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Median-voter Equilibria in the Neoclassical Growth Model under Aggregation*

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Abstract

We study a dynamic version of Meltzer and Richard's median-voter model where agents differ in wealth. Taxes are proportional to income and are redistributed as equal lump-sum transfers. Voting occurs every period and each consumer votes for the tax that maximizes his welfare. We characterize time-consistent Markov-perfect equilibria twofold. First, restricting utility classes, we show that the economy's aggregate state is mean and median wealth. Second, we derive the median-voter's first-order condition interpreting it as a tradeoff between distortions and net wealth transfers. Our method for solving the steady state relies on a polynomial expansion around the steady state.

Keywords: Median voter; Markov equilibrium; time consistency; redistribution; aggregation

JEL classification: *E*61; *E*62; *H*21; *H*23; *H*24

I. Introduction

Income taxes are important determinants of aggregate economic performance, and they are fundamentally endogenous. A widely held belief is that the desire to redistribute is a key explanatory factor in their determination, and furthermore that the amount of redistribution is one of the central elements over which elections are decided. One way to evaluate this theory is to construct a reasonably calibrated macroeconomic model and to compare its politico-economic equilibrium to data. However, dynamic models of political economy are complex objects of analysis,

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and the development of the theory and the associated numerical analysis and empirical methods is still in its infancy. The goal of this paper is to contribute to this methodological development. We consider a theory based on endogenous redistribution between consumers of different wealth: they vote on general income tax rates, with associated equal-per-capita lump-sum transfers. The setting is a standard decentralized version of the neoclassical growth model, and the political aggregation mechanism is majority voting. Voting takes place every period and each consumer votes for the current tax rate that maximizes his or her welfare. We characterize time-consistent Markov-perfect equilibria by solving backwards (and, in the infinite-horizon case, considering the limit of finite-horizon economies).

The two most closely related papers in the literature are Krusell and Ríos-Rull (1999) and Meltzer and Richard (1981). The latter paper describes a static setup where distortionary labor taxes are used to fund transfers. Though conceptually constituting the core of modern median-voter models, Meltzer and Richard's setup is not well-suited for quantitative analysis since it does not deal with the taxation of capital income and the effects that associated distortions cause on the evolution of wealth. Krusell and Ríos-Rull (1999) is a fully dynamic model that, like the present paper, considers the taxation of capital income in a Meltzer–Richard kind of setting. Here we make two additional theoretical contributions. First, we develop an aggregation result that makes feasible the analysis of an arbitrary number of wealth types. Second, we derive a first-order condition for the decisive voter that offers a better understanding of some of the intuitive mechanisms behind tax determination than in the existing literature on dynamic political economy. Unlike Krusell and Ríos-Rull (1999), we also assume that governments choose current tax rates (i.e., have a zero "implementation lag"/no ability at all to commit). In terms of implementation, we also propose a different method for computing equilibria, one that is based directly on the median voter's first-order condition.

Our aggregation characterization is as follows: given consumer preferences of the appropriate form, equilibrium outcomes depend on the mean and the median asset holdings, and on nothing else. This theoretical finding is useful because it simplifies both the theoretical and numerical analysis. With aggregation, so long as taxes do not depend on anything but mean and median wealth, neither can prices nor aggregate quantities: marginal propensities to save and work are equal across all consumers, so aggregates can be arrived at by summing up across individuals. Moreover, the propensities cannot depend on higher moments through taxes, since taxes cannot depend on any other moments.

Because there is no commitment, the first-order conditions for the median voter are conceptually different, and more complex, than first-order conditions from standard optimal control theory. They express how the median voter trades off the marginal benefits against the marginal costs of changes in income taxes. The tradeoff is expressed in terms of distortions to labor–leisure and consumption–savings choices—"gaps"—on the one hand and net transfer effects on the other. By measuring the size of each of the gaps in the calibrated politico-economic equilibrium, one could provide an assessment of which tradeoffs matter most. Prior analysis focusing on first-order conditions in similar contexts includes the work on individual saving under time-inconsistent preferences, where reminiscent first-order conditions have been derived, as in Laibson (1997), and some recent work on dynamic public finance, as in Azzimonti, Sarte and Soares (2006) or Klein, Krusell and Ríos-Rull (2006). Azzimonti, de Francisco, Krusell and Ríos-Rull (2005) survey these methods and their use in different applications.

Numerical solution of models of the kind studied in this paper is not straightforward. The level of capital and the tax rate on income depend on the derivatives of the equilibrium decision rules. This means that one cannot specify a finite set of equations in levels only: levels depend on decision-rule derivatives, which in turn depend on higher-order derivatives of these same decision rules. In this paper, we follow Krusell, Kuruşçu and Smith's (2002) method, which builds on approximating the decision rules with polynomials evaluated at the steady-state point only.

Our focus on Markov-perfect equilibria, i.e., those where the state of the economy consists of payoff-relevant information only, as in e.g. Maskin and Tirole (2001), by design rules out the study of the possible role of "reputation" in influencing political outcomes, whereby voters would collectively make their current voting behavior depend on historical voting/policy outcomes; see e.g. Bernheim and Nataraj (2004). Most existing papers on the topic are based either on computational work without much theoretical characterization or on models which are not quantitatively satisfactory. For example, physical capital accumulation is typically ignored or studied in frameworks that do not allow decreasing returns to capital; see e.g. Alesina and Rodrik (1994), Persson and Tabellini (1994) and Hassler, Rodriguez-Mora, Storesletten and Zilibotti (2003).

The model is set up and the aggregation result proved in Section II. The first-order condition and a special case are then studied in Section III, where we also discuss computation. Section IV concludes.

II. The *T*-period Model

The T-period model is a finite extension of Meltzer and Richard (1981) to include capital. This makes taxation more beneficial, on net, for the median voter: capital is inelastically supplied as the tax is decided upon. One can thus argue that by only focusing on labor income, Meltzer and

Richard ignored a key determinant behind large governments: the existence of a large stock of unequally distributed wealth that can be taxed away and redistributed at low cost. The taxation of capital in an intertemporal setting, however, introduces complexity—taxation at t distorts saving in earlier time periods, and because it redistributes, it influences future taxation as well.

In the present section, we will (i) set up the basic environment in a dynamic context and (ii) discuss aggregation.

The Environment: Exogenous Policy

Environment. There is a set of agents who live for T periods and only differ in initial asset holdings. The asset distribution of the population is described by the vector $\mathbf{A}$. We will assume for simplicity that the number of "types" is finite with measure μ_i for type $i \in \{1, 2, \ldots, I\}$. Population size is normalized to one: $\sum_{i=1}^{I} \mu_i = 1$. Instantaneous utility of each agent is $u(c_t, l_t)$, where c_t and l_t are consumption and leisure in period t. Consumption and leisure both have to be nonnegative.

Assumption 1. *Suppose that* $u(c,l) = f_1(f_2(c,l))$, *where* f_1 *is a power function, logarithmic, exponential or quadratic, and* f_2 *is homogeneous of degree 1.*

Production takes place according to a production function that depends on capital—which depreciates at rate δ—and labor and has constant returns to scale: $Y_t = F(K_t, N_{it})$ (we use capital letters to denote aggregates).

Each consumer has one unit of time, so that $l_{it} + n_{it} = 1$ for all i, where n_{it} denotes the amount of hours worked. We will make assumptions on primitives so that agents' decision problems are strictly concave; hence, all agents of the same type will make the same decisions and we can also write $L_{it} + N_{it} = 1$, where L_{it} and N_{it} reflect common decisions regarding leisure and labor, respectively, of all agents of type i. The aggregate labor input is thus $N_t = \sum_{i=1}^{I} \mu_i N_{it}$.

Equilibrium. In a decentralized economy, consumers buy consumption goods and rent capital and sell their labor services to firms under perfect competition. The rental rate for capital is denoted r and the wage rate w, both in terms of consumption goods in the same period. In addition, we now consider a government which taxes income following the exogenous rule $\Psi_t(\mathbf{A})$—capital and labor income are taxed at the same rate—and makes equal lump-sum transfers T back to all consumers under a balanced budget. The rule $\Psi_t(\mathbf{A})$ will be endogenized below.

In equilibrium, consumers' holdings of assets have to add up to the total capital stock: $\sum_{i=1}^{I} \mu_i A_i = K$. Consumer heterogeneity thus originates in $a_i \neq a_j$ for all $i \neq j$. We define a competitive equilibrium for a given government policy as follows:

Definition 1. *For* $\mathbf{A} \in \mathbb{R}^I$ *and given a sequence of policy rules* $\Psi = \{\Psi_t(\mathbf{A})\}_{t=0}^T$, *a* ***recursive competitive equilibrium*** *(RCE) is a set of price rules* $w(K,N)$ *and* $r(K,N)$ *together with value functions* $V_t(a,\mathbf{A})$ *and allocation rules* $T_t(\mathbf{A}), C_t(a,\mathbf{A})$, $L_t(a,\mathbf{A})$, $H_t(a,\mathbf{A})$ $H_t(\mathbf{A})$ *and* $N_t(\mathbf{A})$ *satisfying the following conditions:*

(1) For all $(t,a,\mathbf{A})$, $(C_t(a,\mathbf{A}), L_t(a,\mathbf{A}), H_t(a,\mathbf{A}))$ *solves*

$$V_t(a,\mathbf{A}) = \max_{(c,l,a')\in B_t(a,\mathbf{A})} u(c,l) + \beta V_{t+1}(a',\mathbf{A}')\ s.t.$$

$$\mathbf{A}' = H_t(\mathbf{A}), \quad \textit{for } K \equiv \sum_{i=1}^{I} \mu_i A_i \textit{ and } N \equiv N_t(\mathbf{A}),$$

$$B_t(a,\mathbf{A}) \equiv \{(c,l,a') : c + a' = a + [ar(K,N) + w(K,N)(1-l)](1-\Psi_t(\mathbf{A})) + T_t(\mathbf{A})\}$$

and $V_{T+1}(a,\mathbf{A}) = 0$.

(2) For all $(t,\mathbf{A})$, $H_t(\mathbf{A}) = \{H_t(A_i,\mathbf{A})\}_{i=1}^I$ *and* $N_t(\mathbf{A}) = \sum_{i=1}^I \mu_i(1 - L_t(A_i,\mathbf{A}))$.

(3) For all (K,N), $w(K,N) = F_n(K,N)$ *and* $r(K,N) = F_k(K,N) - \delta$.

(4) For all $(t,\mathbf{A})$, $T_t(\mathbf{A}) = \Psi_t(\mathbf{A})\,[Kr(K,N_t(\mathbf{A})) + N_t(\mathbf{A})w(K,N_t(\mathbf{A}))]$.

Endogenous Taxes: Politico-economic Equilibrium

We assume that the "median consumer", i.e., the consumer with median wealth holdings, is the politically pivotal voter. In the model with fully endogenous taxes presented below, we assume that taxes in period t are voted on in that period. That is, the median voter *cannot commit to future tax rates.* Since he takes future tax functions as given when choosing the current tax rate (denoted by τ_t), he considers: (i) the effect on current utility and (ii) the effect on capital accumulation and, thus, on future utility. The second of these effects involves how the sequence of equilibrium tax rates given by $\{\Psi_s(\mathbf{A}_s)\}_{s=t+1}^T$ will change, since asset accumulation will change. In particular, τ_t influences $\mathbf{A}_{t+1}$, and hence $\Psi_{t+1}(\mathbf{A}_{t+1})$, $\mathbf{A}_{t+2}, \ldots,$ and so on.

Given that the equilibrium is defined recursively (working from period T and back), it is straightforward to analyze these dynamic effects. Consider a tax choice in period t. The median voter will take future functions as given but will also take into account all the effects mentioned above. Formally, we thus need no additional equilibrium objects for how future variables behave—they are all given by the functions in Definition 1, given the initial conditions $\mathbf{A}_{t+1}$. How, then, is $\mathbf{A}_{t+1}$ determined, for every choice of τ_t? And, in order to find the chosen tax rate, how is median utility influenced? At all t, we thus need to introduce new notation for these "one-period deviations". To that end, we define a one-period equilibrium deviation at t as follows.

Definition 2. *Given a recursive competitive equilibrium with indirect utility function* $V_{t+1}(a, \mathbf{A})$, *a* ***period-t deviation equilibrium*** *is defined as a set of functions* $\tilde{V}_t(a, \mathbf{A}, \tau)$, $\tilde{T}_t(\mathbf{A}, \tau)$, $\tilde{C}_t(a, \mathbf{A}, \tau)$, $\tilde{L}_t(a, \mathbf{A}, \tau)$, $\tilde{H}_t(a, \mathbf{A}, \tau)$, $\tilde{H}_t(\mathbf{A}, \tau)$ *and* $\tilde{N}_t(\mathbf{A}, \tau)$ *as follows:*

(1) For all $(a, \mathbf{A}, \tau)$, $(\tilde{C}_t(a, \mathbf{A}, \tau), \tilde{L}_t(a, \mathbf{A}, \tau), \tilde{H}_t(a, \mathbf{A}, \tau))$ *solves*

$$\tilde{V}_t(a, \mathbf{A}, \tau) \equiv \max_{(c,l,a') \in \tilde{B}_t(a,\mathbf{A},\tau)} u(c, l) + \beta V_{t+1}(a', \mathbf{A}') \ s.t.$$

$$\mathbf{A}' = \tilde{H}_t(\mathbf{A}, \tau), \quad \textit{for } K \equiv \sum_{i=1}^{I} \mu_i A_i \textit{ and } N \equiv \tilde{n}_t(\mathbf{A}, \tau),$$

$$\begin{aligned} \tilde{B}_t(a, \mathbf{A}, \tau) &\equiv \{(c, l, a') : c + a' \\ &= a + [ar(K, N) + w(K, N)(1 - l)](1 - \tau) + \tilde{T}_t(\mathbf{A}, \tau)\}. \end{aligned}$$

(2) For all $(\mathbf{A}, \tau)$, $\tilde{H}_t(\mathbf{A}, \tau) = \{\tilde{H}_t(A_i, \mathbf{A}, \tau)\}_{i=1}^{I}$ *and* $\tilde{n}_t(\mathbf{A}, \tau) = \sum_{i=1}^{I} \mu_i (1 - \tilde{L}_t(A_i, \mathbf{A}, \tau))$.

(3) For all $(\mathbf{A}, \tau)$, $\tilde{T}_t(\mathbf{A}, \tau) = \tau[Kr(K, \tilde{n}_t(\mathbf{A}, \tau)) + \tilde{n}_t(\mathbf{A}, \tau) w(K, \tilde{n}_t(\mathbf{A}, \tau))]$.

We can now state a definition of a Markov-perfect median-voter equilibrium. Let m denote the median type; A_m is thus the median asset holdings.

Definition 3. *A* ***Markov-perfect median-voter equilibrium*** *is a recursive equilibrium and a set of one-period deviation equilibria such that, for all* t *and* $\mathbf{A}$,

$$\Psi_t(\mathbf{A}) = \underset{\tau}{\operatorname{argmax}}\ \tilde{V}_t(A_m, \mathbf{A}, \tau).$$

By definition, we will then also have, for all t and $\mathbf{A}, H_t(\mathbf{A}) = \tilde{H}_t(\mathbf{A}, \Psi_t(\mathbf{A}))$, $N_t(\mathbf{A}) = \tilde{n}_t(\mathbf{A}, \Psi_t(\mathbf{A}))$ and $T_t(\mathbf{A}) = \tilde{T}_t(\mathbf{A}, \Psi_t(\mathbf{A}))$.

Aggregation

The equilibrium outcome for policy, prices and output in this economy generally depends on the entire distribution of assets. However, if the utility function satisfies Assumption 1 and consumers' choices are interior, we will argue that only the asset holdings of the median agent and the aggregate capital stock (mean asset holdings) are relevant state variables. In other words, when the median voter chooses his preferred tax rate he does not need to keep track of how capital is distributed in the population, since equilibrium prices and aggregate quantities do not depend on the distribution either: there is "aggregation".

This theoretical finding is useful because it simplifies both the theoretical and numerical analysis. It is summarized below:

Theorem 1. *Under Assumption 1, if in any competitive equilibrium all agents' solutions are interior, then the politico-economic equilibrium exhibits* ***aggregation****. That is, given values for A_{mt} and K_t, prices and aggregate quantities are independent of the other elements of the asset distribution, $\mathbf{A}_t$.*

The proof proceeds as follows: starting from the last period, we will show that there is aggregation. Then, using an induction argument, we claim that in any T-period economy the politico-economic equilibrium exhibits aggregation. We will now deal first with period T, and we then discuss earlier periods.

In period T, the RCE can be summarized by the first-order conditions for leisure of agents taking prices and taxes as given, their budget constraints, the government budget constraint and the competitive prices. Lemma 1 states that the RCE functions, given an arbitrary tax τ, depend only on one aggregate variable: K.

Lemma 1. *Let $\mathbf{A}$ be the asset distribution at the outset of period T. Under Assumption 1 and given any period-T policy τ, we obtain allocations and prices for the RCE in period T that are equivalent to those characterized by equilibrium outcome functions $r(K, N)$, $w(K, N)$, $\tilde{n}_T(K, \tau)$, $\tilde{T}_T(K, \tau)$, $\tilde{L}_T(a, K, \tau)$ and $\tilde{C}_T(a, K, \tau)$ determined by the following set of functional equations:*

(1) For all (a, K, τ), $\tilde{L}_T(a, K, \tau)$ and $\tilde{C}_T(a, K, \tau)$ solve

$$u_c(c, l)w(K, \tilde{n}_T(K, \tau))(1-\tau) = u_l(c, l)$$

and

$$c = a + [ar(K, \tilde{n}_T(K, \tau)) + w(K, \tilde{N}_T(K, \tau))(1-l)](1-\tau) + \tilde{T}_T(K, \tau),$$

for l and c and are affine in a and such that $\tilde{L}_T(a, K, \tau)/\tilde{C}_T(a, K, \tau)$ is independent of a.

(2) *For all* (K,N), $w(K,N)=F_N(K,N)$ *and* $r(K,N)=F_K(K,N)-\delta$.
(3) *For all* (K,τ), $\tilde{T}_T(K,\tau)=\tau(Kr(K,\tilde{N}_T(K,\tau))+\tilde{n}_T(K,\tau)w(K,\tilde{n}_T(K,\tau)))$.
(4) *For all* (K,τ), $\tilde{n}_T(K,\tau)=1-\tilde{L}_T(K,K,\tau)$.

Lemma 1 implies that for any competitive equilibrium associated to distribution $\mathbf{A}$ and a tax rate τ, one could construct another equilibrium with identical prices and aggregate quantities associated with any other vector $\hat{\mathbf{A}}$ as long as $\hat{K}=\sum_i \mu_i \hat{A}_i=\sum_i \mu_i A_i=K$.

Moving to how τ is chosen, it is simply the value that maximizes the implied $\tilde{V}_T(A_m,K,\tau)$ over τ. Hence, it will depend on A_m in general. Thus, we can write $\tau=\Psi_T(A_m,K)$, since we have now shown that no other aspect of the economy than K or A_m can matter.[1]

Moving to earlier periods, we now need to show that if there is aggregation at any arbitrary period $t+1$, there is also aggregation in period t. The key part of the induction is the fact that $C(a,A_m,K)/L(a,A_m,K)$ is independent of a. In particular, it implies the following.

Lemma 2. *Let* $\mathbf{A}$ *be the asset distribution at the outset of period t. Under Assumption 1, given any policy* τ, *and given the period-*$t+1$ *functions* $L_{t+1}(a,A_m,K)\equiv\tilde{L}_{t+1}(a,A_m,K,\Psi_{t+1}(A_m,K))$ *and* $C_{t+1}(a,A_m,K)\equiv\tilde{C}_{t+1}(a,A_m,K,\Psi_{t+1}(A_m,K))$ *and policy* $\Psi_{t+1}(A_m,K)$, *we obtain allocations and prices for the RCE in period t that are equivalent to those characterized by equilibrium outcome functions* $\tilde{n}_t(A_m,K,\tau)$, $\tilde{T}_t(A_m,K,\tau)$, $\tilde{L}_t(a,A_m,K,\tau)$, $\tilde{H}_t(a,A_m,K,\tau)$ *and* $\tilde{C}_t(a,A_m,K,\tau)$ *determined by the following set of functional equations*:

(1) *For all* (a,A_m,K,τ), $\tilde{L}_t(a,A_m,K,\tau)$, $\tilde{H}_t(a,A_m,K,\tau)$ *and* $\tilde{C}_t(a,A_m,K,\tau)$ *solve*

$$u_c(c,l)w(K,\tilde{n}_t(A_m,K,\tau))(1-\tau)=u_l(c,l),$$

$$u_c(c,l)=\beta u_c(C_{t+1}(a',A'_m,K'),\; L_{t+1}(a',A'_m,K'))[r(A'_m,K')[1-\Psi_{t+1}(A'_m,K')]+1],$$

[1] There is a somewhat subtle issue here, which regards uniqueness. First, there could exist more than one equilibrium for some given τ: multiplicity of competitive equilibria for a given tax rate. Second, if $\tilde{V}(A_m,K,\tau)$ were not to be strictly concave in τ, its maximum might be attained by more than one value. In this case, to "select" among the different possibilities, especially from the perspective of earlier periods, one could let the selection depend on other moments of $\mathbf{A}$ than A_m and K. In the subsequent discussion, to avoid this possible complication we presume uniqueness in both these senses, for time T as well as for earlier time periods.

with $A'_m \equiv \tilde{H}_t(A_m, A_m, K, \tau)$, $K' \equiv \tilde{H}_t(K, A_m, K, \tau)$ *and*

$$c + a' = a + [ar(K, \tilde{n}_t(A_m, K, \tau)) + w(K, \tilde{N}_t(A_m, K, \tau))(1 - l)](1 - \tau) + \tilde{T}_t(A_m, K, \tau),$$

for l, a' *and* c *and are affine in* a *and such that* $\tilde{L}_t(a, A_m, K, \tau)/\tilde{C}_t(a, A_m, K, \tau)$ *is independent of* a.

(2) For all (A_m, K, τ), $\tilde{T}_t(A_m, K, \tau) = \tau(Kr(K, \tilde{n}_t(A_m, K, \tau)) + \tilde{N}_t(A_m, K, \tau)w(K, \tilde{n}_t(A_m, K, \tau)))$.

(3) For all (A_m, K, τ), $\tilde{n}_t(A_m, K, \tau) = 1 - \tilde{L}_t(K, A_m, K, \tau)$.

The proof follows the same steps as those outlined for Lemma 1, with the addition that homotheticity is used also in an intertemporal first-order condition.

It will also be true that, for any asset levels a_1 and a_2 and aggregates A_m and K, agents' relative consumption and leisure levels do not change over time: $\tilde{C}_t(a_1, A_m, K, \tau)/\tilde{C}_t(a_2, A_m, K, \tau) = C_{t+1}(a'_1, A'_m, K')/C_{t+1}(a'_2, A'_m, K')$, where a'_1, a'_2, A'_m and K' are equilibrium savings given a_1, a_2, A_m, K and τ. This fact follows easily from homotheticity and from noting that agents face the same relative prices at any point in time. Relative asset holdings, however, do not necessarily stay constant, but another relative wealth measure does: the relative net-present-value of wealth, which includes any labor and transfer income, present and future.

Finally, as in the period-T case, since the relevant equilibrium functions are independent of any moments of the asset distribution other than A_m and K, the median agent's choice will not depend on anything else either. Thus, we can define $\Psi_t(A_m, K)$ to be the tax rule in the politico-economic equilibrium in period t. By induction, the politico-economic equilibrium exhibits aggregation under endogenous voting when the horizon is finite.

The politico-economic equilibrium is thus a sequence of tax functions $\{\Psi_t^T(A_m, K)\}_{t=0}^T$, where t denotes the time period and T the horizon of the economy, as well as associated allocations and prices that constitute a recursive competitive equilibrium. The equilibrium in the infinite-horizon economy presented below is the limit of the T-period economy equilibrium described in this section:

$$\Psi(A_m, K) \equiv \lim_{T\to\infty} \Psi_t^T(A_m, K).$$

Thus, this definition requires the limit to exist and be independent of t: the function Ψ is stationary. Under what conditions this limit, and hence our voting equilibrium of interest, exists is a topic to be explored in future research. In Section III below, we demonstrate an example where the existence issue is less complex; there, we make use of some parametric restrictions.

III. Characterization

We characterize the equilibrium for an infinite-horizon version of the model so as to ask questions about the long-run level of taxes and inequality. In particular, using our characterization, one can make quantitative assessments.

The Problem of the Median Voter: The Generalized Euler Equation

Income taxes generate well-known distortions to the decisions of the agents, and these are taken into account by the median voter because, by affecting the provision of inputs, they influence prices. On the benefit side, the median agent seeks to use the gap in wealth between himself and the mean agent to obtain transfers.

The median voter chooses taxes taking into account how the winner of the next election will choose taxes tomorrow (the Ψ function). Therefore, when finding the optimal level for the current tax rate τ, he must consider how this will affect average savings in the current period, $\tilde{h}(A_m, K, \tau)$, as well as the savings of the median voter, $\tilde{h}_m(A_m, K, \tau)$, which, by modifying the level of assets that the next incumbent inherits, will influence next period's economic as well as political (tax) outcomes.

The median voter will trade off distortions away from the first-best—gaps—that are introduced by redistributive policies. The infinite-horizon model delivers a first-order condition of the median voter that consists of static gaps (the labor–leisure and the redistribution gaps), and (savings) intertemporal gaps. The first-order condition for the median voter—a "generalized Euler equation", or GEE—can be written as a weighted sum of gaps at different points in time (a sketch of the derivation can be found in the Appendix):

$$\underbrace{GAP_{a_m}\frac{d\tilde{h}_m}{d\tau} + GAP_{l_m}\frac{d\tilde{n}_m}{d\tau} + GAP_{red}}_{t=1}$$

$$+\beta\underbrace{\left[GAP'_{a_m}\frac{d\tilde{h}'_m}{d\tau} + GAP'_{l_m}\frac{d\tilde{n}'_m}{d\tau} + GAP_{red'}\right]}_{t=2}$$

$$+\beta^2\underbrace{\left[GAP''_{l_m}\frac{d\tilde{n}''_m}{d\tau} + GAP_{red''}\right]}_{t=3} = 0, \tag{1}$$

where $\tilde{n}_m(A_m, K, \tau)$ denotes the median agent's labor supply function. Let us now discuss these different "gaps" in turn.

The Labor Gap. The labor gap, GAP_{l_m}, is associated with the distortion on labor supply to the median voter:

$$GAP_{l_m} \equiv wu_{c_m} - u_{l_m}.$$

From the first-order condition of the consumer, this gap is zero if taxes are zero; moreover, in a Pareto optimum this gap must be zero, since w is the marginal product of labor.

The Savings Gap. An increase in the marginal tax, by changing savings—since less time will be spent working and savings under our assumptions will be decreasing in income/wealth: $d\tilde{h}_m/d\tau < 0$—creates an intertemporal distortion, since the presence of next-period taxes on total income will distort savings in the direction of being too low: GAP_{a_m}, defined as $u_{c_m} - \beta u_{c'_m}(1+r')$.

The Redistribution Gap. The other static gap in the median voter's first-order condition is GAP_{red}, which takes into account the labor supply of the mean agent, $\tilde{n}(A_m, K, \tau)$, and measures how an increase in the marginal tax raises "redistribution" each period, and thus utility. In the first period, GAP_{red} reads

$$GAP_{red} = u_{cm}\Bigg\{\underbrace{[r\,(K - A_m) + w\,(\tilde{n} - \tilde{n}_m)]}_{\text{direct}}$$

$$\underbrace{+\,\tau\left[w\frac{d\,[\tilde{n} - \tilde{n}_m]}{d\tau} + \frac{dr}{d\tau}[K - A_m] + \frac{dw}{d\tau}[\tilde{n} - \tilde{n}_m]\right] + \frac{dr}{d\tau}A_m + \frac{dw}{d\tau}\tilde{n}_m}_{\text{indirect}}\Bigg\}.$$

The median agent thus sees a net direct gain from taxation if he has lower asset holdings than the mean agent has. We will, in line with all available data, indeed assume that median asset holdings are lower than average asset holdings. Moreover, the mean agent is richer also in an overall wealth sense, since he only differs from the median agent in his asset holdings (recall that labor productivity, and thus the value of the sequence of time endowments, is equal among agents in the benchmark model). Therefore, if leisure is a normal good, he would buy more leisure and therefore work less than the median agent. This means that the second direct effect of taxation is detrimental to the median agent: he loses, on net, by redistribution of labor income.

The indirect effects include a standard, Meltzer–Richard channel: increased redistribution lowers the gap between the median and mean labor supply, because it moves the net-present-value wealth of the two agents closer to each other, which benefits the median.

The indirect effects also include price effects. Here, a tax increase in the current period will not affect the total capital stock, but it will (if the substitution effect dominates, a presumption which we will maintain in this discussion) reduce work effort, leading to a lower rental rate and a higher wage. The median views both of these positively: the lower rental rate is positive because his asset holdings are lower than mean asset holdings, and the higher wage rate is positive because median labor supply exceeds mean labor supply.

Finally, the last two terms come from a second form of redistribution that occurs: through changes in the composition of income due to price changes. Even in the absence of transfers, a tax increase would lower N and thus increase w. Thus, the median, whose income has a larger wage share in relative terms, sees an increased relative income share. An agent with mean wealth obtains no gain at all from the change in income composition.[2]

It is apparent from the expression that if $A_m = K$, i.e., if the median consumer has wealth that exactly matches mean wealth, the first-order condition is met for a zero tax: the labor–leisure distortion is minimized at this point, there is no change in the net transfer from changing the tax, since the net transfer is always zero in this case, and finally, as argued in the previous paragraph, there is no gain (or loss) to agents with mean asset holdings from changing the composition of income through price changes.

Gaps at Different Moments in Time. Because the increase in τ will induce changes in assets and thus in next-period tax rates, we also have static costs and benefits for the median voter in the next period(s).

Changes in τ, by affecting savings, trigger changes in next-period taxes. The median voter in period one realizes that this will have an effect on redistribution next period ($GAP_{red'}$). The direct effect is then the change in net redistribution (keeping asset holdings constant) due to induced changes in future taxes.

[2] The proof of this statement is as follows. The derivative of $wN + rK = F_n(K, N)N + (F_k(K, N) - \delta)K$ with respect to N equals both $F_{nn}N + F_n + F_{kn}K$ and, due to Euler's theorem since total factor income equals total production for a production function that is homogeneous of degree 1, F_n. This, in turn, means that $F_{nn}N + F_{kn}K$, i.e., the change in total income for the agent with mean asset and labor income only taking price effects into account, must equal zero. Thus, since a tax change operates through the change in N, we have the desired result.

Most of the indirect effects are analogous to those in GAP_{red}. However, an extra term appears because next period's asset holdings are elastic, which in this case is a negative effect of raising current taxes (savings of the median and the mean move closer to each other, thus lowering the net transfer to the median).

The remaining question is why three, and only three, periods appear in the GEE. This can be understood by thinking of the GEE as resulting from a variational experiment. The key insight in this regard is that there are two state variables and only one control in the median voter's maximization problem. Suppose the median agent kept (A_m, K) and (A''_m, K'') fixed and optimally varied the controls in between, as in a parallel of what occurs in a standard dynamic optimization problem. The controls would be τ and τ', or, alternatively, the vector (A'_m, K'). The problem with this experiment is that there are not enough degrees of freedom for a variational experiment: the two controls are completely pinned down by the two end conditions, (A''_m, K'') and cannot be varied beyond that! This is why a variational experiment here has to involve keeping (A_m, K) and (A'''_m, K''') fixed and optimally varying τ, τ' and τ'', where there now is one degree of freedom and utility can be maximized. As a consequence, the GEE must contain terms also dated two periods from the current period.

An Example Economy

We now consider an example economy: $u(c, l) = \alpha \log c + (1-\alpha)\log l$, $\mathrm{F}(K, N)$ is Cobb–Douglas with capital share θ and there is full depreciation. Here, we deviate slightly from the formulation above in that we do not allow for depreciation deduction.

Under this parameterization one can show that although K is important for understanding the level of output, equilibrium taxes and hours worked depend only on the ratio of median to mean assets. Below we label this ratio x. More precisely, we can show the following.

Proposition 1. *The politico-economic equilibrium with aggregation for the parameterized economy is characterized by the evolution of median to mean assets, $\tilde{X}(x, \tau)$, working decisions $\tilde{n}(x, \tau)$ and $\tilde{n}_m(x, \tau)$ for the agents with mean and median wealth, respectively, and tax outcomes $\Psi(x)$ such that*

(1) for all (x, τ), $\tilde{X}(x, \tau)$ solves

$$x' = \frac{\left[(1-\tau)x + \tau - \Psi(x')\right]}{(1-\Psi(x'))},$$

for x', and it satisfies $x = \tilde{X}(x, \Psi(x))$;

(2) for all (x, τ),

$$\tilde{n}(x,\tau) = \frac{(1-\theta)(1-\tau)\alpha}{(1-\tau)(1-\alpha\theta) + (1-\alpha)[\tau - \beta\theta(1-\Psi(\tilde{X}(x,\tau)))]}$$

and

$$\tilde{n}_m(x,\tau) = \frac{(1-\theta) + (1-\alpha)\theta(1-x)(1-\beta)}{1-\theta}\tilde{n}(x,\tau);$$

(3) and, for all x, $\Psi(x)$ *solves*

$$\max_{\tau}\ \alpha \log c_m(x,\tau) + (1-\alpha)\ \log(1-\tilde{n}_m(x,\tau))$$

$$+\frac{\theta\alpha}{1-\theta\beta}\log K + \beta F(\tilde{X}(x,\tau)),$$

where

$$c_m(x,\tau) \equiv \tilde{n}(x,\tau)^{1-\theta}\left\{\left[\theta x + (1-\theta)\frac{\tilde{n}_m(x,\tau)}{\tilde{n}(x,\tau)}\right](1-\tau)\right.$$
$$\left. + \tau - \beta\theta(1-\Psi(\tilde{X}(x,\tau)))\tilde{X}(x,\tau)\right\}$$

and

$$F(x) \equiv \left\{\alpha \log[n(x)^{1-\theta}[(1-\Psi(x))[(\alpha x + (1-\alpha))\theta(1-\beta)\right.$$
$$+ 1 - \theta] + \Psi(x)]] + \log[1 - n_m(x)]^{1-\alpha}$$
$$\left. + \frac{\theta\alpha}{1-\theta\beta}\log[\beta\theta(1-\Psi(x)n(x)^{1-\theta}]\right\}\frac{1}{1-\beta},$$

with $n(x) \equiv \tilde{n}(x, \Psi(x))$ *and* $n_m(x) \equiv \tilde{n}_m(x, \Psi(x))$.

The proposition, which can be used to recover all equilibrium objects, has as a key feature that the ratio of median to mean assets does not change over time.[3] More importantly, as an implication, we obtain that the tax rate does not vary over time. Mean savings are a constant proportion of total output: the savings rate is given by $\beta\theta(1-\Psi(A_m/K))$. Aggregate labor is constant and only depends on the ratio of mean to median capital, via the tax rate. Finally, if the median agent is poorer than the agent with mean wealth (i.e., $x < 1$), he will consume and save less but work more than will the agent with average wealth.

[3] The other functions needed to recover a competitive equilibrium are the savings levels $\tilde{h}(x, K, \tau)$ and $\tilde{h}_m(x, K, \tau)$. They solve the two functional equations $\tilde{h} = \beta\theta(1-\Psi(\tilde{h}_m/\tilde{h}))K^{\theta}\tilde{n}(x,\tau)^{1-\theta}$ and $\tilde{h}_m = \beta\theta K^{\theta}\tilde{n}(x,\tau)^{1-\theta}[(1-\tau)x + \tau - \Psi(\tilde{h}_m/\tilde{h})]$, which for $\tau = \Psi(x)$ delivers $\tilde{h}_m/\tilde{h} = x$.

Despite the simplification that this parametric case allows—essentially, we have a fixed-point problem, through a maximization problem, in a one-dimensional function $\Psi(x)$—we are looking at a nontrivial mathematical problem. We have not been able to find a closed-form solution, so recourse to a numerical solution seems unavoidable. It is possible to consider properties of the equilibrium function locally around $x = 1$, but we move straight to a discussion of numerical methods.

Numerical Analysis

We proceed to find a candidate Ψ function by analyzing the GEE. Because the median voter's first-order condition involves derivatives of Ψ (through the derivatives of the equilibrium decision rules), one cannot solve for steady-state levels independently of solving for higher-order features of these functions.

We propose an algorithm that can be viewed as an extension of linearization: a version of that is outlined in Krusell *et al.* (2002), where it has proven useful both in terms of speed and accuracy.[4] The essential idea behind the method is to approximate the equilibrium function(s) with a polynomial evaluated *at a single point*: the steady-state point. In the general version of the model here, the equilibrium functions we need to find are only the aggregate functions: Ψ, H and N.[5] Thus, a 0th-order approximation would let all the derivatives be zero and the steady state could be found from the system of first-order conditions. A first-order (linear) approximation would involve more unknown parameters—first derivatives in addition to levels—of the three functions. The additional equations needed to pin down these unknown parameters are obtained by partial differentiation, with respect to each argument A_m and K, of each of the equilibrium functional equations. With this procedure, successively higher-order polynomial approximations are thus rather straightforward to derive, and convergence is obtained when the addition of higher orders does not alter the steady state more than by a very small amount. Differentiation of the functional equations is extremely tedious to implement with pencil and paper, but it can be automated using a symbolic math program (one of which is available as part of MATLAB).

For the example model in the previous section, there is really only one unknown function, $\Psi(A_m/K)$, since asset evolution is (implicitly) given as

[4] The method was applied there to a consumption–savings problem under time-inconsistent preferences and has also been employed in optimal-public-expenditure problems without commitment; see Azzimonti *et al.* (2006) or Klein *et al.* (2006).

[5] One really needs $\tilde{H}$ and $\tilde{N}$, but the dependence on τ can be derived based on knowledge of H and N.

a function of Ψ. For illustration, a linear approximation would then read

$$\Psi(A_m/K) = x_0 + x_1 \frac{A_m}{K}. \tag{2}$$

For a numerical example (where $\theta = 0.4$, $\alpha = 0.33$ and $\beta = 0.96$) and the linear approximation, the model predicts taxes that are very high, unless the median/mean asset ratio is very close to one: when this ratio is only 0.95, taxes are almost 100%, and we do not find convergence for ratios lower than 0.95.[6] We also used a global grid-search method and found similar results. It is possible that high inequality simply cannot be a steady state: intuitively, steady states are long-run outcomes, so only a small set of values for median–mean wealth are possible, because large initial inequality would be taxed away over time. The small set of values for A_m/K that are feasible steady states are then associated to high rates of taxation: inequality is "almost" taxed away. However, it may also be that it is particularly difficult in this class of models to find the equilibrium functions numerically. Certainly, more effort devoted to developing numerical tools for solving for (differentiable) Markov-perfect equilibria would be very valuable.

The present paper studies a slightly different setting than the one used in Krusell and Ríos-Rull (1999), who assume that there is an implementation lag for taxes. That is, in their setting, the tax voted on at t is implemented at $t+k$, with $k>0$, making capital income elastic. The absence of an implementation lag in the present work leads to significantly different quantitative results. In particular, the model predicts that only a very narrow range of wealth distributions can be observed as a long-run outcome, and the wealth inequality observed in most developed countries is outside this range. In short, given the large wealth inequality observed, our model predicts that the median voter would tax away most of these differences and that the economy would subsequently converge to a new steady state with much lower inequality. That is, we find that the model is unable to account for the observed combination of taxes and inequality—the marginal benefit to the median voter of further taxation by far exceeds the marginal cost. Thus, we learn that a model that would have greater quantitative success would need a larger cost of taxation or a smaller benefit, like an implementation lag. The nature of implementation lags in actual tax constitutions is not apparent. One implication of the findings here is indeed that we need quantitative measurement of any lags between political decision-making and implementation: these lags really matter quantitatively for what our theories predict.

[6] When the capital share is lower, the asset range for which the program converges is wider.

IV. Concluding Comments

In this paper, we develop finite-horizon models of endogenous redistribution using successive majority voting, and we explore the infinite-horizon version of the setup as well. The analysis demonstrates first that, under assumptions about the utility function that are common in the applied macroeconomic literature, an aggregation result applies: the aggregate politico-economic equilibrium outcomes, i.e., taxes, output, prices, etc., depend on the mean level of assets and on the median asset holdings, and on no other aspect of the asset distribution. This result facilitates tractability considerably; dynamic models with forward-looking, rational agents rapidly become more complex as the number of state variables grows. Thus, it would for example be feasible to study the economy considered here with aggregate productivity shocks and thereby analyze any "political business cycles" arising from median-voter tax determination in a quantitative context.[7]

The aggregation result requires complete markets, and in the present context—which does not have uncertainty—this just means that all agents can borrow and lend at the same rate. Under uncertainty, aggregation would require complete insurance markets. We know, however, from Krusell and Smith (1998), that a setting with idiosyncratic shocks and no insurance markets but precautionary savings using one asset leads to "approximate aggregation". Thus, the finding in the present paper raises hopes that politico-economic equilibria in such a model would also be possible to study without the need to use the entire asset distribution as a state variable. For this to be possible, it seems key to verify that the evolution of the median level of asset holdings not depend on other distribution moments than the mean and median. It would be an important advance if future research found this to be true.

Second, we use first-order conditions of the median voter to interpret how taxes are chosen. We thus show that the tax choice can be viewed as a tradeoff between direct redistribution effects and distortions to the labor–leisure and the consumption–savings choices in three consecutive time periods. One noteworthy point is that the consumption–savings distortion is a consideration for the median voter, even though capital income is inelastic *ex post*; recall that there is no commitment in advance to the tax choice. The reason is that the current tax influences savings from the present to the future, since it influences total resources available.

Third, and finally, we propose a numerical method for finding steady states for the infinite-horizon model. We apply this method, which uses polynomial approximation at the steady-state point, to solve a simple

[7] The addition of an exogenous state variable—aggregate productivity—does make the analysis more difficult but would be entirely feasible.

version of our model. For this model, we find that only a small amount of long-run equilibrium asset inequality can be sustained (assuming parameter values that are close to those used in the macroeconomic literature). This indicates that models that have a chance of generating inequality/tax combinations that resemble those we observe in most developed countries would need different assumptions. As already mentioned, one possibility is that explored in Krusell and Ríos-Rull (1999), namely, that there is an implementation lag for taxes, so that taxes are perceived as more distortionary when they are chosen; in the present paper, taxing capital income involves no distortion *ex post*. Another possibility is that inequality in labor productivity/wages, which is abstracted from here, would improve the quantitative performance of the model. In general, features that make it more costly or difficult to tax, e.g. international capital flight, or less beneficial to redistribute, e.g. because of the costs involved in the transfer system, would be required to improve the quantitative performance of the model.

Appendix

The median voter solves

$$\max_{\tau} u(C(A_m, K, \tau), 1 - \tilde{n}_m(A_m, K, \tau)) + \beta V(A'_m, K')$$

subject to

$$\begin{aligned} C(A_m, K, \tau) = {} & A_m + [A_m r(K, \tilde{n}(A_m, K, \tau)) \\ & + \tilde{n}_m(A_m, K, \tau) w(K, \tilde{n}(A_m, K, \tau))](1 - \tau) \\ & + \tilde{T}(A_m, K, \tau) - A'_m \\ A'_m = {} & \tilde{h}_m(A_m, K, \tau), \quad K' = \tilde{h}(A_m, K, \tau) \end{aligned}$$

and

$$V(A_m, K) = u(C(A_m, K), 1 - n_m(A_m, K)) + \beta V(h_m(A_m, K), h(A_m, K)). \tag{A1}$$

The first-order condition delivers

$$\Upsilon_\tau + \beta\big[V'_{A_m}\tilde{h}_{m\tau} + V'_K\tilde{h}_\tau\big] = 0, \tag{A2}$$

where $\Upsilon_\tau = u_c C_\tau - u_{1-\tilde{n}_m}\tilde{n}_{m\tau}$. To obtain expressions for V'_{A_m} and V'_K, we differentiate equation (A1) with respect to A_m and K and update to obtain

$$V'_i = \Upsilon'_i + \beta[V''_{A_m}\tilde{h}'_{mi} + V''_K\tilde{h}'_i], \tag{A3}$$

where $\Upsilon'_i = u'_c C'_i - u'_{1-\tilde{n}_m}\tilde{n}'_{mi}$ for $i \in \{A_m, K\}$. Notice that the envelope theorem does not eliminate future value-function derivatives in this case; we have a system of three equations, but four unknowns: V'_{A_m}, V'_K, V''_{A_m} and V''_K. In order to obtain the extra equation to complete the system, we can update equation (A2) once (this is possible because it is a functional equation that must hold for all A_m and K). Once the system is solved, we can obtain an expression that is independent of derivatives of the unknown value functions:

$$\Upsilon_\tau + \beta\left[\Upsilon'_{A_m}\tilde{h}_{m\tau} + \Upsilon'_K\tilde{h}_\tau + \Upsilon'_\tau A'_\tau\right] + \beta^2\left[\Upsilon''_{A_m}\tilde{H}'_{m\tau} + \Upsilon''_K\tilde{H}'_\tau + \Upsilon''_\tau A''_\tau\right] = 0, \tag{GEE}$$

where

$$A'_\tau = -\frac{\Omega''_{A_m}\left(\tilde{h}'_{mA_m}\tilde{h}_{m\tau} + \tilde{h}'_{mK}\tilde{h}_\tau\right) + \Omega''_K\left(\tilde{h}'_{A_m}\tilde{h}_{m\tau} + \tilde{h}'_K\tilde{h}_\tau\right)}{\Omega''_{A_m}\tilde{h}'_{m\tau} + \Omega''_K\tilde{h}'_\tau},$$

$$\tilde{H}'_{m\tau} = \xi\Omega''_K, \quad \tilde{H}'_\tau = -\xi\Omega''_{A_m}$$

and

$$A''_\tau = \xi\left(\tilde{h}''_{mK}\Omega''_{A_m} - \tilde{h}''_{mA_m}\Omega''_K\right),$$

with

$$\Omega_i = \tilde{h}_i - \tilde{h}_\tau\frac{\tilde{h}_{mi}}{\tilde{h}_{m\tau}} \quad \text{for } i = A_m, K$$

and

$$\xi = -\tilde{h}'_{m\tau}\frac{\Omega'_{A_m}\tilde{h}_{m\tau} + \Omega'_K\tilde{h}_\tau}{\Omega''_{A_m}\tilde{h}'_{m\tau} + \Omega''_K\tilde{h}'_\tau}.$$

The expression in terms of GAPs can be found using the definition of each gap and rearranging the above GEE equation.

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