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Time-Consistent Public Policy

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In this paper we study how a benevolent government that cannot commit to future policy should trade off the costs and benefits of public expenditure. We characterize and solve for Markov-perfect equilibria of the dynamic game between successive governments. The characterization consists of an inter-temporal first-order condition (a "generalized Euler equation") for the government, and we use it both to gain insight into the nature of the equilibrium and as a basis for computations. For a calibrated economy, we find that when the only tax base available to the government is capital income—an inelastic source of funds at any point in time—the government still refrains from taxing at confiscatory rates. We also find that when the only tax base is labour income the Markov equilibrium features less public expenditure and lower tax rates than the Ramsey equilibrium.

1. INTRODUCTION

In this paper we study how a benevolent government should trade off the costs and benefits of public expenditure. We do this in the context of a neoclassical growth model where consumers value government consumption. Because this framework features dynamic inconsistency, we have to take a stand on whether the government can commit itself to future policy. The view we take here is that it cannot. It follows from this view that the situation is best described as a dynamic game between successive governments. We characterize and solve for Markov-perfect equilibria of this game, focusing on those Markov equilibria that feature differentiable policy functions. This refinement concept is designed to rule out anything resembling a reputation-like mechanism for sustaining "good" equilibria.

The main contributions of this paper are methodological and are applicable to a large set of environments; we provide a new compact characterization of equilibrium, and we also describe a numerical algorithm that can be used to quickly and accurately compute the steady state. However, in applying our methods to a particular environment we also present some results that are specific to that environment and that are new.

The characterization involves deriving a first-order condition for the government that captures the trade-off we are interested in. We call this condition the "generalized Euler equation", or GEE. It reveals how the government optimally trades off tax wedges over time, and this trade-off involves, among other things, the future government's marginal propensity to tax out of its initial capital. The numerical algorithm that we propose is based on local approximations and is described in detail in Appendix A.

There is earlier work on analysing no-commitment outcomes in economies with a long horizon. First, Markov equilibria of the type we are interested in have been studied in Cohen and Michel (1988) and Currie and Levine (1993), who explore linear-quadratic economies, and in some recent papers on taxation and political economy that take the same approach (see footnote 2 below). In such economies, Markov equilibria can be characterized and computed rather easily, since the first-order conditions become linear in the state variable. The problem of unknown derivatives appearing in equilibrium conditions is not nearly as severe as in environments that are not linear-quadratic, since second and higher-order derivatives of decision rules vanish. The drawback, of course, of linear-quadratic settings is that they apply only in extremely special settings. Thus, one has to either give up on quantitative analysis to apply them or accept reduced-form objective functions and/or reduced-form private decision rules. Special functional-form assumptions of other sorts as well can be useful for characterizing Markov-perfect equilibria; see Bassetto and Sargent (2005) for an example involving quasi-linear utility.

There is also a literature both in political economy (Krusell, Quadrini and Ríos-Rull, 1997; Krussell and Ríos-Rull, 1999) and in optimal policy with benevolent governments (Klein and Ríos-Rull, 2003; Klein, Quadrini and Ríos-Rull, 2005) that has used computational methods to find quantitative implications of Markov equilibria for a variety of questions. This work is closely related to the present one, but it has two drawbacks. First, the methods used—essentially, numerical solution of value functions based on linear-quadratic approximations—are of the "black-box" type: they do not deliver interpretable conditions, such as first-order conditions for the key decision maker. The present paper fills this gap. Second, the numerical methods do not deliver controlled accuracy.[1] In contrast, the methods proposed and used here do. An alternative approach to ours can be found in Phelan and Stacchetti (2001), who have looked at environments like those studied in this paper and who have developed methods, based on those in Abreu, Pearce and Stacchetti (1990), to find all "sustainable" equilibria. Their methods, however, do not allow Markov equilibria to be identified and explicitly interpreted, which is what we do here.

Most earlier studies in the literature to which this paper belongs focus on public finance, that is, they take public expenditures as given. Alternatively, they examine how various paths of public expenditures influence growth and business cycles, but do not explicitly discuss the full optimal determination of these paths. Exceptions include some papers using endogenous-growth frameworks, such as Barro (1990), where the commitment solution is time-consistent, and a recent paper by Sarte, Azzimonti-Renzo and Soares (2003), which studies development from the perspective of productive public capital. Another exception is Hassler, Krusell, Storesletten and Zilibotti (2004), which studies a linear-quadratic economy where utility is linear in private and public consumption, but where there are quadratic investment costs.[2] There, the key finding is one of dynamics: paths under commitment (and possibly those without commitment as well) feature oscillating taxes and public-goods provision. Here, in contrast, convergence is monotone. The key reason for the difference is that the present paper uses a standard geometric depreciation structure for capital; Hassler *et al.* focus on human capital and the kinds of physical capital that are more of the one-hoss-shay nature. Moreover, their paper does not allow standard calibration, since the AK production technology used there abstracts from labour and labour taxation. In sum, the Hassler *et al.* and Azzimonti *et al.* papers are the only ones we know of in the literature that compare optimal paths under commitment to those under no commitment. We also know of no other quantitative assessment of the optimal role of public expenditures in the context of a

1. For their economies, Sarte, Azzimonti-Renzo and Soares (2003) compare linear-quadratic approximation with the method used here. They find significant differences for some cases.

2. Related papers in the political-economy literature also discuss government expenditures, which do not have to take the form of redistribution but more generally can be public goods that are not perfect substitutes with private goods; see, for example, Hassler, Mora, Storesletten and Zilibotti (2003) and Hassler, Storesletten and Zilibotti (2007).

standard neoclassical growth framework, and we are not aware of any attempts to approach data on public expenditures using flexible dynamic theory based on microeconomic underpinnings—which we advocate here—as an organizing tool.

Another closely related literature upon which the present work builds quite directly is that analysing dynamic games between successive selves, as outlined in the economics and psychology literature by Strotz (1956), Phelps and Pollak (1968), Laibson (1997), and others. This literature contains the derivation of a GEE, and Krusell, Kuruşçu and Smith (2002) show how to solve it numerically for a smooth decision rule equilibrium. As is elaborated on in Appendix A, the smooth rule can be difficult to find with standard methods, and Krusell *et al.* (2002) resort to a perturbation method of sorts, which we also use here. This method relies on successive differentiation of the GEE. Thus, we view the approach taken here as an adaptation to optimal-policy environments of the tools suggested in these one-agent problems. Finally, there are a couple of recent papers on monetary theory that studies Markov-perfect outcomes using numerical techniques: Díaz-Giménez, Giovannetti, Marimón and Teles (2006) and Martin (2007). We hope that the methods we employ here and those of the papers mentioned will prove to be of general applicability; they could possibly be used for studying optimal monetary policy, dynamic political economy, dynamic industrial organization issues (*e.g.* the durable goods monopoly and dynamic oligopoly), models with impure intergenerational altruism, and so on.

We provide analysis for two sets of economies. The first set is designed to admit analytical solutions. When the horizon is finite, the solution is typically unique, and as the time horizon tends to infinity, the solution tends to a well-defined limit that coincides with the differentiable Markov-perfect equilibrium of the corresponding infinite-horizon economy. This equilibrium has a unique steady state, and convergence to it is monotone.[3] In some examples, however, there are two equilibria with constant tax rates: one with a high rate and one with a low rate. The former is the limit of finite-horizon equilibria; the latter requires an infinite horizon.

The second set of economies is calibrated to match some features of the postwar U.S. economy and solved numerically. Here, the purpose is not to provide an empirical evaluation of the model nor to account for any quantitative facts. Rather, the purpose is to determine the robustness of the analytically obtained results to changes in parameter values in the direction of those that are reasonable given the available data. The computations also serve to illustrate our solution method. In general, numerical solutions can be difficult to obtain in setups like ours. This is because the nature of the equilibrium is quite different from what is mostly studied in macroeconomics. Even finding a steady state—the lowest-order representation of the government's policy function—is difficult, because the level of taxes in a steady state is determined by a condition that involves the future marginal propensity to tax, which is a higher-order feature of the same, unknown policy function. This feature is not present either in standard competitive equilibria or in Ramsey (optimal-tax) equilibria. Our method is based on approximating the policy function and its derivatives evaluated at one point only: the steady state. For the economies we consider, the method performs very well: in the case where an analytical solution exists, the numerical method finds it with very small errors, and in other cases, convergence is fast and the solutions are very close to those obtained with global methods.

The results of our analytical and numerical calculations can be summarized as follows. When capital income is the only tax base, our Markov equilibrium—perhaps surprisingly, since capital income is inelastic *ex post*—does not result in equating the marginal rate of substitution between private and government-provided goods to the marginal rate of transformation. Rather, public goods are underprovided in order to mitigate underinvestment. The steady state represents

3. This contrasts with the non-monotone convergence result in Hassler *et al.* (2004).

a compromise between the distortion of the public/private good margin and the consumption/investment margin.[4] When, on the other hand, labour income is the only tax base and labour supply is elastic, the Markov equilibrium steady-state ratio of public to private goods consumption is smaller than the corresponding Ramsey equilibrium long-run ratio. The reason is that a higher tax rate in period t encourages labour supply in previous periods; this beneficial effect (it is beneficial, since labour supply is suboptimal in equilibrium) is ignored *ex post* by a government without commitment. Finally, if the government is able to tax labour and capital income separately, but faces the constraint that labour cannot be subsidized, then the optimal choice is to set the labour tax equal to 0; see Martin (2006). Thus, this case collapses to the case where only capital income is taxed.

An important conceptual insight coming out of our work concerns the nature of the interaction between successive governments. It turns out that in our framework, the current government does not want to manipulate its successor via the state variable, in the spirit of Persson and Svensson (1989), or as in the case of savers with time-inconsistent preferences. The reason is that, in our environment, it is the constraints, not the preferences, that give rise to time inconsistency. Given the value of tomorrow's state variable, today's government and tomorrow's government agree on what tomorrow's policy should be. What the government would like to do, if it could, is to alter the private sector's expectations of future policy so as to increase savings. It cannot do this directly—it is constrained by private agents forming rational expectations based on equilibrium prices—but its current policy choice will influence private capital accumulation, which has an equilibrium influence on future policy and thus on current expectations. Only in this indirect sense can the government be said to manipulate the state variable to influence future policy.

The outline of the paper is as follows. In Section 2 we describe our baseline environment, in which the only private economic decision is the consumption/savings choice (Section 2.1), define a Ramsey equilibrium (Section 2.2), and then define and discuss our Markov equilibrium (Section 2.3) step by step. The section presents closed-form solutions for specific versions of the model. Section 3 then discusses an extension to our baseline setup where leisure is valued and where there are different possibilities for what tax base might be used. Section 4 discusses the properties of the policies that arise in an environment calibrated to U.S. data where governments do not have access to a commitment technology (Markov policies) and compares them to those that arise both in environments with commitment (Ramsey policies) and in environments where the government has access to lump-sum taxation (Pareto policies). Section 5 concludes. Appendix A includes the description of the computational procedures we use and a treatment of the case with separate tax rates for capital and labour income. Appendix B discusses a definition of equilibrium equivalent to the one proposed in the main body of the paper.

2. THE MODEL

In this section, we describe the specific setup. We then define a benchmark "Ramsey equilibrium"—the solution to an optimal-policy problem where the government can commit to future policies. After that, we proceed toward a definition of a time-consistent equilibrium where the government does not have the ability to commit.

4. If the government were allowed to encourage investment in some other way than by refraining from confiscatory taxation of capital income—say, by providing a lump-sum transfer or by subsidizing labour—it would. This is established and discussed in Martin (2006).

2.1. *The environment*

Our setup is a canonical model of public-goods provision embedded in a neoclassical growth framework. The representative consumer lives forever, and there is a benevolent government with a period-by-period balanced budget and proportional taxation.[5] To begin with, the tax base is total income, and leisure is not valued.

In a competitive equilibrium, households maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t, g_t),$$

subject to

$$c_t + k_{t+1} = k_t + (1-\tau_t)[w_t + (r_t - \delta)k_t],$$

taking the price and tax sequences as given. Firms maximize profits; using a constant-returns-to-scale production function $f(k,l)$, where f is concave, they employ inputs so that w_t and r_t are the marginal products of labour and capital, respectively. The resource constraint in this economy reads

$$c_t + k_{t+1} + g_t = f(k_t, 1) + (1-\delta)k_t.$$

It follows that the government's balanced-budget constraint is

$$g_t = \tau_t[f(k_t, 1) - \delta k_t].$$

We will make use of the following functions:

$$\mathcal{T}(k, g) := g/[f(k,1) - \delta k],$$

and

$$\mathcal{C}(k, k', g) := f(k,1) + (1-\delta)k - k' - g,$$

where the prime symbols denote next-period values. These functions—$\mathcal{C}$ representing consumption as a function of current and next-period capital and the current public expenditure and $\mathcal{T}$ representing the balanced-budget tax rate as a function of current capital and the current public expenditure—are exogenous and will economize on notation significantly.

2.2. *Commitment: the Ramsey problem*

If lump-sum taxes were available, the optimal allocation in this economy would involve two conditions: $u_c(c_t, g_t) = \beta(1 + f_k(k_{t+1}, 1) - \delta)u_c(c_{t+1}, g_{t+1})$ (optimal savings) and $u_c(c_t, g_t) = u_g(c_t, g_t)$ (optimal public expenditures). In our economy lump-sum taxes are assumed not to be available, and the optimal allocation using a proportional income tax is more involved.

We will first assume that the government has the ability to commit to all its future policy choices at the beginning of time. The government's decision problem is therefore to choose a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$ in order to maximize utility, taking into account how the private sector will respond to these taxes. To simplify notation, we will assume that the government

5. In Section 2.3.3 we discuss an economy with a finite horizon.

chooses a sequence of expenditures instead: it chooses $\{g_t\}_{t=0}^{\infty}$.[6] A simple way to describe this problem formally is to choose $\{g_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t u(\mathcal{C}(k_t, k_{t+1}, g_t), g_t),$$

subject to the private sector's first-order condition for savings

$$u_c(\mathcal{C}(k_t, k_{t+1}, g_t), g_t) = \beta u_c(\mathcal{C}(k_{t+1}, k_{t+2}, g_{t+1}), g_{t+1}) \times [1 + (1 - \mathcal{T}(k_{t+1}, g_{t+1}))(f_k(k_{t+1}, 1) - \delta)], \quad (1)$$

for all $t \geq 0$. We refer to the solution of this problem as the Ramsey allocation. As is well known, this solution is typically not time-consistent.

2.3. *No commitment: Markov equilibrium*

The equilibrium concept employed here is the same as that in Krusell and Ríos-Rull (1999) and Krusell, Quadrini and Ríos-Rull (1996). It is inspired by the following informal description of how the public and private sectors interact. In each period, the government moves first, choosing current period policies. Its choices are constrained to depend only on the value of the current period's "state", in our case just the aggregate capital stock; in addition, we only consider equilibria where policy depends *differentiably* on the capital stock. After the government has moved, the private sector chooses its current period action (savings). Since the private sector consists of "small" agents, private-sector agents take future policies as given. The government, however, correctly anticipates how future policy will depend on current policy via the state of the economy.

Our equilibrium concept can be stated in different but equivalent ways. The particular formulation we use here is practical in that it allows for compact statements.[7]

An equilibrium consists of a value function v, a government policy function Ψ, and a savings function h such that, for all k, $g = \Psi(k)$ and $k' = h(k)$ solve

$$\max_{k', g} \{u(\mathcal{C}(k, k', g), g) + \beta\, v(k')\}$$

subject to

$$u_c(\mathcal{C}(k, k', g), g) = \beta\, u_c(\mathcal{C}(k', h(k'), \Psi(k')), \Psi(k')) \cdot \{1 + [1 - \mathcal{T}(k', \Psi(k'))][f_k(k') - \delta]\}, \quad (2)$$

and

$$v(k) \equiv u(\mathcal{C}(k, h(k), \Psi(k)), \Psi(k)) + \beta\, v(h(k)).$$

For this recursive construction to be useful, one needs assumptions on primitives (such as on u, the domain for capital etc.) such that this v also solves the consumer's problem when stated sequentially. Our examples below satisfy these conditions.

6. Formally, letting the government choose g instead of τ can be a problem if more than one tax rate is associated with a given g (which can occur in principle, though not in the environment described here). One could then specify a selection rule singling out the equilibrium with the lower tax or, alternatively, simply state the choice in terms of the tax rate directly.

7. It was suggested to us by Harald Uhlig. See Appendix B for a discussion of the equivalence between this formulation and the one in Krusell and Ríos-Rull (1999).

2.3.1. The GEE. The first-order condition of the government—the GEE—can be derived by using the envelope condition to substitute out the derivative of the value function.[8] Before proceeding with the derivation, we define the inter-temporal tax wedge via the following equation:

$$\eta(k,g,k') := u_c(\mathcal{C}(k,k',g),g)$$
$$-\beta\, u_c(\mathcal{C}(k',h(k'),\Psi(k')),\Psi(k'))\cdot\{1+[1-\mathcal{T}(k',\Psi(k'))][f_k(k')-\delta]\}. \qquad (3)$$

Given an equilibrium (v,Ψ,h), and under some regularity conditions, the implicit function theorem guarantees that there exists a unique function $\mathcal{H}$, defined on some neighbourhood of the steady state, satisfying $\eta(k,g,\mathcal{H}(k,g))\equiv 0$ in that neighbourhood. This function describes the response of the private sector to a "one-shot deviation" on the part of the government in the following sense.[9] If the current capital stock is k, current government consumption is g, and the private sector expects that future government policy will be determined by the equilibrium policy function Ψ, then savings will be given by $\mathcal{H}(k,g)$.

Using this definition of the function $\mathcal{H}$, which implies that $\mathcal{H}_g = -\frac{\eta_g}{\eta_{k'}}$, and $\mathcal{H}_k = \frac{\eta_k}{\eta_k}$, the first-order condition for the government can be written as

$$u_c(-\mathcal{H}_g-1)+u_g+\beta v_k'\mathcal{H}_g = 0,$$

where we have economized on notation by suppressing the functional arguments. The notation v_k' stands for the derivative of v with respect to k (subscripts denote derivatives) evaluated at $\mathcal{H}(k,g)$ (primes denote forward lags). To obtain an expression for it, we differentiate the government's Bellman equation with respect to k. We obtain, noting that $\mathcal{C}_g=\mathcal{C}_{k'}=-1$,

$$v_k = u_c(\mathcal{C}_k-(\mathcal{H}_k+\mathcal{H}_g\Psi_k)-\Psi_k)+u_g\Psi_k+\beta v_k'(\mathcal{H}_k+\mathcal{H}_g\Psi_k)=0.$$

To eliminate the value function, we still need to eliminate v_k'. Using the first-order condition above, we have

$$\beta v_k' = \frac{1}{\mathcal{H}_g}(u_c(\mathcal{H}_g+1)-u_g).$$

Thus, the expression for v_k in terms of primitives and decision rules reads

$$v_k = u_c(\mathcal{C}_k-\mathcal{H}_k)+\frac{\mathcal{H}_k}{\mathcal{H}_g}(u_c(\mathcal{H}_g+1)-u_g).$$

We can now update this expression one period and substitute back into the original first-order condition to obtain our GEE:

$$-u_c[\mathcal{H}_g+1]+u_g-\beta\mathcal{H}_g\left\{u_c'[f_k'+1-\delta-\mathcal{H}_k']+\frac{\mathcal{H}_k'}{\mathcal{H}_g'}(u_c'(\mathcal{H}_g'+1)-u_g')\right\}=0, \qquad (4)$$

where we have also used the definition of $\mathcal{C}$ in terms of primitives. Notice the presence in the GEE of derivatives of policy functions; this makes it a *generalized* Euler equation.

Given the definition of $\mathcal{H}$ in terms of the function η, the GEE simplifies to

$$0=(-u_c+u_g)\eta_{k'}-\eta_g\left[-u_c+\beta u_c'(f_k'+1-\delta)-\beta(-u_c'+u_g')\frac{\eta_k'}{\eta_g'}\right]. \qquad (5)$$

8. It can also be derived by using Bellman's principle to identify a Markov equilibrium with the solution to an appropriate sequence problem. (This is, of course, not the problem that the policy maker actually solves.) The Euler equation of this sequence problem is the GEE.

9. The function $\mathcal{H}$ can be used more directly in the equilibrium definition; see Appendix B.

This is a *functional* equation, a fact that is obscured by the suppression of the functional arguments.

A "wedge" interpretation. The GEE can be rewritten as a linear combination of wedges, the public/private wedge $\gamma := u_g - u_c$ and the savings/consumption wedge $\nu := -u_c + \beta u_c'(1 + f_k' - \delta)$. Thus, rearranging terms we obtain

$$\gamma + \mathcal{H}_g \nu + \beta \mathcal{H}_g \left(-\frac{\mathcal{H}_k'}{\mathcal{H}_g'} \right) \gamma' = 0. \tag{6}$$

Three terms appear: these are the three different "wedges" that are affected by the change in the current tax rate: the current public/private good wedge, the consumption/savings wedge and the next period's public/private good wedge. The weight on the final term can be thought of as the derivative $\frac{dg'}{dg}$ when the household's first-order condition $\eta(k', g', k'') = 0$ is satisfied, and k'' is held constant.

2.3.2. Comparison with the commitment case. In the commitment case, the functions h and Ψ are not applicable, so we need to modify the definition of η that we gave in (3) as follows.[10]

$$\begin{aligned} \widetilde{\eta}(k, k', k'', g, g') &:= u_c(\mathcal{C}(k, k', g), g) \\ &-\beta\, u_c(\mathcal{C}(k', k'', g'), g') \cdot \{1 + [1 - \mathcal{T}(k', g')][f_k(k') - \delta]\}. \end{aligned} \tag{7}$$

With this new definition, the no-commitment GEE becomes, in sequential terms,

$$\gamma_t - \frac{\widetilde{\eta}_{4,t}}{\widetilde{\eta}_{2,t} + \widetilde{\eta}_{3,t} h_{k,t+1} + \widetilde{\eta}_{5,t} \Psi_{k,t+1}} \left(\nu_t - \frac{\widetilde{\eta}_{1,t+1}}{\widetilde{\eta}_{4,t+1}} \beta \gamma_{t+1} \right) = 0. \tag{8}$$

By contrast, the first-order condition for optimal government spending under commitment, which follows from straightforward calculus, is

$$\begin{aligned} &\gamma_t - \widetilde{\eta}_{4,t} D_t \left(\nu_t - \frac{\widetilde{\eta}_{1,t+1}}{\widetilde{\eta}_{4,t+1}} \beta \gamma_{t+1} - \frac{\widetilde{\eta}_{3,t-1}}{\widetilde{\eta}_{5,t-1}} \gamma_t \right) \\ &\quad - \frac{\widetilde{\eta}_{5,t-1} D_{t-1}}{\beta} \left(\nu_{t-1} - \frac{\widetilde{\eta}_{1,t}}{\widetilde{\eta}_{4,t}} \beta \gamma_t - \frac{\widetilde{\eta}_{3,t-2}}{\widetilde{\eta}_{5,t-2}} \gamma_{t-1} \right) = 0, \end{aligned} \tag{9}$$

where $\widetilde{\eta}_{4,t} D_t$ captures a "$\frac{dk_{t+1}}{dg_t}|_{\widetilde{\eta}_t=0}$" effect (the implied change in k_{t+1} that keeps $\widetilde{\eta}_t = 0$):

$$\frac{1}{D_t} := \widetilde{\eta}_{2t} - \widetilde{\eta}_{4t} \frac{\widetilde{\eta}_{3t-1}}{\widetilde{\eta}_{5t-1}} - \widetilde{\eta}_{5t} \frac{\widetilde{\eta}_{1t+1}}{\widetilde{\eta}_{4t+1}}.$$

The first three terms of (9) have direct counterparts in (8).[11] The remaining terms involve, in one way or another, period $t-1$; they represent the impact of current decisions on past expectations and are hence ignored when there is no commitment. Specifically, the fourth term captures

10. By definition of h and Ψ, we obviously have $\eta(k, g, k') \equiv \widetilde{\eta}(k, k', h(k'), g, \Psi(k'))$.

11. Note only that under commitment, future control variables can be chosen directly, so $-\widetilde{\eta}_{4t} \frac{\widetilde{\eta}_{3t-1}}{\widetilde{\eta}_{5t-1}}$ and $-\frac{\widetilde{\eta}_{1t+1}}{\widetilde{\eta}_{4t+1}}$ here replace $\widetilde{\eta}_{3,t} h_{k,t+1}$ and $\Psi_{k,t+1}$, respectively, in the no-commitment GEE.

how the induced change in k_{t+1} alters last period's expectations, thus altering g_t indirectly: it captures "$\frac{dg_t}{dk_{t+1}}|_{\eta_{t-1}=0}$". Finally, terms five through seven are a replica of terms two through four: the latter represent the effects of the change in g_t on k_{t+1} keeping $\eta_t = 0$, whereas the former involve the same kinds of effects, but via how the change in g_t changes k_t keeping $\eta_{t-1} = 0$, that is, via the household's first-order condition last period.

We have stated the commitment GEE in its full dynamic form here; in steady state, of course, it simplifies greatly, and the commitment and no-commitment GEEs are easier to compare. In the steady-state context, the additional terms appearing in the commitment GEE are given some quantitative content in Section 4 below.

2.3.3. An economy that allows a closed-form solution. We will consider an example that illustrates the nature of a Markov-perfect equilibrium and how it relates to the Ramsey solution. So suppose that $u(c, g) = \log c + \gamma \log g$ (so that leisure has no value), $f(k) = k^\theta$, and $\delta = 1$. We will assume that the only tax base is capital income (without any deduction for depreciation), because the two other cases (a tax on labour income and a tax on total income) turn out not to be very interesting: in the former case, the solution coincides with the Pareto optimum, since labour taxes are non-distortionary; in the latter case, the Ramsey optimal solution is time-consistent, and hence it is also a Markov-perfect equilibrium. In this section we allow the time horizon to be finite and assume that the economy exists in periods $t = 0, 1, \ldots, T$.

In this case we have $g_t = \tau_t \theta k_t^\theta$, and $\mathcal{T}(k, g) = \frac{g}{\theta k^\theta}$. Working backwards from the last period, we find that savings rules can be written as

$$k_{t+1} = s_t(1 - \theta\mathcal{T}(k_t, g_t))k_t^\theta, \tag{10}$$

and the savings rates s_t satisfy the recursion

$$s_t = \frac{\beta\theta(1-\tau_{t+1})}{\beta\theta(1-\tau_{t+1}) + (1-\theta\tau_{t+1})(1-s_{t+1})},$$

with $s_T = 0$. Here, savings at t depend on all future tax rates: they decrease in all future tax rates. The reason is that future tax rates decrease discounting, thus making any future income worth more in present terms; this positive wealth effect will increase current consumption and decrease savings. As a result, the commitment solution will not be time-consistent in this case. We will not solve for the commitment solution here—it does not admit closed-form expressions—but we will solve for a time-consistent equilibrium. Optimal government policy can be represented via

$$g_t = \tau_t \theta k_t^\theta.$$

With this in mind, and again working backwards from the final period, we find that the value functions can be written as

$$v_t(k) = A_t \ln k + B_t,$$

where

$$A_t = \theta(1+\gamma)\frac{1-(\beta\theta)^{T-t+1}}{1-\beta\theta},$$

and B_t satisfies the recursion

$$B_t = \ln(1-\theta\tau_t)(1-s_t) + \gamma \ln\theta\tau_t + \beta A_{t+1}\ln s_t(1-\theta\tau_t) + \beta B_{t+1}.$$

Inspecting this expression, one sees that the optimal choice of τ_t does not interact with future taxes, delivering

$$\tau_t = \frac{1}{\theta}\frac{\gamma}{1+\gamma}\frac{1-\beta\theta}{1-(\beta\theta)^{T-t+1}}.$$

With a high enough γ in this case, capital income will not suffice to provide for *ex-post* optimal public consumption levels in the last period, leading to a tax rate above 100%. If this occurs in period T, there will be no savings in any earlier periods, and both c and g will be 0 in all periods but the very first one. With a literally infinite time horizon, however, there will also be an "expectations-driven" equilibrium with savings in this case if $(1-\beta\theta)\gamma/(1+\gamma)/\theta < 1$: if agents—private and public—believe future capital income will not be taxed at high rates, there will be savings.

2.3.4. Strategic policy: does the current government manipulate its successors? The dynamic game played between governments involves a disagreement: the current government would like to see the next government choose a lower tax on income, τ', than it ends up choosing. Does this mean that the current government attempts to "manipulate" the next government in its tax choice? It could influence τ' through its influence on saving, k'. Suppose, for example, that $g' = \Psi(k')$ is increasing. Then the current government might see a reason to increase g a little more, so as to decrease k' and thereby decrease g': it could influence the government expenditure choice next period through savings.

Our GEEs, however, do not directly contain the derivative of the tax policy rule Ψ, as one might think it would. In fact, from our arguments earlier, and the very fact that the government's problem can be written recursively, the successive governments actually agree in one important dimension: *given the value for current savings*, they agree on how to set next period's taxes. That is why the derivative of Ψ does not appear directly in the government's first-order conditions. It appears indirectly, as a determinant of $\mathcal{H}_g$. But this appearance does not reflect strategic behaviour; rather, it simply captures how the effects on private-sector savings of a current change in g depends on how those extra savings will alter next period's tax rate. That is, $\mathcal{H}_g$ reflects how a current tax change influences the *expectations* of private agents and therefore their savings. More precisely, if the tax rate today is changed, how much extra (or less) capital is saved—$\mathcal{H}_g$—depends on how the determination of the expenditure on g' is perceived by the private sector.

To illustrate the role of Ψ in the determination of the savings response, let us compare a "myopic" government to the kind of government we model: a myopic government does not realize that its current taxation behaviour influences future taxes. Suppose that the time-consistent equilibrium has Ψ as an increasing function: the higher the savings today, the higher the government expenditures will be next period. In contrast, the myopic government perceives $\Psi(k)$ to be constant. How, then, would the myopic government's first-order condition look? The answer is that it would look the same, with the one difference that $\mathcal{H}_g$ would be a different number: in terms of our compact equilibrium definition, we have $\mathcal{H}_g = -\frac{\eta_g}{\eta_{k'}}$, and here the denominator (but not the numerator) depends on the derivative of Ψ. Assuming that $u(c,g)$ is additively separable, that $\eta_g > 0$, and that $\eta_{k'} > 0$, one sees that if a change in future government expenditures is ignored, $\eta_{k'}$ would be too high—because of the lowered consumption, and therefore increased future marginal utility value of savings, implied by the higher future tax rate—or too low—because of the lower net-of-tax return from future savings. That is, a myopic government would misperceive $\mathcal{H}_g$, but whether this leads to lower or higher equilibrium taxes is a quantitative question.

3. EXTENSIONS: VALUED LEISURE AND OTHER TAX BASES

Suppose now that leisure is valued: we assume that utility is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1-\ell_t, g_t).$$

We now assume that the tax base is total income. Our equilibrium definition works as before, but one more element is needed: we need to describe the equilibrium labour response to (k, g). The relevant mapping is $\mathcal{L}(k, g)$, which is obtained from the consumer's first-order condition for the labour-leisure choice. Thus,

$$\frac{u_\ell(\mathcal{C}(k, \mathcal{H}(k, g), g), 1-\mathcal{L}(k, g), g)}{u_c(\mathcal{C}(k, \mathcal{H}(k, g), g), 1-\mathcal{L}(k, g), g)} = f_\ell(k, \mathcal{L}(k, g))(1-\mathcal{T}(k, g)), \tag{11}$$

for all (k, g) and the first-order condition for savings (which now contains a leisure argument, but which we will not restate) *jointly* define the functions $\mathcal{H}(k, g)$ and $\mathcal{L}(k, g)$.

The equilibrium conditions now include three functional equations: the private sector's first-order conditions for labour and savings and the government's first-order condition. We go straight to the latter—to the GEE—which can be derived with the same procedure as above. It reads

$$\begin{aligned}&\mathcal{L}_g[u_c f_\ell - u_\ell] + [u_g - u_c] + \mathcal{H}_g[-u_c + \beta u_c'(1 + f_k' - \delta)] \\ &+\beta\mathcal{H}_g\left\{\mathcal{L}_k'[u_c' f_\ell' - u_\ell'] - \frac{\mathcal{H}_k'}{\mathcal{H}_g'}(\mathcal{L}_g'[u_c' f_\ell' - u_\ell'] + [u_g' - u_c'])\right\} = 0,\end{aligned} \tag{12}$$

for all k (again, the arguments of the functions are suppressed for readability). We see a new wedge appearing: $u_c f_\ell - u_\ell$, in the current period as well as in the next. This wedge, which equals $u_c\tau$, must be positive so long as public goods are provided ($\tau > 0$). A current tax increase will thus increase this intra-temporal distortion. Similarly, there will be repercussions through lowered savings on the same wedge in the future, in parallel with the induced effects on future savings.

In a closed-form application of the economy with leisure and taxation of total income, using $u(c, 1-\ell, g) = \alpha \ln c + (1-\alpha)\ln(1-\ell) + \gamma \ln g$ and the same production technology as used in Section 2.3.3, it is straightforward to see that $\Psi(k) = \frac{\gamma}{\alpha+\gamma}(1-\beta\theta)k^\theta \ell^{1-\theta}$, with $\mathcal{L}(k, g) = \frac{\alpha(1-\theta)}{\alpha(1-\theta)+(1-\alpha)(1-\beta\theta)}$ and $\mathcal{H}(k, g) = \beta\theta(1-\mathcal{T}(k, g))k^\theta\ell^{1-\theta}$, solves this functional equation: the tax rate is constant. Here, as above, future taxes do not influence present savings decisions, and present work decisions are not influenced, perhaps because wealth effects are not present: future taxes will lower the net-present-value income for *given* net interest rates (and should increase work effort) but net interest rates go down to exactly cancel the lowering of future income flows.

Below in the quantitative section, we will look at an economy with less than full depreciation of capital and income taxation (*i.e.* the stock of capital is not taxed). There, in contrast, an increase in future taxes on total income would decrease present-value income. This is because the net interest rate would fall by less in percentage terms than would the value of the future labour endowment, and hence current work effort (and savings) would increase.

A similar effect would be present if only labour income were taxed: in that case, there would be no counteracting decrease in the interest rate at all, and increased future tax rates would raise work effort and increase current savings. Hence, in this and the latter kinds of economies, this by-product of future taxation—the induced increases in current work and savings efforts—will counteract the distortions caused by taxation and thus be desirable. In particular, in an economy with labour taxation only the commitment outcome is expected to lead to *higher* taxation than the outcome without commitment, which does not internalize the positive impact of current taxes on past work efforts. The GEE with labour income taxes only becomes

$$\mathcal{L}_g[u_c f_\ell - u_\ell] + [u_g - u_c] \\ +\beta\,\mathcal{H}_g\left\{(\mathcal{L}'_k - \frac{\mathcal{H}'_k}{\mathcal{H}'_g}\mathcal{L}'_g)[u'_c f'_\ell - u'_\ell] - \frac{\mathcal{H}'_k}{\mathcal{H}'_g}[u'_g - u'_c]\right\} = 0. \tag{13}$$

Now consider the case where only (net) capital income can be taxed. Then the GEE becomes

$$[u_g - u_c] + \mathcal{H}_g[-u_c + \beta u'_c(1 + f'_k - \delta)] + \beta\mathcal{H}_g\big(-\frac{\mathcal{H}'_k}{\mathcal{H}'_g}\big)[u'_g - u'_c] = 0. \tag{14}$$

Notice that this is *the same* GEE as in the model without leisure. This does not mean that the equilibrium tax rate is the same—the remaining equilibrium equation elements are different. As in the case without valued leisure, it will not be optimal to go all the way to (statically) optimal public-goods provision.

Finally, if the government can tax labour and capital income at separate rates but faces the constraint that labour cannot be subsidized, then the optimal choice is to set the labour tax equal to zero. The intuition for why the government would like to subsidize labour if it could is that doing so would encourage saving. This case is also discussed in Martin (2006).

4. EQUILIBRIUM POLICY FOR AN ECONOMY CALIBRATED TO POSTWAR U.S. DATA

We proceed next to look at numerical solutions for a selected set of economies with some aggregate statistics that resemble those of the U.S. postwar economy. For the sake of comparison we also provide the optimal policy under the first best (lump-sum taxation) allocation and those implied by a benevolent government that has access to commitment but not to a technology to save resources, that is, the Ramsey equilibrium given a period-by-period balanced-budget constraint.[12]

We specify the per-period utility function of the constant elasticity of substitution class as

$$u(c,\ell,g) = \frac{\left[(1-\alpha_p)(\alpha_c c^\rho + (1-\alpha_c)\ell^\rho)^{\psi/\rho} + \alpha_p g^\psi\right]^{((1-\sigma)/\psi)} - 1}{1-\sigma}. \tag{15}$$

This function reduces to a separable function with constant expenditure shares when $\sigma \to 1$, $\rho \to 0$, and $\psi \to 0$, yielding

$$u(c,\ell,g) = (1-\alpha_p)\alpha_c \ln c + (1-\alpha_p)(1-\alpha_c)\ln\ell + \alpha_p \ln g. \tag{16}$$

Meanwhile, the production function is a standard Cobb–Douglas function with capital share θ: $f(k,l) = A\cdot k^\theta l^{1-\theta}$.

Our parameterization of the baseline economy is also standard. We calibrate the baseline model economy, which is the one with only labour taxes, to have some statistics within the range of U.S. data in the lack-of-commitment economy. So we set the share of GDP that is spent by the government to be slightly under 20%, the capital share to 36%, the investment-to-output ratio to

12. Related insights are also obtained in Stockman (2001). For earlier analysis of a setup without commitment, see Klein and Ríos-Rull (2003), who perform a quantitative analysis of optimal taxation (labour and capital income taxes) for exogenous public expenditures under a period-by-period balanced-budget constraint.

TABLE 1

Baseline model economy

Steady-state statistic	Labour income taxes			Capital income taxes			Total income taxes		
	Type of government			Type of government			Type of government		
	Pareto	Ramsey	Markov	Pareto	Ramsey	Markov	Pareto	Ramsey	Markov
y	1·000	0·700	0·719	1·000	0·588	0·478	1·000	0·669	0·693
k/y	2·959	2·959	2·959	2·959	1·734	1·149	2·959	2·527	2·649
c/y	0·509	0·509	0·573	0·509	0·712	0·688	0·509	0·532	0·587
g/y	0·254	0·254	0·190	0·254	0·149	0·220	0·254	0·265	0·201
c/g	2·005	2·005	3·017	2·005	4·779	3·123	2·005	2·005	2·928
l	0·350	0·245	0·252	0·350	0·278	0·285	0·350	0·256	0·258
τ	—	0·397	0·297	—	0·673	0·821	—	0·334	0·255

a little over 20%, hours worked to about one-fourth of total time, and the capital-to-output ratio to about 3. These choices are common in the macroeconomic literature.

We choose the baseline economy to have logarithmic utility, which makes preference separable (making cross derivatives 0). The parameter values of the baseline economy are given by $\theta = 0{\cdot}36$, $\alpha_c = 0{\cdot}30$, $\alpha_p = 0{\cdot}13$, $\beta = 0{\cdot}96$, $\delta = 0{\cdot}08$, $\rho = 0$, $\psi = 0$, and $\sigma = 1{\cdot}0$.

As indicated, we look for the smooth Markov-perfect equilibrium, and our belief is that it is the limit of finite-horizon equilibria. This is not substantiated with proofs, but we base our belief both on the fact that our numerical algorithms always found only one solution as well as on a theoretical parallel with the cases discussed above and where analytical proofs were possible. There, a "spurious" equilibrium could exist if the limit equilibrium were one with zero savings, and zero production, which was shown to sometimes occur using backwards-induction logic. Here, such a limit equilibrium could not occur, because the taxation in the last period will not lead to zero savings in preceding periods: taxing at strictly less than 100% is enough to cover public expenditure needs, given our calibration. Specifically, γ is set low enough that the desired public expenditure is not excessive in this sense.

4.1. *The baseline economy with different tax regimes*

We now look at the steady states of the baseline economy under three different benevolent governments that we label Pareto, Ramsey and Markov. These labels refer to a government with commitment and access to lump-sum taxation (Pareto); a government restricted by a period-by-period balanced-budget constraint and to the use of distortive taxation, both one with access to commitment (Ramsey) and one without such access (Markov, because we look at the Markov equilibrium). Table 1 reports the steady-state allocations of these three economies under three different assumptions about the tax base.

4.1.1. Taxes on labour income. The absence of capital income taxes ensures that the steady-state interest rate is equated to the rate of time preference, yielding an equal capital-to-output ratio in all economies. Comparing the Pareto and the Ramsey economies, we see the effect of distortionary labour taxation. The Pareto economy delivers the optimal allocation, while the Ramsey economy has a distortionary tax that discriminates against produced goods and in favour of leisure. As a result, leisure is significantly higher in the Ramsey economy than in the Pareto economy, and because of this and the equal rate of return, the steady-state stock of capital and output are much lower in the Ramsey economy. However, the ratio between private and public consumption is the same in both economies given that this margin is undistorted. This latter feature is a special implication of the functional form that we have chosen, and it relies on preferences being separable in all three goods and on being of the CRRA class with respect to consumption.[13]

When we look at the behaviour of the Markov economy, we see two things: first, qualitatively, the distortion introduced by the tax on labour is also present in this economy, inducing more leisure and less consumption (both private and public) than in the Pareto economy; and second, the ratio between private and public consumption is not the same as in the other economies (where it was equal to the relative share parameter in preferences). Recall that from equation (13) the optimal policy of the Markov case amounted to striking a balance between achieving the first best in terms of equating the marginal utility of the private and public good and the distortion that the labour tax induces on the leisure-private consumption margin. This balance does not imply setting the margin between the public and the private good to 0. Indeed, the term $u_g - u_c$ is positive in the Markov case, making the second term of equation (13) positive and the first one negative.

The difference with the Ramsey case can perhaps be best described by the fact that the Ramsey policy maker takes into account the fact that a tax hike at t not only lowers labour supply at t but also raises it at $t-1$ and indeed at any previous periods.[14] In contrast, a Markov policy maker treats the latter as a bygone and hence chooses lower tax rates. To see more clearly how this mechanism operates, we computed an economy that is initially in the steady state generated by the Markov policy, but which acquires a commitment technology in period 0. The labour income tax then starts at 21·6% and then converges monotonically to the new steady-state value of 39·7%. In period 5 it is 31·8%. This gradual increase is easy to understand: for any t, the more periods that have preceded t, the more periods there are in which a tax hike in t will encourage labour supply.

4.1.2. Taxes on capital income. The tax on capital income is, in general, very distortionary. The Ramsey government understands this and, therefore, reduces future taxes so as to mitigate the distortionary effect. However, since no other tax base is available here, the result is that the ratio of private to public consumption is much lower than in the unconditional first best. The Markov government, however, does not see the current tax as distortionary at all, since capital is already installed when the government chooses the tax rate: capital is inelastically supplied.

The Markov government, however, understands that the government that follows one period later will distort the allocation significantly and is therefore willing to attempt to transfer resources into the future to increase future consumption. For this reason, it does not tax capital so as to set the private-to-public consumption ratio at the first-best level. The ability of the Markov government to influence the future choices is, of course, smaller than that of the Ramsey government, and as a result, its capital tax rate is higher, and capital and output are lower.

Another interesting feature of this case is that leisure is the lowest in the Pareto case, even when there is no tax on labour. With the preferences of this model economy, in any market implementation, the household's choice of leisure can be decomposed into two parts. One part is what would be chosen if all income were labour income—it equals $1-\alpha_c$ exactly, independently of the wage (in this case 0·7). The other part comes from the amount of additional income the household has, so that leisure is increasing in that additional income. In the Pareto economy, the lump-sum tax levied is larger than the amount of capital income, inducing the household to enjoy less leisure than 0·7, while in all the other economies, the after-tax capital income

13. This is a simple implication of the first-order conditions of the Ramsey problem when written in primal form.

14. For the case of inelastic labour supply, the technical nature of this kind of consideration is discussed in Section 2.3.2.

is always positive, which accounts for why workers enjoy leisure of more than 0·7 in those economies.

4.1.3. Taxes on total income. With respect to the case of a tax on total income, a couple of points are worth stressing.

First, the Ramsey government sets the ratio of private to public consumption to its unconditionally optimal level. Partly because of the special nature of the preferences used in this model economy, the distortions that affect the inter-temporal margin and the consumption leisure margin do not affect the private-to-public-consumption margin. From the point of view of the Markov government, however, this is not the case. An uncommitted policy maker does not take into account that today's taxes increase yesterday's incentives to work, and in addition, it wishes to increase savings by taxing less today, and these effects induce a smaller government sector. This result is perhaps surprising because one might have guessed that a Markov government, which views its taxes as less distortionary than does the Ramsey government, would tax more.[15]

4.1.4. Comparisons across tax regimes. The different tax regimes provide a useful illustration for the mechanisms governing the time-consistent (Markov) equilibria. In the Markov equilibria, capital income taxation is not distortionary *ex post*, since it is like a lump-sum tax. However, the tax base is quite small, since capital income is much smaller than labour income.[16] In contrast, labour taxes are distortionary, and the tax base is larger. Finally, total income taxes have the highest tax base, and they are as distortionary as the labour income tax rate for the same tax rate or less distortionary for the same revenue.

The results reveal, as expected, that the larger the role of capital income taxes (which implies an ordering with capital income first, followed by total income and last labour income), the lower the stock of capital, and hence the lower is output. Second, hours worked vary as expected, but rather little, across environments. Third, and more surprisingly, we see that the ratio of private consumption to public consumption is the highest in the capital-tax economy. One might have expected that the government, since it considers taxes to be non-distortionary, would allocate current resources optimally across these goods, thus equating the marginal utility of public and private consumption (which is what the Pareto government does). This, however, does not occur because the government in the capital-income economy understands that the next government will tax capital heavily (more heavily, indeed, than what this government would like). Thus, in an effort to move resources into the future, it sacrifices current public consumption. Note also that the private-to-public consumption ratio closest to the first best is that of the total income-tax economy.

5. CONCLUDING REMARKS

What is the significance of the assumption of no commitment? Our hunch is that governments are much more than machines implementing past decisions. Whether or not our hunch is correct, however, what we hope to accomplish here is simply to take another step toward deriving implications that one may eventually be able to test against alternatives, such as that based on believing that governments have full commitment. Since Kydland and Prescott first pointed to the

15. We also conducted sensitivity analysis with regard to the preference specification—in particular, the elasticity parameters between the three arguments in utility—and obtained intuitive changes in the results. These results are available from the authors on request.

16. Note that because the tax base excludes depreciation, the tax base of a capital income tax is not a constant fraction of GDP.

time-inconsistency problem, most of the attempts to deal with it have been attempts to fully overcome the problem. In short, the idea has been to introduce (full or partial) commitment through other mechanisms: "rules" (*e.g.* Kydland and Prescott, 1977), delegation (*e.g.* Rogoff, 1985), a richer set of policy instruments with built-in irreversibilities (such as long-maturity bonds, which, by assumption, cannot be defaulted upon; see, for example, Lucas and Stokey, 1983), and so on. To us, it is not clear that these alternatives are feasible. Finally, it is possible to use reputational equilibria to argue—see Chari and Kehoe (1990)—that good outcomes are feasible without explicit commitment, assuming that the time horizon is infinite and that agents are sufficiently patient. Here we simply wonder what might occur if reputation mechanisms fail. In addition, in contrast with what we assume, governments may not be benevolent, or they may be torn between constituencies with conflicting goals, and the political process itself, as well as markets, may be less than perfect. However, before proceeding to such arguably more realistic setups, one needs to understand the underlying basics of policy choice over time when commitment is lacking even under benevolence and no frictions other than those implied by taxation itself. Hopefully, the methods we use in this paper help in this respect.

Throughout this paper, we have assumed that the government must balance its budget in each period. Given that we insist on a lack of commitment on the part of the government, it is not hard to motivate why no positive debt can be issued. With our solution concept, the government would immediately default on any debt, and the private sector, anticipating this, would not buy it at any positive price. However, if the government could accumulate positive *assets*, it would do so, as quickly as possible, up to the point where it can live off the interest income without taxing at all. A very closely related result is established in Azzimonti-Renzo, Sarte and Soares (2006). A less closely related, but, nevertheless, similar result is found in Reis (2006), who shows that if households can default on debt, capital taxes—but not necessarily labour taxes—converge to zero in the best sustainable plan. Domínguez (2007) also has a result along similar lines. Can limits on asset accumulation be defended? In a slightly richer environment, we think so. Governments might have reason to believe that their successors have different preferences with respect to the amount or composition of public spending from their own. In this situation, it would be unwise to accumulate assets that might be squandered by a successor.[17] Formally looking at such an environment would be an interesting extension of the current work.

The numerical algorithm that we propose, though it works very well for the purposes of the present paper, is not intended to be the last word. Rather, we hope that our contribution will encourage more work in the development of numerical methods for solving models in political economy and optimal policy without commitment. Indeed, this should be a fertile area for future research, both because we think it is important and because so little is known about it, especially in contrast to what is known about efficient methods of solving versions of the standard growth model. The presence of derivatives of unknown functions in what we call the GEE (equation 5) means that the insights from the large (and still growing) literature on numerical solutions to the growth model are not sufficient for solving problems similar to those of this paper.

APPENDIX A. NUMERICAL ALGORITHM

A.1. *The functional equations*

Recall the two first-order conditions: the one for the private sector,

$$0 = u_c - \beta\, u_c' \,[1 + (1 - \mathcal{T}')\,(f_k' - \delta)], \tag{17}$$

17. This idea is explored, in a very different environment from ours, by Persson and Svensson (1989).

and the one for the government,

$$0 = (-u_c + u_g)\,\eta_{k'} - \eta_g \left[-u_c + \beta\, u'_c\;(f'_k + 1 - \delta) - \beta\,(-u'_c + u'_g)\,\frac{\eta'_k}{\eta'_g} \right]. \tag{18}$$

These are functional equations: they hold for all k. The derivatives of η are derived from the definition of η in Section 2.3; they are

$$\eta_k = u_{cc}\,\mathcal{C}_k \tag{19}$$

$$\eta_g = u_{cc}\,\mathcal{C}_g + u_{cg} \tag{20}$$

$$\begin{aligned}\eta_{k'} = {} & u_{cc}\,\mathcal{C}_{k'} + u'_c[1-\mathcal{T}']\,f'_{kk} - u'_c[f'_k - \delta][\mathcal{T}'_k + \mathcal{T}'_g\,\Psi'_k] \\ & + \{1 + [1-\mathcal{T}']\,[f'_k - \delta]\}\,\{u_{cc}[\mathcal{C}'_k + \mathcal{C}'_{k'}\,h'_k + \mathcal{C}'_g\,\Psi'_k] + u_{cg}\Psi'_k\}.\end{aligned} \tag{21}$$

If we substitute equations (19)–(21) into equations (17) and (18) we obtain a system of two equations that we can write compactly as

$$0 = \xi^p\{k, h(k), \Psi(k), h[h(k)], \Psi[h(k)]\} \tag{22}$$

$$0 = \xi^g\{k, h(k), \Psi(k), h[h(k)], \Psi[h(k)], h'_k(k), \Psi'_k(k)\}. \tag{23}$$

Global computation of a solution to the pair of functional equations could be operationalized in a number of ways, including postulating flexible parameterized functional forms for h and Ψ and requiring that the functional equations hold exactly on an appropriately chosen grid, or that the error to these equations be minimized over a large number of grid points. Here, however, we will solve only for steady states, and thus a simple generalization of a linearization method can be used.

A.2. *The steady state*

A steady state is a pair of values k^* and g^* such that the two functional equations are satisfied when setting $k = k' = k''$ and $g = g'$. Doing this yields

$$0 = \xi^p(k^*, k^*, g^*, k^*, g^*) \equiv \xi^{p*}(k^*, g^*) \tag{24}$$

$$0 = \xi^g(k^*, k^*, g^*, k^*, g^*, h^*_k, \Psi^*_k) \equiv \xi^{g*}(k^*, g^*, h^*_k, \Psi^*_k). \tag{25}$$

Using this compact form, we see two equations and four unknowns: the vector of steady-state values for k and g and the first derivatives of their associated decision rules evaluated at the steady state: $(k^*, g^*, h^*_k, \Psi^*_k)$. This means that levels cannot be solved for without knowing derivatives.

The method we use to solve for a steady state is outlined for a simpler problem in Krusell *et al.* (2002). In short, it relies on a successive set of approximations to the decision rules that are polynomial functions and that use only steady-state information.

The algorithm builds on (i) constructing a set of local approximations of order m—here, m-order polynomials—to the functions h and Ψ; (ii) denoting these approximations $\varphi^{p,m}(k)$ and $\varphi^{g,m}(k)$, respectively, solving for the steady state given m; and (iii) increasing m until the steady state changes by less than some convergence criterion. We now show in more detail how such an algorithm is implemented.

1. When $m = 0$, the functions φ are constants. With two equations—(24) and (25)—and two unknowns (using the fact that the derivatives are 0) there is typically a unique solution. Denote the implied steady state $\{k^0, g^0\}$.
2. For $m = 1$, the functions φ are linear, yielding $k' = \varphi_0^{k,1} + \varphi_1^{k,1}\,k$ and $g = \varphi_0^{g,1} + \varphi_1^{g,1}\,k$; this means that all derivatives of order 2 and above are 0 and that the functions are entirely specified by their levels and derivatives at the steady state. Now the four unknowns necessitate four equations. We thus keep the equations from the previous step and differentiate each of these with respect to k; this is valid (assuming differentiability), since the equations have to hold for all k. Thus, we have four equations and four unknowns. Imposing the steady-state condition and substituting k by $\frac{\varphi_0^{k,1}}{1-\varphi_1^{k,1}}$, g by $\frac{\varphi_0^{g,1}}{1-\varphi_1^{g,1}}$, $h_k(k)$ by $\varphi_1^{k,1}$, and $\Psi_k(k)$ by $\varphi_1^{g,1}$ we have

$$0 = \xi^p \tag{26}$$

$$0 = \xi^g \tag{27}$$

$$0 = \xi^p_k + \varphi_1^{k,1}\,\xi^p_{k'} + \varphi_1^{g,1}\,\xi^p_p + \left(\varphi_1^{k,1}\right)^2\,\xi^p_{k''} + \varphi_1^{g,1}\,\varphi_1^{k,1}\,\xi^p_{g'} \tag{28}$$

$$0 = \xi^g_k + \varphi_1^{k,1}\,\xi^g_{k'} + \varphi_1^{g,1}\,\xi^g_g + \left(\varphi_1^{k,1}\right)^2\,\xi^g_{k''} + \varphi_1^{g,1}\,\varphi_1^{k,1}\,\xi^g_{g'}, \tag{29}$$

where equations (28) and (29) use the fact that $h_{kk}(k)$ and $\Psi_{kk}(k)$ are 0 because these functions are assumed to be linear at this stage of the iteration. In this equation system, the ξ functions and their derivatives, of course, depend on the four unknowns, and a non-linear solver has to be used to deliver the unknowns, and hence $\{k^1, g^1, h_k^1, \Psi_k^1\}$.

3. Turning to $m = 2$, there are six unknowns that are uniquely determined by the values of h and Ψ and their first two derivatives at a given point. The six equations are the four equations from the previous step plus those that result from differentiating the last two equations once more with respect to k.
4. The procedure is repeated until the steady-state values for k and g (and possibly some low-order derivatives, if local dynamics are also an object of study) change by a small amount.

Two specific additional comments are in order. First, to differentiate the first-order condition (multiple times) one can either use numerical differentiation or use symbolic differentiation using a package like MAPLE. The latter imposes no bound on the number of derivatives that can be computed; numerical derivatives of high order are hard to obtain with precision.

Second, to solve the non-linear equation system at step m, which involves $2(m+1)$ equations and unknowns, one can, of course, use brute force. However, it is also possible to use an inherent recursivity in the system. This recursivity, however, requires computing not the coefficients in the polynomials for h and Ψ but the associated sequence of derivatives. In terms of these derivatives, (i) the first equation always contains two levels and no derivatives; (ii) the next two equations contain the two levels and two first-order derivatives; (iii) the next two equations contain the two levels, the two first-order derivatives and the two second-order derivatives; and so on until the last equation, which contains no new higher-order derivatives, since these are assumed to be 0. Thus, one guesses on, say, k^m and then uses the first equation to solve for g^m, the next two to solve for the two first-order derivatives, the following two to solve for the two second-order derivatives, and so on until all the non-zero derivatives have been calculated; the last equation remains, and it has to be satisfied, which is ensured by iteration on the initial choice k^m. Thus, at no stage is it necessary to simultaneously solve more than two equations in two unknowns with this recursive method.

The solutions reported in our quantitative section have been compared to solutions obtained with global methods for solving for the fixed-point decision rules. In particular, when Chebyshev polynomials were used, we obtained steady states and derivatives of decision rules at steady state that are very close (up to the third decimal point) to those obtained using our steady-state-based method. Moreover, global plots show that these rules are also close on a much larger domain for the state variable; thus, the steady-state-based methods are not just efficient but, at least for this environment, appear to deliver reliable global decision rule features as well. This is not a surprise, perhaps, since if decision rules are functions that are analytic (as are those in our closed-form examples), information at one point can, with standard polynomial expansions, be used to provide accurate approximations of the functions far from this point. Details of these comparisons are available upon request from the authors.

APPENDIX B. TWO EQUIVALENT DEFINITIONS OF EQUILIBRIUM

An alternative way of defining the equilibrium without commitment is the following, using some of the notation of Section 2.3. An equilibrium is a value function $v(k)$, a differentiable policy function $\Psi(k)$, and a savings function $\widetilde{\mathcal{H}}(k,g)$ satisfying the following conditions.

1. For all k,
$$\Psi(k) \in \underset{g}{\operatorname{argmax}}\{u(\mathcal{C}(k,\mathcal{H}(k,g)),g)+\beta v(\mathcal{H}(k,g))\}.$$
2. For all k and g,
$$u_c(\mathcal{C}(k,\widetilde{\mathcal{H}}(k,g),g),g) \equiv$$
$$\beta\, u_c(\mathcal{C}(\widetilde{\mathcal{H}}(k,g),\widetilde{\mathcal{H}}(\widetilde{\mathcal{H}}(k,g),\Psi(\widetilde{\mathcal{H}}(k,g))),\Psi(\widetilde{\mathcal{H}}(k,g))),\Psi(\widetilde{\mathcal{H}}(k,g)))$$
$$\{1+[1-\mathcal{T}(\widetilde{\mathcal{H}}(k,g),\Psi(\widetilde{\mathcal{H}}(k,g)))][f_k(\widetilde{\mathcal{H}}(k,g))-\delta]\}.$$
3. For all k,
$$v(k)=u(\mathcal{C}(k,\widetilde{\mathcal{H}}(k,\Psi(k)),\Psi(k)))+\beta v(\widetilde{\mathcal{H}}(k,\Psi(k))).$$

(Also, $\widetilde{\mathcal{H}}$ must satisfy the boundary condition $\widetilde{\mathcal{H}}(0,0)=0$.)

Meanwhile, the equilibrium defined in Section 2.3 is a value function $v(k)$, a differentiable policy function $\Psi(k)$ and a savings function $h(k)$ such that $h(0)=0$ and such that, for all k, $g=\Psi(k)$ and $k'=h(k)$ solve

$$\max_{g,k'}\{u(\mathcal{C}(k,k',g))+\beta v(k')\},$$

subject to

$$\eta(k, g, k'; h, \Psi) = 0,$$

and, for all k,

$$\upsilon(k) = u(\mathcal{C}(k, h(k), \Psi(k))) + \beta\upsilon(h(k)),$$

where η is as defined in Section 2.3 except that we have made explicit the dependence on h and Ψ.

These definitions are equivalent in the following sense. If υ, $\widetilde{\mathcal{H}}$ and Ψ are an equilibrium in the first sense, then υ, Ψ, and h are an equilibrium in the second sense if one defines h via $h(k) := \widetilde{\mathcal{H}}(k, \Psi(k))$.

Also, if υ, h and Ψ are an equilibrium in the second sense, we can define the function $\mathcal{H}$ as in Section 2.3, $\eta(k, g, \mathcal{H}(k, g)) \equiv 0$ and $\mathcal{H}(0, 0) = 0$; then $(\upsilon, \mathcal{H}, \Psi)$ are an equilibrium in the first sense. To see why this equivalence holds, notice that the constraints $k' = \widetilde{\mathcal{H}}(k, g)$ and $\eta(k, g, k') = 0$ are in fact the same. Obviously, if the regularity conditions required by the implicit function theorem apply, the function $\mathcal{H}$ of Section 2.3.1 corresponds exactly to the function $\widetilde{\mathcal{H}}$ defined above.

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