Energy-Saving Technical Change

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We estimate an aggregate production function with constant elasticity of substitution between energy and a capital/labor composite using U.S. data. The implied measure of energy-saving technical change appears to respond strongly to the oil-price shocks in the 1970s and has a negative medium-run correlation with capital/labor-saving technical change. Our findings are suggestive of a model of directed technical change, with low short-run substitutability between energy and capital/labor but significant substitutability over longer periods through technical change. We construct such a model, calibrate it based on the historical data, and use it to discuss possibilities for the future.

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I. Introduction

Resource scarcity is a recurring theme in the public debate, in particular when it comes to our energy sources. In this paper we estimate an aggregate production function including capital, labor, and fossil energy using postwar aggregate U.S. data on quantities and input prices. We then use the function we arrive at, along with the historical data, in order to shed light on a number of issues: (i) the nature of saving on the different input sources, including energy-saving; (ii) how one might expect energy costs to evolve in the future; and (iii) implications for the long-run growth rate of consumption.

We restrict attention to aggregate functions with a constant elasticity of substitution between two inputs: (fossil) energy, on the one hand, and a capital-labor composite, on the other. We also restrict the capital-labor composite to be of the Cobb-Douglas form—based on the observation that the relative capital and labor shares have been close to constant over this period. Based on annual data, we estimate the elasticity of substitution between energy and the capital-labor composite to be very near zero. This finding is rather robust. Thus, in the short run, it appears very difficult to substitute across these input factors.

Our estimated function implies that we can back out a historical energy-saving technology series, along with one for the saving on the capital-labor composite. These series have rather striking features. In particular, it appears (i) that the oil price increases in the 1970s were followed by a large and persistent increase in energy-saving and (ii) that there is a marked medium-run negative co-movement between energy-saving and capital/labor-saving. These observations suggest that our economy directs its R&D efforts to save on inputs that are scarce, or expensive, and away from others. We thus interpret our findings as aggregate evidence of "directed technical change", as discussed by Hicks (1932) and studied by Kennedy (1964) and Dandrakis and Phelps (1966) a long time ago and, much more recently, by Acemoglu (2002). These findings are consistent with available disaggregated analysis, such as Popp (2002) and Aghion et al. (2012).

Motivated by these findings, in the second part of the paper we set up a growth model with a non-renewable resource where the energy input is fixed in the short run but can be changed over time by directed, input-augmenting technical change. In line with the evidence in the first part of the paper, we assume that higher growth rates of energy-augmenting technology comes at the expense of lower growth rates of technologies that augment the other factors of production. The model has a number of implications not found in more standard growth models. First, it generates stationary income shares despite the fact that we have assume no substitutability in the short run. Second it can produce "peak oil", i.e., a period of increasing fossil fuel use. The latter is observed in data but is difficult to produce in more standard models. We use this model to address the issue of whether technological progress may allow a balanced growth with increasing consumption also if production requires use of a natural resource that exists in limited supply, like fossil fuel. Of course, the answer depends on the nature of R&D and how it allows us to save on different inputs. We use our historical data, in particular the extent of the tradeoff between the two kinds of technological change, to calibrate our model. This, of course, is a leap of faith, since historical R&D patterns and returns might not be informative of the future, but it is an exercise that at least is disciplined empirically. We find that balanced growth with (low) positive consumption growth is possible. A key result of the exercise is also that the income share of oil in the model is uniquely determined by the elasticity between the two growth rates, allowing us to assess its future path. The empirical relation points to an elasticity not far from unity, implying an energy share in balanced growth not far from one half, which is ten times that observed over the postwar period. The high oil share suggests huge returns from developing alternative energy sources, and we therefore examine a model with an alternative source, interpreted either as coal or as a renewable. Here we find a smaller long-run energy share—just below 30%—and long-run consumption growth of almost 2%.

Particularly relevant references for the current paper can be found in the research following the oil-price shocks of the 1970s. In particular Dasgupta and Heal (1974), Solow (1974), and Stiglitz (1974, 1979) analyzed settings with non-renewable resources and discussed possible limits to growth as well as intergenerational equity issues. Relatedly, Jones's (2002) textbook on economic growth has a chapter on non-renewable resources with quantitative observations related to those we make here. Recently, a growing concern for the climate consequences of the emission of fossil CO_2 into the atmosphere has stimulated research into in the supply of, but also demand for, fossil fuels as well as alternatives; see, e.g., Acemoglu et al. (2011). The recent literature, as well as the present paper, differ from the earlier contributions to a large extent because of the focus on endogenous technical change, making use of the theoretical advances from the endogenous-growth literature.[1]

We begin the analysis with a discussion of production functions in Section

[1]See, e.g., Aghion and Howitt (1992).

II, whereafter we carry out our estimation in Section III. Section IV then develops the model of directed energy-saving technical change. Section V concludes.

II. Aggregate data and aggregate production functions

The objective is to use a parsimoniously specified aggregate production function as a lens through which we can interpret and analyze the macroeconomic data on the use of energy (and other, more standard, inputs). What kinds of functions are appropriate? The central issue is how much substitutability is allowed across inputs, and the extent of substitutability of course depends on the time horizon considered.[2] We will use yearly data in our analysis, though we will also be concerned with longer-horizon perspectives.

We begin the analysis by examining the Cobb-Douglas aggregate production function, where the elasticity is one across all inputs. This function has been a focal point in the neoclassical growth literature, as well as in business-cycle analysis, primarily because it fits the data on capital and labor shares rather well, without structural or other changes in the parameters of the function. More importantly, however, it was also proposed in influential contributions to the literature on non-renewable resources; see, e.g., Dasgupta and Heal (1980). Since then it has also been employed heavily, including in Nordhaus's DICE and RICE models of climate-economy interactions (see, e.g., Nordhaus and Boyer, 2000). Our examination will

[2]Early contributions are Hudson and Jorgenson (1974) and Berndt and Wood (1975) who both use time-series data and estimate the substitutability of energy with other inputs. Both find energy to be substitutable with labor and complementary to capital. Griffin and Gregory (1976) instead use pooled international data and find capital and energy to be substitutes. They argue that there data set better captures the long-run relationships. None of these early studies cover the time of the oil price shocks.

suggest that the Cobb-Douglas function is not appropriate at shorter horizons. That suggestion then leads us to allow a more general formulation. Toward the end of the paper, and in the context of endogenous, directed technical change, we will return to the issue of whether the elasticity between energy and other inputs might still be unitary on a longer time horizon.

A. Cobb-Douglas

The Cobb-Douglas function in capital, labor, and energy reads

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha-\theta} E_t^{\theta}, \tag{1}$$

where A_t is a time-dependent technology parameter and the parameters α and ν are constants. An implication of the Cobb-Douglas specification is that separate trends in factor-augmenting technological change cannot be identified using data on output and inputs; hence the single technology shifter A_t.

The assumption of perfect competition in the input market yields the result that the income shares of all the production factors are constant. As already pointed out, this is approximately correct in U.S. data for capital and labor, even on an annual basis, but is it true for energy? To examine this issue, we maintain a U.S. perspective. We abstract from non-fossil sources of energy such as nuclear power and renewable energy.[3] This appears a reasonable abstraction, because fossil fuel is and has been the dominant source of energy throughout the whole sample period. According to the Energy Information Administration, fossil fuels constituted 91 percent of

[3]Furthermore, substitution of other energy sources for fossil fuel is part of the process whereby the economy responds to changing fossil-fuel prices, a process we want to estimate.

the total energy consumption in 1949 and are around 85 percent in 2008 (see table 1.3 in the Annual Energy Review, 2008). Looking at energy's share of income is thus our first task.

Output gross of energy expenditure is defined as $Q_t \equiv Y_t$+net export of fossil fuel, where Y_t is real GDP denoted in chained (2005) dollars. The data on fossil-energy use, E, and prices in chained (2005) dollars, P, are both taken from the U.S. Energy Information Administration.[4] Specifically, we construct a composite measure for fossil energy use from U.S. consumption of oil, coal, and natural gas. Similarly, a composite fossil-fuel price is constructed from the individual prices of the three inputs. Since transportation services provided by the private use of cars and motorcycles are not included in GDP, fossil-fuel use for these purposes is deducted from total fossil fuel use when E is constructed.[5] The method for constructing the two composites is described in the Appendix. Energy's share of output is then defined as EP/Q.

Figure 1 shows the evolution of fossil energy's share of income as well as the fossil-fuel price. As can be seen, fossil energy's share of income is highly correlated with its price and it is not constant. Specifically, the share starts out around three percent in 1949 and then decreases somewhat up to the first oil price shock when it increases dramatically. The share then

[4]The GDP data is taken from the Bureau of Economic Analysis at http://www.bea.gov/histdata/Releases/GDP_and_PI/2009/Q4/Third_March-26-2010/Section1ALL_xls.xls. For the USEIA reference, see tables 1.3 and 3.1, respectively, available at http://www.eia.gov/totalenergy/data/annual/. The Net export of fossil fuel is taken from the Annual Energy Review 2011, Table 3.9.

[5]Since housing services is included in GDP, we include fossil fuel used for heating residential houses in E. Including the use of petroleum consumption for passenger cars but not the output of private transportation services would likely bias our results since the short-run demand for fossil fuel for private transportation is fairly elastic (see Kilian, 2008). The data on petroleum consumption is taken from the Federal Highway Administration, *Highway Statistics Summary to 1995*, table VM-201a at http://www.fhwa.dot.gov/ohim/summary95/section5.html.

falls drastically between 1981 and the second half of the nineties and then finally increases again. The share does not seem to have an obvious long-run trend, implying that the possibility that the unitary elasticity is a good approximation for the very long run cannot be excluded. For the medium term, however, the data seems hard to square at least with an exact Cobb-Douglas production function.

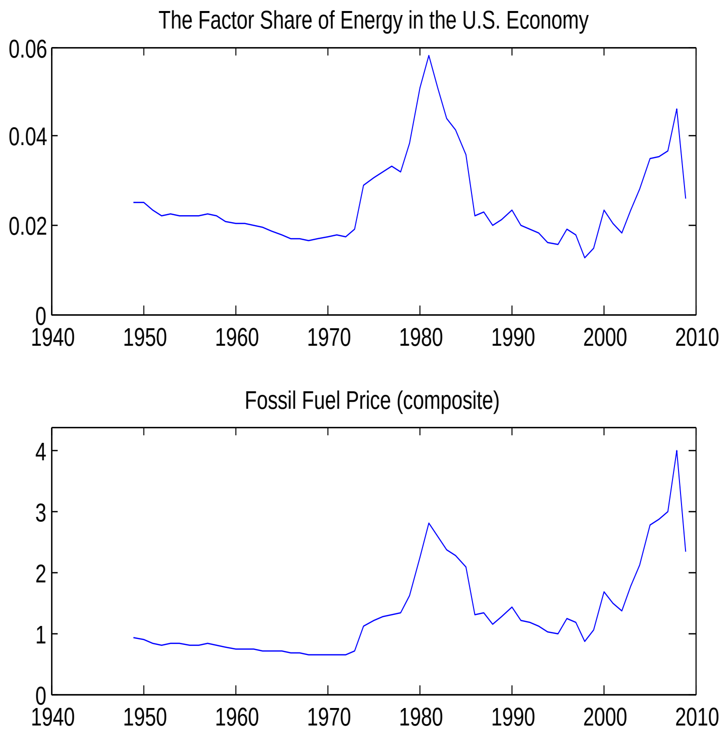

Figure 1. Fossil energy share and its price

B. A nested CES production function

A slightly broader class of production functions is offered by the following nested CES production function:

$$(2) \qquad Y_t = \left[(1-\gamma)\left[A_t K_t^{\alpha} L_t^{1-\alpha}\right]^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left[A_t^E E_t\right]^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where L is labor, A capital/labor-augmenting technology, A^E fossil energy-augmenting technology and ε the elasticity of substitution between capital/labor and fossil energy. γ is a share parameter. It should be pointed out, however, that the specific nesting of capital, labor and energy in (2) is not important for our results. In fact, all the results below still hold with alternative specifications where the elasticity of substitution between capital and energy is allowed to differ from the elasticity of substitution between labor and energy.[6]

The first argument in the production function is a Cobb-Douglas composite of capital and labor, ensuring that the relative shares of capital and labor inherits their properties from the usual Cobb-Douglas form used in growth studies. The second argument, again, is energy. Note that when $\varepsilon = \infty$, the Cobb-Douglas composite and fossil energy are perfect substitutes, when $\varepsilon = 1$, the production function collapses to being Cobb-Douglas in all input arguments; and when $\varepsilon = 0$ the Cobb-Douglas composite and energy are perfect complements, implying a Leontief function in the capital-labor composite and energy.

Note also that this nested CES function implies that there are two distinct factor-augmenting technical change series. Equation (2) can thus be

[6] All the results are, for instance, similar with the alternative specification $Y_t = \left(\left[(1-\gamma)\left[A_t K_t\right]^{\frac{\varepsilon-1}{\varepsilon}} + \gamma\left[A_t^E E_t\right]^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right)^{\alpha} L_t^{1-\alpha}$. This specification requires annual data on capital's share of output, which we compute as $r_t K_t / Q_t = 1 - W_t L_t / Q_t - P_t E_t / Q_t$. The data used to compute the wage share is taken from the Bureau of Economic Analysis, and it includes data on self-employment income.

used to identify the evolution of the two technology trends A and A^E: A is capital/labor-saving and one is energy-saving. In the next section, we will formally estimate these trends along with the key elasticity parameter ε. However, let us first start with how one could use a procedure similar to that in Solow (1957), provided one knew the correct production function. In his seminal paper, Solow shows how to measure technology residuals with a general production function with two key assumptions: perfect competition and constant returns to scale. To measure factor-specific technology residuals, which we aim for here, one needs to make more assumptions on the production function.[7] Specifically, we assume that the production function takes the form given by (2).

For the estimation below, we impose $\alpha = 0.3$, as we know that it will fit the data on the relative capital/labor shares well. The elasticity parameter ε will be estimated, but for now assume that we knew its value, along with γ.

Under perfect competition in input markets, marginal products equal factor prices, so that labor's and energy's shares of income are respectively given by

$$L_t^{Share} \equiv (1-\alpha)(1-\gamma)\left[\frac{A_t K_t^{\alpha} L_t^{1-\alpha}}{Q_t}\right]^{\frac{\varepsilon-1}{\varepsilon}} \tag{3}$$

and

$$E_t^{Share} \equiv \gamma\left[\frac{A_t^E E_t}{Q_t}\right]^{\frac{\varepsilon-1}{\varepsilon}}. \tag{4}$$

[7]This strategy has been used in many applications; see, e.g., Krusell et al. (2000) and Caselli and Coleman (2006).

Equations (3) and (4) can be rearranged and solved directly for the two technology trends A_t and A_t^E. This delivers

$$A_t = \frac{Q_t}{K_t^{\alpha} L_t^{1-\alpha}} \left[\frac{L_t^{Share}}{(1-\alpha)(1-\gamma)} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

and

$$A_t^E = \frac{Q_t}{E_t} \left[\frac{E_t^{Share}}{\gamma} \right]^{\frac{\varepsilon}{\varepsilon-1}} . \tag{6}$$

Note that with ε and γ given, and with data on Q_t, K_t, L_t, E_t, L_t^{Share} and E_t^{Share}, equations (3) and (4) give explicit expressions for the evolution of the two technologies. Clearly, the parameter γ is a mere shifter of these time series and will not play a role in the subsequent analysis. The key parameter, of course, is ε.

The goal of the exercises to follow is to gauge what a reasonable value of ε is, then to use equations (5) and (6), and finally to study the properties of these two implied time series: what their trends are and, more importantly, how they co-vary and relate to the energy price.

III. Estimation

We now estimate the elasticity ε, together with some other parameters, directly with a maximum-likelihood approach. The idea behind the estimation is perhaps simplistic but, we think, informative for our purposes: we specify that the technology series are exogenous processes of a certain form and then estimate the associated parameters along with ε. The technology processes have innovation terms and the maximum likelihood procedure, of course, chooses these to be small. Hence, the key assumption behind the estimation is to find a value of ε such that the implied technology series behave smoothly, or as smoothly as the data allows. This may not be appropriate for other kinds of series but it seems reasonable precisely to require that changes in technology are not abrupt.

As for the particular specification of our technology processes, we also choose parsimony: we require that they be stationary in first differences and have iid, but correlated, errors. Other formulations, allowing moving trends or serially correlated errors, will undoubtedly change the details of the estimates we obtain, but they are unlikely to change the broad findings. As a support for this position, and after showing the results of the estimation, we will discuss implications of radically different values for ε: why they produce series for technology that are not smooth at all.

A. *The technology processes*

Our formulation is

$$\begin{bmatrix} a_t \\ a_t^E \end{bmatrix} - \begin{bmatrix} a_{t-1} \\ a_{t-1}^E \end{bmatrix} = \begin{bmatrix} \theta^A \\ \theta^E \end{bmatrix} + \begin{bmatrix} \varpi_t^A \\ \varpi_t^E \end{bmatrix}, \tag{7}$$

where $a_t = \log(A_t)$, $a_t^E = \log(A_t^E)$ and $\boldsymbol{\varpi}_t \equiv \begin{bmatrix} \varpi_t^A \\ \varpi_t^E \end{bmatrix} \sim N\left(\mathbf{0}, \Sigma\right)$.

Dividing equations (5) and (6) by their counterparts in period $t-1$ gives

$$\frac{A_t}{A_{t-1}} = \frac{Q_t}{K_t^{\alpha} L_t^{1-\alpha}} \frac{K_{t-1}^{\alpha} L_{t-1}^{1-\alpha}}{Q_{t-1}} \left[\frac{L_t^{Share}}{L_{t-1}^{Share}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{8}$$

and

$$\text{(9)} \qquad \frac{A_t^E}{A_{t-1}^E} = \frac{Q_t}{E_t}\frac{E_{t-1}}{Q_{t-1}}\left[\frac{E_t^{Share}}{E_{t-1}^{Share}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Taking logs of (8) and (9) and using (7) in these expressions gives allows us to write the system as

$$\text{(10)} \qquad \mathbf{s}_t = \boldsymbol{\theta} - \frac{\varepsilon}{\varepsilon-1}\mathbf{z}_t + \boldsymbol{\varpi}_t,$$

where

$$\mathbf{s}_t \equiv \begin{bmatrix} \log\left(\frac{Q_t}{K_t^{\alpha}L_t^{1-\alpha}}\right) - \log\left(\frac{Q_{t-1}}{K_{t-1}^{\alpha}L_{t-1}^{1-\alpha}}\right) \\ \log\left(\frac{Q_t}{E_t}\right) - \log\left(\frac{Q_{t-1}}{E_{t-1}}\right) \end{bmatrix} \quad \text{and} \quad \mathbf{z}_t \equiv \begin{bmatrix} \log L_t^{Share} - \log L_{t-1}^{Share} \\ \log E_t^{Share} - \log E_{t-1}^{Share} \end{bmatrix}.$$

The log-likelihood function is now given by

$$\text{(11)} \qquad \begin{gathered} l\left(\mathbf{s}|\boldsymbol{\theta},\varepsilon,\Sigma\right) = \\ -\tfrac{N}{2}\log|\Sigma| - \tfrac{1}{2}\sum_{t=1}^{N}\left(\mathbf{s}_t - \left(\boldsymbol{\theta} - \tfrac{\varepsilon}{\varepsilon-1}\mathbf{z}_t\right)\right)^T \Sigma^{-1}\left(\mathbf{s}_t - \left(\boldsymbol{\theta} - \tfrac{\varepsilon}{\varepsilon-1}\mathbf{z}_t\right)\right) \\ + \, const. \end{gathered}$$

B. Results

Maximization of (11) with respect to $\boldsymbol{\theta}$, ε, and Σ gives the estimated parameters straightforwardly. The data for output, energy, and its price was discussed above. Data on the labor force is taken from the Bureau of Labor Statistics, whereas data on the capital stock as well as the data required to compute labor's share of income are taken from the Bureau of

Economic Analysis.[8]

The results of our estimation are displayed in Table 1.

Table 1—Estimated parameters

θ^A	θ^E	ε
0.0132	0.0136	0.0044
(0.0021)	(0.0030)	(0.0127)

Standard errors in parenthesis

There are two noteworthy features here. One is that the technology trends are both positive and of very similar, and a priori reasonable, magnitude. The second, and more important, point is that the elasticity of substitution between the capital/labor composite and energy is very close to, and in fact not significantly different from, zero. Thus, the CES function that fits the annual data on shares best—where we again emphasize that the estimation procedure penalizes implied technology series that are not smooth—is essentially a Leontief function.

Turning to the estimated covariance matrix, it is given by

$$\Sigma = 10^{-3} * \begin{bmatrix} 0.28 & -0.03 \\ -0.03 & 0.48 \end{bmatrix}.$$

Thus, the energy-saving shocks are somewhat more volatile than are capital/labor-saving technology shocks, and the two shocks are somewhat negatively correlated in the annual data.

[8]We use online data from the BLS on the labor force. The data is available at http://www.bls.gov/webapps/legacy/cpsatab1.htm#a1.f.1. For the capital stock, we use online-data on the net stock of private non-residential fixed assets available at http://www.bea.gov/histdata/Releases/FA/2009/AnnualUpdate_August-17-2010/Section4ALL_xls.xls (table 4.2). Labor's share of income is calculated as (compensation of employees / (compensation of employees + private surplus - proprietors' income)) and is taken from BEA, National Accounts 2008:Q1, table 11100.

What features of the data make our estimation select a value for ε close to 0? As a way of interpretation and before looking at implications of our estimates, we first examine how higher values for the elasticity would change the implied technology series. We then return to our estimates.

C. A high elasticity of substitution

The Cobb-Douglas function, with its unitary elasticity, does not allow the two technology series to be separately identified. For any non-unitary value, however, this is possible. We thus set $\varepsilon = 0.8$. The evolution of the implied fossil energy-saving technology is presented in Figure 2. Clearly, the series implied by a high ε is a rather nonsensical one if interpreted as technology. It features large jumps and dives; it increases by more than 50,000 percent between 1980 and 1998 and decreases by more than 7,600 percent between 1998 and 2009.

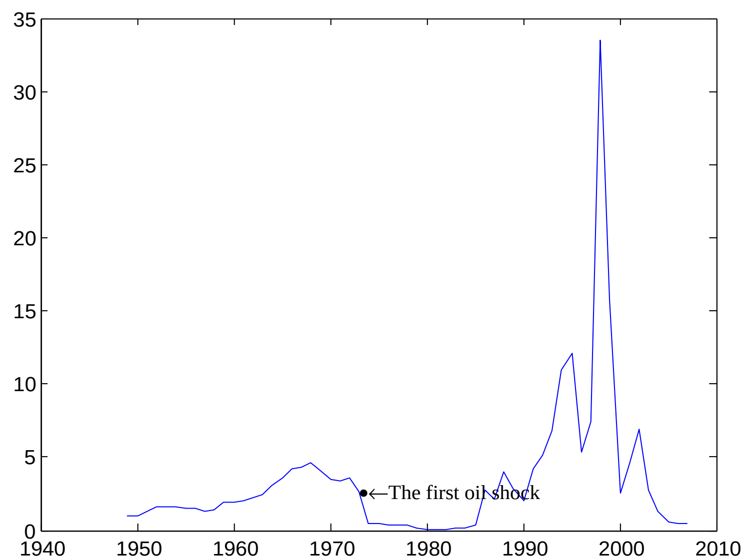

Figure 2. Energy-saving technology with an elasticity of 0.8

To understand this result, take the first oil shock as an example. The price increase in 1973 would then, with a fairly high elasticity, call for much lower energy consumption in U.S. business. However, this fall in consumption is not observed in the data. According to the model, this must then be because of a huge fall in the energy-specific technology value: oil is now much more expensive, but it is also less efficient in producing energy services, the net result of which is a rather constant level of oil use. Similarly, when the price falls in the nineties, an increased demand for energy is expected, but again, this does not take place in the data. The reason, from the perspective of the model, must be that the energy-specific technology increases sharply—by 350 percent in just one year. The mean annual growth rate in A^E during the period with the chosen elasticity is negative (-1.42 percent) and the standard deviation is very high (62.8 percent).[9]

When thinking about the implications of the price movements for fossil fuel, note that the exact reason for the large volatility of fossil fuel prices is not important. This is key since there are two very different, and contrasting, interpretations of the large movements in oil prices. Barsky & Kilian (2004), in particular, argue that the conventional view, i.e., that events in the Middle East and changes in OPEC policy are the key drivers of oil-price changes, is incorrect. Rather, they contend, the price changes were engineered by U.S. administrative changes in price management. Be that as it may, what is important here is simply that firms actually faced the prices we use in our analysis. In conclusion, at least from the perspective of the assumptions of the theory, it is not possible to maintain as high an elasticity of substitution between capital/labor and energy as 0.8. In fact,

[9]The capital/labor-augmenting technology is highly volatile too when ε is high, but the effects of high substitutability are larger on the energy-saving technology.

it takes much smaller εs to make the volatile, non-technology-like features go away—only values below $\varepsilon \sim 0.05$ make the series settle down. We now turn to what those series look like for the near-Leontief case.

D. Low elasticity: implications

The near-Leontief case, i.e., ε close to zero is rather robust in that once the substitution elasticity is in this low range, the features of the technology series do not vary noticeably. So consider now instead a low elasticity and set $\varepsilon = 0.02$, a value slightly higher than the estimated one, but yielding almost identical output. The implied energy-saving technology trend is presented in Figure 3. The figure shows the path for fossil energy-saving technology A^E. As is evident, we observe a smooth, increasing, and overall reasonable-looking graph for fossil energy-specific technology. The mean growth rate is 1.49 percent and the standard deviation is 2.37 percent.[10]

The figure also shows separate trends lines before and after the first oil-price shocks: 1949–1973 and 1973–2009. Clearly, the technology series appears to have a kink around the time of the first oil price shock. In fact, the growth rate is 0.15 percent per year up to 1973 and 2.44 percent per year after 1973. The fact that the kink occurs at the time of the first oil price shock suggests that the higher growth rate in the technology is an endogenous response to the higher oil price.

Figure 4 displays both the fossil energy-saving technology and the oil price and suggests that the energy-saving technology seems to respond to price changes at even higher frequencies. The figure specifically shows that from 1949 up the first oil shock in 1973, the price decreases and at the same time,

[10]The energy-saving technology does not change much for elasticities in the interval $(0, 0.05)$.

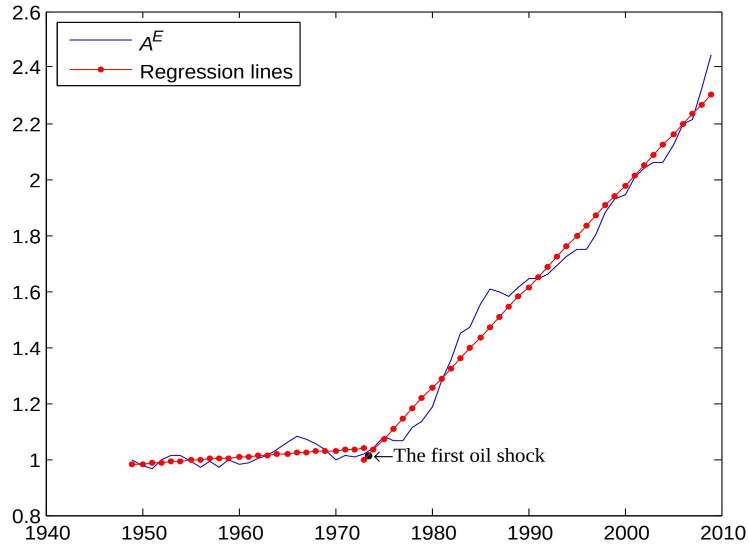

FIGURE 3. ENERGY-SAVING TECHNOLOGY WITH AN ELASTICITY OF 0.02

the energy-specific technology grows only slowly at a rate of 0.15 percent per year. After the large price increase in 1973, the technology grows at the significantly higher rate of 3.69 percent/year up to 1981. However, as the price starts decreasing between 1982 and 1998, the growth rate in the energy-saving technology also slows down to 1.80 percent/year. The price then increases from around 1996 up to 2009 and the growth rate in the technology increases to a rate of 2.35 percent/year. In the last period of our data, the fossil fuel price decreases significantly. If the relation is robust, this can be expected to have a negative effect on the growth in energy-saving technology.

What does a low elasticity imply for the evolution of the capital/labor-augmenting technology? The series for A is plotted the solid line in Figure 5, alongside the A^E series. A too is smooth and increasing graph and very much looks like the conventional total-factor productivity (TFP) series. The

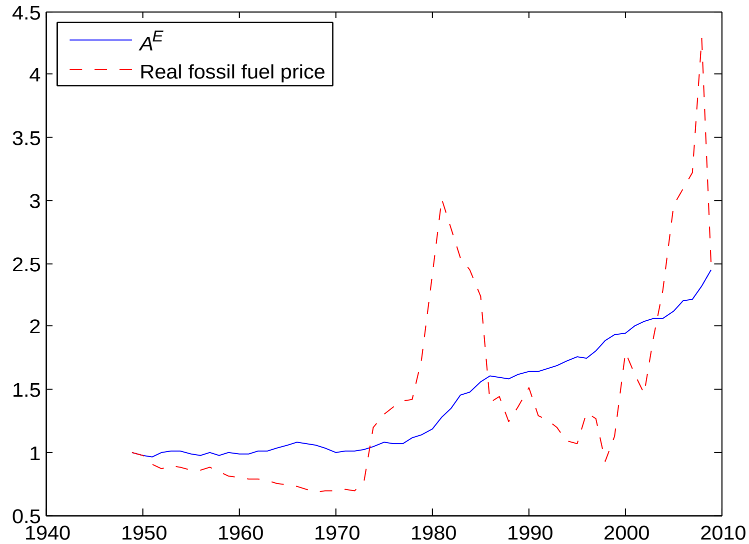

Figure 4. Energy-saving technology and the energy price compared

mean growth rate in A is 1.31 percent and the standard deviation is 1.68 percent. When we compare it to the A^E series, we see that the two series nearly mirror each other.[11] In the beginning of the period, the capital/labor-augmenting technology series grows at a relatively fast rate, whereas the growth rate for the energy-augmenting technology is relatively slow. This goes on until around 1970, i.e., somewhat just before the first oil price shock. After 1970, the energy-augmenting technology grows at a faster rate and the growth rate for the capital/labor-augmenting technology slows down. This continues up to the mid-1980s. Hence, the much-discussed productivity slowdown coincides with a faster growth in the energy-saving technology.

Our impressions that there is a medium-run negative correlation between the two technology series are formalized by applying an HP filter to the two series, taking out the cyclical component, and regressing the trend growth

[11]Both series have been normalized so that the initial value is 1.

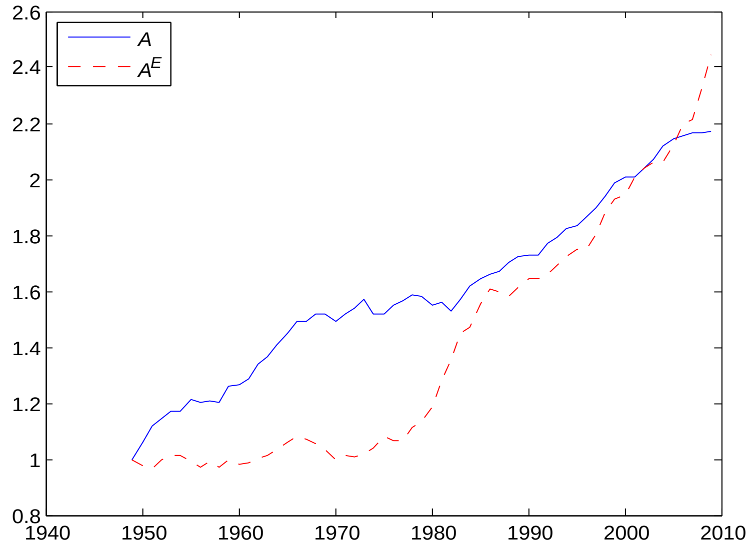

FIGURE 5. ENERGY- AND CAPITAL/LABOR-SAVING TECHNOLOGIES COMPARED

of energy-augmenting technology g_{A^E}, on the trend growth of the capital-augmenting technology g_A.[12] We find the relation

$$g_{A^E} = 2.26 - 1.23 g_A, \tag{12}$$

which is significant at the 1% level. This result can be compared with Popp (2003), who uses expenditure on energy R&D and non-energy R&D and finds a correlation coefficient of -0.41.

At this point, let us take stock of what we have found. Rather simple, though in our view revealing, features of the share and the price data are suggestive of (i) an aggregate CES technology in a capital/labor composite and energy that is close to Leontief on an annual horizon and (ii) of implied movements in factor-saving technologies that are negatively correlated. To

[12] λ is set to 100 which is standard for annual data.

us, this points in the direction of a model of directed technical change. In the least complex fashion of illustrating this, one envisions a fixed R&D resource that can be allocated between two sectors, and where allocating a larger share to one sector implies a faster growth rate in that sector, as a result of which the other sector's growth rate will deteriorate. Of course, this interpretation also indicates that there are substantial costs associated with improving energy efficiency, since a higher energy efficiency will come at the cost of lower growth of capital/labor-efficiency. In order to explore this mechanism, in the next section we construct a formal model of directed technical change. This model can be thought of as a putty-clay model in which the substitution possibilities between capital/labor and energy are fixed ex post but chosen (optimally, or otherwise) ex ante.

IV. A model of directed technical change

The previous sections suggest that energy saving and capital/labor saving, captured as shift parameters in an aggregate production function, respond to incentives. In this section, we formalize that idea, allowing us to fully work out the logic of directed technical change. One of the implications of the analysis, as we shall see, is that the long run will feature a constant energy share, thus giving a much higher long-run elasticity between energy and capital/labor. We also use the model to carry out quantitative exercises that rely on the empirical relationships—in particular the negative medium-run correlation between the growth in energy-saving and in capital/labor-saving—obtained in the first section of our paper.

Our analysis of directed technical change allows us to address two additional issues. One of these regards "peak oil". Clearly, oil use has increased over time and since it is in finite supply it must peak at some point and then fall. The issue here is that standard models do not predict a peak-oil pattern: they predict *falling* oil use from the beginning of time. By standard models, here, we refer to settings relying on the classic work on nonrenewable resources in Dasgupta and Heal (1974) and with high substitutability between oil and capital/labor. A simple case makes the point: with standard preferences (logarithmic utility, discounted in a standard way over time), Cobb-Douglas production, and full depreciation of capital between periods, along with the assumption that oil is costless to extract, oil use will fall at the rate of utility discount. Our perspective on this result, which is robust to moderate changes in all the assumptions, is that a much lower elasticity between oil and capital/labor may allow a peak oil result, a possibility we examine after laying out the model.

A second issue we address with the model, given its constant long-run share of energy costs, is just how large this share will be quantitatively. Our benchmark model assumes that energy only comes from oil. This case is easily analyzed and interesting, despite its unrealistic nature, since it makes the "need for alternative energy sources" rather clear: the oil share must, under reasonable parameterizations of the model, rise over time and eventually become quite high. This oil-only case then naturally leads into a second case, that with either a more abundant fossil-fuel source (say, coal) or an alternative, non-fossil energy source, and we conclude by showing that such a model predicts a more modest long-run share for energy costs.

A. The setting

In order to keep the model simple and transparent, we assume log utility, literal Leontief production, full depreciation, and zero-cost fossil-fuel extraction. These assumptions are not that unreasonable if the time period is 10 years and the time horizon is not too long. We focus on the planning solution and sidestep any issues coming from suboptimal policy with regard to R&D externalities, monopoly power due to patents as well as the market power in the energy sector. We also ignore the (global) climate externality, which might still be of concern in this context even though the damages to the U.S. economy have been estimated to be small. Clearly, these are important issues and we are studying them in related work, but we believe that they are not of primary concern for our main points here.[13]

The representative consumer thus derives utility from a discounted sum of the logarithms of consumption at different dates. This is also the objective function of the planner, i.e.,

$$\sum_{t=0}^{\infty} \beta^t \log C_t.$$

The period resource constraint is

$$C_t + K_{t+1} = \min \left\{ A_t K_t^{\alpha} L_t^{1-\alpha}, A_t^E E_t \right\},$$

where the right-hand side specifies a Leontief production function. The

[13]See Nordhaus and Boyer (2000) and Golosov et al. (2012).

growth rates of the technology trends are assumed given by

$$\frac{A_{t+1}}{A_t} \equiv 1 + g_{A,t} = f(n_t) \tag{13}$$

$$\frac{A^E_{t+1}}{A^E_t} \equiv 1 + g_{A^E,t} = f^E(1 - n_t), \tag{14}$$

where we interpret n as the share of a fixed amount of R&D resources that is allocated to enhancing the efficiency of the capital/labor bundle. The functions f and f^E are increasing. Of course, an increase in A (A^E) is equivalent to a decrease in the Leontief input requirement coefficient for the capital/labor (energy) bundle. By changing n, the planner can direct technical change to either of the two. When A grows at a different rate than A^E, the requirement of energy relative to the requirement of the capital/labor bundle changes. Thus, factor substitutability exists in the long run. Finally, we assume that energy comes from a fossil-fuel source satisfying

$$R_{t+1} = R_t - E_t \in [0, R_t],$$

where R_t is the remaining stock of oil in ground. We also restrict labor supply so that $l_t = 1$ for all t. This can be thought of as "full employment". We let k denote capital henceforth, as in capital per unit of labor.

B. Planning problem

We focus on interior solutions such that capital is fully utilized. This requires initial conditions where capital is not too large, in which case it could be optimal to let some capital be idle for some time. In a deterministic model with full depreciation and forward-looking behavior, less than full utilization can only occur in the first period.[14] Due to solutions being interior, we replace the Leontief production function by the equality

$$A_t k_t^{\alpha} = A_t^E E_t \tag{15}$$

and let the planner maximize

$$\max \sum_{t=0}^{\infty} \beta^t \log(A_t k_t^{\alpha} - k_{t+1}).$$

Condition (15) will be referred to as the "Leontief condition". In addition, the planner must respect the constraints

$$\sum_{t=0}^{\infty} E_t \leq R_0, \tag{16}$$

and (13)–(14).

Let the multipliers on the constraints be $\beta^t \lambda_t$ on the Leontief condition (15), κ on the resource constraint (16) and $\beta^t \mu_t$, and $\beta^t \mu_t^E$, respectively for the two R&D constraints (13)–(14). The first-order conditions are: for k_{t+1}, the Euler equation

$$\frac{1}{(1-s_t)A_t k_t^{\alpha}} = \beta \left(\frac{1}{(1-s_{t+1})A_{t+1} k_{t+1}^{\alpha}} - \lambda_{t+1} \right) \alpha A_{t+1} k_{t+1}^{\alpha-1}; \tag{17}$$

for E_t,

$$\beta^t \lambda_t A_t^E = \kappa;$$

[14]In a model with stochastic shocks, we could have reoccurring periods of less than full capital utilization.

for A_{t+1} and A^E_{t+1}, respectively,

$$\mu_t = \beta\left[\left(\frac{1}{(1-s_{t+1})A_{t+1}k^{\alpha}_{t+1}} - \lambda_{t+1}\right)k^{\alpha}_{t+1} + \mu_{t+1}f(n_{t+1})\right]$$

and

$$\mu^E_t = \beta\left[\lambda_{t+1}E_{t+1} + \mu^E_{t+1}f^E(1-n_{t+1})\right];$$

and, for n_t,

$$\mu_t A_t f'(n_t) = \mu^E_t A^E_t (f^E)'(1-n_t).$$

Here, s_t is the saving rate out of output. Of course, in a recursive formulation, optimal savings depend on the vector of state variables. Here, we solve the model sequentially and present the solution as a time series.

Solving For Saving Rates Conditional On Energy Use

The second first-order condition above can be used to solve for λ_t in a useful way:

$$\lambda_t = \kappa\frac{1}{A_t k^{\alpha}_t}\hat{E}_t,$$

where we have defined $\hat{E}_t \equiv \beta^{-t}E_t$. If this expression is inserted into the Euler equation we obtain

$$\frac{s_t}{1-s_t} = \alpha\beta\left(\frac{1}{1-s_{t+1}} - \kappa\hat{E}_{t+1}\right). \tag{18}$$

Using $\hat{s}_t \equiv s_t/(1-s_t)$ and the algebraic identity $\frac{1}{1-s} = 1 + \frac{s}{1-s}$, we can rewrite this equation as

$$\hat{s}_t = \alpha\beta\left(1 + \hat{s}_{t+1} - \kappa\hat{E}_{t+1}\right).$$

Solving this forward yields

$$\hat{s}_t = \frac{\alpha\beta}{1-\alpha\beta} - \kappa \sum_{k=0}^{\infty} (\alpha\beta)^{k+1} \hat{E}_{t+k+1}. \tag{19}$$

Thus, given a sequence $\{\hat{E}_t\}$ and a value of κ, this equation uniquely, and in closed form, delivers the full sequence of savings rates. We can see from this equation that the more energy is used in the future (in relative terms), the lower is current $\hat{s}$, implying a lower current saving rate. The intuitive reason for this is that more capital requires more fossil fuel and/or more energy efficiency. This limits the value of accumulating capital and more so, the more scarce is fossil fuel. If fossil fuel were not scarce, $\kappa = 0$ and $\hat{s} = \frac{\alpha\beta}{1-\alpha\beta} \Rightarrow s = \alpha\beta$ since the model in this case collapses to the textbook Solow model.

Solving For Energy Use And R&D

From the original Euler equation, we have that

$$\beta \left(\frac{1}{(1-s_{t+1})A_{t+1}k_{t+1}^{\alpha}} - \lambda_{t+1} \right) k_{t+1}^{\alpha} = \frac{1}{\alpha A_{t+1}} \frac{k_{t+1}}{(1-s_t)A_t k_t^{\alpha}} = \frac{s_t}{\alpha(1-s_t)A_{t+1}}.$$

Inserted into the first-order condition above for A_{t+1}, we obtain

$$\mu_t = \frac{s_t}{\alpha(1-s_t)A_{t+1}} + \beta\mu_{t+1} f(n_{t+1}).$$

Multiplying this equation by A_{t+1} and defining $\hat{\mu}_t \equiv \mu_t A_{t+1}$, we obtain

$$\hat{\mu}_t = \frac{s_t}{\alpha(1-s_t)} + \beta\hat{\mu}_{t+1} = \frac{\hat{s}_t}{\alpha} + \beta\hat{\mu}_{t+1}. \tag{20}$$

Given a sequence of saving rates, this equation can be solved for the $\hat{\mu}_t$

sequence. Iterating forward on (20) is also a discounted sum; it delivers

$$\hat{\mu}_t = \frac{1}{\alpha} \sum_{k=0}^{\infty} \beta^k \hat{s}_{t+k}.$$

Recalling that $\hat{\mu}_t$ is the current marginal value of capital-augmenting technology, it is intuitive that the more is saved into the future, the higher is the value of $\hat{\mu}_t$. Using equation (19), we can write this directly in terms of the $\hat{E}$ sequence. By direct substitution and slight simplification we obtain

$$\hat{\mu}_t = \frac{\beta}{(1-\beta)(1-\alpha\beta)} - \frac{\kappa}{\alpha} \sum_{k=0}^{\infty} \beta^k \sum_{j=0}^{\infty} (\alpha\beta)^{j+1} \hat{E}_{t+1+k+j}. \tag{21}$$

Similarly, from the expression for λ_t above, we can write

$$\lambda_{t+1} E_{t+1} = \kappa \frac{E_{t+1}}{A_{t+1} k_{t+1}^{\alpha}} \hat{E}_{t+1} = \kappa \frac{\hat{E}_{t+1}}{A_{t+1}^E},$$

where the last equality follows from the Leontief assumption. Inserted into the first-order condition for A_{t+1}^E, we obtain

$$\mu_t^E = \beta \left[\kappa \frac{\hat{E}_{t+1}}{A_{t+1}^E} + \mu_{E,t+1} f^E (1 - n_{t+1}) \right].$$

Multiplying this equation by A_{t+1}^E and defining $\hat{\mu}_t^E \equiv \mu_t^E A_{t+1}^E$, we obtain

$$\hat{\mu}_t^E = \beta \left[\kappa \hat{E}_{t+1} + \hat{\mu}_{t+1}^E \right].$$

Given a sequence of normalized fossil fuel uses, this equation can be solved for the $\hat{\mu}_t^E$ sequence. It can also be solved forward:

$$\hat{\mu}_t^E = \kappa \sum_{k=0}^{\infty} \beta^{k+1} \hat{E}_{t+k+1}. \tag{22}$$

In parallel to the case of capital-augmenting technology, the more energy is used in the future and the larger is fossil fuel scarcity, the higher is the marginal value of energy-enhancing technology.

Finally, rewriting the first-order condition for n_t above in terms of the redefined multipliers, we arrive at

$$\hat{\mu}_t \frac{f'(n_t)}{f(n_t)} = \hat{\mu}_t^E \frac{(f^E)'(1-n_t)}{f^E(1-n_t)}.$$

From this equation we can solve for n_t uniquely for all t, given the values of the multipliers, so long as f is (strictly) increasing and (strictly) concave (either the monotonicity or the concavity need to be strict). Using the expressions above for the multipliers, we obtain

$$\frac{(f^E)'(1-n_t)f(n_t)}{f^E(1-n_t)f'(n_t)} = \frac{\frac{\beta}{\kappa(1-\beta)(1-\alpha\beta)} - \frac{1}{\alpha}\sum_{k=0}^{\infty}\beta^k \sum_{j=0}^{\infty}(\alpha\beta)^{j+1}\hat{E}_{t+1+k+j}}{\sum_{k=0}^{\infty}\beta^{k+1}\hat{E}_{t+k+1}}. \tag{23}$$

This equation enables us to solve for n_t directly as a function of the future values of $\hat{E}$. The more energy is used in the future, the lower is current n, i.e., the more R&D labor is allocated to energy-saving now. It is straightforward to solve this equation numerically for transition dynamics. However, one can characterize long-run growth features more exactly.

C. *Balanced growth*

On a balanced path, the saving rate is constant and so, from the Euler equation, $\hat{E}_t$ is constant. Recalling the definition $\hat{E}_t \equiv \beta^{-t} E_t$, this means that $E_t = (1-\beta)R_t$ and $R_t = \beta^t R_0$ implying $\hat{E} = (1-\beta)R_0$. The saving rate s_t the balanced growth path is constant and $\hat{s}_t = \frac{s_t}{1-s_t} = \frac{\alpha\beta(1-\kappa\hat{E})}{1-\alpha\beta}$.

The Leontief condition holds for all t when E falls at rate β, so that

$$\frac{A_{t+1}}{A_t}\left(\frac{k_{t+1}}{k_t}\right)^{\alpha} = \frac{A^E_{t+1}}{A^E_t}\beta,$$

implying (since n_t has to be constant)

$$f(n)\frac{k^{\alpha}_{t+1}}{k^{\alpha}_t} = f^E(1-n)\beta.$$

A constant saving rate implies, if k is to grow at a constant rate, that k grows at the gross rate $f(n)^{\frac{1}{1-\alpha}}$. Thus, we have

$$f(n)f(n)^{\frac{\alpha}{1-\alpha}} = f^E(1-n)\beta$$

or

$$f^E(1-n) = \beta^{-1} f(n)^{\frac{1}{1-\alpha}}. \tag{24}$$

This equation, on the one hand, describes a positive relation between the growth rates of A and A^E: $1 + g_{A^E} = \beta(1+g_A)^{\frac{1}{1-\alpha}}$. The positive relation comes from the two inputs growing hand-in-hand due to the short-run substitution elasticity between capital/labor and oil being zero. On the other hand, equation (24) allows us to solve for n, and therefore the point on the positively-sloped curve, which in turn implies a long-run growth rate of consumption for the economy. This, however, requires a specification and calibration of f^E and f. We will carry out a calibration below.

The solution for n can be used in equation (23) to solve for the steady-state value for κ. Using the finding that $\hat{E}$ is constant, (23) becomes

$$\frac{(f^E)'(1-n)f(n)}{f^E(1-n)f'(n)} = \frac{1-\kappa\hat{E}}{\kappa\hat{E}(1-\alpha\beta)}. \tag{25}$$

Thus, given the n from equation (23), we obtain $\kappa\hat{E}$ from equation (25). We then also obtain the saving rate, as specified above, since it depends on $\kappa\hat{E}$. Notice that since $\kappa\hat{E}$ is pinned down independently of initial conditions and $\hat{E} = (1-\beta)R_0$, any increase in R_0 just lowers the κ by the same percentage amount on a balanced growth path. Of course, a balanced path from time 0 also requires that the initial vector (k_0, A_0, A_0^E, R_0) satisfy

$$A_0 k_0^\alpha = A_0^E (1-\beta) R_0;$$

if and only if this equation is met will the economy grow in a balanced way all through time.

Let us finally find the energy share and the value of the fossil fuel resource in balanced growth. This is straightforward to derive using the fact that the current shadow price of the energy resource is $\beta^{-t}\kappa$. Dividing by the marginal utility of consumption to get a price in consumption units and multiplying by $\frac{E_t}{Y_t}$ gives the energy share;

$$E_t^{Share} = \frac{\beta^{-t}\kappa}{u'(c_t)} \frac{E_t}{Y_t} = \beta^{-t} \frac{\frac{\kappa}{u'(c_t)}}{A_t^E}$$

and since $u'(c_t) = \frac{1}{(1-s)A_t^E E_t}$, this is

$$E^{Share} = \kappa (1-s) \hat{E}.$$

Using this in the Euler equation (18) along a balanced growth path yields

$$\begin{aligned} s &= \alpha\beta \left(1 - (1-s)\kappa\hat{E}\right) \\ &= \alpha\beta \left(1 - E^{Share}\right). \end{aligned}$$

Thus, $\frac{E^{Share}}{1-s} = \kappa\hat{E}$ and $\alpha\beta = \frac{s}{1-E^{Share}}$. Using (25), we obtain

$$\frac{(f^E)'(1-n)f(n)}{f^E(1-n)f'(n)} = \frac{1-E^{Share}}{E^{Share}}, \tag{26}$$

which delivers the energy share as a function of n. The steady-state income share of energy is thus fully determined by technological constraints implied by the R&D functions. In particular, it does not depend on the amount of oil left in the ground (but does rely on oil use going to zero asymptotically at rate β).[15]

D. Calibration and long-run predictions

To obtain a long-run value for n, we need information on f and f^E, according to equation (23). The natural restrictions to place on these functions are in the past data, as described above. In particular, we observed a negative medium-run correlation between the growth rates of A and A^E variables and, more precisely, the regression equation (12) details not only the rela-

[15]Utility functions with different curvature yield oil use going to zero at a different rate. Our calculations, however, show the departures from logarithmic utility to have relatively minor effects on this rate and on the long-run oil share.

tion between the two growth rates but also an intercept. In fact, as we shall show, for our main questions in this section we do not need to go beyond this information and find specific functional forms for f and f^E.

So first jointly consider the positive relation (24) between g_A and g_{A^E} and the negative one (12) given by our data; these are depicted in Figure 6 below. The upward-sloping line (24) is drawn for $\alpha = 0.3$ and $\beta = 0.99^{10}$—these are relatively standard values in the growth literature. The estimated downward-sloping line (12), intuitively, is informative of the functions f and f^E: the growth rates g_A and g_A^E in our theory are related through equations (13)–(14), which imply

$$1 + g_{A^E} = f^E(1 - f^{-1}(1 + g_A)). \tag{27}$$

Our linear regression can be thought of as a linearization of equation (27). Using the linearization, we can immediately use the graph for predicting the long-run values of the two growth rates, since these will simply be given by the intersection of the two lines in the figure. We see that the curves intersect at $g_A = 0.0074$ and $g_{A^E} = 0.021$ implying a long-run growth rate of consumption of 1.07% per year. That is, our theory implies that despite oil running out, consumption will grow in the long run, albeit at a relatively modest rate.

Let us finally determine the long-run value of the oil share of output. Equation (27) can be differentiated to obtain

$$-\frac{dg_{A^E}}{dg_{A^E}} = \frac{(f^E)'(1-n)f(n)}{f^E(1-n)f'(n)} = \frac{1 - E^{Share}}{E^{Share}}.$$

Now notice that this expression coincides with the left-hand side of equation

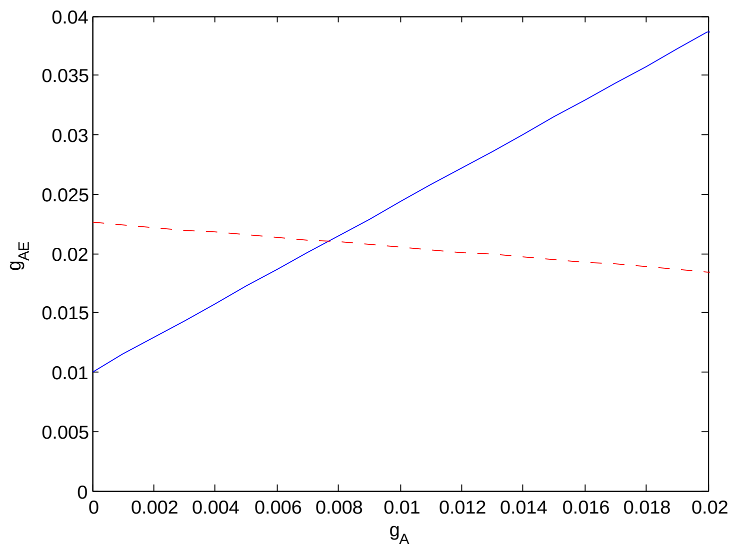

Figure 6. Quantitative determination of the long-run growth rates

(26), whose right-hand side equals $(1 - E^{Share})/E^{S}hare$. Hence, this being a steady-state relation, the steady-state income share of energy can be determined if we use the linearization above. The regression coefficient of g_{A^E} on g_A (the slope in Figure 6) was found to be -1.23, implying a long-run fossil-fuel income share of 45%. This is a large number, in fact so large that it would be hard to imagine that significant resources would not be spent on developing alternative sources of energy. For this reason, we look at such a possibility below. However, one might wonder whether the value around 40% we obtain is robust. As noted above, this share is only determined by the shape of the f and f^E functions, and to obtain an income share more in line with historically observed values, we need a much higher slope than -1.23: it would have to be around -20, i.e., one unit less of growth in A would deliver 20 units more of growth in g_A^E. This is only possible if our estimate based on historical data is off by more than an order of magnitude.

E. Peak oil?

In this section the aim is to calculate the transition path for our economy. If the initial value of A (relative to A^E) is low enough, the idea is then that the economy would mainly accumulate A initially, with an accompanying increase in E: a peak-oil pattern. To use the model for quantitative predictions, we specify the R&D functions to be

$$\begin{aligned} f(n_t) &= Bn_t^{\phi}, \\ f^E(1-n_t) &= B_E\left(1-n_t\right)^{\phi}. \end{aligned} \tag{28}$$

Inverting the first function, substituting, and taking logs yields

$$1 + g_{A^E} = \ln B_E + \phi \ln \left(1 - e^{\frac{1+g_A - \ln B}{\phi}}\right)$$

The three parameters of these functions are calibrated using the relation between the two technological growth rates found in the first part of the paper. Specifically, we use the average growth rates of the two technology series for the pre-oil crisis (1946–73), the oil crisis (1974–84), and the post-oil crisis (1985–2009). These six observations allow us to identify the three parameters of the R&D functions and the three unobserved values of n. In addition we use $\alpha = 0.3$ and $\beta = 0.99^{10}$.

The model is easily solved numerically. We find that if A_0 and k_0 are set to 45 and 55 percent of their balanced-growth values, respectively, we obtain five decades of increasing fossil fuel use. During the transition, E has to grow fast to keep up with the quickly increasing capital and capital/labor-saving technology growth. The results are plotted in Figure 7 below.

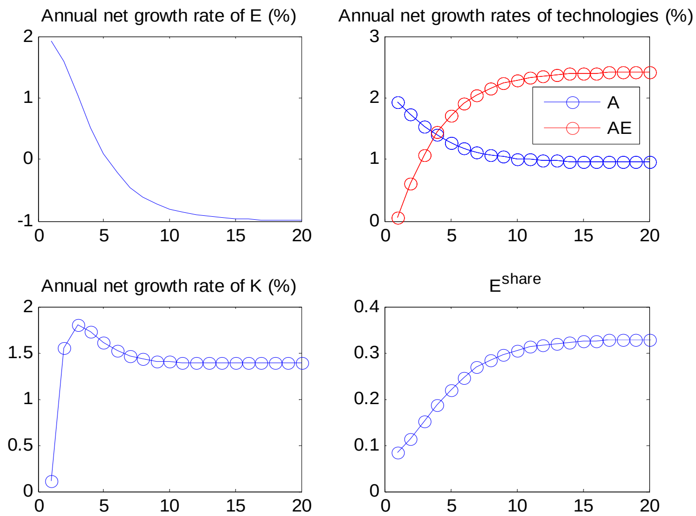

Figure 7. Transition

F. An alternative energy source

Our setting above relies on oil being the only source of energy. Today, oil accounts for a little less than half U.S. energy supply. However, the amount of oil remaining in the ground is rather limited, at least compared to the amount of available coal. We can also expect the emergence of energy-efficient non-fossil alternatives, especially given our estimates above of a very high long-run oil share of output. In this section, we consider an alternative to oil. To simplify the analysis, we abstract from oil entirely. The main purpose of the section is to assess what the long-run value of the energy share will be in this alternative setting.

Our modeling of the alternative energy source—we will refer to it as "coal" but it could equivalently be thought of as a non-fossil resource—is simply to assume that labor used in final production could alternatively be used in the production of an energy source, again denoted E, with at a constant marginal cost per unit of coal. Thus, $E = \xi l$, where l is the amount of labor used in energy production and $1 - l$ is used in the production of the final good. Output is then given as follows:

$$c_t + k_{t+1} = \min \left\{ A_t k_t^{\alpha} (1 - l_t)^{1-\alpha}, A_t^E \xi l_t \right\}.$$

Note the contrast with oil: coal is costly to extract, whereas we considered oil not to be. This is a rough approximation but captures a key aspect of these energy sources: the largest part of the market price for oil is a Hotelling rent rather than a production cost, whereas the reverse is true for coal.[16]

It is straightforward to solve the coal model along the same lines as in Sections IV.B–IV.C. We refer the interested reader to an online appendix for the details. A key property of the model for the present purposes is that the oil share will simply equal $w_t l_t / y_t = (1 - \alpha) \frac{1-l_t}{l_t} / (1 + (1 - \alpha) \frac{1-l_t}{l_t})$.[17] On a balanced path, l_t will converge, and its solution will depend on the parameters of the research technologies. Here, we will assume that the R&D technology for advancing coal-saving is the same as that for oil, so that the tradeoff between coal-saving and capital/labor-saving again can be measured by the downward-sloping line in Figure 6. With the parameter values used in that context, we obtain a long-run energy share of 29%, which is less than for oil but still large. Along with this share, we found the growth of

[16]See Hotelling (1931). Clearly, the correspondence here between the physical classification of fossil fuel into oil and coal and the economic distinction is not perfect. In particular, some sources of non-conventional oil have extraction costs that are large relative to their price. Such oil is, in economic terms, to be considered equivalent to coal.

[17]This can be derived from the firm's problem, which involves maximizing $A_t k_t^{\alpha} l_{1t}^{1-\alpha} - w_t(l_{1t} + l_{2t})$ subject $A_t k_t^{\alpha} l_{1t}^{1-\alpha} = \xi A_t^E l_{2t}$.

A to be 1.3% and that of A^E to be 1.9%, resulting in consumption growth of 1.9%. Clearly, a fuller model of energy supply would include several energy sources simultaneously and gradual phasing out of oil/phasing in of alternatives. However, the long-run features of such a model would likely not differ markedly from those examined in this section.

V. Concluding remarks

In this paper we have estimated an aggregate production function on historical U.S. data in order to shed light on how the economy has dealt with the scarcity of fossil fuel. The evidence we find strongly suggests that the economy directs its efforts at input-saving so as to economize on expensive, or scarce, inputs. We also used this evidence to inform estimates of R&D technologies, allowing us to make some projections into the future regarding energy use and, ultimately, consumption growth. Our analysis is quite stylized but we believe that it captures both qualitatively and quantitatively important aspects of energy resource scarcity.

Our paper calls for several different kinds of elaboration and follow-up. One regards the postwar data and, in particular, a joint explanation of prices and quantities; here, we have treated the movements in the oil/energy price as an exogenous driver of quantities. The different hypotheses of the large price hikes all have an exogenous component to them but one would clearly need to go one step further. Another important simplification in our analysis here is our modeling of energy and different sources of energy. Clearly, it is desirable to add some richness here and, although we guess that a more realistic structure would not drastically change our conclusions here, we definitely cannot rule it out. Our modeling of technical progress is also extremely simple; for example, we do not consider spillovers across the different kinds of factor saving, and we have not grounded our calibration in microeconomic estimates of R&D functions. It would be very valuable to take several more steps in this direction, especially when several energy sources are considered jointly. Finally, our results indicate large long-run energy shares, even for the case of coal/renewables, suggesting that the economy might respond with an increase in the total amount of R&D. It would be valuable to model this channel too.

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