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Investment-Specific R&D and the Decline in the Relative Price of Capital

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The relative price of capital has declined at a rapid rate in the postwar period.This article provides a candidate explanation for this relative price decline—research and development that are embodied in new, more efficient investment goods. The model mimics the secular aspects of the data, and it has the property that the long-run growth rate of consumption is nontrivially determined as a function of the R&D efforts. Because growth is driven by investment in durable goods in the present model, it seems natural to assume that R&D is product-specific and that the firms producing these goods are long-lived profit centers that internalize the dynamic gains from R&D. A result of this assumption is that the growth rate in the decentralized economy is too low: the so-called business stealing effects that may cause the equilibrium growth rate to be too high in other models is internalized here in the form of planned obsolescence.

Keywords: investment, R&D, planned obsolescence

JEL classification: D92, O30, O32, O40

1. Introduction

The relative price of equipment capital has fallen at a high rate in the postwar period: the measures provided recently for the United States show an annual rate of decline of more than 3 percent (see Gordon, 1990). When the price decline is identified as investment-specific (exogenous) technological change and compared to residual, sector-neutral productivity change, these measures imply that the investment-specific developments account for more than 50 percent of total consumption growth during this period.[1] Given that these sector-specific movements seem so important for aggregate growth, we need to ask, What are the origins of the price decline? In this article I explore one candidate's answer to this question. I take the view, first of all, that the price decline does reflect investment-specific technological change. Second, I hypothesize that this technological change is a result of explicit R&D decisions on the level of the private firm.

The story considered here is based on the ongoing development of new and better equipment technologies, such as the recent advances we have observed in information processing, telecommunication, transportation, robotization, and so on. The formal analysis in this article summarizes these phenomena in a vintage-capital version of the standard growth model (which is in the spirit of that in Solow, 1959) in which new vintages have endogenous efficiency levels. Capital comes in different types—monopolistic competition is assumed—and there is an R&D decision regarding how much to upgrade the efficiency of each type of capital from one vintage to the next. Along the balanced growth path of the model, the markups on these capital goods are constant in percentage terms. Thus, the model of equilibrium technology choice developed in this article suggests that the rate of price decline for investment goods in the data indeed should be interpreted as the growth rate of investment-specific technology.

Because growth is driven by investment in durable goods in the present model, it seems natural to assume that R&D is product specific and that the firms producing these goods are long-lived profit centers that internalize at least part of the dynamic gains from R&D and product development. This is an alternative to the typical assumption in the literature (see Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; and Segerstrom, 1991), where these dynamic returns typically are regarded as pure externalities.

A result of assuming that R&D is product specific is that the growth rate in the decentralized economy is too low: the so-called business stealing effects that may cause the equilibrium growth rate to be too high in other models becomes internalized here. This phenomenon can be referred to as planned obsolescence, which is often associated with the kinds of capital-embodied technological change that I study in a stylized way in this article.

Section 2 presents the environment and defines equilibrium, and Section 3 characterizes symmetric, balanced-growth equilibria. Section 4 compares the decentralized to the centralized outcomes and shows that the long-run equilibrium growth rate is always too low.

2. The Model

2.1. The Economic Environment

There is one consumption good per period, and there is a representative, infinitely lived consumer with additively separable preferences over this good. The discount factor is $\beta < 1$, and the period utility function u satisfies $u(c) = \lim_{v\to\sigma} \frac{c^{1-v}-1}{1-v}$, where $\sigma > 1$. The consumer is also endowed with L time units each period but does not value leisure.

The technology available for producing final output y satisfies

$$y = \left(\int \mathbf{k}^{\alpha}\right) m^{1-\alpha}, \tag{1}$$

where $\mathbf{k}$ is a vector of capital services, and m is the amount of labor employed in this sector.[2] There is a given interval of capital services available at all moments in time, $\mathbf{k}_t : [0, \mu] \to \mathbf{R}_+$, and the index j is used for specific capital types. (Boldfaced fonts are used throughout to denote positive, real-valued functions on the domain $[0, \mu]$.)

Investment, i, is produced one for one from final output and builds capital according to

$$\mathbf{k}_{j,t+1} = (1-\delta)\mathbf{k}_{jt} + \mathbf{T}_{j,t+1}\mathbf{i}_{jt}, \tag{2}$$

where $\mathbf{T}_{jt}$ denotes the efficiency level for capital of type i at time t, and $\mathbf{i}_{jt}$ is the investment good at t used for capital of type j.[3] Note that the physical depreciation rate in this model, δ, is not equal to the rate of "economic depreciation," $1-(1-\delta)\mathbf{T}_{jt}/\mathbf{T}_{j,t+1}$, which increases when higher quality capital arrives and which is endogenous in this model. The resource constraint for output, finally, reads $c + \int \mathbf{i} \leq y$.

The key variable in the analysis is the sequence $\{\mathbf{T}_t\}_{t=0}^{\infty}$. Greenwood, Hercowitz, and Huffman (1988) and Greenwood, Hercowitz, and Krusell (1997b) analyze environments where this sequence is an exogenous stochastic process used as the driving force behind economic growth as well as economic cycles. These economies are convex and admit a standard decentralization.

Here, $\mathbf{T}_{jt}$ is modeled as a result of explicit R&D decisions, and all the aspects of the research technology will be embedded in the description of how $\mathbf{T}_{jt}$ is accumulated. I use a simple parametric form that captures some of the main ingredients of the R&D technology:

$$\mathbf{T}_{j,t+1} = \mathbf{T}_{jt}^{\gamma} \bar{T}_t^{1-\gamma} H(\mathbf{n}_{jt}), \tag{3}$$

where $\mathbf{n}_j$ is an amount of labor used for R&D in sector j, $\bar{T}$ is the average technology level across product varieties, and the parameter γ is in [0,1]. Given current technology levels, R&D output is strictly increasing in labor, but it is bounded and the marginal productivity is declining: $H' > 0$, $H < b$ for some $b > 1$, and $H'' < 0$. Also, $H'(0) = \infty$ to ensure interior solutions. Last, to ensure that the maximization problems of the R&D units are well-defined, the bound on b has to be related to preferences and technology: I assume $b < \beta^{\frac{\alpha-1}{\alpha}}$.

Unless $\gamma = 0$, there are dynamic returns to R&D from the point of view of a single capital variety. In the decentralized economy, each product variety will be produced by only one firm, and $\gamma > 0$ will therefore also mean that these dynamic returns to R&D are internalized. Further when $\gamma < 1$, there is a spillover across capital technologies: an R&D advance in technology j will also have positive impact on the technology advance for other products. In summary, γ is a measure of the *relative* importance of the dynamic, product-specific returns to R&D. Also, since $H' > 0$, it is clear that there are increasing returns within each given capital variety (that is, taking the technology levels of other varieties as given). These increasing returns are stronger, and more *dynamic*, the higher the value of γ.

Finally, labor is perfectly substitutable across its different uses: $\int \mathbf{n}_t + m_t \leq L$.

2.2. Equilibrium

The decentralized economy consist of utility-maximizing consumers, a perfectly competitive sector with a large number of final good-producing firms, and an interval $[0, \mu]$ of capital-producing firms. I assume that there is monopolistic competition in the latter sector: there is one producer per type of capital. The monopolistic competitors sell capital services to the sector producing final goods. They all compete for the same labor and for the same investment goods and consequently take the corresponding prices as given, whereas they perceive a downward-sloping inverse demand function for their own good. The monopolistic firms will typically be making positive profits in equilibrium, and one way to formally describe the market form is that there are infinite patents for each type of capital. The infinite-patent assumption is motivated in part as a shortcut (it avoids explicit modeling of those costs that in practice give existing producers an edge over potential entrants and that explain why firms are long-lived) and in part as an alternative benchmark to the standard assumption that research firms are entirely short-lived and do not internalize the dynamic returns to research.

The consumer's and the final-output-producing firm's maximization problems are straightforward. The consumer maximizes utility subject to $\sum_{t=0}^{\infty} p_t c_t \leq \sum_{t=0}^{\infty} p_t w_t L + \int \Pi$, where p_t is the Arrow-Debreu price of consumption at t, $p_t w_t$ is the price of labor at t, and Π_j are the profits from firm j. The firm maximizes the static function $y - \int \mathbf{r}\mathbf{k} - wm$, where $\mathbf{r}_j$ is the rental price of capital, leading to marginal product conditions $\mathbf{r}_j = \alpha \mathbf{k}_j^{\alpha-1} m^{1-\alpha}$, which is capital-producing firm j's inverse demand for its product, and $w = (1-\alpha)\int \mathbf{k}^{\alpha} m^{-\alpha} = (1-\alpha)y/(m/\mu)$ (note that m/μ is productive labor per type of capital, since μ is the number of varieties).

The monopolist producing capital of type j chooses $\{\mathbf{r}_{jt}, \mathbf{k}_{j,t+1}, \mathbf{n}_{jt}, \mathbf{i}_{jt}, \mathbf{T}_{j,t+1}\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} p_t \mathbf{r}_{jt} \mathbf{k}_{jt} - \sum_{t=0}^{\infty} p_t [w_t \mathbf{n}_{jt} + \mathbf{i}_{jt}]$$

subject to the inverse demand function and the R&D technology. When the former is substituted in, this maximization problem can thus be written

$$\max_{\{\mathbf{k}_{j,t+1}, \mathbf{n}_{jt}, \mathbf{T}_{j,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} p_t \left(\alpha \mathbf{k}_{jt}^{\alpha} m_t^{1-\alpha} - w_t \mathbf{n}_{jt} - \frac{\mathbf{k}_{j,t+1} - (1-\delta)\mathbf{k}_{jt}}{\mathbf{T}_{j,t+1}} \right) \tag{4}$$
$$\text{s.t. } \mathbf{T}_{j,t+1} = \mathbf{T}_{jt}^{\gamma} \bar{T}_t^{1-\gamma} H(\mathbf{n}_{jt}) \ \forall t,$$

with $\mathbf{k}_{j0}$ and $\mathbf{T}_{j0}$ given. The maximized value is denoted Π_j.

An equilibrium for this economy is defined as a set of prices, quantities, and profits such that all agents maximize and markets for goods and labor clear.

3. Symmetric, Balanced-Growth Equilibria

A balanced-growth, or steady-state, equilibrium is one in which all variables grow at constant rates. A *symmetric* equilibrium is one in which all firms have the same technology levels over time. The focus in this article is on symmetric steady states. There are other outcomes of interest; in particular, when the dynamic, product-specific returns to R&D are strong enough, there may be asymmetric steady-state equilibria, in which large and small capital firms coexist. I study this kind of implication in another paper (Krusell, 1995), where the R&D is in nondurable intermediate goods. Here, since the focus is on capital and its role in growth, I assume that parameters are such that asymmetric steady states cannot exist.[4]

I start by deriving some key properties of symmetric steady states using first-order conditions. I thereafter show how long-run R&D is determined, provide an existence result, and perform comparative-statics exercises.

Since the focus is now on a symmetric equilibrium, it simplifies matters to drop all j subscripts and boldfaced fonts and to use $\bar{T} = T$ in equilibrium. Denoting the growth rate of variable x by g_x, it is clear, first, that $g_m = g_n = 0$. Second, it follows from the technological assumptions that

$$g_T = H(n), \;\; g_k = g_i g_T, \;\text{ and } g_c = g_i = g_k^{\alpha}.$$

The consumer's first-order condition implies that the gross real interest rate, $p_t/p_{t+1} \equiv R_t$, equals g_c^{σ}/β on the balanced path.

Moreover, the competitive firm's first-order conditions implies that $g_r = g_k^{\alpha-1}$ and that $g_w = g_k^{\alpha}$. These growth-rate conditions together imply that $g_c = g_k^{\alpha} = H(n)^{\theta}$, where $\theta \equiv \alpha/(1-\alpha)$, so it is possible to summarize by expressing all growth rates and the interest rate as functions of n, the steady-state level of R&D:

$$g_c = g_i = g_w = H(n)^{\theta}, g_k = H(n)^{1/(1-\alpha)}, g_r = H(n)^{-1}, \text{ and } R = (1/\beta)H(n)^{\sigma\theta}. \tag{5}$$

Note that the balanced-growth equilibrium of the model shares many features with that of the standard neoclassical growth model: consumption, investment, and real wages grow at a common rate. However, the capital stock, which is now measured in efficiency units, grows at a higher rate since the technological change here is "embodied" in capital, or "investment specific." And whereas the standard Solowian model has a constant relative price of capital, the present model has a steady-state decline in this price/rental rate along the balanced-growth path iff $H(n) > 1$—that is, there can only be a sustained relative price fall when the R&D technology is such that there is sustained (endogenous) growth in the capital-specific technology. Thus, note that although there will be a markup in this framework, the markup is constant in percentage terms over time, and the relative price falls at a constant rate. This framework thus supports the long-run accounting results in Greenwood, Hercowitz, and Krusell (1997a), although they use a fully competitive framework: their results rely on identifying the rate of investment-specific technological progress with the inverse of the growth rate of the relative price, and that identity is a fact in the steady state of this model.

The determinants of the relative price fall can be found by studying the monopolist's problem (4). To derive Euler equations for this problem, it is convenient to first maximize over the capital sequence. This gives k_{t+1} as a function of T_{t+1} and T_{t+2}:

$$k_{t+1} = \alpha^{2(1+\theta)} m_{t+1} u_t^{-(1+\theta)} T_{t+1}^{1+\theta}, \tag{6}$$

where u_t is the *user cost of capital*—that is,

$$u_t = R_t - (1-\delta)\frac{T_{t+1}}{T_{t+2}}.$$

Notice that the user cost of capital includes the capital loss due to the fact that next period's capital is more efficient than current capital (assuming $T_{t+2} > T_{t+1}$).

Equation (6) illustrates the effect of technology and, indirectly, of R&D, on the choice of how much capital to produce. In particular, the choice of k_{t+1} is affected by technology in the following ways. First, the technology level T_{t+1} gives it a direct boost by lowering its production cost. Second, with $\delta < 1$, the choice of k_{t+1} is also affected by the relative technology levels at $t+1$ and $t+2$. In particular, if T_{t+2} is high relative to T_{t+1}, the production of k_{t+1} is discouraged. This is the *planned obsolescence* effect: the producer knows that a side-effect of research at $t+1$ (which makes T_{t+2} high) is to lower the value of date $t+1$ capital—that is, future research lowers the value of producing capital today.[5]

Turning the attention to how to solve for technology levels—that is, how to optimally invest in R&D—it is convenient to substitute the choices of capital levels from (6) back into the objective. This leads to a net cash flows at time t, v_t (measured in terms of goods at t), satisfying

$$\begin{aligned} v_t \;=\; & \alpha^{1+2\theta} m_t u_{t-1}^{-\theta} T_t^{\theta} - w_t n_t \\ & -\alpha^{2(1+\theta)} m_{t+1} u_t^{-(1+\theta)} T_{t+1}^{\theta} + (1-\delta)\alpha^{2(1+\theta)} m_t u_{t-1}^{-(1+\theta)} T_t^{1+\theta} / T_{t+1} \end{aligned}$$

for $t > 0$.[6] Substantial simplification can now be achieved by first restating v_t as $v_t = v_{1t} + v_{2t}$, where

$$v_{1t} = \alpha^{1+2\theta} m_t u_{t-1}^{-\theta} T_t^{\theta} \left(1 + \alpha u_{t-1}^{-1} \frac{T_t}{T_{t+1}} (1-\delta)\right) - w_t n_t$$

and

$$v_{2t} = -\alpha^{2(1+\theta)} m_{t+1} u_t^{-(1+\theta)} T_{t+1}^{\theta}.$$

Since v_{2t} contains variables dated one period later than v_{1t}, it is helpful to collect terms with the same dating: rewrite the present value of all cash flows, $\sum_{t=0}^{\infty} p_t(v_{1t} + v_{2t})$, as $\sum_{t=0}^{\infty} p_t \hat{v}_t$, where

$$\hat{v}_t = v_{1t} + (p_{t-1}/p_t) v_{2,t-1}$$

for $t > 0$ and $\hat{v}_0 = v_{10}$. After collecting terms, the new cash-flow variable thus simply equals

$$\hat{v}_t = (1-\alpha)\alpha^{1+2\theta} m_t u_{t-1}^{-\theta} T_t^{\theta} - w_t n_t. \tag{7}$$

To maximize with respect to the R&D labor input, we thus notice that the technology variable T_{t+1} affects cash flows in a number of different ways. The costs are twofold:

- *Cost 1:* Raising T_{t+1} requires R&D resource expenses at t (through $w_t n_t$ in $\hat{v}_t$).
- *Cost 2:* Raising T_{t+1} increases the user cost of k_t—that is, makes this capital obsolete (through the term u_{t-1} in $\hat{v}_t$).

The second of these costs thus represents the planned obsolesecence effect of research. The benefits from research are threefold:

- *Benefit 1:* Raising T_{t+1} has a direct positive impact on $\hat{v}_{t+1}$ lowering the costs of producing k_{t+1}.
- *Benefit 2:* Raising T_{t+1} decreases the future user cost u_t (in $\hat{v}_{t+1}$) by making capital used at $t+1$ closer in efficiency to the next vintage of capital.

- *Benefit 3:* Raising T_{t+1} creates a dynamic gain from research by raising the productivity of any future research efforts. This can also be expressed as lowering the research effort at $t+1$ needed to maintain a given level of T_{t+2} (working through the term $w_{t+1}n_{t+1}$ in $\hat{v}_{t+1}$).

This last dynamic gain from present research reflects an important feature of this model: the research firm internalizes some of the beneficial effects of current research by lowering the future costs of further researching its own product line. Of course, this effect is present only when $\gamma > 0$. Similarly, as in other models, there is also an externality associated with research in that other firms can partly build their research on yours (so long as $\gamma < 1$).

The above cost-benefit considerations have their formal counterpart in the first-order condition obtained by setting the derivative of the present value of cash flows with respect to each period's value of T_{t+1} (for $t \geq 1$) equal to zero.[7] Multiplying this first-order condition by T_{t+1}/p_{t+1} and rearranging then gives

$$\begin{aligned}&\alpha^{2(1+\theta)} m_{t+1} u_t^{-\theta-1} R_t T_{t+1}^{\theta} - \alpha^{2(1+\theta)} m_t u_{t-1}^{-\theta-1} R_t T_t^{\theta} (1-\delta) \frac{T_1}{T_{t+1}} \\ &\quad = w_{t+1} \left(R_t \frac{w_t}{w_{t+1}} \frac{H(n_t)}{H'(n_t)} - \gamma \frac{H(n_{t+1})}{H'(n_{t+1})} \right), \quad t > 1. \end{aligned} \tag{8}$$

Equation (8) (a second-order difference equation in T_t from the point of view of the monopolist) thus incorporates the benefits and costs from above, and it is central in determining equilibrium research efforts in this model.[8]

It is now possible to find the balanced-growth research level and, thus, the economy's long-run growth rate by combining the stated first-order conditions and the resource constraints with the expressions for how the growth rates of variables depend on research efforts. Therefore, substitute the market-clearing condition for labor, $L = m + \mu n$, the growth-rate formulas (equation (5)), and the expression for the wage rate into (8), and simplify to obtain[9]

$$\begin{aligned}&\alpha\beta\theta(L/\mu - n) H(n)^{-(2+\sigma\theta)} (H(n)^{1+\theta} - (1-\delta)) H'(n) \\ &\quad = (1 - \beta(1-\delta) H(n)^{-1-\sigma\theta})(1 - \gamma\beta H(n)^{(1-\sigma)\theta}). \end{aligned} \tag{9}$$

Equation (9), which has only one unknown, n, is the core of the characterization of symmetric, balanced-growth paths for this model.

Existence and uniqueness of solutions to (9) depend in part on what is assumed about $H(0)$—that is, on what happens to technologies in the absence of research efforts. If it is assumed that $H(0) \geq 1$, so that there is no "forgetting," then existence and uniqueness are straightforward to obtain: under the added condition that $\delta > (1+\theta)/(2+\sigma\theta)$, one can show that the left-hand side of (9) is strictly decreasing in n (that the right-hand-side is increasing in n follows from $H(n) \geq 1$).[10] The appendix states and proves the formal existence and uniqueness result.

The comparative statics exercises, which can be obtained by manipulating (9), involve changes in the parameters $\gamma, \beta, \alpha, \delta, L$, and μ. Under the assumption ensuring uniqueness—$\delta > (1+\theta)/(2+\sigma\theta)$—the left-hand side of (9) is decreasing in n and the right-hand side is increasing in n, so it is straightforward to analyze the effects of variations in parameters by determining how they alter each side of the equation.

Since γ appears only on the right-hand side of (9) and lowers the value of this side for any given value of n, it follows that the long-term growth rate must be increasing in γ. In other words, the larger the fraction of the dynamic returns to R&D that is internalized, the higher the long-term growth rate.

As in standard growth models, long-run growth is increasing in β, the degree of patience. However, unlike in standard growth models, the effects of the rate of physical depreciation δ are not clearcut: although a low depreciation rate raises the return to capital accumulation and thus works to increase the growth rate, it also increases the extent to which planned obsolescence hampers technological development, thus working in the opposite direction.

The effect of α, capital's share of income, is not transparent since the left-hand side of (9) is not monotone in α. This reflects, on the one hand, that a high α raises the return to capital accumulation, since it raises the marginal product of capital. On the other hand, a high α also makes the demand for capital services more elastic, thus lowering markups and the profits from accumulation (when $\alpha = 1$, capital goods are perfect substitutes).

The effects of the size of the economy, as measured by the size of the labor force L, are clearcut and standard for growth models building on increasing returns: the larger the economy is, the more research labor is in effect available, so the faster is growth. For the same reason, the effects of the exogenous amount of variety, μ, are the reverse: more variety means more types of capital goods and thus less research labor available for each capital type.

4. Optimal Allocations

The economy studied has two sources of distortions—monopoly power and technology spillovers. In this section I briefly comment on properties of optimal allocations for our economy. The problem of the planner is to choose a distribution of R&D at each moment in time and to allocate final output between consumption and investment into the different capital goods to maximize

$$\sum_{t=0}^{\infty} \beta^t u \left(\left(\int \mathbf{k}^{\alpha} \right) \left(L - \int H^{-1} \left(\frac{\mathbf{T}_{t+1}}{\mathbf{T}_t^{\gamma} \bar{T}_t^{1-\gamma}} \right) \right)^{1-\alpha} - \int \frac{\mathbf{k}_{t+1} - (1-\delta)\mathbf{k}_t}{\mathbf{T}_{t+1}} \right). \quad (10)$$

Differentiating with respect to $\mathbf{k}_t(i)$ and $\mathbf{T}_t(i)$ and eliminating the multipliers yields a dynamic system of necessary conditions that, as in the case of equilibria, is consistent with symmetric, balanced growth.[11]

It is straightforward to develop these necessary conditions into the following equation, which characterizes the set of symmetric, optimal balanced-growth paths:

$$\begin{aligned} &\beta\theta(L/\mu - n)H(n)^{-(2+\sigma\theta)}(H(n)^{1+\theta} - (1-\delta))H'(n) \\ &\quad = (1 - \beta(1-\delta)H(n)^{-1-\sigma\theta})(1 - \beta H(n)^{(1-\sigma)\theta}). \end{aligned} \quad (11)$$

First, we observe that equation (11) is independent of γ. This is because when all firms are alike and the externalities are internalized by the planner, the relative importance of the firm-specific and general returns to R&D do not matter. This can be compared to equation

(9), which determines the set of symmetric balanced-growth equilibria. In the latter, growth is higher when γ is larger. Note the other difference between the steady-state equations for the optimal and decentralized solutions: the latter equation has an α multiplying the left-hand side. This is the effect of monopoly power, and it makes the equilibrium steady-state growth rate lower than optimal.

The two sources of distortion—externalities and monopoly power—both lead to a growth rate that is too low in equilibrium. This contrasts the existing literature on R&D races in equilibrium growth models (see, i.e., the presentation in Grossman and Helpman, 1991), where the relative ranking of the equilibrium and the optimal growth rates is nontrivial. The latter result is due to the so-called business-stealing effect: when an innovation replaces an existing idea, there is a loss in value to those controlling the old idea. Here, the same phenomenon is present—new models of equipment replace old machines—but all the business-stealing is internalized. Instead I refer to the phenomenon here as planned obsolescence.

As far as economic policy aimed at improving on the decentralized outcome, it is easy to show that simple proportional investment subsidies, income subsidies, or consumption subsidies can be implemented so that the symmetric balanced-growth equilibrium path coincides with the optimal symmetric balanced-growth path.

Appendix

To show existence, it is necessary to show that there is a research level n that solves (9) and also to demonstrate that the monopolist maximizes on the given path. This is nontrivial here, since the objective function of the monopolist is not concave (unless $\gamma = 0$—that is, unless there is no dynamic, product-specific returns to R&D).

To this end, define $R^t(z_t, z_{t+1})$, where $z_t \equiv T_t/\bar{T}_t$, as the period return function of the monopolist's objective when this is expressed with the constant discount factor λ in a context when all the other agents are on a balanced-growth path ($\bar{T}_t = T_0 H(n)^t$ and so on). Then $R^t(z_t, z_{t+1}) = R(z_t, z_{t+1}) \equiv a_1(1 - a_4 z_t/z_{t+1})^{-\theta} z_t^{\theta} - a_2 \tilde{H}^{-1}(z_{t+1}/z_t^{\gamma})$ for $t \geq 1$ and $R^0(z_t, z_{t+1}) \equiv a_3 + (1-\delta)k_0/z_{t+1} - a_2 \tilde{H}^{-1}(z_{t+1}/z_t^{\gamma})$, where the a_i's are constants that are functions of primitives.[12] Also, define $\mathbf{S}$ as the recursive constraint correspondence in the monopolist's problems—that is, $\mathbf{S} \equiv \{(z_t, z_{t+1}) \in \mathbf{R}_+^2 : z_t/z_{t+1}^{\gamma} = [H(0)/H(n), b/H(n))\}$. The following condition can now be used to show that the monopolist indeed maximizes by setting $z_{t+1} = 1$ when $z_t = 1$ for all t:

Assumption: *R, R^0, and $\mathbf{S}$ are such that*

$$(1,1) \in \arg\max_{(x,x') \in \mathbf{S}} R(x,x') - R(1,1) - R_1(1,1)(x-1) - R_2(1,1)(x'-1) \text{ and}$$

$$(1,1) \in \arg\max_{(1,x') \in \mathbf{S}} R^0(1,x') - R^0(1,1) - R_2^0(1,1)(x'-1).$$

This assumption says that the period return function is everywhere below its tangent hyperplane at the hypothesized optimum (1,1), and it is a weaker condition than requiring R^t to be globally concave.

We now have

Theorem: *Under the assumptions made above, together with $H(0) \geq 1$, there exists a symmetric, balanced-growth equilibrium. If $\delta > (1+\theta)/(2+\sigma\theta)$, then there is only one such equilibrium.*

Proof: For existence, note that equation (9) determining the long-run research effort has to have a solution, since its left-hand side is continuous in n and goes from ∞ to 0 as n moves from 0 to L/μ, whereas its right-hand side is continuous and strictly positive for all values of n. Furthermore, given the above assumptions, it is straightforward to employ an alteration of the proof used in Krusell (1995) to show that the first-order conditions are sufficient for a maximum.

For uniqueness, observe that the right-hand side is a product of two positive factors, both of which are increasing in n. The left-hand side is decreasing in n provided that $H(n)^{-2(2+\sigma\theta)}(H(n) - (1-\delta))$ is decreasing. Taking derivatives of this expression and simplifying, we obtain

$$H(n)^{-2-\theta(\sigma-1)}[-1+\theta(1-\sigma)+(1-\delta)(2+\sigma\theta)H(n)^{-1+\theta}],$$

which is negative if $(1-\delta)(2+\sigma\delta) < 1-\theta(1-\sigma)$ since $H(n) \geq 1$. This inequality can be rewritten as $\delta > (1+\theta)/(2+\sigma\theta)$. ■

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Notes

1. See Greenwood, Hercowitz, and Krusell (1997a) for details.
2. Time subscripts are suppressed whenever they are not essential.
3. It is possible to think of $\mathbf{i}_{jt}(1-\delta)^{(s-t)}$ as the number of undepreciated machines of vintage t and of $\mathbf{i}_{jt}\mathbf{T}_{j,t+1}(1-\delta)^{(s-t)}$ as the corresponding stock of vintage t capital measured in efficiency units. Given the above law of motion, it is clear that the vintages aggregate according to $\mathbf{k}_t \equiv \mathbf{i}_{t-1}\mathbf{T}_t + (1-\delta)\mathbf{i}_{t-2}\mathbf{T}_{t-1} + (1-\delta)^2\mathbf{i}_{t-3}\mathbf{T}_{t-2} + \cdots + (1-\delta^t)\mathbf{k}_0$.
4. Since the condition on parameters is not transparent without an explicit discussion of asymmetric steady states, I do not state it here; suffice it to say a key part of this condition is that γ be "large."
5. For another example of a model with an explicit analysis of the dynamics of planned obsolescence, see Pesendorfer (1995).
6. For $t = 0$, the expression is different in that k_0 is not a function of technology levels but given exogenously.
7. It is straightforward to show that the solution has to be interior.
8. In terms of the labeling above, the first term on the left-hand side of (8) combines Benefit 1 and Benefit 2 and the second term on the left-hand side is Cost 2; the terms on the right-hand side represent, in order, Cost 1 and Benefit 3.

9. Note that the balanced-growth wage rate can be written as $w_t = (1-\alpha)\mu k_t^{-\alpha} m^{-\alpha} = (1-\alpha)\mu\alpha^{2\theta} u^{-\theta} T_t^{\theta}$, where $u = R - (1-\delta)/H(n)$.
10. Existence does not require particular restrictions on δ; the lower bound, moreover, is not a necessary condition for uniqueness. In particular, calibration to yearly U.S. data, which would require a δ lower than this bound, gives a unique steady state.
11. For large values γ, asymmetric balanced growth paths are possible here as well.
12. With T_0 normalized to 1, we have $a_1 = AH(n)$, $a_2 = H(n)w$, $a_3 = H(n)\alpha k_0^{\alpha}(L - \mu n)^{1-\alpha}$, and $a_4 = \beta(1-\delta)H(n)^{-(\sigma\theta+1)}$.

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