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Economic Theory

Money and insurance in a turnpike environment*

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Summary. We study the effects of introducing a feasible insurance market into the spatial separation model of money described in Mitsui and Watanabe (1989). We show that the insurance contract may or may not drive out money. We also show that, depending on the degree of risk aversion, the additional market can reduce welfare for all agents, increase welfare for all agents, or increase welfare for some agents and reduce it for others.

1 Introduction

In monetary economics there is a class of models where money is the only asset and money is used to smooth consumption in the presence of idiosyncratic income uncertainty, e.g. Lucas (1980), Bewley (1980). From the perspective of an individual agent, valued fiat money can be viewed as an asset for self insurance, and as such its presence improves welfare. In the turnpike environment of Townsend (1980), the uniqueness of the monetary asset has been derived from an explicit consideration of the economy's spatial and informational structure. In this context it is worth examining how robust the results concerning existence of an equilibrium with valued fiat money and its welfare implications are with respect to the introduction of additional means of insurance. Our work is based on a variation of Townsend's (1980) turnpike model, due to Mitsui and Watanabe (1989). In this environment, valued fiat money provides insurance from the perspective of the aggregate economy since its presence reduces the idiosyncratic risk which agents face. We show that no equilibrium with valued fiat money may exist, or that welfare in an equilibrium with valued fiat money may decrease, if some additional insurance contract becomes available. The insurance contract we consider can be implemented given the restrictions imposed by the environment.

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Mitsui and Watanabe (1989) present a simple yet fully specified model in which the environmental assumptions call for an asset like money. Their monetary equilibrium has the attractive features that money is traded and that there simultaneously exists another, higher-yielding asset. Although these should be basic properties of a model of money, to formulate general equilibrium models which have these properties has proven hard without imposing unattractive assumptions on agents' behavior or on the equilibrium concept employed. We construct an insurance scheme which can be implemented in the environment described by Mitsui and Watanabe. Due to the features of the environment, the insurance scheme is partial in the sense that only a subset of agents have access to the insurance market at any point in time. The insurance opportunities available to the agents are left unexploited in the equilibrium considered by Mitsui and Watanabe.

We show that for a subset of the parameter space the introduction of the insurance market drives out money: if insurance is allowed, there is no equilibrium with valued money, whereas there is an equilibrium without valued money and with insurance. For another subset of the parameter space, valued fiat money and insurance coexist. For this case we show that the introduction of insurance into an economy with valued money can decrease welfare for all agents in the economy. This result is a general equilibrium effect: the welfare losses due to a lower equilibrium return on money outweigh the benefits from the insurance.

The implications for welfare should be contrasted to earlier work by Hart (1975), who shows in a Radnerian setup with incomplete markets that the addition of markets can reduce welfare: the equilibrium with the new asset added is Pareto dominated by the equilibrium without it. Our work has the advantage that the precise incompleteness considered can be explained from first principles. Although it is well-known that in economies without a complete set of markets "anything can happen", we believe that our example is attractive in that it offers a concrete and plausible economic interpretation of the result.[1]

The explicit spatial environment presented by Mitsui and Watanabe is admittedly artificial. The constraints implied by it, however, offer an appealing interpretation when observing the economy from the outside. At a given point in time, each agent has access to an investment project. There are good projects and bad projects, and which type of project a given agent has access to is determined randomly. Agents with bad projects cannot directly invest their wealth in other agents' projects. If money is available, however, these agents will not operate their projects and instead hold their wealth in the form of money. In an indirect sense,

[1] In recent work Engineer and Bernhardt (1991) come to similar conclusions, also in the context of a monetary turnpike economy. They analyze an environment with many goods per period and some additional locational restrictions within the period where money is used as a medium of exchange to avoid inefficient barter trading patterns. In the economy they describe, it is possible that prohibiting barter in favor of pure monetary exchange improves overall welfare. This restriction on the set of possible means of payment can be interpreted as a shut-down of certain specific markets. Barter can be welfare reducing because it lowers the real value of money within each period, i.e. the relative price of money and consumption goods within each period. The rate of return on money in their environment remains independent of the presence or absence of barter.

then, these agents use the good technology, but cannot fully exploit the high return. If in addition partial insurance is available, agents with high return projects can insure against the possibility that their projects turn bad the next period. In this case they would no longer operate their project and hold their wealth, including insurance compensation, in the form of money. The insurance is partial since agents who currently have bad projects cannot insure against the possibility that their project continues to give a low return.

Section 2 presents the economy and defines our equilibrium concept for different contractual settings. Section 3 contains existence and uniqueness theorems, and Section 4 contains a discussion of the welfare properties of equilibria. Section 5 concludes.

2 The economy

2.1 *The environment*

The economy is formulated in discrete time and has a large number (continuum) of islands upon each of which reside a large number (continuum) of infinitely lived agents. At the beginning of every time period the status of each agent, whether an agent has to move or not, is decided in a random, exogenous way. The event that a given person has to move at the end of the period is independent of all other events, and has probability $1 - p$. Stayers will be indexed by 1 and leavers by 0. Leavers are spread out across islands in such a way that the probability that two persons who once lived on the same island will meet again on another island is zero. Furthermore, the allocation of leavers among islands is such that the population size on each island remains constant over time. No trade can occur between islands within a period nor can communication between islands take place.

There is one consumption good in the economy for which preferences are additively separable over time with time preference parameter β, $0 < \beta < 1$. The period utility function is of the constant relative risk aversion type:

$$U(c) = \frac{c^{1-\alpha} - 1}{1 - \alpha}.$$

When $\alpha = 1$, preferences are logarithmic.

Each agent is endowed with w_0 units of the consumption good at $t = 0$, and has access to a linear storage technology with stochastic returns: for a stayer, the return is g, and for a leaver the return is b, where $b < g$.[2] Hence, when the moving shock is realized for an agent at the beginning of period t, the return on the agent's own investment technology between t and $t + 1$ is also known. Note that there are

[2] Mitsui and Watanabe let stayers face a positive probability of also getting the low return b on their intertemporal technology. This will make stayers with low returns lend to those with high, and the asset used for this purpose will yield a higher return than money does in their monetary equilibria. The equilibrium allocation is the same whether or not stayers face an uncertain return, and for simplicity we therefore assume that they do not.

endowments only of the time zero good, so savings are necessary in order to get future consumption in this economy.

All realizations of random variables are public information within islands.

2.2. *Equilibrium arrangements*

We will consider stationary equilibria for economies with and without money, and with and without insurance. We only consider equilibria for monetary economies where the asset money actually has positive value in the equilibrium. Since the equilibria without insurance are described in Mitsui and Watanabe, our exposition here will focus mainly on the cases with insurance. When comparisons are relevant, we will state their assumptions or results as parallel to ours, and we will use primes to make notational distinctions.

Throughout, the timing is the following: agents receive their moving and technology shock in the beginning of every period. In a given period, agents do not meet before their shocks are realized. Later in the same period, markets open whereafter consumption takes place. Moving takes place at the very end of the period.

The unconstrained Pareto optimal allocation (full optimum) involves never using the low return technology, and making consumption independent of the status of an agent. Agents who leave an island do not use the low return storage technology but leave their endowment behind to be invested in the high return storage technology. When leavers get to a new island, they receive the appropriate return. Consequently the agents solve a deterministic optimization problem and choose a constant savings ratio. The resulting allocation, however, cannot be supported by a decentralized equilibrium with credit arrangements because there is no communication between islands and because agents never meet again after they leave an island.

We consider an insurance scheme which is feasible given the environmental constraints. At a given point in time stayers on a particular island insure each other against having to leave the next period. At the market in period t, people who are stayers engage in contracts promising some payoff which is contingent on their status in the beginning of period $t+1$. At $t+1$, payments are made according to the by then realized and publicly observed status. We call this type of insurance partial insurance. Partial insurance is feasible since an agent's status is public information on an island. A leaver at t would also want to insure against his becoming a leaver at $t+1$, but there is no one to engage in insurance with. Other leavers at t on the island will go to different islands, and on the new island no one is encountered until the shock at $t+1$ is realized. Thus full insurance is not feasible. In the equilibria described by Mitsui and Watanabe, the possibility of partial insurance is ignored.

As in Mitsui and Watanabe, we restrict attention to stationary equilibria for the economies considered. A stationary equilibrium is defined to be an equilibrium where the rates of return, in particular the real return on money, are time independent. In an economy with money and partial insurance, an equilibrium will be stationary only if the distribution of wealth between stayers and leavers satisfies

certain constraints. In particular, the fraction of wealth held by each group is an endogenously determined constant.

For an agent facing time independent rates of return the optimization problem is to choose $\{w_{t+1}, c_t, d_t\}$ to solve

(P) $\max E_0\left(\sum_{t=0}^{\infty} \beta^t U(c_t)\right)$ subject to $0 \leqq c_t \leqq w_t$ and

$w_{t+1} = g(w_t - c_t) - d_t w_t$ if the agent is a stayer at t and at $t+1$,

$w_{t+1} = g(w_t - c_t) + \frac{p}{1-p} d_t w_t$ if the agent is a stayer at t and a leaver at $t+1$

$w_{t+1} = r(w_t - c_t)$ if the agent is a leaver at t

w_0 given.

E_0 denotes the expected value conditional on the information available at 0, which includes the agent's status. The initial endowment w_0 is exogenously given. Wealth (alternatively, the endowment) at the moment of trading in the market at t after receipt of the return on asset holdings and the payment or receipt of insurance entered into at $t-1$ is denoted w_t. The amounts of insurance payments and receipts made immediately before the opening of the markets are $d_t w_t$ and $\frac{p}{1-p} d_t w_t$, that is the insurance contract has zero expected value at t. The return on savings for leavers is denoted r. In the nonmonetary economies, r is equal to the low return b, and in the monetary economies r is the endogenously determined return on money.

Our analysis of (P) will be based on the study of the corresponding functional equation (FE):

$$V_1(w) = \max_{\theta_1 \in [0,1], d} \left\{ U((1-\theta_1)w) + \beta \left[pV_1([g\theta_1 - d]w) + (1-p)V_0\left(\left[g\theta_1 + \frac{p}{1-p}d\right]w\right)\right]\right\} \tag{FE}$$

$$V_0(w) = \max_{\theta_0 \in [0,1]} \{U((1-\theta_0)w) + \beta[pV_1(r\theta_0 w) + (1-p)V_0(r\theta_0 w)]\}.$$

$V_1(w)$ and $V_0(w)$ are the value functions of stayers and leavers, respectively, and θ_1 and θ_0 are the corresponding savings fractions.

In the Appendix, we prove that under Assumptions 1 and 2 below, a solution to (FE) is also a solution to (P).

Assumption 1. $\beta(pg^{1-\alpha} + (1-p)b^{1-\alpha}) < 1.$

Assumption 2. $\beta g^{1-\alpha} < 1.$

The first assumption is the same as the one used in Mitsui and Watanabe. The second is used only when the utility function has less than logarithmic curvature, i.e. for $\alpha < 1$.

The problem (P′) faced by agents in the environment without insurance is like (P) with the additional restriction that insurance payments are zero at all times.

In the Appendix, we verify the following:[3]

Lemma 1. *Under Assumptions* 1 *and* 2, (FE) *has the solution*

$$V_1(w) = \frac{a_1^\alpha w^{1-\alpha}}{1-\alpha} - \frac{1}{(1-\alpha)(1-\beta)} \tag{1}$$

$$V_0(w) = \frac{a_0^\alpha w^{1-\alpha}}{1-\alpha} - \frac{1}{(1-\alpha)(1-\beta)},$$

where a_1 *and* a_0 *are given by the (unique) solution to*

$$a_1 = 1 + (\beta g^{1-\alpha})^{1/\alpha}(pa_1 + (1-p)a_0)$$

$$a_0 = 1 + (\beta r^{1-\alpha})^{1/\alpha}(pa_1^\alpha + (1-p)a_0^\alpha)^{1/\alpha}. \tag{2}$$

Furthermore, the optimal values of the controls are

$$\theta_1 = \frac{a_1 - 1}{a_1}$$

$$\theta_0 = \frac{a_0 - 1}{a_0}$$

$$d = \frac{a_0 - a_1}{pa_1 + (1-p)a_0}(1-p)g\theta_1. \tag{3}$$

Note from the expression for d in (3) that stayers compensate leavers if $a_0 > a_1$ and vice versa. A consequence of the following lemma is that when $\alpha < 1$, leavers will actually compensate stayers.

Lemma 2. $a_1 < (>) a_0$ *if* $\alpha > (<) 1$.

Proof of Lemma 2. Since $g > r$, $(\beta g^{1-\alpha})^{1/\alpha} < (>) (\beta r^{1-\alpha})^{1/\alpha}$ if $\alpha > (<) 1$. Moreover, it follows from convexity (concavity) of x^α that $(pa_1 + (1-p)a_0) < (>) (pa_1^\alpha + (1-p)a_0^\alpha)^{1/\alpha}$ if $\alpha > (<) 1$. The assertion is now immediate from (2).

The solution in the case without insurance is similar: (1) and the relevant parts of (3) are the same, and (2) is replaced by

$$a_1 = 1 + (\beta g^{1-\alpha})^{1/\alpha}(pa_1^\alpha + (1-p)a_0^\alpha)^{1/\alpha}$$

$$a_0 = 1 + (\beta r^{1-\alpha})^{1/\alpha}(pa_1^\alpha + (1-p)a_0^\alpha)^{1/\alpha}. \tag{2'}$$

Total wealth for an economy $\bar{w}_t$ is given by the sum of total wealth held by stayers and leavers, $\bar{w}_t = \bar{w}_{1t} + \bar{w}_{0t}$. To obtain the transition equation of aggregate wealth, we have to obtain the transition equations for its components first. We can restrict ourselves to the transition equations for total wealth of stayers and leavers, since an agent's optimal accumulation rule in a stationary equilibrium implies

[3] The limit of the value function as α converges to 1 coincides with the value function in Mitsui and Watanabe (Lemma 1, p. 127). In this case, the optimal decision is to obtain no insurance.

saving and insuring constant fractions of wealth. In fact, for a monetary economy with insurance, a stationary equilibrium uniquely determines the fraction of wealth held by stayers.

From the agents who are stayers at t, a fraction p will continue to be stayers at $t+1$. Given their period t wealth holdings, $\bar{w}_{1t}$, and their savings and insurance decisions, θ_1 and d, they will contribute $(g\theta_1 - d)p\bar{w}_{1t}$ to the wealth of stayers at $t+1$. Analogously a fraction p of leavers at t will become stayers at $t+1$. Given their period t wealth holdings, $\bar{w}_{0t}$, and their savings decision, θ_0, they will contribute $r\theta_0 p\bar{w}_{0t}$ to the wealth of stayers at $t+1$. Total wealth of stayers at $t+1$ is then given by

$$\bar{w}_{1t+1} = (g\theta_1 - d)p\bar{w}_{1t} + r\theta_0 p\bar{w}_{0t}. \tag{4a}$$

Total wealth of leavers at $t+1$ is determined analogously and evolves according to

$$\bar{w}_{0t+1} = \left(g\theta_1 + \frac{p}{1-p}d\right)(1-p)\bar{w}_{1t} + r\theta_0(1-p)\bar{w}_{0t}. \tag{4b}$$

Equations (4) imply that the total wealth for the entire population, $\bar{w}_t$, follows

$$\bar{w}_{t+1} = g\theta_1\bar{w}_{1t} + r\theta_0\bar{w}_{0t}. \tag{5}$$

Mitsui and Watanabe study the case where $d = 0$ and (5) becomes

$$\bar{w}_{t+1} = \frac{1}{p}\bar{w}_{1t+1} = \frac{1}{1-p}\bar{w}_{0t+1}. \tag{6}$$

Equation (6) implies that total wealth grows at the same rate for stayers as for leavers, independently of the initial shares of wealth held by the two groups. In fact, the shares will adjust in the first period to p and $1-p$, respectively, and then remain at those values. Any constant growth rate is consistent with these shares. In the case with insurance, however, the situation is different because of the insurance transfers between the groups. It is easy to see that a constant growth rate of wealth, $\lambda \equiv \bar{w}_{t+1}/\bar{w}_t$, is only possible if the share of wealth held by stayers, $s \equiv \bar{w}_{1t}/\bar{w}_t$, is constant and (λ, s) is the unique solution to the system given by the stationary wealth process for (4):

$$\lambda = p(g\theta_1 - d) + pr\theta_0\frac{1-s}{s}$$

$$\lambda\frac{1-s}{s} = (1-p)\left(g\theta_1 + \frac{p}{1-p}d\right) + (1-p)r\theta_0\frac{1-s}{s}. \tag{7}$$

This of course means that unless the initial share of wealth is just right, constant growth cannot occur from the beginning of time. For tractability, we restrict our attention to equilibria that are stationary, and in the case with insurance and money this requires a precise initial value $s_0 = s$.

We are now in a position to provide a complete definition for stationary equilibria of nonmonetary economies.[4]

[4] Note that the equilibrium definitions we present restrict attention to situations in which the solution to the functional equation also is a solution to the original problem.

Definition. A *stationary equilibrium for a nonmonetary economy with partial insurance* is a policy rule for savings and insurance, (θ_1, θ_0, d), solving the optimization problem (FE) for $r = b$.

Remark. The equilibrium elements can be summarized by the vector $(a_1, a_0, \theta_1, \theta_0, d)$ satisfying (2) and (3).

For some given initial distribution of wealth, wealth in the nonmonetary equilibrium evolves according to (7). A *stationary equilibrium for a nonmonetary economy without insurance* is defined in the obvious, parallel way.

In the case of the monetary economy, apart from the restriction to balanced growth paths for the prices and the aggregate variables, one equation needs to be added to account for market clearing in the money market. The demand for real wealth by movers is equal to the real value of money:

$$\theta_0 \bar{w}_{0t} = q_t M, \tag{8}$$

where q_t is the price of money at t, and M is the stock of money. Equation (8) implies that

$$\frac{\bar{w}_{0t+1}}{\bar{w}_{0t}} = \frac{q_{t+1}}{q_t}, \tag{9}$$

where the right hand side is the return on money. For stationary equilibria this return is time independent. Therefore the stationary growth rate for the wealth of leavers is constant. The wealth transition Equations (4) then imply that the growth rate of total wealth λ and the fraction of wealth held by stayers s are constant and satisfy Equations (7).

We now provide complete definitions for stationary equilibria of economies with partial insurance.

Definition. A *stationary equilibrium for a monetary economy with partial insurance* is a policy rule for savings and insurance, (θ_1, θ_0, d), solving the optimization problem (FE) for $r = r_m$; a growth rate λ of aggregate wealth and a fraction of aggregate wealth s held by stayers satisfying (7); and a real return on money r_m satisfying the money market clearing condition (9) and the inequalities $b < r_m < g$.

Remark. The equilibrium elements can be summarized by the vector $(a_1, a_0, \theta_1, \theta_0, d, \lambda, s, r_m)$ satisfying Equations (2) and (3) with $r = r_m$, (7), (9), and $b < r_m < g$.

A *stationary equilibrium for a monetary economy without insurance* is defined in a similar way, and as noted, an important difference is that such an equilibrium has $s = p$, independently of other variables.

3 Existence and uniqueness of equilibrium

This section contains statements of the main existence and uniqueness results. All proofs, except those relevant to the economies without insurance, are contained in the Appendix.

In order to prove existence of nonmonetary equilibria, we need no more assumptions or arguments than those already made: no equilibrium market clearing conditions are involved (except the implicit market clearing in insurance contracts). We therefore have:

Theorem 1. *Under Assumptions 1 and 2, there exists a stationary equilibrium for a nonmonetary economy with partial insurance.*

Theorem 1′. *Under Assumption 1, there exists a stationary equilibrium for a nonmonetary economy without insurance.*

In order to establish existence and uniqueness of monetary equilibria, but also for the welfare comparisons in the next section, we need the following:

Lemma 3(3′). *When $\alpha > (<) 1$, a_1 and a_0 are decreasing (increasing) functions of r.*

The lemma implies, for $\alpha > 1$, that the optimal savings rate decreases as r increases, since $\theta_i = 1 - \frac{1}{a_i}$. This is a standard result for the case of constant relative risk aversion preferences: when the coefficient of relative risk aversion is higher than 1 (when there is more than logarithmic curvature and goods at different times are relatively weak substitutes), the income effect of an increase in the relative price of goods today dominates the substitution effect, and the savings rate decreases. When $\alpha < 1$, the substitution effects dominates.

To prove existence of a monetary equilibrium with partial insurance, we need one additional assumption:

Assumption 3. $H(b) > 0$, where

$$H(r) \equiv g\theta_1(r)\left\{\frac{pa_1(r)}{\bar{a}(r)} + \frac{(1-p)a_0(r)}{\bar{a}(r)} \cdot \frac{p\theta_0(r)}{1-(1-p)\theta_0(r)}\right\} - r.$$

In $H(r)$, $\bar{a} \equiv pa_1 + (1-p)a_0$, and the dependence on r reflects the dependence of the optimal choices on r. The corresponding assumption for the case without insurance is

Assumption 3′. $G(b) > 0$, where $G(r) \equiv pg\theta_1(r) + (1-p)r\theta_0(r) - r$.

We are now ready to state the existence results:

Theorem 2. *Under Assumptions 1, 2, and 3 there exists a stationary equilibrium for a monetary economy with partial insurance.*

Theorem 2′. *Under Assumptions 1, 2, and 3′ there exists a stationary equilibrium for a monetary economy without insurance.*

Assumption 3 can be a nontrivial strengthening of Assumption 3′. In particular when $\alpha > 1$, there are parameter values such that Assumption 3′ is satisfied, but not Assumption 3, that is such that an equilibrium with valued money exists only if no insurance is available.

The uniqueness results are the following:

Theorem 3. *Under the assumptions of Theorem 2, and if in addition $\alpha \geqq 1$, the stationary equilibrium for a monetary economy with partial insurance is unique.*

Theorem 3'. *Under the assumption of Theorem 2', and if in addition $\alpha \geqq 1$, the stationary equilibrium for a monetary economy without insurance is unique.*

4 Welfare properties of equilibria

Hart (1975) shows that in economies with incomplete markets the introduction of additional markets need not always be welfare improving. An unattractive feature of his approach is the ad-hoc specification of which markets exist and which markets are to be added. In contrast we start out with an explicit description of the economic environment, and consider adding an asset which is consistent with that environment. It should be emphasized that the environment was originally not intended for dealing with the problem of adding markets into economies with incomplete markets. We now discuss the welfare effect of the addition of an insurance market to the turnpike economy, by comparing stationary equilibria. In particular we show how the welfare changes depend on the degree of risk aversion. The case where the coefficient of relative risk aversion is greater than one and the case where it is less than or equal to one are discussed separately. We do this because the way the savings rate depends on the interest rate, and the behavior towards risk, are qualitatively different for the two cases. We start with the case where the degree of risk aversion is greater than one. First the nonmonetary economy is discussed. This economy can be dealt with using partial equilibrium analysis, since agents' decisions do not affect intertemporal prices. For the nonmonetary economy the introduction of insurance increases welfare. We then proceed to the general equilibrium analysis of the monetary economy. Here the introduction of insurance can decrease welfare for all agents. We conclude with some comments on the second case where the degree of risk aversion is less than or equal to one. Here, introducing insurance unambiguously increases welfare for both the nonmonetary and the monetary economy.

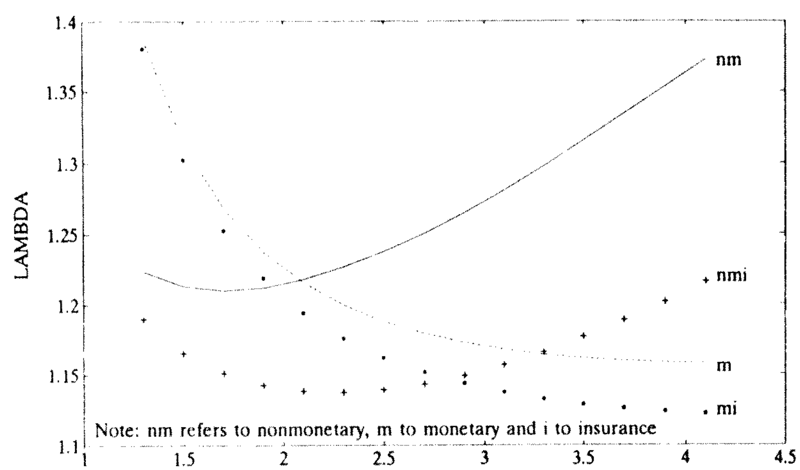

Fig. 1. Average growth rates.

Case 1. $\alpha > 1$

The introduction of an insurance contract into the nonmonetary economy enlarges the set of feasible allocations for each agent. This results in an unambiguous increase in welfare. The growth rate of total wealth, although not a determinant of welfare in the nonmonetary economy, is also affected by the introduction of insurance. This occurs via two main channels: changes in savings rates, and wealth transfers between stayers and leavers.

First, from the definition of the value function, Equation (1), it follows that the welfare increase from insurance is reflected in a decrease of the value function coefficient a_i, cf. Fig. 2.[5] From Equation (3) it then follows that the savings rates for stayers and leavers decline.[6] This tends to decrease the growth rate of total wealth in the economy.

Second, from Lemma 2 and Equation (3) it follows that when $\alpha > 1$, the equilibrium insurance contract implies asset transfers from agents who stay to agents who leave. This means that wealth is transferred from agents with a high rate of return on wealth but a low savings rate to agents with a low rate of return on wealth and a high savings rate. The net-effect of the change in savings rates and the wealth transfers, for the parameters values we have looked at, is a negative one: the growth rate of total wealth declines. Figure 1 shows that the relation between

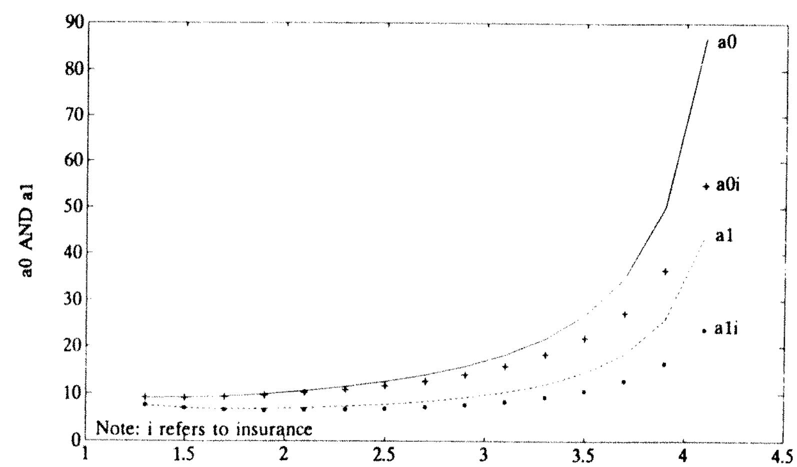

Fig. 2. Non-monetary economies.

[5] The parameters, other than the coefficient of risk aversion α, for Figs. 1 through 3, are set at the values $\beta = 0.9$, $b = 0.8$, $g = 2.0$, and $p = 0.5$.

[6] Savings rates decline for the following reason. The introduction of insurance increases indirect utility from wealth in the next period, relative to utility from present period consumption. This can be interpreted as an increase in the time preference rate, which is formally equivalent to an increase in the rate of return. For $\alpha > 1$, the wealth effect dominates the substitution effect for this intertemporal price change, and present period consumption increases, that is the savings rate declines.

the degree of risk aversion and the growth rate of the nonmonetary equilibrium is not monotone.

Introducing insurance into a monetary economy has quite different effects.[7] In the nonmonetary economy the reduction of the growth rate of total wealth has no repercussions on the rates of return agents can obtain. For the monetary economy, the return on money, however, is equal to the growth rate of total wealth in the economy. But as in the case without money, introducing insurance lowers this growth rate and therefore the return on money. This tends to increase the savings rates and reduce welfare, and hence counteracts the partial equilibrium effect of introducing insurance.

Over time all agents face the same chances of being a leaver or stayer, and are therefore qualitatively equally affected by the two opposing welfare effects of insurance. In any given period, however, an agent is either a leaver or a stayer. As a leaver an agent experiences mainly the negative effect, since the rate of return on money declines and insurance is not available for leavers. As a stayer an agent experiences mainly the positive effect, since the high rate of return is exogenous and the agent can insure against being a leaver in the next period. Figure 3 plots the value function coefficients of agents in a monetary economy against the degree of risk aversion α, all other parameter values fixed. For the range of α considered, leavers are always worse off in the economy with insurance. For low values of α,

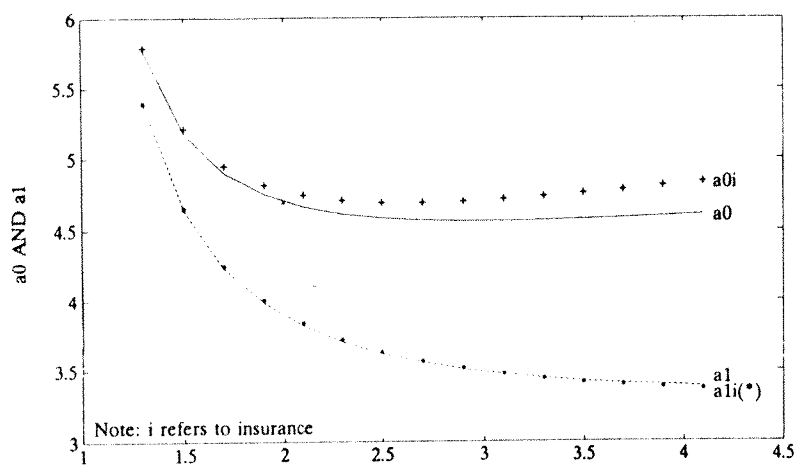

Fig. 3. Monetary economies.

[7] We have noted in Sect. 2.2 that in a stationary equilibrium for an insurance economy the relative size of asset holdings by stayers and leavers is uniquely determined at the time when markets open. When we consider the introduction of an insurance market into the monetary economy, we assume that the distribution of assets in the monetary economy is such that this condition is satisfied. This is possible since this magnitude is not determined for a monetary economy.

stayers are also worse off. The negative effect of potentially lower rates of return in the future dominates the positive effect of insurance. For higher values of α, that is with more risk aversion, the insurance option becomes more valuable and the welfare for stayers is higher with insurance.

Note that in all cases the relative welfare loss is higher for leavers than for stayers. In addition Figs. 2 and 3 show that for the parameter values considered, the absolute welfare changes due to the introduction of insurance are small compared with the ones due to the introduction of money.

Case 2. $\alpha = 1$ *and* $\alpha < 1$

The presence of insurance markets is irrelevant for logarithmic period utility functions, $\alpha = 1$. In this case savings rates are constant and independent of the assets available, and an agent's optimal choice is to obtain no insurance, $d = 0$. For period utility functions with less than logarithmic curvature, $\alpha < 1$, the introduction of insurance increases welfare in the nonmonetary and the monetary economy.

Introducing insurance into the nonmonetary economy enlarges the set of feasible allocations for each agent and welfare increases unambiguously. In contrast to Case 1 agents increase their savings rates if insurance is available.[8] This has a positive effect on the growth rate of total wealth. Again contrary to Case 1, the equilibrium insurance contract implies wealth transfers from agents with a low savings rate and low rate of return on wealth (leavers) to agents with a high savings rate and high rate of return on wealth (stayers). This implies an unambiguous increase in the growth rate of total wealth.

To summarize Case 2 for $\alpha < 1$, introducing insurance into a nonmonetary economy increases welfare and the growth rate of total wealth. In a monetary economy the higher growth rate means a higher rate of return on money, which reinforces the increase in welfare. Thus the effect on welfare of the introduction of insurance into a monetary economy is positive for all agents.

5 Concluding comments

At this point we want to caution against drawing unwarranted conclusions from our results on the welfare implications of different market structures in a monetary economy. Although the results indicate that the opening of an additional insurance market can have an adverse effect on welfare, this does not tell us anything about what a government can or should do. Any discussion of policy experiments, such as the closing/opening of markets, presupposes the existence of an enforcement mechanism. Such a mechanism has to be based on certain features of the environment. In this case it would first be necessary to investigate what other policies could be implemented given these additional features of the environment. Similarly, a study of monetary policy in the Mitsui and Watanabe environment would require a more careful specification of the abilities of the hypothesized monetary authority.

[8] The change in the savings rate can be explained by the same reasoning as in footnote 6. This time the substitution effect dominates the wealth effect and therefore the savings rate increases.

Appendix

The first task is to prove that under Assumptions 1 and 2 a solution to (FE) is also a solution to (P). For this purpose, we will use Theorem 9.12 from Stokey and Lucas (1989; p. 274). In order for the theorem to be applicable here, it remains to verify that the boundedness assumption in b) of the theorem is satisfied. To do this, we distinguish between two cases: $\alpha > 1$ and $\alpha \leqq 1$. When $\alpha > 1$, the value function is bounded above by zero. If Assumption 1 is met, there always exists a policy involving $d = 0$ at all times for which we know from the analysis without insurance that the boundedness assumption is satisfied. If $\alpha \leqq 1$, we first note that (P) gives a lower value than if all agents, i.e. stayers as well as leavers, can insure. The value achieved with such insurance, in turn, must be lower than if, in addition, $r = g$, i.e. than if there is no risk. For this case, we know from concavity that $d_t = 0 \forall t$ is optimal, and that the value is bounded if Assumption 2 is satisfied. We have therefore established boundedness also for $\alpha \leqq 1$.

Proof of Lemma 1. It is straightforward to verify that (1) and (3) satisfy the functional equation so long as (2) is satisfied. From the first equation of (2) a_1 can be solved for as

$$a_1 = c_0 + c_1 a_0, \tag{A1}$$

where $c_0 \equiv (1 - p(\beta g^{1-\alpha})^{1/\alpha})^{-1}$ and $c_1 \equiv \dfrac{1-p}{p}(1 - p(\beta g^{1-\alpha})^{1/\alpha})^{-1} \cdot p(\beta g^{1-\alpha})^{1/\alpha}$, provided, since both a_1 and a_0 have to be greater than 1,

$$p(\beta g^{1-\alpha})^{1/\alpha} < 1. \tag{A2}$$

We get $a_0 = f(a_0)$, where $f(a_0) \equiv \{a_0^{-1} + (\beta r^{1-\alpha})^{1/\alpha}(p[(c_0/a_0) + c_1]^\alpha + 1 - p)^{1/\alpha}\} \cdot a_0$, about which it is easy to show: a) that $f(1) > 1$; b) that if $a_0 = f(a_0)$, then $0 < f'(a_0) < 1$; and c) that $f(a_0)/a_0$ converges to a number less than one as $a_0 \to \infty$, provided

$$\beta r^{1-\alpha}(pc_1^\alpha + 1 - p) < 1. \tag{A3}$$

Hence, with (A2) and (A3) satisfied it can be concluded that there is a unique solution to (2). (A3), when c_1 is substituted for, becomes

$$p\beta g^{1-\alpha} \frac{(1-p)^\alpha \beta r^{1-\alpha}}{(1 - p(\beta g^{1-\alpha})^{1/\alpha})^\alpha} + (1-p)\beta r^{1-\alpha} < 1. \tag{A4}$$

Assumption 1 implies that (A4) is satisfied if

$$p(\beta g^{1-\alpha})^{1/\alpha} + (1-p)(\beta r^{1-\alpha})^{1/\alpha} < 1. \tag{A5}$$

(A5), in turn, is implied by Assumption 1 when $\alpha > 1$, since in this case x^α is a convex function. When $\alpha < 1$, we first note that $(\beta g^{1-\alpha})^{1/\alpha} < \beta g^{1-\alpha}$ is implied by Assumption 2. Second, the same assumption and $r < g$ imply that $(\beta r^{1-\alpha})^{1/\alpha} < \beta r^{1-\alpha}$. Together, these two observations show that the left-hand side of (A5) is less than $\beta(pg^{1-\alpha} + (1-p)r^{1-\alpha})$, which is less than one from Assumption 1. It furthermore follows from (A5) being satisfied that also (A2) is met. We have therefore established that under Assumptions 1 and 2, and when $\alpha \neq 1$, there is a unique solution to (FE). The case of logarithmic preferences has the property that no insurance is used and thus we can use the proof provided in Mitsui and Watanabe.

Remark. We conjecture, but have not yet been able to verify, that (A2) and (A4) are sufficient conditions for the solution to (FE) to also be a solution to (P).

Proof of Lemma 3. Expressing (2) as $a_0 = f(a_0, r)$, it follows that $da_0/dr = \dfrac{\partial f/\partial r}{1 - \partial f/\partial a_0}$. This expression will have the same sign as its numerator, since the denominator is positive from b) of the proof of Lemma 1. The numerator has the sign of $1 - \alpha$.

Proof of Theorem 2. If s is eliminated from (7) and it is observed that $g\theta_1 - d = g\theta_1(a_1/\bar{a})$ and that $g\theta_1 + \dfrac{p}{1-p} d = g\theta_1(a_0/\bar{a})$, some simplifications give, as the remaining equilibrium condition on r,

$$1 = \frac{g}{r}\theta_1(r)\{x(r) + y(r)[1 - x(r)]\}, \tag{A6}$$

where $x(r) \equiv \dfrac{p\theta_0(r)}{p\theta_0(r) + 1 - \theta_0(r)} \in (0,1)$ and $y(r) \equiv \dfrac{pa_1(r)}{\bar{a}(r)} \in (0,1)$. For $r = g$ the right-hand side of (A6) is less than one. Assumption 3 states that for $r = b$, the right-hand side is bigger than one. It follows from continuity that there is a solution to (A6) between b and g.

Proof of Theorem 3. It is sufficient to prove that the factor in braces on the right-hand side of (A6) is decreasing in r, since $\theta_1(r) = 1 - \dfrac{1}{a_1(r)}$ is decreasing in r from Lemma 3. Differentiation w.r.t. r gives

$$(1-y)x' + (1-x)y' = (1-x)(1-y)\left\{\frac{x'}{1-x} + \frac{y'}{1-y}\right\}$$
$$= (1-x)(1-y)\left\{\frac{p^{-1}x^2\theta_0^{-2}a_0^{-2}}{1-x}a_0' - \frac{p(1-p)c_0\bar{a}^{-2}}{1-y}a_0'\right\}.$$

It remains to check the sign of

$$\frac{p^{-1}x^2\theta_0^{-2}a_0^{-2}}{1-x} - \frac{p(1-p)c_0\bar{a}^{-2}}{1-y} = (pa_0)^{-1}\left\{\frac{x^2\theta_0^{-2}a_0^{-1}(p\theta_0 + 1 - \theta_0)}{1-\theta_0} - (p^2c_0)/\bar{a}\right\},$$

where the equality follows from use of the definitions of $x(r)$ and $y(r)$. The factor in braces becomes, after noting that $1 - \theta_0 = 1/a_0$ and, again, use of the definition of $x(r)$,

$$p^2\left\{\frac{1}{p\theta_0 + 1 - \theta_0} - c_0/\bar{a}\right\} = p^2\frac{\bar{a}^{-1}}{p\theta_0 + 1 - \theta_0}\{\bar{a} - c_0(p\theta_0 + 1 - \theta_0)\}$$

$$= p^2 \frac{\bar{a}^{-1}}{p\theta_0 + 1 - \theta_0}\{(pc_1 + 1 - p)a_0 - (1 - p)c_0 a_0^{-1}\}$$

$$= p^2 \frac{(\bar{a}a_0)^{-1}}{p\theta_0 + 1 - \theta_0}(pc_1 + 1 - p)\left\{a_0^2 - \frac{(1 - p)c_0}{pc_0 + 1 - p}\right\}.$$

Since the definitions of c_0 and c_1 imply that $(1 - p)c_0 = pc_1 + 1 - p$, and since $a_0 > 1$, the last expression is greater than zero. Finally, $a_0' < 0$ from Lemma 3 now implies that the sought derivative is negative, and we have established that so long as $\alpha > 1$, the right-hand side of (A6) is decreasing in r.

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