, and we denote them by r and w, respectively. As noted above, we assume that the capital stock is the only asset. We can additionally allow the consumers to borrow and lend using one period bonds in zero net supply, subject to a borrowing limit. In such a case, it is easy to show that the return on holding bonds must be equal to the return on holding capital, so that the net return on bonds in steady state is the same as that on capital. In the following, we impose a borrowing constraint at zero; i.e., that the net asset holdings for an individual cannot be negative.

We show in the appendix (Proposition 2) that in the steady-state equilibrium of this model, prices and the aggregate allocation are identical to those that obtain in the model that includes markets for lotteries (i.e., insurance markets for idiosyncratic risk). In this sense, the lack of complete markets in this setting has no aggregate implications. It follows that the aggregate effects of tax and transfer schemes in this model are identical to those in the complete-markets model, and are therefore well understood.[6] However, it turns out that understanding the nature of individual decision rules in this context is very important to understanding how the addition of frictions will affect aggregate outcomes, and so in the next subsection we explore the properties of steady-state equilibrium decision rules. A nice feature of this model is that we can obtain a full analytic characterization of the steady-state decision rules.

3.2. Decision rules in the steady-state equilibrium

If r is the rental rate on capital, then the effective real rate of return on assets is $1+r-\delta$. The only individual state variable for a household is the level of assets that they have at the beginning of the period, which we denote by a. The dynamic programming problem for a household is then given by

$$V(a) = \max\left\{\max_{a'} \log((1+r-\delta)a + (1-\tau)w + T - a') - d(1) + \beta V(a'),\right.$$
$$\left.\max_{a'} \log((1+r-\delta)a + T - a') - d(0) + \beta V(a')\right\}$$

subject to

$$a' \geqslant 0.$$

Given that preferences are assumed to be separable between consumption and leisure, the Euler equation for this problem is

$$\frac{1}{c} = \beta(1+r-\delta)\frac{1}{c'}$$

and is independent of the labor/leisure decision, where c' is next period's consumption. Thus, for aggregate consumption to be constant, it is necessary that $1+r-\delta = 1/\beta$. In what follows we will replace $1+r-\delta$ by $1/\beta$.

We further assume that $\beta > \frac{1}{2}$. Proposition 1, which is stated formally and proven in the appendix, characterizes the value function and decision rules for the above problem. Here we focus on a diagrammatic representation of the results and some intuition. As noted, this is of significant interest since an understanding of the decision rules in the frictionless context will be very useful in understanding how the introduction of frictions in the next section influences decision rules.

[6] Although the aggregate behavior of the incomplete-market economies is the same as that of the complete-market economy, the individual behavior of employment and asset dynamics may look very different.

Figs. 1, 2 and 3 depict the decision rules for the work–leisure choice, the asset choice, and consumption. Intuitively, we can interpret the result as three different types of behavior, depending on the level of wealth, with two "buffer zones" in between. When the wealth level is very low ($a \leqslant \underline{a}$: "work" region), the consumer always works, and the asset level remains constant over time. In this region, a higher wealth level means a higher level of consumption. In contrast, when the wealth level is very high ($a \geqslant \bar{a}$: "leisure" region), the consumer never works, but the asset level again remains constant over time. As in the previous case, a higher wealth level means a higher consumption. A worker who starts in either of these two regions of wealth will have the same values for h, a, and c forever. It also follows that each of these regions are absorbing states, in that once in either of these regions, an individual will forever remain in them.

Next we consider the case in which the wealth level is intermediate ($a \in [a_*, a^*]$: "indifference" region). In this region the consumer is indifferent between working and not working in the current period. In general, the consumer will move between periods of work and periods of leisure, but consumption remains constant independently of the current work decision. During a period of work, the consumer accumulates assets, and during a period of leisure, the consumer runs them down. In this region, the consumption level is constant across different wealth levels, but the wealth level changes over time. Many different dynamic work/leisure patterns for the consumer are possible, though this will be clearer once we discuss the role of the "buffer zones."

Between the "work" region and the "indifference" region and between the "indifference" region and the "leisure" region, there are "buffer zones." Starting from these zones, the asset level always moves towards the "indifference" region. Workers who start with wealth levels in the "indifference" region can enter these buffer zones, but they are always brought back to the "indifference" region. They will never leave the interval consisting of the "indifference" region and the two buffer zones, so that the region $(\underline{a}, \bar{a})$ is also an absorbing state. In the buffer zones, the consumption level is the same as in the "indifference" region. Although the labor decision is not determined inside the region of indifference, if a given household were to repeatedly choose to not work (or work), then they would eventually transit to the buffer zone that lies below (above) the indifference region, and at this point the labor supply decision becomes determinate until they once again enter the indifference region. It follows that the equilibrium places some discipline on the number of consecutive periods of employment or nonemployment, but apart from this places relatively few restrictions on the nature of individual employment histories.

It is of interest to consider the issue of how large the various regions are, and how they respond to changes in various parameters. One can show that if β tends to one, the relative size of the buffer zones (compared to the size of the indifference zone) tends to zero. This finding is intuitive. In a continuous time model the buffer zones would not exist; instead we would simply have reflecting barriers on either end of the indifference region. One interpretation of the case where β tends to one is that we are making each period very very short, and hence we approach the continuous time result.

Although we will not pursue the issue further here, we think it is interesting to note that the above characterization has some interesting implications for empirical work that seeks to uncover various labor supply elasticities. Specifically, the fact that current labor supply does not respond to an increase or decrease in wealth is potentially not at all informative regarding the overall effect on labor supply. The associated changes in labor supply may occur in the future instead of contemporaneously. Another interesting implication of the above characterization is that a wealth transfer has very different effects on consumption, depending on the initial level of wealth. When the wealth level is very low or very high, a (small) transfer in wealth increases the worker's consumption level, but has no impact on hours of work. However, the consumption level is unaffected by a (small) transfer if the wealth level is in the "indifference" region. There, an increase in wealth is perfectly absorbed by an increase in leisure time, though as just noted, not necessarily contemporaneously. In other words, there is no wealth effect on consumption in this region. Similarly, the effect of a wealth transfer on labor supply is also very dependent on the initial wealth holdings and can be very nonlinear in the amount of the transfer.

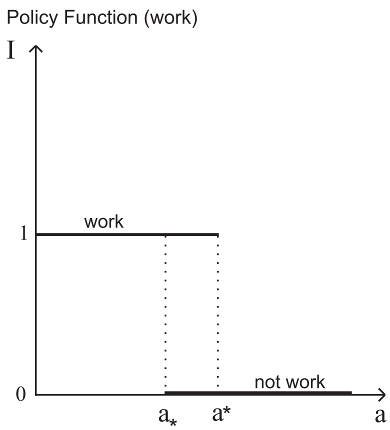

Fig. 1. Decision rules for work–leisure choice.

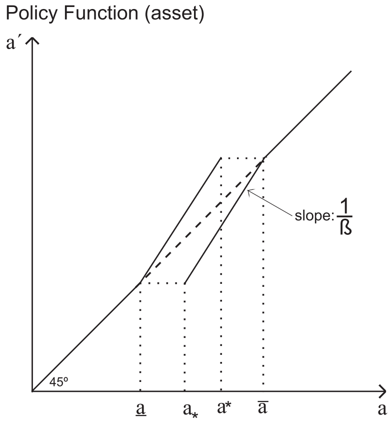

Fig. 2. Decision rules for asset choice.

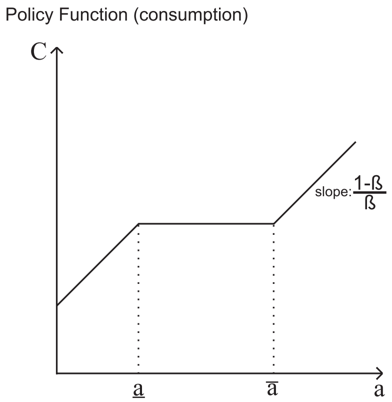

Fig. 3. Decision rules for consumption.

For example, a positive wealth transfer can push a household from the region of always working to the indifference region, but for smaller wealth transfer there might not be any effect.

It is also of interest to look ahead slightly and anticipate how the addition of frictions will influence decision rules. Of particular interest is the indifference region. The key feature of decision rules in this region is that the individual is indifferent between working and not working in any given period. Depending upon the size of the indifference region, this indifference applies not only for the current period, but also for some number of periods in the future as well. Intuitively, depending upon the size of frictions relative to the indifference region, frictions may not matter much at all for such an individual. Frictions imply that it might take several periods for an individual to find a job even after they have decided that they would like to work. But if the time it takes to find a job is small relative to the number of periods over which they are indifferent between working and not working, then the friction will have little effect. Conversely, in order for frictions to matter to the individual, the frictions must be large relative to the size of the indifference region.

4. Frictions in the benchmark economy

We now introduce frictions to the labor market. Our approach to modeling frictions is in the spirit of the island model of Lucas and Prescott (1974), though our environment differs from theirs in some respects. In particular, we will assume that there are two islands, one of which we call the "production island," and the other of which we will call the "leisure island." The production island is endowed with an aggregate production function that is the same as that considered in the benchmark model in the previous section. We introduce frictions by assuming that workers cannot freely move between the two islands. In particular, if a worker supplies labor in period t (i.e., resides on the production island in period t), then with probability σ he or she will begin the next period on the leisure island, and with probability $1-\sigma$ will begin the next period on the production island. At the beginning of period $t+1$, any individual who either did not supply labor in period t (i.e., lived on the leisure island in period t) or was sent to the leisure island at the end of period t, will be sent to the production island with probability λ_w. These workers, plus any workers who resided on the production island in period t

and were not sent to the leisure island, all have the opportunity to supply labor in period $t+1$. All other workers do not have the opportunity to supply labor in period $t+1$. Loosely speaking, σ is the exogenous separation rate, and λ_w is the exogenous job arrival rate. Note that given this formulation, the frictionless model in the previous section corresponds to the case with $\lambda_w = 1$, since if all workers always have a job offer, then separations are irrelevant.[7] The key feature of this economy relative to the benchmark model is that workers do not always have the opportunity to work. This manifests itself in two different ways. First, if an individual chooses to not work in period t, then it is not certain that he or she will have the opportunity to work in period $t+1$. Second, even if an individual works in period t, he or she is not guaranteed an opportunity to work in period $t+1$.

Once again we will focus on a steady-state equilibrium. We assume the same market structure as before, i.e., markets for output, labor and capital services in each period, in addition to a one-period bond. We again denote steady-state values for the wage and rental rate for capital services as w and r. A worker's state consists of his or her location at the time that the labor supply decision needs to be made, and the level of asset holdings. Let the value function for a worker in the productive island be $W(a)$, the value function for a worker who begins the period on the leisure island but before the job offer realization be $S(a)$, and the value function for a worker who does not work for the current period (nonemployed worker) be $N(a)$. Then, the Bellman equation for an individual who has the opportunity to work and chooses to work is

$$W(a) = \max_{c,a'} \log(c) - d(1) + \beta \mathrm{E}[\sigma S(a') + (1-\sigma)\max\{W(a'), N(a')\}]$$

subject to

$$a' = (1+r-\delta)a + (1-\tau)w + T - c$$

and

$$a' \geqslant 0.$$

The worker who begins a period on the leisure island has the Bellman equation given by

$$S(a) = \lambda_w \max\{W(a), N(a)\} + (1-\lambda_w)N(a).$$

And an individual who does not work, either because he or she did not have the opportunity or chose not to, has a Bellman equation given by

$$N(a) = \max_{c,a'} \log(c) - d(0) + \beta S(a')$$

subject to

$$a' = (1+r-\delta)a + T - c$$

and

$$a' \geqslant 0.$$

The firm and the government problems are formulated the same way as in the previous section, so we do not repeat them here.

The economy with frictions does not permit as sharp an analytical characterization as was possible for the frictionless model. However, some properties can be established. For example, the decision rule for whether to work has a reservation property with regard to asset holdings. In particular, if it is not optimal for a worker with asset holdings a to work, then any individual with assets greater than a will also find it not optimal to work. Similarly, if an individual with asset holdings a finds it optimal to work, then any individual with asset holdings less than a will choose to work given the opportunity.

Note that adding frictions to the model serves to break the indeterminacy result that we found for the frictionless model. There we found a region of asset holdings for which the individual was indifferent regarding current labor supply, but with frictions this region shrinks to a single point. An important quantitative issue is that there may still be a region in which the individual is very close to indifference, so that even when the individual strictly prefers to work this period, it may not matter much to them.[8]

In contrast to the frictionless model, it will not be the case that $1/\beta = 1+r-\delta$ in the steady-state equilibrium. The presence of frictions implies that individuals face idiosyncratic income risk, and as is standard in models with idiosyncratic income risk and incomplete markets, we will have greater accumulation of capital.

[7] In contrast to standard matching models in which the frictions are endogenously determined, we assume them to be exogenous. While our analysis can assess the extent to which the presence of frictions influences the effect of tax changes, it does not directly assess the extent to which the answer might be affected by allowing frictions to also change in response to the change in taxes. Nonetheless, our results do suggest when this latter effect might be important.

[8] Idiosyncratic variation in either productivity or preferences can also serve to break the indifference in the frictionless model. Rogerson and Wallenius (2007) use age varying productivity or disutility from working to generate determinate labor supply patterns over the lifecycle. Chang and Kim (2006) use idiosyncratic productivity shocks to generate determinate labor supply patterns.

5. Implications of frictions: quantitative results

In this section we analyze how labor market frictions impact the answer to a simple tax experiment. Specifically, we consider tax policies of the form described earlier, in which the government levies a constant proportional tax on labor earnings and uses the proceeds to fund a uniform lump-sum transfer to all individuals, subject to a period-by-period balanced budget rule. We examine how the presence of frictions affects the response to a tax changes of a given magnitude.

5.1. Calibration

In this subsection we describe how we calibrate the model. We will consider different levels of frictions, but for our benchmark calibration we will also calibrate the parameters that characterize the frictions. We set the period length equal to one month. Many of the parameters can be calibrated using standard methods. Specifically, we choose values for β, α, and δ so as to match three targets: a capital share of 0.3, an investment to output ratio of 0.2, and a 4% annual rate of return to capital. This gives $\beta = 0.9967$, $\alpha = 0.3$ and $\delta = 0.0067$. We set $\tau = 0.30$ as the tax rate, consistent with measured values of the current average effective tax rate on labor income for the US. For the two parameters that capture the extent of frictions we set $\lambda_w = 0.2$ and $\sigma = 0.02$. These values are consistent with the transition probabilities between unemployment and employment in the CPS data.[9] We normalize $d(0) = 0$ and set $d(1) = -2.3 \times \log(1 - 1/3)$. From these, we obtain that 66% of the population is working in the steady state of the benchmark, which is similar to the employment to population ratio in the United States.

As noted above, we will also consider economies with different levels of frictions in order to assess the importance of frictions for the answer to a specific policy question. Specifically, we will consider various values of λ_w, holding σ constant. In these economies with different values of λ_w we recalibrate all of the other parameters of the model so as to match the same aggregate targets, i.e., capital's share of income, the investment to output ratio, the real rate of return to capital, and the employment rate.

One of the economies that we study is the frictionless benchmark economy. When considering the frictionless model we assume that all the workers start from the wealth level in the "indifference" region. As noted earlier, there is a large indeterminacy in the decision rules of the workers in the "indifference" region. However, in the presence of frictions this indeterminacy does not exist—instead there are reservation asset levels. If we want to think of the frictionless economy as the limit of the economy with frictions as the level of frictions tend to zero then it seems reasonable to focus on decision rules for the frictionless economy which also impose a reservation asset rule, and so we do this when solving for the equilibrium of the frictionless model. We let the threshold asset holdings be denoted by $\tilde{a}$: work when $a \leqslant \tilde{a}$ and take leisure when $a > \tilde{a}$. Although there are many values of $\tilde{a}$ that are consistent with optimal decision rules, there is only one choice of $\tilde{a}$ that is consistent with the steady-state aggregate values of K and H. It is typically the case that K is increasing in $\tilde{a}$, so $\tilde{a}$ can be pinned down by the aggregate value of K.

We consider four values for λ_w: 1.0 (no frictions), 0.3, 0.2, and 0.1. Although the targets used in the calibration are the same across all economies, the economies do differ along some dimensions that are not targeted. Table 1 shows the values of capital–labor ratio K/H for the various calibrated economies.

Given that aggregate employment is the same across all four economies, these differences reflect differences in capital accumulation. Note that this value is the same for $\lambda_w = 1.0$ and 0.3, but that it increases as λ_w is decreased further. Given the literature on precautionary savings (see e.g., Huggett, 1993; Aiyagari, 1994), it is intuitive that K/H increases as frictions increase, since greater frictions lead to greater uncertainty in the individual income process, and therefore additional precautionary savings. What the table tells us is that this effect only becomes quantitatively noticeable when frictions are quite large, since even when $\lambda_w = 0.3$ the amount of capital accumulated is effectively identical to that in the frictionless economy. Key to this result is the fact that in our calibrated economy, individuals only want to work roughly two-thirds of the time. This makes it easy for the individuals to accommodate some frictional nonemployment. For example, if an individual in the frictionless economy were simply told that he or she would not be allowed to work every 10th period, this would have no effect on his or her accumulation of assets.

It is also of interest to examine how individual employment histories vary with the extent of frictions. Table 2 presents the average duration of employment and nonemployment spells in steady-state equilibrium for the four different economies.

The final two columns report the duration of employment and nonemployment spells that would result if there were only frictional nonemployment. Interestingly, the average duration of nonemployment spells in all of the economies with frictions is exactly that which would emerge if all nonemployment were frictional. It follows that individuals in these economies effectively never turn down an employment opportunity. To understand this result, note that any individual who has spent one period not working must necessarily have asset holdings below the reservation asset level. If they are

[9] See Hobijn and Şahin (2007, Table 3). They report that the transition rate from unemployment to employment is on average 20% for 1976–2005. Consistent with this, we set $\lambda_w = 0.2$ for our benchmark calibration. Hobijn and Şahin also report that employment to unemployment transition rate is on average 1.6% for the same sample period. Since $\lambda_w = 0.2$ fraction of unemployed workers find jobs in the same period, we set $\sigma = 0.02$ which is consistent with a transition rate of 1.6%.

Table 1
Capital– labor ratio

	K/H
$\lambda_w = 1.0$	128.7
$\lambda_w = 0.3$	128.7
$\lambda_w = 0.2$	129.0
$\lambda_w = 0.1$	131.8

Table 2
Average duration of employment and nonemployment

	Equilibrium		Frictional nonemployment only	
	Employment	Nonemployment	Employment	Nonemployment
$\lambda_w = 1.0$	1.9	1.0	–	–
$\lambda_w = 0.3$	6.4	3.3	71.4	3.3
$\lambda_w = 0.2$	9.6	5.0	62.5	5.0
$\lambda_w = 0.1$	19.1	10.0	55.6	10.0

nonemployed because of job loss, then their assets at the time of job loss must have been below the reservation level, and spending one period without working will have reduced them further. If, on the other hand, they became nonemployed by choice, by virtue of having spent a period in unemployment they will necessarily have reduced their asset holdings.[10] Because the steady-state employment rate is above 0.5, it must be that one period of unemployment necessarily pushes their assets below the reservation level. It follows that no one who has spent one period unemployed ever turns down a job opportunity. One might conjecture that if individuals never turn down job offers, then employment must be determined by the frictions. However, this table shows that employment spells are much shorter than those that would obtain in an economy in which nonemployment was only due to frictions. In other words, labor supply considerations are very much at work in equilibrium even though all unemployed workers will always accept an offer to work.

The table also shows that a key impact of frictions is to change the nature of individual employment histories. In particular, as frictions increase, the average duration of both employment and nonemployment spells increase. Average employment durations respond to changes in λ_w even though the probability of job loss is constant across these economies. The reason for this is that when λ_w is low, a worker knows that if they choose not to work today, it may be several periods before they get another opportunity to work. Since they need to have sufficient assets to provide for consumption during this nonemployment spell, they need to work longer to accumulate more assets before choosing to not work in an economy with low λ_w.

It is also of interest to examine the difference in asset distributions across the four economies. This is done in Figs. 4 and 5. As one might expect from the previous intuition, as frictions increase, the asset distributions become more spread out.

5.2. Results

In this subsection we report the effects of changes in taxes on steady-state allocations for the four economies that differ in the extent of frictions.

We begin by examining the effects on aggregate employment. Table 3 presents the results. Recall that the calibration set $\tau = 0.30$, so that by construction, employment is the same for all four economies for this value of the tax rate.

The table reveals a striking asymmetry. If taxes increase from 0.30 to 0.45, then the effect on aggregate employment is independent of the level of frictions in the economy: in all cases the employment rate drops from 0.66 to 0.52, implying a decrease in hours worked of over 20%. Similarly, if taxes were reduced to 0.15, then the employment rate increases in all cases, and the increase is virtually identical across the four economies. However, if taxes were reduced to 0, then the increase in employment varies quite substantially across the economies, with the increase in employment being a decreasing function of the level of frictions. For example, when taxes are reduced from 0.15 to 0.00, the increase in aggregate hours is almost 25% when $\lambda_w = 1.0$, but less than 8% when $\lambda_w = 0.1$.

[10] Of course, a worker who suffered a job loss at the end of period t but who would have chosen not to work in period $t+1$ will not accept a job opportunity in period $t+1$ even if offered.

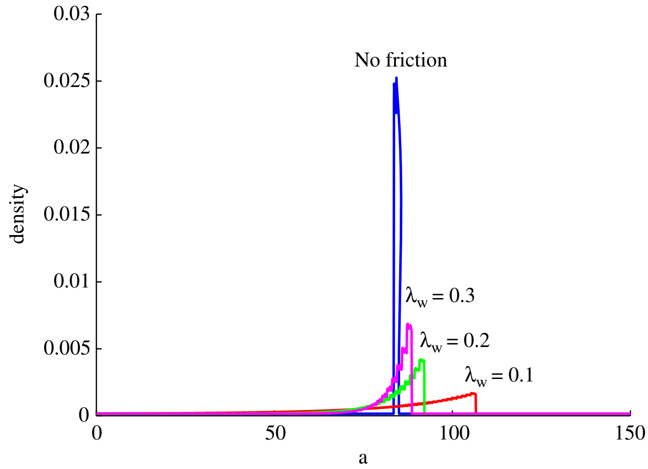

Fig. 4. Wealth distributions for employed workers, $\tau = 0.30$.

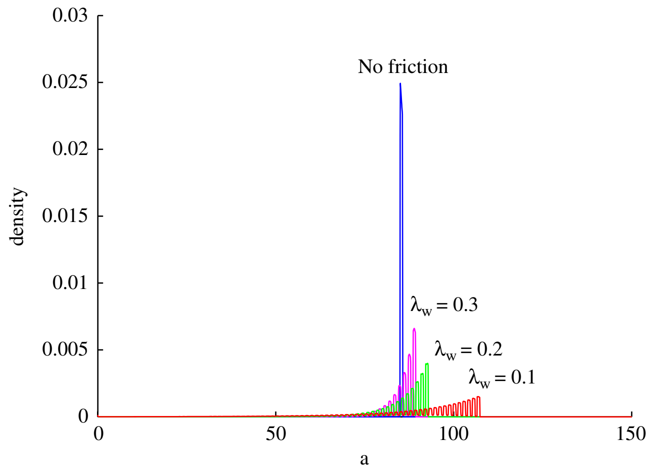

Fig. 5. Wealth distributions for nonemployed workers, $\tau = 0.30$.

Table 3
Aggregate employment H

	$\tau = 0.00$	$\tau = 0.15$	$\tau = 0.30$	$\tau = 0.45$	$\bar{n}$
$\lambda_w = 1.0$	0.99	0.80	0.66	0.52	0.00
$\lambda_w = 0.3$	0.92	0.80	0.66	0.52	0.03
$\lambda_w = 0.2$	0.91	0.80	0.66	0.52	0.07
$\lambda_w = 0.1$	0.85	0.79	0.66	0.52	0.15

To understand these results it is instructive to note that if all consumers choose to work whenever they have the opportunity (i.e., there is frictional nonemployment only), then the nonemployment rate evolves according to

$$n_{t+1} = (1-\lambda_w)n_t + \sigma(1-\lambda_w)(1-n_t).$$

Note in this expression that a worker who separates at the end of period t will not necessarily be nonemployed in period $t+1$, since they will still have probability λ_w of obtaining the opportunity to work in period $t+1$. The above expression implies that at the steady state

$$\bar{n} = \frac{\sigma(1-\lambda_w)}{\sigma(1-\lambda_w)+\lambda_w}.$$

Table 4
Average duration of employment/nonemployment

	$\tau = 0.00$	$\tau = 0.15$	$\tau = 0.30$	$\tau = 0.45$
$\lambda_w = 1.0$	15.4/1.0	4.0/1.0	1.9/1.0	1.1/1.0
$\lambda_w = 0.3$	40.6/3.3	13.1/3.3	6.4/3.3	3.6/3.3
$\lambda_w = 0.2$	49.7/5.0	20.0/5.0	9.6/5.0	5.3/5.0
$\lambda_w = 0.1$	55.5/10.0	38.1/10.0	19.1/10.0	10.7/10.0

This value is reported in the last column in Table 3. Looking at the numbers in the final column, the following pattern emerges. As long as actual nonemployment is not too close to frictional nonemployment, then the level of frictions is effectively irrelevant for the response of aggregate employment to changes in taxes. But as the two values become closer, frictions start to have an effect. Interestingly, however, it is not the case that frictions matter only if the frictions bind in terms of the maximal steady-state employment rate. For example, consider a reduction of τ from 0.15 to 0.00 in the $\lambda_w = 1.0$ and 0.3 economies. When $\tau = 0.15$, both economies have employment rates of 0.80. As taxes are decreased from 0.15 to 0.00, the frictionless economy has employment increase from 0.80 to 0.99. The maximal steady-state employment rate in the $\lambda_w = 0.3$ economy is 0.97. But the employment rate in the $\lambda_w = 0.3$ increases only to 0.92 when taxes drop to 0.00, substantially below the maximal level dictated by the frictions. Note that whether we are considering raising taxes from 0.00 to 0.15 or lowering taxes from 0.15 to 0.00, the elasticity of employment with regard to taxes is less in the $\lambda_w = 0.3$ economy than in the $\lambda_w = 1.0$ economy.

To summarize, a key implication of these results is that in economies where individuals do not desire to work in every period, there is scope for a great deal of substitution between frictional nonemployment and voluntary nonemployment. In this case the level of frictions are not very relevant for how the economy responds to permanent tax changes. If however, the level of nonemployment approaches that of frictional nonemployment, then the level of frictions matter, and in particular, responses to permanent tax changes will be less in economies with greater frictions.

It is also of interest to ask how individual employment dynamics change as we change taxes, and how these changes are affected by the level of frictions. Table 4 shows the combinations of average employment and nonemployment durations for each of the economies as we change taxes.

The table shows that all of the adjustment take place along the employment duration margin. As was true in the calibrated economies, nonemployment durations are completely dictated by the arrival rate of employment opportunities, and in all cases a worker who has been nonemployed for at least one period will always work if presented with the opportunity.

6. Conclusion

This paper analyzes a model that features frictions, an operative labor supply margin, and incomplete markets. While much has been learned about models with frictions that do not feature an operative labor supply margin as well as about models that feature operative labor supply but no frictions, little is known about models with both features. We have argued that an important goal is to determine for which issues these various features are quantitatively important. The analysis carried out here is only a first step. In particular, we have only considered a model with homogeneous individuals, and the only experiments that we considered were permanent changes in the size of tax and transfer systems. Nonetheless, we feel that the analysis has provided several important findings. For example, there is substantial scope for substitution between voluntary and frictional nonemployment in our model. This creates the possibility that incorporating frictions into the analysis may have little or no impact on the aggregate effects of some policies. If employment is close to the maximal amount allowed given the extent of frictions, then incorporating frictions does affect the aggregate response of the economy to changes in policy. In particular, the effects on aggregate employment will be diminished. The effect of frictions was found to be highly nonlinear, suggesting that one cannot determine whether frictions are important without consideration of what type of policy change is being considered, and what the initial equilibrium is. We also found that caution must be used in interpreting job acceptance decisions to infer the relevance of labor supply considerations. In our calibrated economies, all individuals who have been unemployed for at least one period will accept any opportunity to work, but aggregate employment is far from the level that would result if labor supply considerations were not relevant and employment were dictated only by frictions.

The model we develop can also be used for addressing other questions; more generally, it is a good vehicle for gauging to what extent frictions and/or labor supply considerations matter quantitatively for the answers. Already in the present setting, one could perform other comparative statics exercises. As for the case of the policy change we considered here, due to the nonlinearity of the model, some of these questions may have quite different answers depending on the exact details of the experiments. For other issues, such as for example the analysis of how average hours worked are influenced by a permanent increase in productivity, we suspect that the answer does not depend on the degree of frictions; for this question, whether there are frictions or not, we expect very small effects under the preference specification adopted here, since substitution and income effects cancel each other out.

Of particular interest here is the extent to which adding frictions to the indivisible labor model might mute the size of employment fluctuations in response to productivity shocks. Additionally, the nature of employment histories at the individual level may be very different, so that incorporating frictions and incomplete markets might provide a consistent framework in which we understand a larger set of observations. For example, the indivisible labor model without frictions might not do a good job of accounting for the patterns we see in individual employment histories, and adding frictions may help in this dimension. The framework might also have very interesting implications in the context of heterogeneous agents. We offer one example here in the context of business cycles. Hall (2005) and Shimer (2005) argue that layoff rates cannot drive cyclical employment rate fluctuations because they produce an inconsistency with the Beveridge curve. But if one considers a more general model it may be that layoffs are the dominant source of fluctuations for some groups while labor supply considerations are the dominant source of fluctuations for other groups, and that by combining these the aggregate Beveridge curve is well behaved. Loosely speaking, one might imagine that for many prime aged workers it is the increase in layoff rates coupled with search frictions that accounts for much of their cyclical fluctuations in employment, while for younger and less attached workers layoffs play very little role.

Appendix

In the following, let $D \equiv d(1) - d(0)$.

Proposition 1. *In the steady-state equilibrium, the value function has the following form:*

$$V(a) = \begin{cases} \dfrac{\log\big(((1-\beta)/\beta)a + (1-\tau)w + T\big) - d(1)}{1-\beta} & \text{if } a < \underline{a}, \\[2ex] \dfrac{\log\big((1-\tau)w/D\big)}{1-\beta} - \dfrac{d(1)+1}{1-\beta} + \dfrac{\beta D((1-\tau)w+T) + (1-\beta)Da}{\beta(1-\tau)w(1-\beta)} & \text{if } \underline{a} \leqslant a < \bar{a}, \\[2ex] \dfrac{\log\big(((1-\beta)/\beta)a + T\big) - d(0)}{1-\beta} & \text{if } \bar{a} \leqslant a. \end{cases}$$

A worker's decision rules for asset accumulation and the work– leisure choice are summarized by the following five cases that depend on the level of the current asset a. Let I be indicator function which takes 1 *when working and* 0 *when not working.*

- *Case* 1: *When* $a \leqslant \underline{a}$:

$$a' = a,$$

$$c = \frac{1-\beta}{\beta}a + (1-\tau)w + T,$$

and

$$I = 1.$$

- *Case* 2: *When* $a \in (\underline{a}, a_*)$:

$$a' = \frac{1}{\beta}a + (1-\tau)w + T - \frac{(1-\tau)w}{D},$$

$$c = \frac{(1-\tau)w}{D},$$

and

$$I = 1.$$

- *Case* 3: *When* $a \in [a_*, a^*]$:
Indifferent between

$$a' = \frac{1}{\beta}a + (1-\tau)w + T - \frac{(1-\tau)w}{D},$$

$$c = \frac{(1-\tau)w}{D},$$

and

$$I = 1;$$

and

$$a' = \frac{1}{\beta}a + T - \frac{(1-\tau)w}{D},$$

$$c = \frac{(1-\tau)w}{D},$$

and

$$I = 0.$$

- *Case* 4: *When* $a \in (a^*, \bar{a})$:

$$a' = \frac{1}{\beta}a + T - \frac{(1-\tau)w}{D},$$

$$c = \frac{(1-\tau)w}{D},$$

and

$$I = 0.$$

- *Case* 5: *When* $a \geqslant \bar{a}$:

$$a' = a,$$

$$c = \frac{1-\beta}{\beta}a + T,$$

and

$$I = 0.$$

The thresholds on a are defined as

$$\underline{a} = \frac{\beta((1-\tau)w - D((1-\tau)w + T))}{(1-\beta)D},$$

$$\bar{a} = \frac{\beta((1-\tau)w - DT)}{(1-\beta)D},$$

$$a_* = \underline{a} + \beta(1-\tau)w,$$

and

$$a^* = \bar{a} - \beta(1-\tau)w.$$

Note that $\bar{a} - \underline{a} = \beta(1-\tau)w/(1-\beta)$. *Also note that* $\bar{a} > a^* > a_* > \underline{a}$ *holds.*

Proof. We proceed by the "guess and verify" method. It turns out that the borrowing constraint does not bind (it can be verified easily from the solution), thus we ignore the constraint in the following analysis. First, we conjecture that the value function takes the form described above. To verify our conjecture, we need to solve the optimization problem on the right-hand side (RHS) and see if we obtain the decision rules defined above and $V(a)$ on the left-hand side (LHS).

Note that the conjectured value function $V(a)$ is (weakly) concave, so the two maximization problems inside are both concave programming problems. Also note that it is continuous and differentiable everywhere (there are no kinks). So we can solve each optimization problem (for working and not working) one by one using the first-order conditions (FOCs), and compare the values. Details are filled in later.

- Case 1: When $a \leqslant \underline{a}$:
  - First optimization (working): From the FOC, $a' = a$ follows. Therefore, from the budget constraint, $c = ((1-\beta)/\beta)a + (1-\tau)w + T$ follows. Thus, the value from working is

$$W(a,1) = \log\left(\frac{1-\beta}{\beta}a + (1-\tau)w + T\right) - d(1) + \beta\frac{\log\big(((1-\beta)/\beta)a + (1-\tau)w + T\big) - d(1)}{1-\beta}$$
$$= \frac{\log\big(((1-\beta)/\beta)a + (1-\tau)w + T\big) - d(1)}{1-\beta}.$$

- Second optimization (not working): From the FOC, $a' = a - \beta(1-\tau)w$ follows. Therefore, $c = ((1-\beta)/\beta)a + \beta(1-\tau)w + T$. Thus, the value from not working is

$$W(a,0) = \log\left(\frac{1-\beta}{\beta}a + \beta(1-\tau)w + T\right) - d(0) + \beta\frac{\log\big(((1-\beta)/\beta)a + \beta(1-\tau)w + T\big) - d(1)}{1-\beta}.$$

One can show (see the details later) that for $a \in [0, \underline{a}]$, it is always the case that $W(a,1) > W(a,0)$.

- Case 2: When $a \in (\underline{a}, a_*)$:
  - First optimization (working): From the FOC,

$$a' = \frac{1}{\beta}a + (1-\tau)w + T - \frac{(1-\tau)w}{D}$$

and thus

$$c = \frac{(1-\tau)w}{D}$$

follows. Therefore, the value from working is

$$W(a,1) = \frac{\log\big((1-\tau)w/D\big)}{1-\beta} - \frac{d(1)+1}{1-\beta} + \frac{\beta D((1-\tau)w+T) + (1-\beta)Da}{\beta(1-\tau)w(1-\beta)}.$$

  - Second optimization (not working): From the FOC, $a' = a - \beta((1-\tau)w)$ follows. Thus, $c = ((1-\beta)/\beta)a + \beta w$. Thus, $c = ((1-\beta)/\beta)a + \beta(1-\tau)w + T$. Thus, the value from not working is

$$W(a,0) = \log\left(\frac{1-\beta}{\beta}a + \beta(1-\tau)w + T\right) - d(0) + \beta\frac{\log\big(((1-\beta)/\beta)a + \beta(1-\tau)w + T\big) - d(1)}{1-\beta}.$$

One can show (see the details later) that for $a \in (\underline{a}, a_*)$, it is always the case that $W(a,1) > W(a,0)$.

- Case 3: When $a \in [a_*, a^*]$:
  - First optimization (working): From the FOC,

$$a' = \frac{1}{\beta}a + (1-\tau)w + T - \frac{(1-\tau)w}{D}$$

and thus

$$c = \frac{(1-\tau)w}{D}$$

follows. Therefore, the value from working is

$$W(a,1) = \frac{\log\big((1-\tau)w/D\big)}{1-\beta} - \frac{d(1)+1}{1-\beta} + \frac{\beta D((1-\tau)w+T) + (1-\beta)Da}{\beta(1-\tau)w(1-\beta)}.$$

  - Second optimization (not working): From the FOC,

$$a' = \frac{1}{\beta}a + T - \frac{(1-\tau)w}{D}$$

and thus

$$c = \frac{(1-\tau)w}{D}$$

follows. Therefore, the value from not working is

$$W(a,0) = \frac{\log\big((1-\tau)w/D\big)}{1-\beta} - \frac{d(1)+1}{1-\beta} + \frac{\beta D((1-\tau)w+T) + (1-\beta)Da}{\beta(1-\tau)w(1-\beta)}.$$

Thus, $W(a,1) = W(a,0)$ and the agent is indifferent.

- Case 4: When $a \in (a^*, \bar{a})$:
  - First optimization (working): From the FOC, $a' = a - \beta(1-\tau)w$ follows.
Thus, $c = ((1-\beta)/\beta)a + (1-\beta)(1-\tau)w + T$. Thus, the value from working is

$$W(a,1) = \log\left(\frac{1-\beta}{\beta}a + (1-\beta)(1-\tau)w + T\right) - d(1) + \beta\frac{\log\big(((1-\beta)/\beta)a + (1-\beta)(1-\tau)w + T\big) - d(0)}{1-\beta}.$$

  - Second optimization (not working): From the FOC,

$$a' = \frac{1}{\beta}a + T - \frac{(1-\tau)w}{D}$$

and thus

$$c = \frac{(1-\tau)w}{D}$$

follows. Therefore, the value from not working is

$$W(a,0) = \frac{\log\big((1-\tau)w/D\big)}{1-\beta} - \frac{d(1)+1}{1-\beta} + \frac{\beta D((1-\tau)w+T)+(1-\beta)Da}{\beta(1-\tau)w(1-\beta)}.$$

One can show (see the details later) that for $a \in (a^*, \bar{a})$, it is always the case that $W(a,0) > W(a,1)$.

- Case 5: When $a \geqslant \bar{a}$:
  - First optimization (working): From the FOC, $a' = a - \beta((1-\tau)w)$ follows. Thus, $c = ((1-\beta)/\beta)a + (1-\beta)(1-\tau)w + T$. Thus, the value from working is

$$W(a,0) = \log\left(\frac{1-\beta}{\beta}a + (1-\beta)(1-\tau)w + T\right) - d(1) + \beta\frac{\log\big(((1-\beta)/\beta)a + (1-\beta)(1-\tau)w + T\big) - d(0)}{1-\beta}.$$

  - Second optimization (not working): From FOC, $a' = a$. Therefore, from the budget constraint, $c = ((1-\beta)/\beta)a + T$ follows. Thus, the value from not working is

$$\begin{aligned} W(a,1) &= \log\left(\frac{1-\beta}{\beta}a + T\right) - d(0) + \beta\frac{\log\big(((1-\beta)/\beta)a + T\big) - d(0)}{1-\beta} \\ &= \frac{\log\big(((1-\beta)/\beta)a + T\big) - d(0)}{1-\beta}. \end{aligned}$$

  One can show (see the details later) that for $a \in [\bar{a}, \infty)$, it is always the case that $W(a,0) > W(a,1)$.

This completes the proof. □

Details: Here, we fill in the details of the proof. In particular, we check two things for each case (in addition to the obvious ones).

1. That we are taking the FOCs in the right region of $V(a')$ in each optimization for a'.
2. The work/leisure inequality.

In the following, we check them one by one.

- *Case* 1: Clearly, the FOCs (both working and not working) are taken in the first region of the value function. Now we check that $W(a,1) > W(a,0)$. This is equivalent to showing that

$$\log\left(\frac{1-\beta}{\beta}a + (1-\tau)w + T\right) - d(1) > \log\left(\frac{1-\beta}{\beta}a + \beta(1-\tau)w + T\right) - (1-\beta)d(0) - \beta d(1).$$

  That is,

$$\log\left(\frac{((1-\beta)/\beta)a + (1-\tau)w + T}{((1-\beta)/\beta)a + \beta(1-\tau)w + T}\right) > (1-\beta)D.$$

  Since the LHS is decreasing in a, it is sufficient to show that this holds when $a = \underline{a}$. Using the expression for $\underline{a}$ and rearranging, the inequality we need to show becomes

$$-\log(1-(1-\beta)D) > (1-\beta)D.$$

  Since $-\log(1-x) > x$ for any $x > 0$, the result follows.

- *Case* 2: First check the FOCs for each optimization.
  First optimization: To show: a' is in $[\underline{a}, \bar{a}]$. This can be checked by the expression of a' and the fact $a \in (\underline{a}, a^*)$, as the following shows.
  It turns out (with some algebra) that $a \in (\underline{a}, a^*)$ corresponds to $a' \in (\underline{a}, \underline{a} + (1-\tau)w)$. Since $\beta > \frac{1}{2}$, $\beta((1-\tau)w)/(1-\beta) > (1-\tau)w$. Thus, $\underline{a} + (1-\tau)w < \underline{a} + \beta(1-\tau)w/(1-\beta) = \bar{a}$.
  Second optimization: $a' < \underline{a}$ can easily be seen from the expression for a'.
  Now, we check that $W(a,1) > W(a,0)$. We need to show that

$$\log\left(\frac{(1-\tau)w}{D}\right) - d(1) - 1 + \frac{\beta D((1-\tau)w+T)+(1-\beta)Da}{\beta(1-\tau)w} > \log\left(\frac{1-\beta}{\beta}a + \beta(1-\tau)w + T\right) - (1-\beta)d(0) - \beta d(1).$$

That is,

$$-(1-\beta)D-1+\frac{D((1-\tau)w+T)}{(1-\tau)w}+\frac{(1-\beta)D}{\beta((1-\tau)w)}a>\log\left(\frac{(1-\beta)D}{\beta(1-\tau)w}a+\frac{D(1-\beta)}{(1-\tau)w}+\frac{D\beta(1-\tau)w+DT}{(1-\tau)w}\right).$$

Both sides are equal when $a=a_*$. Thus, to show the claim, we only need to show that the slope of the RHS, as a function of a, is larger than the slope of the LHS for $a\in(\underline{a},a_*)$. The slope of the LHS is

$$\frac{(1-\beta)D}{\beta(1-\tau)w}$$

and the slope of the RHS is

$$\frac{(1-\beta)D}{\beta(1-\tau)w}\times\frac{1}{f(a)},$$

where

$$f(a)\equiv\frac{(1-\beta)D}{\beta(1-\tau)w}a+\frac{D(1-\beta)}{(1-\tau)w}+\frac{D\beta(1-\tau)w+DT}{(1-\tau)w}.$$

For $a\in(\underline{a},a_*)$, $f(a)\in(1-D(1-\beta),1)$. Thus the slope of the RHS is always larger.

- *Case* 3: We only need to check that we are in the right region in the optimizations.
  First optimization: Check that $a'\in[\underline{a},\bar{a}]$. From the expression for a' and $a'\in[a_*,a^*]$, $a'\in[\underline{a}+(1-\tau)w,\bar{a}]$ follows.
  Second optimization: Check that $a'\in[\underline{a},\bar{a}]$. From the expression for a' and $a'\in[a_*,a^*]$, $a'\in[\underline{a},\bar{a}-(1-\tau)w]$ follows.
- *Case* 4: First, check the FOCs.
  First optimization: $a'\geqslant\bar{a}$ can easily be seen from the expression for a'.
  Second optimization: To show: $a'\in(\underline{a},\bar{a})$. This can be checked by the expression for a' and the fact $a\in(a^*,\bar{a})$, as the following shows.
  It turns out (with some algebra) that $a\in(a^*,\bar{a})$ corresponds to $a'\in(\bar{a}-((1-\tau)w),\bar{a})$. Since $\beta>\frac{1}{2}$, $\beta((1-\tau)w)/(1-\beta)>(1-\tau)w$. Thus, $\bar{a}-((1-\tau)w)>\bar{a}-\beta((1-\tau)w)/(1-\beta)=\underline{a}$.
  Now, we check that $W(a,0)>W(a,1)$. We need to show that

$$\log\left(\frac{(1-\tau)w}{D}\right)-d(1)-1+\frac{\beta D((1-\tau)w+T)+(1-\beta)Da}{\beta(1-\tau)w}>\log\left(\frac{1-\beta}{\beta}a+(1-\beta)(1-\tau)w+T\right)-(1-\beta)d(1)-\beta d(0).$$

That is,

$$\beta D-1+\frac{D((1-\tau)w+T)}{(1-\tau)w}+\frac{(1-\beta)D}{\beta(1-\tau)w}a>\log\left(\frac{(1-\beta)D}{\beta(1-\tau)w}a+\frac{D(1-\beta)(1-\tau)w}{(1-\tau)w}+\frac{DT}{(1-\tau)w}\right).$$

Both sides are equal when $a=a^*$. Thus, to show the claim, we only need to show that the slope of the RHS, as a function of a, is smaller than the slope of the LHS for $a\in(a^*,\bar{a})$. The slope of the LHS is

$$\frac{(1-\beta)D}{\beta(1-\tau)w}$$

and the slope of the RHS is

$$\frac{(1-\beta)D}{\beta(1-\tau)w}\times\frac{1}{g(a)},$$

where

$$g(a)\equiv\frac{(1-\beta)D}{\beta(1-\tau)w}a+\frac{D(1-\beta)(1-\tau)w}{(1-\tau)w}+\frac{DT}{(1-\tau)w}.$$

For $a\in(a^*,\bar{a})$, $g(a)\in(1,1+D(1-\beta))$. Thus the slope of the RHS is always smaller.

- *Case* 5: Clearly, the FOCs (both working and not working) are taken in the third region of the value function. We check that $W(a,0)>W(a,1)$. This is equivalent to showing that

$$\log\left(\frac{1-\beta}{\beta}a+T\right)-d(0)>\log\left(\frac{1-\beta}{\beta}a+(1-\beta)(1-\tau)w+T\right)-(1-\beta)d(1)-\beta d(0).$$

That is,

$$\log\left(\frac{((1-\beta)/\beta)a+T}{((1-\beta)/\beta)a+(1-\beta)(1-\tau)w+T}\right)>-(1-\beta)D.$$

Since the LHS is increasing in a, it is sufficient to show that this holds when $a=\bar{a}$. Using the expression for $\bar{a}$ and rearranging, the inequality we need to show becomes

$$-\log(1+(1-\beta)D)>-(1-\beta)D.$$

Since $-\log(1+x)>-x$ for any $x>0$, the result follows.

Proposition 2. *Consider a complete-markets version of our model where an employment lottery is available. The aggregate values of K and H are identical between this complete-markets model and the incomplete-markets model in which all individuals have assets in the indifference region.*

Proof. We start by characterizing equilibrium for the incomplete-markets economy. In equilibrium, K is equal to the sum of the individual asset holdings a and H is equal to the mass of individuals who decide to work. From the FOCs of the firm,

$$r = \alpha\left(\frac{K}{H}\right)^{\alpha-1}$$

and

$$w = (1-\alpha)\left(\frac{K}{H}\right)^{\alpha}$$

hold. From the worker's Euler equation,

$$\frac{1}{\beta} = 1 + \alpha\left(\frac{K}{H}\right)^{\alpha-1} - \delta \tag{1}$$

holds.

Integrating across all the workers' budget constraints we have that

$$K = \frac{1}{\beta}K + w(1-\tau)H + T - C. \tag{2}$$

Given that the government balances its budget each period, we have $\tau wH = T$.

Further, assume that this economy has workers only in the $(\underline{a}, \bar{a})$ region. Then, everyone has consumption given by $(1-\tau)w/D$, and

$$K = \frac{1}{\beta}K + (1-\alpha)\left(\frac{K}{H}\right)^{\alpha}H - \frac{(1-\tau)(1-\alpha)\left(K/H\right)^{\alpha}}{D} \tag{3}$$

holds. We can obtain K and H by solving (1) and (3). T can then be calculated as $T = \tau wH$.

When an employment lottery is available in a complete-market setting, in the steady state a worker solves the following problem:

$$V(a) = \max_{a',\lambda} \log((1+r-\delta)a + \lambda(1-\tau)w + T - a') - \lambda d(1) - (1-\lambda)d(0) + \beta V(a'),$$

where λ is the employment probability. The FOC for the asset choice yields

$$\frac{1}{c} = \beta(1+r-\delta)\frac{1}{c'}$$

thus again, in steady state, $1/\beta = (1+r-\delta)$ has to hold. Since the firm's FOC is identical to the incomplete-market case, Eq. (1) has to hold in this model.

Note that $\lambda = H$ in equilibrium. Summing up the budget constraint in this economy yields the same equation as (2). The FOC for λ yields

$$(1-\tau)w\frac{1}{c} = D,$$

therefore

$$c = \frac{(1-\tau)w}{D}.$$

Note that this is identical to the incomplete-market case. Thus, Eq. (3) holds in the complete-market economy. Since K and H solve (1) and (3) in both economies, the solution has to be identical. □

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