

# Consumption Risk Sharing with Private Information and Limited Enforcement

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## Abstract

We study consumption risk sharing when individual income shocks are persistent and not publicly observable, and individuals can default on contracts at the price of financial autarky. We find that, in contrast to a model where the only friction is limited enforcement, our model has observable implications that are similar to those of an Aiyagari (1994) self-insurance model and therefore broadly consistent with empirical observations. However, some of the implied effects of changes in policy or the economic environment are noticeably different in our model compared to self-insurance.

## 1 Introduction

This paper studies the quantitative implications of a general equilibrium model of consumption risk sharing where there is private information about earnings

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(which are persistent) and enforcement of contracts is limited in the sense that consumers can walk away from a dynamic contract, giving rise to period-by-period participation constraints as well as truth-telling constraints. Our main finding is that the empirically testable implications of our model are similar—though not identical—to those of a Bewley (1977) and Aiyagari (1994) model, and are therefore broadly in line with key features of the data. This contrasts with the implications of models where the only friction is limited enforcement, as in Krueger and Perri (2006). On the other hand, the response to an introduction of a compulsory social insurance program financed by an income tax is similar to the Krueger-Perri model, and in contrast to the Bewley-Aiyagari model.

The spirit of our exercise is quite similar to that of Krueger and Perri (2006) and Krueger and Perri (2011).<sup>1</sup> We explicitly model the frictions that lead to imperfect risk sharing, as opposed to simply assuming that markets are exogenously incomplete. The question is, how do the frictions affect the implied effects of policy interventions or changes in the environment? It seems to us that this question needs to be answered in the context of a model with observable implications which are broadly in line with the facts. In this respect, as documented by Broer (2013), models with only limited enforcement fall short. Specifically, such models imply a much stronger left skew of log consumption than of log earnings, and that consumption responds extremely asymmetrically to earnings increases and decreases; both these features are strongly counterfactual. This feature comes from the fact that, in these models, consumption always drifts down when the participation constraint does not bind, and then jumps up when it does bind, which happens whenever earnings increase sufficiently. This gives rise to an extreme and very counterfactual degree of left skewness in both consumption and log consumption.

In contrast, our model with private information and limited enforcement (PILE model) implies that consumption will gradually drift up when income is high as well as gradually drift down when income is low. This means that our model

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<sup>1</sup>Kinnan (2014) also tests limited commitment and hidden income model using data from Thailand. Unlike us, her hidden income model only allows for i.i.d. income shocks.

avoids the counterfactual implications of limited enforcement (LE) models such as those in Krueger and Perri (2006) or Krueger and Uhlig (2006). Indeed, the observable implications of our model are quite similar, though not identical, to those of the self insurance (SI) models of Bewley (1977) and Aiyagari (1994).<sup>2</sup> In this sense our model provides a more rigorous foundation of this property than SI models do. Though our model is not able to deliver log-normality of consumption, it avoids the counterfactual implication that log consumption is much more skewed to the left than log earnings are.

In order to compare the implications of our PILE model with those of SI and LE models in more detail, we examine the consequences of the following interventions: introduction of a social insurance program financed by income taxes, introduction of a social insurance program financed by consumption taxes, and reduction in idiosyncratic risk. We find that, in response to changes in income taxes, our model is even more pessimistic than Krueger and Perri (2011): public insurance always crowds out private insurance at least by the same amount. This is because truth-telling constraints make public and private insurance perfect substitutes whenever income is unobserved both by the government and by insurance providers. Moreover, because public insurance makes the outside option of financial autarky more attractive, in contrast to private insurance, a rise in income redistribution typically implies a rise in consumption risk by more than fully crowding-out private insurance. On the other hand, we find that the implications of a change in consumption taxes, and of a reduction in idiosyncratic risk, turn out to be very similar to those in an SI model, and consumption volatility in our PILE model decreases. Our model thus does not exhibit the property discussed in Krueger and Perri (2011), where lower income volatility can cause a rise in consumption risk by making the punishment of default—financial autarky—less severe.

To characterize the properties of the equilibrium, we follow the recursive formulation of Thomas and Worrall (1990) and Fernandes and Phelan (2000), but go

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<sup>2</sup>See also Huggett (1993), Huggett (1997), and many others.

several steps further. We show that the dynamic program for solving for the optimal insurance contract can be separated into two subproblems, each solving for the optimal insurance contract conditional on the current shock. The subprograms are completely independent of each other, which simplifies the computation. Moreover, conditional on the current shock, the allocations are independent of the previous period's shock, in contrast to the Fernandes-Phelan recursive formulation. The difference is that we dispense with the notion of the promised utility of the truth-teller and promised utility of the deviator (threat utility), and simply keep track of the promised utility for the low and high types. That turns out to be important, because previous shock is only important to the extent that it identifies the truth-teller and the deviator. Our recursive formulation is in this respect closely related to Doepke and Townsend (2006), who study dynamic incentive contracts in an environment with both moral hazard and hidden information and condition explicitly on the realization of the current shock, and also have policy functions independent of the previous period's shock. We also provide a sharper characterization of the state space (the set of feasible vectors of promised utilities). In the presence of the participation constraints, we provide a simple expression for the bounds of the state space, and provide a sharp characterization of the lower left-hand corner of the feasible set. Moreover, we show that in the absence of limited enforcement constraints, the state space is a convex cone whose bounds we characterize analytically.

In addition to the papers mentioned already, our work is also related to the work of Allen (1985) and Cole and Kocherlakota (2001) who study the efficient allocations in economies similar to ours under the assumption that individuals can privately save. They show that under certain conditions the efficient allocations can be decentralized in a competitive equilibrium with a risk-free bond, and thus provide explicit microfoundations for the Bewley-Aiyagari SI model.<sup>3</sup> We view our work as complementary to theirs. Our results show that hidden savings may be sufficient, but not necessary, to obtain quantitatively similar results in dynamic

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<sup>3</sup>See also Doepke and Townsend (2006), Attanasio and Pavoni (2011), Golosov and Tsyvinski (2007) and Ales and Maziero (2009) for further results.

private information economies and Bewley-Aiyagari economies.

Many recent papers study efficient allocations in related dynamic environments, where the agents have private information about their productivity rather than incomes, as in Mirrlees (1971). For example, Farhi and Werning (2011) and Golosov et al. (2016) consider a dynamic Mirrlees economy where private productivity shocks are persistent. The interpretation of efficient allocations is different from ours, however, in that they are typically interpreted as optimal capital and income taxes, while we focus on private insurance contracts, which may or may not lead to efficient allocations. On the technical side, these papers typically use the first-order approach (see Kapička, 2013 and Pavan et al., 2014). Although potentially useful also in our environment, we do not use the first-order approach in this paper.

The paper is organized as follows. Section 2 lays out the model framework and establishes some important theoretical results. In Section 3, we describe how to characterize the optimal insurance contract recursively in the case where earnings can take only two possible values. Section 4 compares the observable implications of our model to those of other models of consumption risk sharing and to key features of US micro-data. Section 5 considers the effects of changes in the environment or policy. Section 6 concludes.

## 2 The PILE Economy

We analyze an economy with three types of agents: a continuum of households (agents) facing idiosyncratic shocks, competitive insurance providers (principals), and a government. The agents have private information (about their earnings) and contracts are subject to limited enforcement, and we will henceforth call it the PILE (“private information with limited enforcement”) economy.

**Household.** Each household lives forever in discrete time. The households max-

imize the expected utility of consumption

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t U(c_t) \right], \quad (1)$$

where the subjective discount factor satisfies  $0 < \beta < 1$  and where  $U : \mathbb{R}_+ \rightarrow \mathbb{R}_-$  is increasing, differentiable, and is such that  $U(c) < 0$  for all  $c \geq 0$  and  $\lim_{c \rightarrow \infty} U(c) = 0$ .<sup>4</sup> We further restrict the utility function as follows:

**Assumption 1** *The function  $\varphi(c) := -\frac{U''(c)}{U'(c)} \cdot c$  is strictly positive and nonincreasing in  $c$ .*

For instance, this assumption is satisfied for any CRRA or CARA utility function.

Idiosyncratic productivity shocks follow a finite-state Markov chain with state space  $\{y^1, y^2, \dots, y^N\}$  where  $y^1 < y^2 < \dots < y^N$ , with the probability of transiting from state  $i$  to state  $j$  denoted by  $\pi_{ij}$ . We adopt the notation  $\mathcal{N} = \{1, 2, \dots, N\}$  and simplify the analysis by assuming that the average shock equals one. The probability of a sequence of shocks  $h^t = (i_0, i_1, \dots, i_t)$  given an initial shock level  $j$  is denoted by  $\pi^t(h^t|j)$ . The set of possible sequences  $h^t$  is denoted by  $\mathcal{N}^t$ . The endowment shocks are private information of the agents, with the exception of the initial “seed” value  $i_{-1}$ , which is given and known to everyone. While this is certainly a strong assumption, income is not perfectly observable even by the tax authorities: Johns and Slemrod (2010) estimate that individuals underreport their income by 13.7 percent on average, and about 57 percent for non-farm proprietors’ income. The estimates are large for other countries too, see Feldman and Slemrod (2007). Given that private insurance providers have less tools to audit individuals’ incomes than the tax authority, individuals will be able to misreport their incomes even more.

The amount consumed is also private information of the households. We assume, however, that all the goods consumed must be purchased through anonymous

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<sup>4</sup>The assumption that the utility is bounded from above by 0 is a convenient normalization. The results easily extend to utilities bounded from above by a constant, and to utilities unbounded from above.

market transactions. We also assume that the households are not allowed to save. As usual, by Ricardian equivalence, this assumption is identical to the assumption that agents are allowed to have savings, but savings are observable.

Each period, the households report their current shock. The report is observed both by the government and the insurance providers. We denote a history of reports until period  $t - 1$  as  $h^{t-1}$  and the same history followed by a report of income  $y^i$  as  $(h^{t-1}, i)$ .

**Government.** The government taxes income and consumption as follows. It chooses an income tax schedule  $s^y = \{s_t^y(h^t)\}_{t=0}^\infty$ , where  $s_t^y(h^t)$  is the tax that must be paid by an agent with reported history  $h^t$ . In addition, the government imposes a proportional tax on consumption  $s^c = \{s_t^c\}_{t=0}^\infty$ , where  $s_t^c$  is the tax rate in period  $t$ . The government chooses a consumption tax rate independent of individual histories. This is due to the fact that individual consumption is unobservable, but anonymous market transactions can only be taxed at a flat tax rate common to everyone. One can imagine (without explicitly modeling the idea) that agents purchase their consumption goods from many stores, none of which can observe the agents' total purchases, and each of which has many customers. In this environment, in order to collect the consumption tax, the government only needs to observe sales by each store as opposed to individual purchases, meaning that individual consumption remains private information. Throughout the paper we analyze government policies  $s = (s^y, s^c)$  that are exogenously given, constrained only by the government budget constraint, specified below.

**Insurance Providers.** There is a large number of insurance providers, who compete with each other by offering, at time  $t = 0$ , mutually agreeable insurance contracts with households. An insurance contract is a transfer program  $\tau = \{\tau_t(h^t)\}_{t=0}^\infty$ . The agents cannot save on their own, and so the principal's transfer, together with government policies, completely determines the consumption of an agent reporting her shock truthfully:

$$c_t(h^t; \tau_t, s_t) = \frac{y^{h^t} - s_t^y(h^t) + \tau_t(h^t)}{1 + s_t^c}.$$

Let  $\mathbf{c}(\boldsymbol{\tau}, \mathbf{s}) = \{c_t(h^t; \tau_t, s_t)\}_{t=0}^\infty$  is a sequence of the agent's consumption, specified below. Define also  $\mathbf{u}(\boldsymbol{\tau}, \mathbf{s}) = \{u_t(h^t; \tau_t, s_t)\}_{t=0}^\infty$  to be a sequence of period utilities, where  $u_t(h^t; \tau_t, s_t) = U(c_t(h^t; \tau_t, s_t))$ . Henceforth, we will work with period utility rather than period consumption.

Insurance providers evaluate a transfer policy  $\boldsymbol{\tau}$  according to the profit function

$$P_{i-1}(\boldsymbol{\tau}) = - \sum_{t=0}^{\infty} \sum_{h^t \in \mathcal{N}^{t+1}} q_t \tau_t(h^t) \pi^t(h^t | i_{-1}). \quad (2)$$

where  $\mathbf{q} = \{q_t\}_{t=0}^\infty$  are the intertemporal prices of consumption. The households rank transfer policies  $\boldsymbol{\tau}$  according to the lifetime utility function

$$V_{i-1}(\boldsymbol{\tau}, \mathbf{s}) = \sum_{t=0}^{\infty} \sum_{h^t \in \mathcal{N}^{t+1}} \beta^t u_t(h^t; \tau_t, s_t) \pi^t(h^t | i_{-1}).$$

Let  $\boldsymbol{\tau}(h^t) := \{\tau_{t+j}((h^t, \tilde{h}^j))\}_{j=0}^\infty$  be a continuation of the transfer policy after history  $h^t$ , and  $\mathbf{s}(h^t) := \{s_{t+j}((h^t, \tilde{h}^j))\}_{j=0}^\infty$  be a continuation of the government policy after history  $h^t$ . The definition of  $V$  implies that  $V_i(\boldsymbol{\tau}(h^t), \mathbf{s}(h^t))$  is the continuation lifetime utility for an agent who reported  $h^t$  and had a last period shock  $i$ .

Without loss of generality, it is assumed that the transfer policy induces agents to tell the truth. Consider an agent who receives income  $y^i$ , but reports  $y^j$  instead. Her current consumption gain is  $\delta_t^{ij} = (1 + s_t^c)^{-1}(y^i - y^j)$ , and the utility gain is

$$\psi(u, \delta_t^{ij}) = U[\delta_t^{ij} + U^{-1}(u)].$$

The function  $\psi$  is nonpositive, increasing in  $u$  with  $\psi(0, \delta) = 0$ , and differentiable with  $\psi'(0, \delta) = 1$ . For the more relevant case of  $\delta > 0$  it also satisfies  $\psi(-\infty, \delta) = U(\delta)$ ,  $\psi(u, \delta) > u$  if  $u < 0$  and  $\psi'(-\infty, \delta) = 0$  and is, under Assumption 1, strictly convex in  $u$ .

Following Fernandes and Phelan (2000), we impose the temporary incentive constraints, where only one period deviations are permitted. The temporary incen-



tive constraint is

$$\begin{aligned}
& u_t((h^{t-1}, i); \tau_t, s_t) + \beta V_i[\boldsymbol{\tau}((h^{t-1}, i)), \mathbf{s}((h^{t-1}, i))] \\
& \geq \psi[u_t((h^{t-1}, j); \tau_t, s_t) + \delta_t^{ij}] + \beta V_i[\boldsymbol{\tau}((h^{t-1}, j)), \mathbf{s}((h^{t-1}, j))] \\
& \forall i, j \in \mathcal{N}, \forall h^{t-1} \in \mathcal{N}^{t-1}
\end{aligned} \tag{3}$$

While the insurance providers are fully committed to the transfer policy, the agents are free to walk away from the contract at the end of each period.<sup>5</sup> Let  $V_i^{\text{AUT}}(\mathbf{s}(h^t))$  be the expected utility from moving to autarky after history of reports  $h^t$ , given that the current period shock is  $y^i$ . Define  $\mathbf{V}^{\text{AUT}}(\mathbf{s}) = \{V_i^{\text{AUT}}(\mathbf{s}(h^t))\}_{t \geq 0, i \in \mathcal{N}}$ . In defining the value of autarky to depend on government policies we implicitly assume that the government has an advantage over the private insurance providers in that the agents cannot avoid paying their taxes even in autarky.<sup>6</sup> To prevent the agents from walking away, the transfer policy has to satisfy the following limited enforcement constraint:

$$V_i(\boldsymbol{\tau}(h^{t-1}), \mathbf{s}(h^{t-1})) \geq V_i^{\text{AUT}}(\mathbf{s}(h^{t-1})) \quad \forall i \in \mathcal{N}, \forall h^{t-1} \in \mathcal{N}^t, \tag{4}$$

Our benchmark definition of autarky is that neither consumption taxes nor income taxes can be avoided, and so agents in autarky have the expected utility given by

$$V_{i-1}^{\text{AUT}}(\mathbf{s}) = \max_{\{\kappa_j\}_{j \in \mathcal{N}}} \sum_{j \in \mathcal{N}} \left[ U\left(\frac{y^j - s_0^y(\kappa_j)}{1 + s_0^c}\right) + \beta V_j^{\text{AUT}}(\mathbf{s}(\kappa_j)) \right] \pi_{i-1, j}, \tag{5}$$

with  $V_i^{\text{AUT}}(\mathbf{s}(h^{t-1}))$  being the continuation of the value of autarky after a history

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<sup>5</sup>Note that our timing seems, *prima facie*, to depart from the convention of previous studies, such as Krueger and Perri (2011) or Krueger and Perri (2006), that agents default after observing their income but before transfers are paid. This conventional timing assumption is natural in an environment with full information, where the main incentive to default results from the ability to avoid paying transfers in periods of high income. When income is unobserved, however, agents take the joint decision of default and reporting income. This implies agents would optimally never default before transfer payments, as reporting a low income realisation and defaulting after the corresponding transfer payments are received yields strictly higher utility. Our timing assumption imposes this optimal behaviour, thus simplifying the problem.

<sup>6</sup>History dependence enters the value of autarky only through the tax system.

$h^{t-1}$ . Note that the incentive compatibility constraint (3) does not necessarily apply in autarky, and the deviating agents may thus misreport their type to the government. We will also consider an alternative value of autarky  $V^{\text{AUT}}$  equal to minus infinity, which corresponds to a situation where agents cannot default at all. Other alternatives are possible as well.

**Aggregates.** In the aggregate, consumption has to equal available resources in each period, i.e.

$$\sum_{h^t \in \mathcal{N}^{t+1}} [c_t(h^t; \tau_t, s_t) - y^{h^t}] \pi^t(h^t | i_{-1}) = 0 \quad \forall t \geq 0, \quad (6)$$

and the government policy  $s$  must satisfy the following budget constraint:

$$\sum_{t=0}^{\infty} \sum_{h^t \in \mathcal{N}^{t+1}} q_t [s_t^c c_t(h^t; \tau_t, s_t) + s_t^y(h^t)] \pi^t(h^t | i_{-1}) = 0. \quad (7)$$

**Equilibrium.** The timing is as follows. At the beginning of period  $t$ , the agent observes current period earnings, makes a report to the principal, and receives (or pays) the appropriate transfer from (or to) the principal. At the end of the period, before knowing the next period's realization of earnings, the agent may choose to opt out of the contract and remain in autarky as of the next period. For a given tax policy  $s$  and autarky values  $V^{\text{AUT}}(s)$ , the *competitive equilibrium* is given by a transfer policy  $\tau$  and prices  $q$  such that (i)  $\tau$  maximizes the insurance provider's profits  $P_{i_{-1}}(\tau)$  subject to the incentive constraint (3) and the limited enforcement constraint (4) taking  $q$  and  $V^{\text{AUT}}(s)$  as given, (ii) the insurance providers yield zero profits, i.e.  $P_{i_{-1}}(\tau) = 0$ , (iii) the resource constraint (6) holds, and (iv) the government budget constraint (7) holds.

## 2.1 The Effects of Government Policies: Theory

As we shall now show, consumption taxes and income taxes have a very different impact on the equilibrium consumption of the agent. Unlike consumption taxes, the role of income taxes is very limited as they matter only by changing the value of autarky. Conditional on the value of autarky, income taxes do not affect the equilibrium allocation since they will be perfectly offset by the transfers specified by the optimal private insurance contract.

**Proposition 1.** *Suppose that  $\tau$  and  $q$  is an equilibrium given tax policy  $(s^y, s^c)$*

and autarky values  $V^{\text{AUT}}(s^y, s^c)$ . Let  $\tilde{s}^y$  be another income tax policy. Then  $\tilde{\tau} = \tau + \tilde{s}^y - s^y$  and  $q$  is an equilibrium given tax policy  $(\tilde{s}^y, s^c)$  and autarky values  $V^{\text{AUT}}(s^y, s^c)$ .

**Proof.** Suppose that  $\tau$  and  $q$  is a competitive equilibrium given  $(s^y, s^c)$  and  $V^{\text{AUT}}(s^y, s^c)$ . Consider an alternative policy  $(\tilde{s}^y, s^c)$ . Consumption implied by this alternative policy and a transfer policy  $\tilde{\tau}$  is

$$c_t(h^t; \tilde{\tau}_t, (\tilde{s}_t^y, s_t^c)) = \frac{y^{ht} - \tilde{s}_t^y(h^t) + \tilde{\tau}_t(h^t)}{1 + s_t^c} = \frac{y^{ht} - s_t^y(h^t) + \tau_t(h^t)}{1 + s_t^c} = c_t(h^t; \tau_t, (s_t^y, s_t^c)).$$

Thus, the agents rank  $\tau$  under  $s^y$  identically to  $\tilde{\tau}$  under  $\tilde{s}^y$ . In addition, if  $\tau$  satisfies (3) and (4) under  $s^y$  and  $V^{\text{AUT}}(s^y, s^c)$  then  $\tilde{\tau}$  satisfies (3) and (4) under  $\tilde{s}^y$  and  $V^{\text{AUT}}(s^y, s^c)$ . The insurance provider's profits are

$$\begin{aligned} P_{i-1}(\tilde{\tau}) &= - \sum_{t=0}^{\infty} \sum_{N^{t+1}} q_t \tilde{\tau}_t(h^t) \pi^t(h^t | i_{-1}) \\ &= P_{i-1}(\tau) - \sum_{t=0}^{\infty} \sum_{N^{t+1}} q_t \frac{\tilde{s}_t^y(h^t) - s_t^y(h^t)}{1 + s_t^c} \pi^t(h^t | i_{-1}) \\ &= P_{i-1}(\tau), \end{aligned}$$

where the last equality follows from the government budget constraint (7) and equality between  $c(\tau, (s^y, s^c))$  and  $c(\tilde{\tau}, (\tilde{s}^y, s^c))$ . Hence, the insurance providers rank  $\tau$  under  $s^y$  identically to  $\tilde{\tau}$  under  $\tilde{s}^y$ . Moreover, if  $\tau$  yields zero profits under  $s^y$  then  $\tilde{\tau}$  yields zero profits under  $\tilde{s}^y$ . Since  $c = \tilde{c}$ , the resource constraint continues to hold. Hence,  $\tilde{\tau}$  and  $q$  is an equilibrium given  $(\tilde{s}^y, s^c)$  and  $V^{\text{AUT}}(s^y, s^c)$ .

■

The result thus shows that direct effects on consumption are perfectly offset by the equilibrium insurance policy. In this sense, the government is subject to the same private information friction as are the private insurance providers. The income tax affects the economy only indirectly, through changes in the autarky values. The indirect effects will in general be nontrivial. If, however,  $V^{\text{AUT}}$  is minus infinity, then the enforcement constraints are never binding, and the indirect

effects are zero. Equilibrium consumption is then independent of the income tax policy.

To obtain sharper results, we will now specialize the income tax policy to be history independent. That is, we assume that  $s_t^y(h^{t-1}, i)$  is independent of  $h^{t-1}$ . We will say that an income tax policy  $\tilde{s}^y$  is *more redistributive* than an income tax policy  $s^y$  if it imposes a lower minimum tax burden than an alternative tax policy,

$$\min_{i \in \mathcal{N}} \tilde{s}_t^y(i) \leq \min_{i \in \mathcal{N}} s_t^y(i) \quad \forall t \geq 0.$$

For example, suppose that the income tax is affine,  $s_t^y(i) = -\eta_0 + \eta_1 y^i$ . Then an increase in  $\eta_1$  accompanied by a corresponding increase in  $\eta_0$  generates a more redistributive policy. The next proposition shows that history independent income tax policies that are more redistributive cannot increase welfare. To that end, define  $V^*(s^y, s^c)$  as the expected utility in a competitive equilibrium given the tax policy  $(s^y, s^c)$  and values of autarky  $V^{\text{AUT}}(s^y, s^c)$ .

**Proposition 2.** *Suppose that income tax policies  $s^y$  and  $\tilde{s}^y$  are history independent, and  $\tilde{s}^y$  is more redistributive than  $s^y$ . Then  $V^*(\tilde{s}^y, s^c) \leq V^*(s^y, s^c)$ .*

**Proof.** Since  $s^y$  and  $\tilde{s}^y$  are history independent, the agents in autarky maximize utility by minimizing tax liabilities in every state of the world. Let  $c^{\text{AUT}}(\tilde{s}^y, s^c)$  be consumption in autarky under  $(\tilde{s}^y, s^c)$  and  $c^{\text{AUT}}(s^y, s^c)$  be consumption in autarky under  $(s^y, s^c)$ . They are given by

$$c^{\text{AUT}}(h^t; \tilde{s}_t^y, s_t^c) = \max_{i \in \mathcal{N}} \frac{y^{h^t} - \tilde{s}_t^y(i)}{1 + s_t^c} \geq \max_{i \in \mathcal{N}} \frac{y^{h^t} - s_t^y(i)}{1 + s_t^c} = c^{\text{AUT}}(h^t; s_t^y, s_t^c),$$

where the inequality follows from the assumption that  $\tilde{s}^y$  is more redistributive than  $s^y$ . Let  $\hat{V}^*$  be the expected utility in an equilibrium given  $(\tilde{s}^y, s^c)$  and  $V^{\text{AUT}}(s^y, s^c)$ . Then  $V^{\text{AUT}}(\tilde{s}^y, s^c) \geq V^{\text{AUT}}(s^y, s^c)$ , which in turn implies that  $\hat{V}^* \geq V^*(\tilde{s}^y, s^c)$ . Finally, it follows from Proposition 1 that  $\hat{V}^* = V^*(s^y, s^c)$ . Combining,  $V^*(s^y, s^c) \geq V^*(\tilde{s}^y, s^c)$ . ■

The result is an implication of a more general principle: whenever a change in the

income taxes tightens the limited enforcement constraint, it will decrease welfare, because the direct effects are zero by Proposition 1. By the same token, if a change in income taxes neither increases nor decreases the minimum tax liability, it will have no effect on welfare. This result is similar to that obtained by Krueger and Perri (2011) where a more progressive income tax increases the value of autarky, which in turn reduces the extent of private risk sharing. However, Proposition 1 implies that the results in a private information environment differ markedly from the results in Krueger and Perri (2011) in one aspect: Income taxes affect the value of autarky only through the value of the minimum tax liability, while in Krueger and Perri (2011), all the other aspects of a history independent income tax matter as well.

In contrast to the income tax, a consumption tax matters in two different ways. Like the income tax it affects the value of autarky, but it also directly affects the incentive constraint. As seen from the right-hand side of the incentive constraint (3), any hidden earnings are taxed by a consumption tax before being consumed. When an agent claims to have a low endowment but in fact has a high endowment, the consumption tax ensures that the agent is only able to enjoy a fraction of the difference between the low and the high endowment. In this sense, the consumption tax can bypass the information friction and reduces the gains from misreporting income. Indeed, in the limit as the consumption tax goes to infinity, perfect insurance is attained. This is arguably an implausible feature of our framework. However, this implausibility only becomes really severe as the consumption tax becomes very high. The point is this: As transfers and consumption taxes rise beyond any bound, the temptation to engage in black market activity eventually becomes overwhelming, and at that point our model becomes implausible because it rules out black market activity entirely. The purpose of the present work, however, is not to analyze optimal policy, and therefore there is no need to take seriously the implications of extremely high consumption taxes. What we may quite reasonably assume is that there is some finite cost of engaging in black market activities, which is sufficiently high as to be prohibitive for the reasonable levels of the consumption tax that we consider. We will return to the analysis of

consumption taxes in section 5.

### 3 Recursive Formulation with Two Shock Values

For computational purposes it is essential to provide a recursive representation of the optimal insurance contract. In order to provide such a representation, we begin by simplifying the government policies by insisting that both the consumption tax rate  $s^c$  and the income tax are history and time independent. Given Proposition 2, we can confine our attention to a constant lump-sum income tax (or, more plausibly, transfer)  $s^y$ . We also assume that the intertemporal price  $q$  is constant over time. We also restrict our attention to two shock values  $y^1 < y^2$  and write  $\delta = (1 + s^c)^{-1}(y^2 - y^1)$ . We assume that the stochastic process is persistent: the probability of getting a low shock for a previously low type ( $\pi_{11}$ ) is higher than for a previously high type ( $\pi_{21}$ ):

**Assumption 2**  $\pi_{11} \geq \pi_{21}$ .

Define an *allocation rule* by  $(u, w) = \{u_1, u_2, w_1^1, w_1^2, w_2^1, w_2^2\}$ , where  $u_i$  is the current utility of an  $i$ -type agent who truthfully reports her shock,<sup>7</sup> and  $w_j^i$  is the continuation utility of type  $i$  who reports to be of type  $j$ . The temporary incentive constraints require that a truthfully reporting agent's type is utility maximizing:

$$u_2 + \beta w_2^2 \geq \psi(u_1, \delta) + \beta w_1^2 \tag{8a}$$

$$u_1 + \beta w_1^1 \geq \psi(u_2, -\delta) + \beta w_2^1. \tag{8b}$$

We also require that choosing autarky at the end of the current period cannot be optimal either for the truthtelling agents, or for the agents misreporting their

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<sup>7</sup>For each allocation rule the associated transfer rule defined via  $\tau_i = (1 - s^c)^{-1}(U^{-1}(u_i) - y^i - s^y)$ . Given a transfer rule, the transfer function  $\tau$  can be defined recursively. We will also simplify notation by keeping the dependence on  $s^y$  and  $s^c$  implicit from now on.

types:

$$w_j^1 \geq V_1^{\text{AUT}} \quad j = 1, 2 \quad (9a)$$

$$w_j^2 \geq V_2^{\text{AUT}} \quad j = 1, 2, \quad (9b)$$

where the value of autarky (5) is now

$$V_i^{\text{AUT}} = \left[ U \left( \frac{y^1 - s^y}{1 + s^c} \right) + \beta V_1^{\text{AUT}} \right] \pi_{i1} + \left[ U \left( \frac{y^2 - s^y}{1 + s^c} \right) + \beta V_2^{\text{AUT}} \right] \pi_{i2}. \quad (10)$$

In each period, an insurance provider is restricted to deliver a lifetime utility  $v_1$  to a previously low type, and  $v_2$  to a previously high type. The pair  $(v_1, v_2)$  is an element of a state space  $\mathcal{V} \subseteq \mathbb{R}^2$ , which we characterize below. The promise keeping constraints are:

$$v_i = (u_1 + \beta w_1^1) \pi_{i1} + (u_2 + \beta w_2^2) \pi_{i2}, \quad i = 1, 2. \quad (11)$$

Finally, we require that the continuation utility pair belongs to the state space, defined in the next section:

$$(w_i^1, w_i^2) \in \mathcal{V} \quad i = 1, 2. \quad (12)$$

An allocation rule  $(u, w)$  is said to *implement* a promised utility pair  $(v_1, v_2)$  if it satisfies the constraints (8), (11), (9) and (12).

### 3.1 The Set of Implementable Utilities

We now define and characterize the state space, or the set of feasible utilities  $\mathcal{V} \subseteq \mathbb{R}^2$ . It is defined as the set of all promised utility pairs  $v = (v_1, v_2)$  that are implementable by some allocation rule, i.e. that there exists an allocation rule that is incentive compatible, satisfies the promise-keeping constraints, the limited enforcement constraints, and such that its continuation utilities are again



in the set  $\mathcal{V}$ :

$$\mathcal{V} = \{(v_1, v_2) \in \mathbb{R}^2 \mid \exists(u, w) \text{ s.t. (8), (11), (9) and (12) holds}\}.$$

The following proposition shows that  $\mathcal{V}_i$  is convex whenever the utility exhibits decreasing relative risk aversion, and the incentive constraint on the low type is not binding.

**Proposition 3.** *If Assumption 1 holds and the incentive constraint (8b) is slack, then  $\mathcal{V}$  is convex.*

**Proof.** See Appendix A.

A useful way of characterizing the state space  $\mathcal{V}$  is to characterize its lower and upper bounds. Define these bounds by minimizing and maximizing the utility of the high type by keeping the utility of the low type fixed:

$$\begin{aligned} \underline{V}(v_1) &= \min \{v_2 \mid (v_1, v_2) \in \mathcal{V}\}, \\ \bar{V}(v_1) &= \max \{v_2 \mid (v_1, v_2) \in \mathcal{V}\}. \end{aligned}$$

We show that the set  $\mathcal{V}$  is a subset of a pointed cone defined by a 45 degree line and a line with a slope  $\pi_{21}/\pi_{11} < 1$ :

**Proposition 4.** *The upper and lower bounds of the set  $\mathcal{V}$  satisfy*

$$\begin{aligned} \underline{V}(v_1) &\geq v_1 \\ \bar{V}(v_1) &\leq \frac{\pi_{21}}{\pi_{11}}v_1, \end{aligned}$$

*In addition,  $\underline{V}(V_1^{\text{AUT}}) = V_2^{\text{AUT}}$ .*

**Proof.** See Appendix A.

The grey set in Figure 1 is a typical set of feasible utilities  $\mathcal{V}$ . The details of the proof of Proposition 4 are relegated to Appendix A, and we only sketch an outline of the proof here. The intuition behind the lower bound is that the deviator always has the option of pretending to be of the low type. If he does so, he consumes all

the transfers of the lower type. However, since his past endowment is higher and Assumption 2 holds, he can secure himself a lifetime utility greater than the utility of the truth-teller  $v_1$ . Note that if the shocks are i.i.d.,  $\pi_{21}/\pi_{11} = 1$  and the cone shrinks to a line with a slope of 1. That is, the deviator's utility is then always the same as the truth-teller's utility.<sup>8</sup> The upper bound can be obtained by ignoring the incentive compatibility constraint and solving for the resulting relaxed upper bound. Note also that the characterization of the lower bound is independent of the normalization that the utility function is bounded from above by zero, while the characterization of the upper bound is not.<sup>9</sup>

Proposition 4 also shows that the value of the lower bound is the autarkic value of the deviator  $V_2^{\text{AUT}}$ . Remarkably, the result holds even given that the constraint (9b) is not explicitly imposed. Since the lower bound is increasing, it follows that once the constraint (9a) is imposed, it is never optimal to deviate jointly by misreporting in the current period and then choosing autarky and so constraint (9b) can be ignored:

**Proposition 5.** *The constraint (9b) is slack.*

**Proof.** Proposition 4 shows that  $\underline{V}(V_1^{\text{AUT}}) = V_2^{\text{AUT}}(s)$  without the constraint (9b) being imposed. Since  $\underline{V}(v_1)$  increases in  $v_1$ , if  $(w_i^1, w_i^2) \in \mathcal{V}$  then  $w_i^2 \geq \underline{V}(w_i^1) \geq V_2^{\text{AUT}}$ , whenever  $w_i^1 \geq V_1^{\text{AUT}}$ . ■

If the value of autarky is minus infinity, then the set of implementable utilities simplifies further. The lower bound is now a straight line, as illustrated by the black cone in Figure 1:

**Proposition 6.** *If  $V_1^{\text{AUT}} = -\infty$  then  $\underline{V}(v) = v$ .*

**Proof.** Consider an allocation that assigns  $u_1 = u_2 = 0$  and  $w_i^i = \frac{v}{\beta}$ . This allocation is trivially incentive compatible since it is independent of the report. It also delivers  $v_2 = v_1$ . Since the value of autarky is minus infinity, inequalities (9) trivially hold. Hence  $\underline{V}(v) \leq v$ . Given that  $\underline{V}(v) \geq v$  by Proposition 4, it follows

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<sup>8</sup>This is the case analyzed by Thomas and Worrall (1990), whose state space is one-dimensional.

<sup>9</sup>If the utility is bounded from above by a constant, then that constant becomes the apex of the cone.

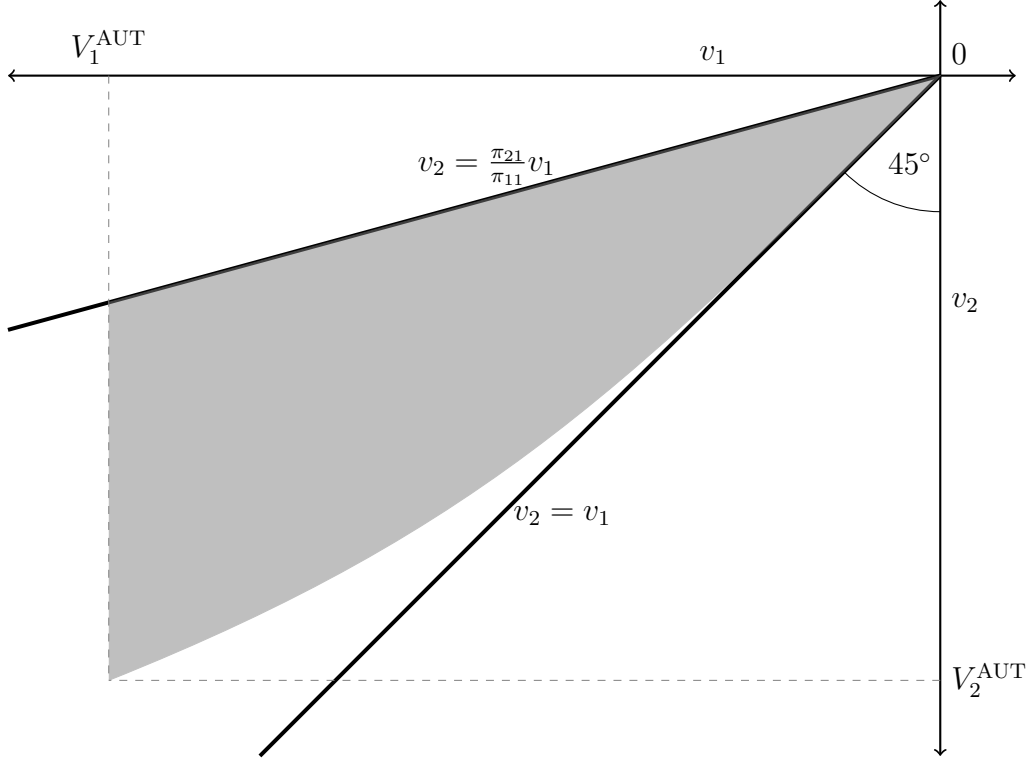


Figure 1: The Set of Feasible  $(v_1, v_2)$  Pairs

that  $\underline{V}(v) = v$ . ■

Finally, the properties of the set of implementable utilities imply the following result that lifetime utilities conditional on the current shock are monotonically increasing in the current shock:

**Proposition 7.** *If  $(u, w)$  implements  $(v_1, v_2) \in \mathcal{V}$  then  $u_2 + \beta w_2^2 > u_1 + \beta w_1^1$ .*

**Proof.** The incentive constraint (8a) implies that

$$u_2 + \beta w_2^2 \geq \psi(u_1, \delta) + \beta w_1^2 \geq \psi(u_1, \delta) + \beta \underline{V}(w_1^1) \geq u_1 + \beta w_1^1,$$

where the second inequality follows from the definition of  $\underline{V}$ , and the third one from Proposition (4) and the properties of  $\psi$ . ■

### 3.2 A Separation Property

We now rearrange the incentive constraints (8) and the promise keeping constraints (11), and show that the problem exhibits the following separation property: Given  $(v_1, v_2) \in \mathcal{V}$ , one can solve for the allocation rule in the current low state independently of the allocation rule in the current high state. The profit maximization problem then separates into two independent ex-post subproblems. In addition, we will show that the optimal allocation rule is independent of the previous state. Both ex-post subproblems have a very symmetric structure, and to highlight it, define  $\hat{u}_2 = \psi(u_2, -\delta)$  to be the period utility of a deviator who receives a high shock, but reports a low shock. The promise keeping constraints (11) constitute a set of linear equations in  $(u, w)$  and one can write them as

$$\mu_1(v_1, v_2) = u_1 + \beta w_1^1 \tag{13a}$$

$$\mu_2(v_1, v_2) = \psi(\hat{u}_2, \delta) + \beta w_2^2, \tag{13b}$$

where  $\mu_1$  and  $\mu_2$  are lifetime utilities conditional on the current shock (ex-post utilities). They are linear in  $v = (v_1, v_2)$ , and are given by  $\mu(v) = \pi^{-1}v$ , with  $\pi^{-1}$  being the inverse of the transition matrix. One can in turn rewrite the incentive constraints (8) as follows:

$$\mu_2(v_1, v_2) \geq \psi(u_1, \delta) + \beta w_1^2 \tag{14a}$$

$$\mu_1(v_1, v_2) \geq \hat{u}_2 + \beta w_2^1. \tag{14b}$$

Note that the constraints (13a) and (14a), as well as the constraints (9) and (12) contain only the allocation rule for the current low state  $(u_1, w_1^1, w_1^2)$ . Similarly, the constraints (13b), (14b), (9) and (12) contain only the allocation rule for the current high state  $(u_2, w_2^1, w_2^2)$ . The objective function of the insurance provider is additively separable in the allocation rule for the low and high state. Thus, one can choose the allocation rule for each state independently of each other.

Furthermore, the fact that the allocations can be chosen conditionally on the current state implies that the previous state is no longer relevant. Hence the maxi-

mization problem can be written independently of the previous state  $i_-$ . Denote the profit function conditional on the current state  $i$  by  $Q_i(v_1, v_2)$ . It satisfies the following Bellman equation:

$$Q_1(v_1, v_2) = \max_{u_1, w_1^1, w_1^2} \{y^1 - U^{-1}(u_1) + qP_1(w_1^1, w_1^2)\} \text{ s.t. (9), (12), (13a), (14a)}$$

$$Q_2(v_1, v_2) = \max_{\hat{u}_2, w_2^1, w_2^2} \{y^2 - U^{-1}(\hat{u}_2) + qP_2(w_2^1, w_2^2)\} \text{ s.t. (9), (12), (13b), (14b),}$$

where  $P_i(v_1, v_2)$  is the expected profit function conditional on the current shock  $i$ :

$$P_i(v_1, v_2) = \pi_{i1} Q_1(v_1, v_2) + \pi_{i2} Q_2(v_1, v_2).$$

Each of the two subproblems are in effect ex-post problems, once the current shock has been realized. Moreover, the subproblems are symmetric: the two constraints (13a), (14a) have the same functional form as (13b), (14b), but differ with respect to whether they hold as weak inequality, or as equality.

The fact that the optimal allocation rule is independent of the previous shock may appear puzzling. In an alternative recursive formulation in Fernandes and Phelan (2000), the optimal allocation rule exhibits dependency on the previous shock. The key "trick" in our reformulation of the problem is that we dispense with the notion of a promised utility of the truth teller and a promised utility of a deviator, and simply keep track of promised utilities for the low and high type. Although this may seem to be merely a notational change, it has one important and nontrivial consequence: the low and high types are independent of the previous period's shock, while the identity of a truth teller and a deviator depends on it: If a previous shock was high then the deviator has low type, while if the previous shock was low, the deviator has high type. More formally, let  $\tilde{u}_i(v, \hat{v}, i_-)$  be the current utility as a function of a truth teller's utility  $v$  and deviator's utility  $\hat{v}$ , as in Fernandes and Phelan (2000). Then the current utility function is symmetric in the sense that  $\tilde{u}_i(v, \hat{v}, 1) = u_i(v, \hat{v})$  and  $\tilde{u}_i(v, \hat{v}, 2) = u_i(\hat{v}, v)$ , for  $i = 1, 2$ . An analogous symmetry applies for the continuation utilities as well.

Our reformulation of the dynamic program is closely related to the recursive for-

mulation in Doepke and Townsend (2006), that conditions explicitly on the ex-post utilities  $\mu_1$  and  $\mu_2$  and delivers policy functions that depends only on  $\mu$  and the current shock. Our formulation thus combines the more conventional Fernandes and Phelan (2000) recursive formulation with some of the benefits and computational simplifications of the Doepke and Townsend (2006) recursive formulation.

### 3.3 General Equilibrium

We now define, for the purpose of approaching the data, a stationary competitive equilibrium. Relative to Section 2 we modify it to allow for aggregate capital accumulation and impose stationarity. We assume that output is produced by competitive firms using an aggregate production function  $Y = AK^\theta N^{1-\theta}$ , where  $K$  is the aggregate capital stock,  $N = \mathbb{E}[y] = 1$  is aggregate labor supply, and  $A$  is total factor productivity. The capital stock depreciates at rate  $\delta_K$ . Capital is owned by the insurance providers who rent it to firms at a competitive price  $q^{-1} + \delta_K - 1$ . This means that consumers in autarky receive no capital income, while consumers outside of autarky receive returns from capital indirectly, through transfer payments from financial intermediaries. Wages, however, including those received in autarky, are determined by the size of the aggregate capital stock.<sup>10</sup>

A stationary competitive equilibrium is characterized by an intertemporal price of consumption  $q$ , wage  $w$  and a distribution of the promised utilities and current shock  $(v_1, v_2, i)$  such that (i) the allocation rule  $(u, w)$  maximizes the profits of the financial intermediary taking prices  $(q, w)$  and autarky values as given, (ii) firms maximize profits, i.e.

$$q^{-1} + \delta_K - 1 = A\theta K^{\theta-1} \tag{15a}$$

$$w = A(1 - \theta)K^\theta, \tag{15b}$$

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<sup>10</sup>This externality means that there is no reason to believe that the competitive equilibrium is constrained efficient. We owe this insight to Abraham and Cárceles-Poveda (2009).

(iii) the resource constraint holds,

$$C + \delta_K K = AK^\theta, \tag{16}$$

where  $C = \mathbb{E}[c]$  is aggregate consumption, (iv) the government budget constraint balances,

$$s^c C + s^y = 0, \tag{17}$$

and (v) the distribution is stationary. Note that, because of free entry, the profits of financial intermediaries are zero in present value terms as of period zero, but they may be nonzero in each period in the stationary equilibrium. Nevertheless, because profits are zero in present value terms as of period zero, it does not matter who owns the insurance companies, and so we do not need to specify that. Note also that the capital stock is owned by the intermediaries—consumers are not allowed to own capital.

## 4 Risk Sharing Properties Compared to US Data and Other Models

In this section, we compare the quantitative implications of our theory to US micro-data, and to the implications of two other popular models of consumption risk sharing: Bewley-Aiyagari (SI) type models where the agents can only self-insure through borrowing and saving<sup>11</sup> and Krueger-Perri (LE) type models where the insurance contracts are limited only by the enforcement constraints.<sup>12</sup>

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<sup>11</sup>See Bewley (1977), Aiyagari (1994) and Huggett (1993).

<sup>12</sup>See Krueger and Perri (2004), Krueger and Perri (2006), and Krueger and Uhlig (2006).

## 4.1 Earnings and Consumption in US Data

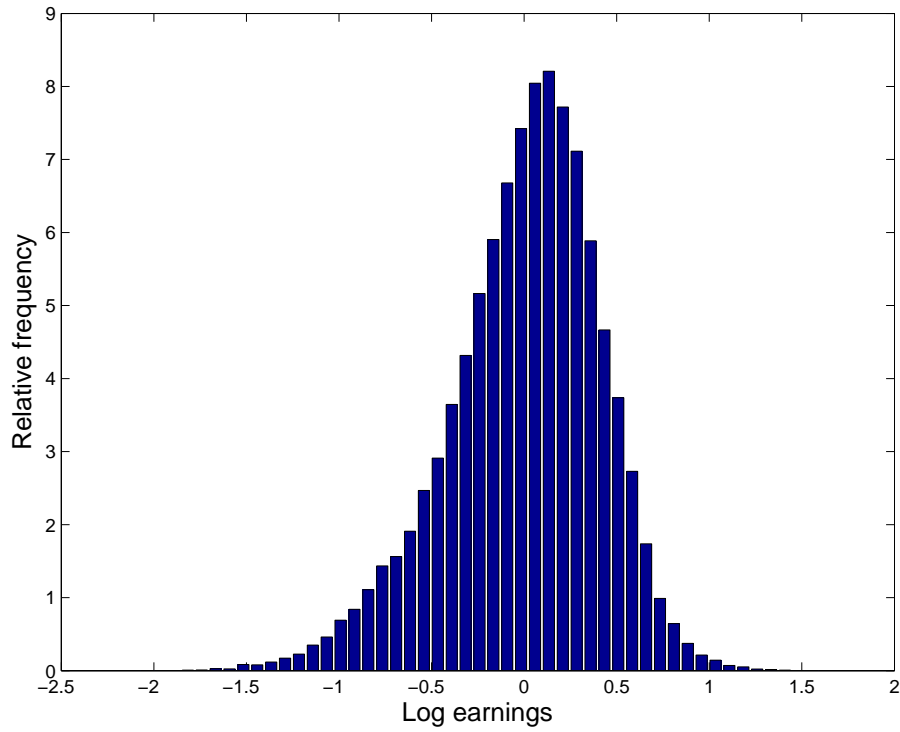
The data sources used for the estimation of the income process and of the key moments of consumption risk-sharing are, respectively, the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX), the main sources of income and consumption data for the United States. Our variable definitions and sample selection closely follow Heathcote et al. (2010).

**Earnings.** The appropriate measure of earnings for our model are household earnings net of taxes and government transfers, but excluding private transfers. Our consumption measure is expenditure on non-durables, and the frequency of measurement is annual. For earnings, we use a PSID sample from 1967 to 2002 that is selected according to the same principles as that of Sample C in Heathcote et al. (2010). This means that we exclude observations with values of earnings that are zero or negative and also that we exclude observations with positive labour earnings but zero hours worked or where earnings and hours are such as to imply an hourly wage less than half the minimum wage for the relevant year. In addition, extreme values were deleted in order to focus on what might be called ordinary households. Specifically, we delete, for each year, the bottom and top 2.5 percentiles.

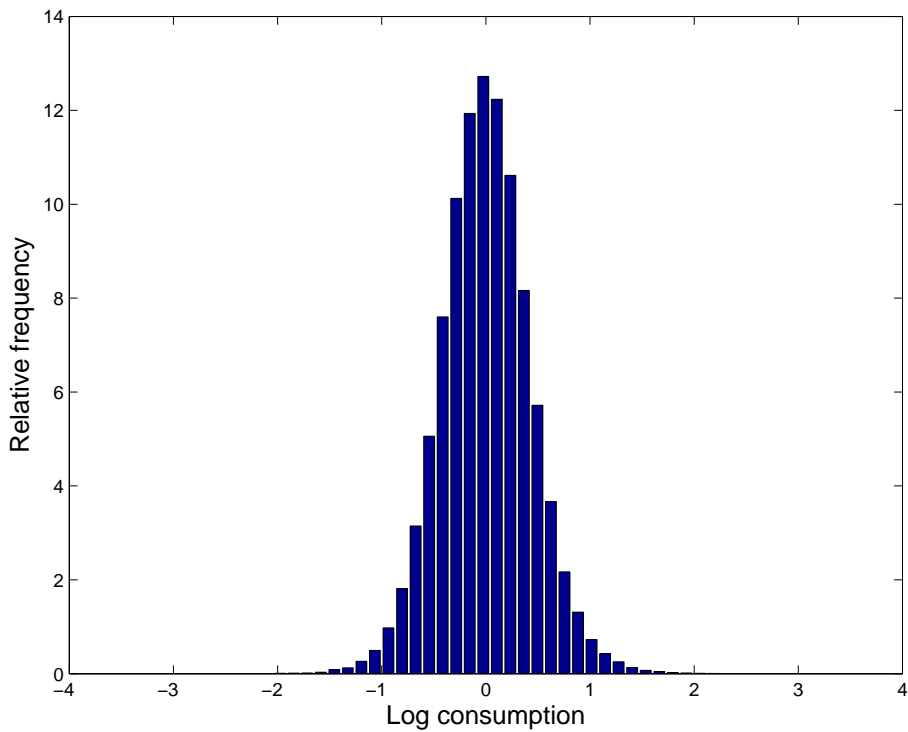
For both consumption and earnings we identify the idiosyncratic component as the residuals from a first-stage regression on a set of observable household characteristics  $Z_{i,t}$  known by household  $i$  at time  $t$ . Specifically, we consider  $Z_{i,t}$  to comprise dummies describing whether the household is a married couple, a single man or a single woman and whether the adult members of the household have more than 12 years of education, time dummies and a polynomial in the age of the head of household. The covariates account for about 40 percent of the total variance of earnings in our PSID sample. A histogram of the residuals from the first-stage regression can be seen in Figure 2.

Our approach to estimating the earnings process is designed to capture the key statistical properties in micro data and to produce an earnings process that is





(a) Histogram for residual log earnings in the PSID 1967-2002



(b) Histogram for residual log consumption in the CEX 1980-2003

Figure 2: The empirical distribution of earnings and consumption residuals

consistent with our theoretical framework. We obtain the earnings process for each household  $i$  by decomposing the residual idiosyncratic component  $\ln y_{i,t}$  of log earnings (whose unconditional mean is zero by construction) into three components,

$$\ln y_{i,t} = \alpha_i + z_{i,t} + x_{i,t},$$

where  $\alpha_i$  is the *permanent* (unchanging) component,  $z_{i,t}$  is the *persistent*, and  $x_{i,t}$  is the *transitory* (i.i.d.) component. We treat the purely transitory component as measurement error and the permanent component as inherently uninsurable. Therefore the relevant component for our risk sharing analysis is the persistent component. It satisfies

$$z_{i,t-1} = \rho z_{i,t-1} + \varepsilon_{i,t},$$

where  $\varepsilon_{i,t}$  is an i.i.d. normal shock. The parameters that characterize the income process are the variances of the shocks  $\sigma_\varepsilon^2$ ,  $\sigma_x^2$  and  $\sigma_\alpha^2$ , the autocorrelation of the persistent shock  $\rho$  and the third central moments of  $\alpha_i$ ,  $x_{i,t}$  and  $\varepsilon_{i,t}$ . Appendix B describes in detail how we estimate these moments from PSID data using a GMM procedure.

Table 1 reports the values of the standard deviation, skewness and autocorrelation of the persistent earnings component  $z$ . Interestingly, the persistent component of earnings has a significant left (negative) skew. The implied variance of persistent shocks  $\varepsilon$  is very similar to that reported in Heathcote et al. (2010) who, however, assume these shocks to have permanent effects, and allow their variance to change over time.<sup>13</sup>

**Consumption.** For our CEX sample that comprises the years 1980 to 2006 we perform a very similar sample selection as in the PSID.<sup>14</sup> Consumption residuals

<sup>13</sup>A defence of the approach used here can be found in Klein and Telyukova (2013). The only difference between the approach used here and theirs is that we estimate skewness parameters in addition to parameters governing second moments. For more on the issues involved in estimating earnings processes, see Manovskii et al. (2015).

<sup>14</sup>Specifically, we drop households where neither head nor spouse are between 25 and 60 years of age, or where either head or spouse receives a wage smaller than half the minimum, and where the head works less than 260 hours per year. We exclude income outliers as in the PSID sample, and households whose responses we deem unreliable because their reported race or sex change, or because they report becoming less educated or age too fast or become younger over time. Since

Table 1: Estimated moments

Persistent earnings component	
$\sigma_z$	0.329
skew $z$	-0.576
$\rho$	0.925
$\sigma_\varepsilon^2$	0.125

in CEX data (displayed in the second panel of Figure 2) have a standard deviation that is only about 17% smaller than that of the CEX earnings measure, as reported in Table 2. A striking feature of the data is that the cross-sectional distribution of (log) consumption is much more symmetric than that of earnings; see Figure 2. In fact, (log, residual) earnings are somewhat skewed to the *left*, with a skewness coefficient (the third central moment divided by the cube of the standard deviation) of about  $-0.531$ <sup>15</sup>, similar to that for the persistent component reported in Table 1. The fact that log consumption is distributed more symmetrically than earnings was first noted by Battistin et al. (2009) for the United Kingdom and is also documented for Canada in Brzozowski et al. (2010).

A common measure of the degree of risk sharing is the regression coefficient of (residual) consumption changes on (residual) earnings changes. As pointed out in Gervais and Klein (2010), the structure of the CEX presents some difficulties in estimating this statistical moment in U.S. data, due to the fact that consumption and income are not measured for the same time periods. Using their approach to estimate this coefficient in a consistent manner on our sample yields a value of 0.22.<sup>16</sup> Using a more straightforward, but invalid, OLS approach, the number is about 0.079. This is evidence of significant risk sharing: consumption does not respond very strongly to earnings changes.

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we found the coefficient in a regression of (residual) consumption changes on (residual) earnings changes to be very sensitive to the inclusion of unstable households, we also exclude households whose composition changes between the four quarterly interviews, for example because couples divorce or children are born.

<sup>15</sup>Source: PSID.

<sup>16</sup>Notice that (i) the Gervais and Klein (2010) approach used here delivers a consistent estimate of the regression coefficient of *quarterly* consumption changes on *quarterly* earnings changes and that (ii) the value that Gervais and Klein (2010) themselves report for this coefficient, based on a broader sample than that used here, is 0.15.

## 4.2 Alternative Models

We now compare the risk sharing properties that arise in the benchmark equilibrium of our PILE model to those from alternative models. First, we investigate a version of the Krueger-Perri economy where agents can write state-contingent contracts but the lack of contract enforcement implies participation constraints that endogenously limit risk sharing. We call this model the LE (“limited enforcement”) model. Second, we investigate a version of the Bewley-Aiyagari economy where agents can smooth consumption only by saving and borrowing using a non-contingent riskless asset. We investigate two versions of the Bewley-Aiyagari economy: one where the agents face a natural borrowing limit, which we call the SIN (“self-insurance with natural borrowing limit”) economy and one where the borrowing limit is the maximum borrowing limit that prevents the agents from defaulting tomorrow as in Zhang (1997), which we call the SILE (“self-insurance with limited enforcement”) model.

In formulating the alternative models, we continue to assume that the government policy  $s$  consists of a time invariant lump-sum income tax (transfer)  $s^y$ , and a consumption tax rate  $s^c$ . The government budget constraint (17) thus continues to hold. We also keep the preferences (1) and the aggregate resource constraint (16) unchanged. Since the recursive formulations are easier in both Krueger-Perri and Bewley-Aiyagari, we write the models for an arbitrary number of shocks  $N$ .

**Krueger-Perri LE model.** To highlight the effect of limited information on the equilibrium allocation in the PILE model, we contrast it to an environment where intermediaries offer insurance contracts under a limited enforcement constraint, but with full information about agents’ income histories. Specifically, consider a version of the benchmark environment in which competitive insurance providers maximize expected profits as defined in Equation (2) by investing in capital and offering insurance contracts to the agents subject to the limited enforcement constraint. For comparability with previous contributions we also adopt the standard timing assumption in LE models, that agents can choose the outside option be-

fore any transfers are made (while in our PILE model the agent can only choose to move to autarky *after* current-period transfers). The value of autarky is

$$V_i^{\text{AUT}} = U\left(\frac{y^i - s^y}{1 + s^c}\right) + \beta \sum_{j \in \mathcal{N}} V_j^{\text{AUT}} \pi_{ij}. \quad (18)$$

The insurance providers and the government have full information about income histories, and can disregard the incentive constraint (3).

A *stationary competitive equilibrium* of the LE environment is characterized by an interest rate  $R = q^{-1}$ , a wage  $w$  and a distribution of promised utilities and the current shock  $(v, i)$  such that (i) the distribution is stationary, (ii) the allocation rule  $(u, w)$  maximizes the profits of the financial intermediary taking prices  $(q, w)$  and autarky values as given, (iii) firms maximize profits, i.e. (15) holds, (iv) the resource constraint (16) holds, and (v) government budget constraint (17) holds.

**Bewley-Aiyagari SIN and SILE models.** Agents maximize utility (2) by choosing non-contingent assets  $b_t$  every period subject to the following budget constraint

$$(1 + s^c)c_i + b'_i = y^i - s^y + Rb,$$

taking as given the interest rate  $R$  and a borrowing limit  $\underline{b}$  such that  $b'_i \geq \underline{b}$ . We study two versions of the economy distinguished by the value of  $\underline{b}$ . First, we look at the SIN (“self-insurance with natural borrowing limit”), where  $\underline{b}$  equals the natural borrowing limit  $\underline{b}^{\text{N}}$ , given by

$$\underline{b}^{\text{N}} = -\frac{y^1}{R - 1}.$$

Second, we look at an economy where  $\underline{b}$  equals the maximum level of borrowing today  $\underline{b}^{\text{LE}}$  such that agents do not prefer to default in any income state tomorrow (Zhang (1997)):

$$\underline{b}^{\text{LE}} = \min\{b \mid V_i(b) \geq V_i^{\text{AUT}} \quad \forall i \in \mathcal{N}\}$$

where  $V_i(b)$  is the expected lifetime utility of an agent with assets  $b$  and income  $y_i$  given tax policy  $s$  and  $V_i^{\text{AUT}}$  is the value of autarky, as defined in (18). We call this model the SILE (“self-insurance with limited enforcement”) model. Note that as in our PILE model, we assume that default excludes agents from any saving or borrowing in the future. Thus agents that default consume their after-tax income forever.

A *stationary competitive equilibrium* in the SIN economy is characterized by a borrowing limit  $\underline{b}^N$ , an interest rate  $R = q^{-1}$ , a wage  $w$  and a distribution of assets and the current shock  $(b, i)$  such that (i) the distribution is stationary, (ii) households maximise their utility (2) taking interest rates and wages  $(q, w)$  as well as borrowing limits  $\underline{b}^N$  as given, (iii) firms maximize profits, i.e. (15) holds, (iv) the resource constraint (16) holds, and (v) government budget constraint (17) holds. A stationary competitive equilibrium in the SILE economy is defined analogously, with  $\underline{b}^{\text{LE}}$  replacing  $\underline{b}^N$ .

### 4.3 Calibration and Benchmark Results

We calibrate the two earnings levels  $y^1$  and  $y^2$  and two transition probabilities  $\pi_{11}$  and  $\pi_{22}$  to match the estimated standard deviation, skewness and autocorrelation of the persistent component of earnings  $z_{i,t}$  (see Table 1), and to normalize the average earnings to one. The procedure yields  $y^1 = 0.614$ ,  $y^2 = 1.218$ ,  $\pi_{11} = 0.952$  and  $\pi_{22} = 0.973$ . In the benchmark economy we set both tax rates  $s^y$  and  $s^c$  to zero. Agents’ period utility function is of the constant relative risk aversion form

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

with  $\sigma = 2$ . In our benchmark calibration to annual data, we choose an interest rate  $R = q^{-1}$  equal to 1.04. We assume that firms operate a Cobb-Douglas technology with capital share parameter  $\theta = 0.36$  and choose a value for the depreciation rate  $\delta$  consistent with a stationary capital-output ratio equal to 3, yielding  $\delta_K = 0.08$ . We choose a total factor productivity parameter  $A$  to normalize wages

$w$  to 1 by setting  $A(1 - \theta)K^\theta = 1$ . We then calibrate the benchmark value for the discount factor  $\beta$  to be such that the resource constraint (16) holds. Our benchmark calibration delivers  $\beta = 0.958$ .

To calibrate the LE, SILE and SIN economies, we keep the benchmark earnings process, the technology parameters  $A$  and  $\theta$ , as well as the coefficient of relative risk aversion  $\sigma$  and the benchmark interest  $R = 1.04$  unchanged. For the discount factor  $\beta$ , we look at two different specifications. In the first, *general equilibrium exercise*, we target the same level of aggregate consumption as in the PILE economy. In other words, we choose the value of the discount factor to clear the resource constraint (16). That is, like in the benchmark calibration of the PILE economy, we are looking for a stationary general equilibrium by choosing discount factors that yield an interest rate of  $R = 1.04$  in the three alternative models. This exercise yields  $\beta = 0.957$  in the SIN economy and  $\beta = 0.953$  in the SILE economy. (Because the interest rate is the same in each case, the capital stock is, too.) In the LE economy, as will be discussed below, this exercise yields full insurance implying  $\beta = R^{-1}$ .

In the second, *partial equilibrium exercise*, we keep the value of  $\beta = 0.958$  from the PILE economy, maintain the interest rate fixed at  $R = 1.04$  and solve for the optimal allocations without requiring the resource constraint (16) to clear. There are two reasons why we also look at this second set of partial equilibrium moments. First, the relative values of interest rate and discount factor have been identified as a key determinant of the degree of risk sharing. Comparing the key features of the models also at identical values of  $\beta$  and  $R$  makes the analysis, in our view, more robust to any misspecification of the asset supply that could affect our calibration of  $\beta$  (e.g. due to the abstraction from any open economy considerations). Second, the LE model predicts perfect insurance in a general equilibrium, implying  $\beta R = 1$ . Looking at a partial equilibrium where  $\beta R < 1$  allows us to also discuss the key moments of the income and consumption distribution in the LE model.

**Risk sharing.** The first two columns of Table 2 show the standard deviation of log consumption relative to disposable income  $\sigma_{\ln c}/\sigma_{\ln y}$  and the regression coefficient of percentage consumption changes on percentage earnings changes  $\gamma$  as derived in Gervais and Klein (2010), and compare their empirical values with values in the four models. The standard deviation of log consumption in the PILE model is about 81.3 percent of the standard deviation of earnings, and the regression coefficient of (annual) consumption changes on (annual) income changes is 0.220. Both values are close to their empirical counterparts.

Table 2: Aggregate Moments

	$\sigma_{\ln c}/\sigma_{\ln y}$	$\gamma$	skew $\ln c$
Data (CEX)	0.826 (0.00460)	0.210 <sup>†</sup> (0.0391)	0.061 (0.0200)
<i>General Equilibrium</i>			
PILE	0.813	0.220	-0.940
LE	N/A	0	N/A
SIN	1.385	0.361	-1.823
SILE	0.873	0.406	-0.904
<i>Partial Equilibrium</i>			
LE	0.094	0.030	-3.680
SIN	1.316	0.308	-1.673
SILE	0.950	0.280	-0.785

$\sigma_{\ln c}/\sigma_{\ln y}$  is the standard deviation of log consumption relative to disposable income. The parameter  $\gamma$  is the regression coefficient of percentage consumption changes on percentage earnings changes. skew  $\ln c$  is skewness of log consumption.

<sup>†</sup>Quarterly coefficient estimated using the method in Gervais and Klein (2010). The more standard, but misspecified, annual coefficient is 0.0693, with a standard deviation of 0.00807.

The LE economy, on the other hand, predicts perfect risk sharing in general equilibrium. This implies a regression coefficient  $\gamma = 0$ , as consumption is independent of income shocks. The standard deviation of consumption is not defined, as the stationary distribution of the model is usually not unique with perfect insurance.<sup>17</sup> The partial equilibrium version of the LE model delivers delivers

<sup>17</sup>Full insurance is a consequence of the assumption that there are only two income states. With a more general income process including purely transitory shocks, as in Krueger and Perri (2006), the model yields very strong but not perfect risk sharing.

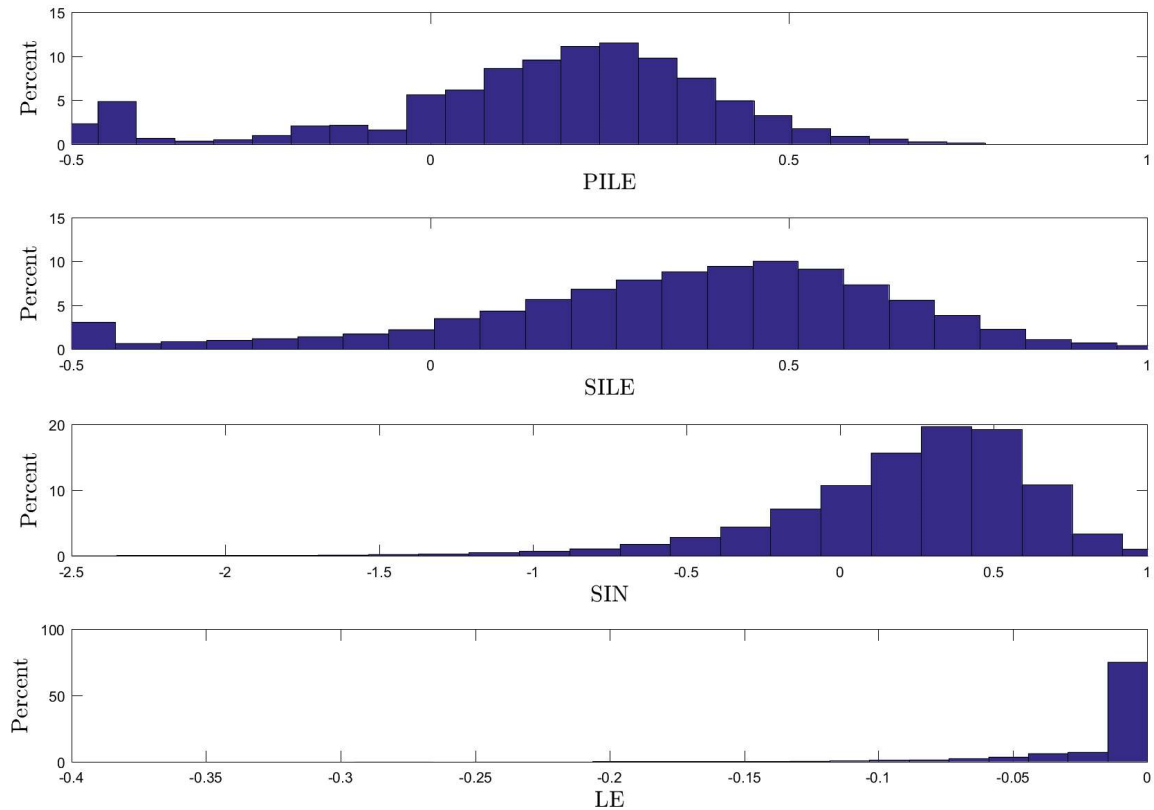


imperfect consumption insurance, but both consumption dispersion and a value of  $\gamma$  are an order of magnitude smaller than in the data or the other models, with relative standard deviation of consumption of only 9.4 percent, and the insurance coefficient  $\gamma = 0.030$ .

For the SIN economy, the standard deviation of log consumption in the stationary distribution in general equilibrium is 1.385, about 25 percent higher than that of earnings, while the regression coefficient of consumption changes on earnings changes is 0.361, more than 1.6 times the value in the data. While the natural borrowing limit in the SIN is more than 10 times annual labour income, no-default borrowing limit in the SILE allows the agents to borrow only up to 15 percent of their annual labour income. As a result, compared to the SIN economy, the SILE economy exhibits a lower discount factor, smaller relative standard deviation of consumption (0.873 vs. 1.385), and an even higher insurance coefficient of 0.406. The partial equilibrium exercise, where we keep the discount factor equal to the calibrated value in our PILE model, has only a small effect on the standard deviation of log consumption but reduces the regression coefficient to a value of around 0.3 in both models. Overall, the results suggest that the risk-sharing properties in the PILE economy are closer to the empirical evidence than the risk-sharing properties in other models, although the SILE economy is not far off as well (especially in partial equilibrium).

A cautionary note is in order. The values of the regression coefficients are not directly comparable to the quarterly regression coefficient that we report in Section 4.1, for two reasons. One is that the empirical figure is based on quarterly data and the model is annual. The other is a bit more subtle. For computational reasons, we model income as a stationary two-state Markov chain. In order estimate that statistical model on the data, we begin by taking out a transitory component, choosing to think of it as measurement error. Therefore, if we take seriously the idea that income has a persistent and a transitory component, and that the transitory component is in fact measurement error, then a fair comparison between model and data should involve adding on a transitory measurement error component to simulated income before estimating the regression coefficient

of consumption changes on income changes.<sup>18</sup>



Histograms of the cross-sectional distribution of log consumption in the PILE, SILE, SIN and LE model in the partial equilibrium with common  $\beta = 0.958$  and  $R = 1.04$ .

Figure 3: Distribution of log consumption across models

**Skewness of Consumption** The last column of Table 2 shows the skewness of log consumption skew  $\ln c$ , another measure that we use to compare all models. The skewness of log consumption in the PILE model is  $-0.940$ , which is somewhat greater in magnitude than the skewness of log earnings, but significantly smaller than in the data (0.065). The other models do no better, especially the LE economy, with an even stronger negative skewness of  $-3.680$ . Consumption skewness in the SILE economy ( $-0.904$ ) is comparable to skewness in the PILE economy,

<sup>18</sup>If we do that, the estimated coefficient is reduced in the PILE economy from 0.22 to 0.03 because of attenuation bias. The reduction for other economies is similar.

while in the SIN economy consumption is much more skewed (-1.823).

Figure 3 presents histograms of the stationary equilibrium distribution of log consumption. The most striking feature is its bimodal character in both PILE and SILE economy: while higher consumption values are distributed in a bell-like pattern, there is bunching in the left tail at the lower bound where participation-constraints of low income individuals are binding. To understand the shape of the stationary distribution in the PILE economy, note that, consumption drifts up when earnings are high, and drifts down when they are low. The downward drift, however, is bounded by the lower earnings level where the participation constraint binds. The upward- and downward drift, together with the approximately constant probability of experiencing a change in income status are behind the bell-like shape of the right-hand side of the stationary consumption distribution. The binding lower limit, at which low-income individuals remain until they experience a high income shock, explains the bunching at the lower bound.<sup>19</sup> Interestingly, the qualitative features of this distribution in the SILE economy are similar to the PILE economy—both are a combination of a roughly bell-shaped upper part with a left tail that has bunching at a lower bound. The natural limit of the SIN model, in contrast, where future consumption equals zero with strictly positive probability, is never binding for any consumer, as this would imply infinite marginal utility due to Inada conditions.

The last panel of Figure 3 shows the main empirical difficulties of the LE economy. A high consumption level when the participation constraint binds for high income individuals together with downward drift in consumption whenever the participation constraint does not bind for the low type implies bunching of all high-income individuals at the upper bound, and a distribution of consumption with finite support and a probability mass function that geometrically declines as consumption declines.

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<sup>19</sup>Note that this bunching is a common feature of limited commitment models with a small number of income states. In models with more income states, one may still have a partial insurance without bunching Broer (2013), which is why we think this feature is due to our results from our stylized income process and not an inherent feature of the model.

**Policy functions: PILE versus SILE.** Since the PILE and SILE economies provide similar results, we investigate the differences in the policy functions in those two models. In Figure 4 we compare the consumption function of the agents in the PILE economy and in the SILE economy. We plot the consumption as a function of the financial intermediary’s costs, and compare it with the consumption functions in the Bewley economy, as a function of the assets.<sup>20</sup> Since the policy functions in our economy are a function of both  $v_1$  and  $v_2$ , we simplify the plots by showing the policy functions for the cost-minimizing value of the deviator’s utility. Specifically, let  $v_i^*(v)$  be the cost minimizing deviator’s promised utility if the previous period’s shock was  $i$  and the truth-teller’s promised utility is  $v$ . That is we plot  $c_i(v, v_1^*(v))$  and  $c_i(v_2^*(v), v)$  against the costs  $P_i(v, v_1^*(v))$  and  $P_i(v_2^*(v), v)$  for  $i = 1, 2$ , and compare it to the consumption function  $c_i(b)$  in the SILE economy. We see that for low and intermediate cost levels the optimal policy functions in the SILE economy are very close to the optimal policy functions in the PILE economy when the previous period’s shock is equal to the current period shock (e.g.  $c_1(b)$  in the SILE economy is close to  $c_1(v, v_1^*(v))$  in the PILE economy). Given high persistency of the shocks, this turns out to be the quantitatively relevant case. The policy functions diverge for higher asset/cost levels, where the policy functions in the PILE economy turn out to be generally flatter than in the SILE economy, reflecting a higher degree of insurance.

## 5 Effects of Interventions

In this section, we consider the implications of two types of intervention: (i) a compression of the distribution of individual earnings, reducing the standard deviation by 10 percent, and (ii) a social insurance policy consisting of a lump-sum transfer payment financed by a proportional consumption tax. We concentrate on consumption taxes, rather than income taxes because, according to Proposition

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<sup>20</sup>Assets in the Bewley economy are comparable to costs in our economy. They both determine the costs of delivering certain lifetime utility in a given market structure (complete markets in our economy, and incomplete markets in the Bewley economy.)

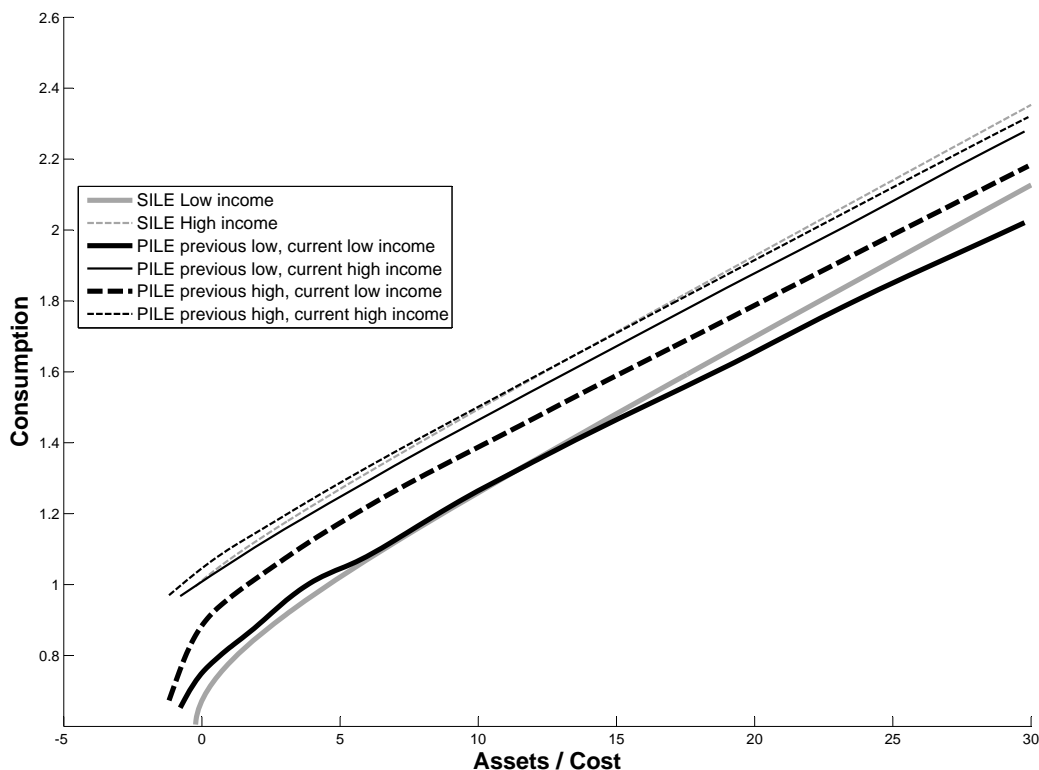


Figure 4: Policy functions in PILE and SILE economies.

2.1, there is very little rationale for income taxes in our PILE economy: a tax reform that increases the degree of redistribution decreases welfare. For each intervention, we compare the effect on risk sharing and also average steady state welfare in general equilibrium. We fix  $\beta$  at the value that ensured market clearing in each model's benchmark version (corresponding to those in the *general equilibrium* exercise in section 4.3), and let the interest rate  $R$  adjust in response to each intervention in order to maintain market clearing in all models. It is worth noticing that the aggregate capital stock, and hence the equilibrium wage, changes as  $R$  is adjusted. In the case of the social insurance policy, we also ensure that the transfer  $s^y$  is such that the government budget constraint balances.

A priori, we would expect both an earnings compression and a lump-sum income transfer financed by consumption taxes to translate into less volatile consumption

and thus to increase average steady state welfare. When risk sharing is limited because of frictions that inhibit information flow or contract enforcement, however, it is crucial to take into account how any intervention interacts with those frictions. The main question we ask here is therefore the following: to what extent does the intervention in question undermine private insurance, in the context of our PILE model as well as other models of risk sharing? Given that the LE economy exhibits perfect insurance in the benchmark case, we only compare the PILE economy with SILE and SIN economies.

## 5.1 Compression of the Earnings Process

We first consider the effects of a 10 percent contraction in the support of labour productivity relative to the benchmark, leaving both mean productivity and transition probabilities unchanged. We focus on the resulting change in both the standard deviation of log consumption, and on a simple measure of steady-state welfare  $\bar{c} = U^{-1}(\mathbb{E} U(c))$ , equal to the constant level of consumption that yields utility equal to the average expected utility across the stationary distribution of agents.

Table 3: Effects of 10 % income compression

	$\Delta\sigma_{\ln c}$	$\Delta\bar{c}$	$\Delta K$
PILE	-6.390	0.645	-0.648
SIN	-3.173	1.472	-1.200
SILE	-7.279	0.832	-1.914

Percentage change in the standard deviation of log consumption  $\Delta\sigma_{\ln c}$ , in the permanent consumption equivalent of average expected utility  $\Delta\bar{c}$ , and in the aggregate capital stock  $\Delta K$  in response to a 10% reduction in the cross-sectional income dispersion.

Table 3 presents the results of the earnings compression exercise. In line with the results in Krueger and Perri (2006), the SILE economy predicts a strong reduction in the dispersion of consumption by 7.279%, about three quarters of the reduction in income dispersion. The corresponding increase in average welfare, however, is only 0.832%, as the reduction in income volatility reduces precautionary savings, and thus (through a rise in the equilibrium interest rate) the capital stock. The

response in the SIN economy differs from that in the SILE model for two reasons. First, higher minimum income loosens the natural borrowing limit, which acts to spread out the stationary consumption distribution, and thus counteracts the effect of the income compression, leading to a decline in the standard deviation of log consumption of only 3.173%, about half as large as in the SILE economy. Second, the reduction in interest rates is smaller in the general equilibrium of the SIN economy. Aggregate capital, and thus output and consumption, therefore falls less than in the SILE economy, implying a larger welfare increase of 1.472%. The PILE model predicts fall in the dispersion of consumption of -6.390%, slightly less than in the SILE economy, and a fall in capital of 0.648%, smaller than in both self-insurance economies. Welfare increases by 0.645%.<sup>21</sup>

## 5.2 Consumption Taxes

We now consider the effects of introducing a proportional consumption tax  $s^c$  whose revenue is used to finance a lump-sum transfer  $s^y$ . We vary the consumption tax from zero (the benchmark economy) to 50 percent. Figures 5 and 6 show how consumption dispersion and average steady state welfare change with respect to their benchmark values ( $s^y = s^c = 0$ ) as consumption taxes increase.

The first striking result is the similarity of the solid and dashed grey lines: changes in consumption dispersion and in average steady state welfare induced by rising consumption taxes are very similar in the PILE and SILE economy. Specifically, consumption dispersion falls approximately linearly in both models, with a reduction in the standard deviation of log-consumption equal to between 70 to 80 percent of that in log after-tax incomes at all tax levels. So crowding out of consumption taxes is small. This implies an also approximately linear increase in average steady state welfare, as Figure 6 shows. In line with the results for the income compression, consumption dispersion falls by less in the SIN economy.

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<sup>21</sup>Since our measure of welfare does not take into account transitional dynamics of the consumption distribution and the aggregate capital stock is reduced in all cases, the computed increase is likely to be a lower bound on the actual welfare increase that includes transition changes.

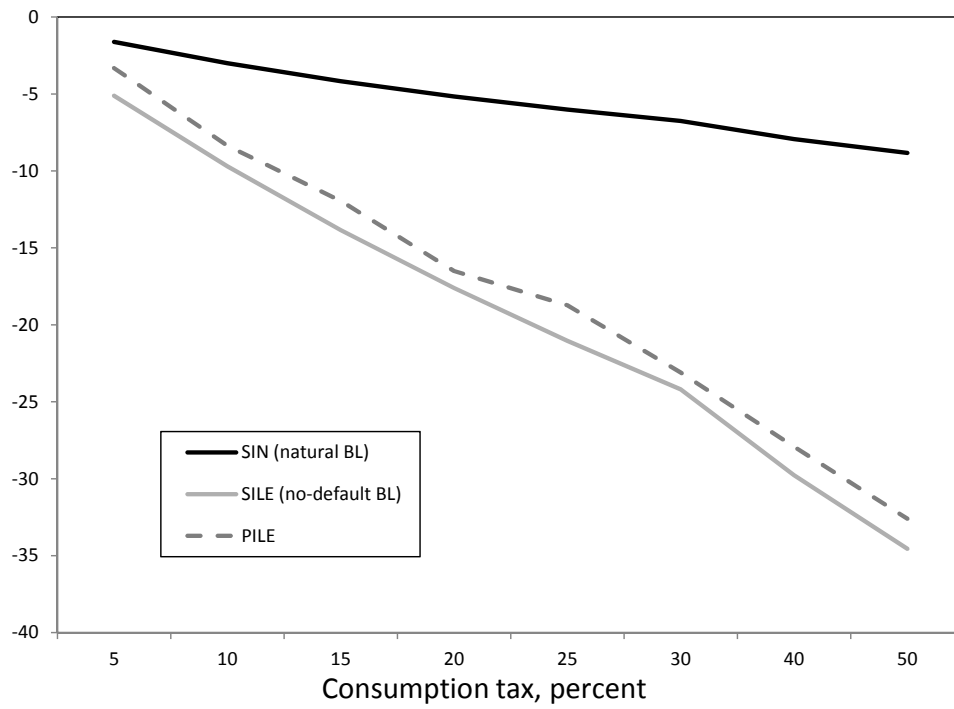


Figure 5: Consumption tax: changes in standard deviation of log-consumption  $\sigma_{\ln c}$ , relative to no tax

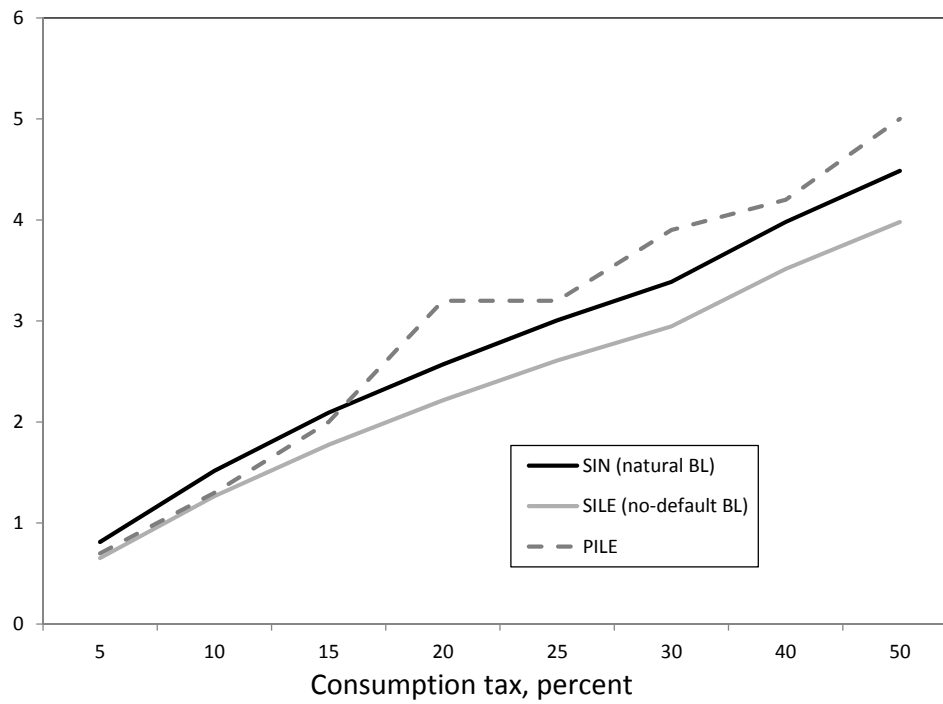


Figure 6: Consumption tax: changes in steady-state welfare  $\bar{c}$ , relative to no tax



However, aggregate capital and output also falls less, leading to an increase in steady-state welfare that is very similar to those in the SILE and PILE economies.

It is useful to also compare our results with the results in Krueger and Perri (2011). In the Krueger-Perri economy both the consumption tax and the income tax affect the equilibrium allocations similarly. In both cases, they only matter to the extent that they affect the limited enforcement constraints. By making the outside option of financial autarky more attractive, and thus tightening the participation constraint, consumption and income taxes can potentially end up increasing consumption risk, crowding out private insurance more than one-for-one. In the PILE economy, the effects of an income tax are very similar to the Krueger-Perri LE economy. But for consumption tax there is an additional channel through which changes in the consumption tax affect equilibrium consumption insurance: they affect the gains from deviations. Higher consumption tax reduces the gains from deviations, and increases the efficiency of the equilibrium insurance contract. This in turn decreases consumption dispersion leading to results that are more in line with the Bewley-Aiyagari SILE economy, rather than with the Krueger-Perri LE economy.

## 6 Concluding Remarks

In this paper, we study the implications of a model of dynamic risk sharing under private information about earnings and limited enforcement of contracts. We find that such a model delivers observable implications that are much more palatable than those of its parent environments. Models of efficient consumption risk sharing with private information alone imply immiserization; models with limited enforcement alone imply that consumption is either constant, drifts down or leaps up. With the two frameworks combined, the implications are much more reasonable; more in line with the data and in fact rather similar to those of Bewley-Aiyagari economies.

Meanwhile, the mechanisms are of course quite different. This led us to inves-

tigate the effects of hypothetical interventions as implied by our model, and to compare them with the implications of other models of dynamic risk sharing in the literature. Our main findings were as follows. Perhaps our starkest result is that any social insurance scheme that relies on income taxes cannot work in our private information and limited enforcement (PILE) model; anything useful that such a scheme accomplishes would be undone by private insurance companies. However, funding through consumption taxes can work very well, provided that the government can observe sales.

Moreover, we find that redistributive consumption taxes and a compression of the after-tax earnings process in our PILE model translate into falls in consumption dispersion and increases in welfare that are large and similar to those in a SILE economy. In other words, while the PILE model is pessimistic about the redistributive potential of income taxes by construction, it predicts only moderate crowding out of private insurance in response to redistributive consumption taxation or a fall in income inequality.

## Appendix A: Proofs

**Proof of Proposition 3.1.** The promise keeping constraint (11), the threat keeping constraint (9) and the left hand side of the incentive compatibility constraint (8a) are all linear in  $u$  and  $w$ . The right-hand side of (8a) is convex in  $u$  and  $w$  when  $\delta > 0$ . The constraint (8b) is slack by assumption. Hence the constraints form a convex set. ■

Before proving Proposition 4, we show several preliminary results. The lower bound can be written as

$$\underline{V}(v_1) = \min_{u,w} \pi_{21}(u_1 + \beta w_1^1) + \pi_{22}(u_2 + \beta w_2^2)$$

subject to

$$v_1 = \pi_{11}(u_1 + \beta w_1^1) + \pi_{12}(u_2 + \beta w_2^2) \quad (11-1)$$

$$u_2 + \beta w_2^2 \geq \psi(u_1, \delta) + \beta w_1^2 \quad (8a)$$

$$u_1 + \beta w_1^1 \geq \psi(u_2, -\delta) + \beta w_2^1 \quad (8b)$$

$$w_i^1 \geq V_1^{\text{AUT}} \quad (9a)$$

$$w_i^2 \geq V_2^{\text{AUT}} \quad (9b)$$

$$w_i^2 \geq \underline{V}(w_i^1) \quad (19)$$

The constraints 11 (for  $i = 1$ ), 8 and 9 are repeated only for easier reading.<sup>22</sup> The constraint (19) is a requirement that the continuation on the contract along the lower bound is not below the lower bound, and is implied by 12.<sup>23</sup>

Standard dynamic programming arguments show that  $\underline{V}$  is continuous and bounded from above by 0 and from below by  $V_2^{\text{AUT}}$ . Other properties of interest are proven in a series of lemmas.

**Lemma 1** *Along the lower bound, the constraint (8a) holds with equality.*

**Proof.** Suppose not. Assume first that  $u_1 < 0$ . Increase  $u_1$  by  $\epsilon_1 > 0$  and decrease  $u_2$  by  $\epsilon_2 = \frac{\pi_{11}}{\pi_{12}}\epsilon_1$ . The constraint (11-1) continues to hold. Since constraint (8a) was slack, it continues to hold for  $\epsilon_1$  small enough. Constraint (8b) is relaxed, while constraints (9a), (9b) and (19) continue to hold. The objective function decreases by  $\pi_{11} \left( \frac{\pi_{22}}{\pi_{12}} - \frac{\pi_{21}}{\pi_{11}} \right) \epsilon_1 > 0$ .

If  $u_1 = 0$  and  $w_1^1 < 0$  then increase  $w_1^1$  by  $\epsilon_1/\beta > 0$ . If (19) holds with equality, increase  $w_1^2$  so that  $w_1^2 \geq \underline{V}(w_1^1)$  continues to hold. Since constraint (8a) was slack,  $w_1^1$  and  $w_1^2$  can be increased for  $\epsilon_1$  small enough. Decrease again  $u_2$  by  $\epsilon_2 = \frac{\pi_{11}}{\pi_{12}}\epsilon_1$ . Then constraints (9a), (9b) and (19) continue to hold, and the objective function

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<sup>22</sup>For simplicity, we set  $s^c = s^y = 0$  to reduce notation. The result does not depend on this assumption.

<sup>23</sup>One should in principle also include a requirement that the continuation on the contract along the lower bound is not above the upper bound. That constraint will not be binding, however, and is omitted.

again decreases.

If both  $u_1 = w_1^1 = 0$  then the right-hand side of (8a) is zero, implying that the left-hand side  $u_2 + \beta w_2^2$  must be zero as well. ■

**Lemma 2**  $w_1^2 = \underline{V}(w_1^1)$ .

**Proof.** Suppose that  $w_1^2 > \underline{V}(w_1^1)$ . Decreasing  $w_1^2$  is then feasible, and relaxes the constraint (8a). Since  $w_1^2$  does not appear in any of the remaining constraints, nor the objective function, decreasing  $w_1^2$  is optimal. ■

#### Proof of Proposition 4.

We first show that  $\underline{V}(v) \geq v$ . Consider a truncated problem where the agents live only for  $T + 1 < \infty$  periods  $t = 0, \dots, T$ . Let  $(u_{i,t}, w_{i,t+1}^j)$  be an allocation of the truncated problem in period  $t = 0, \dots, T$ , with  $w_{i,T+1}^j = 0$  by truncation. Let also  $\underline{V}_t^{(T)}(v)$  be the lower bound of the truncated problem in period  $t = 0, \dots, T$ . The incentive constraint (8a) in the last period  $T$  implies that  $u_{2,T} \geq \psi(u_{1,T}, \delta) \geq u_{1,T}$ . If  $u_{i,T}$  is an allocation along the lower bound then Assumption 2 implies that

$$\underline{V}_T^{(T)}(v) = \pi_{21}u_{1,T} + \pi_{22}u_{2,T} \geq \pi_{11}u_{1,T} + \pi_{12}u_{2,T} = v.$$

Now assume that  $\underline{V}_{t+1}^{(T)}(v) \geq v$  for some  $t \leq T - 1$ . The incentive constraint (8a) in period  $t$  implies

$$u_{2,t} + \beta w_{2,t+1}^2 \geq \psi(u_{1,t}) + \beta w_{1,t+1}^2 \geq \psi(u_{1,t}) + \underline{V}_{t+1}^{(T)}(w_{1,t+1}^1) \geq u_{1,t} + \beta w_{1,t+1}^1.$$

If  $u_{i,t} + \beta w_{i,t+1}^i$  is an allocation along the lower bound then, again by Assumption 2,

$$\begin{aligned} \underline{V}_t^{(T)}(v) &= \pi_{21}(u_{1,t} + \beta w_{1,t+1}^1) + \pi_{22}(u_{2,t} + \beta w_{2,t+1}^2) \\ &\geq \pi_{11}(u_{1,t} + \beta w_{1,t+1}^1) + \pi_{12}(u_{2,t} + \beta w_{2,t+1}^2) = v. \end{aligned}$$

By induction,  $\underline{V}_t^{(T)}(v) \geq v$  for all  $t = 1, \dots, T$ . Since  $T$  was arbitrary we have

$$\underline{V}(v) = \lim_{T \rightarrow \infty} \underline{V}^{(T)}(v) \geq v.$$

We now show that  $\bar{V}(v) \leq \frac{\pi_{21}}{\pi_{11}}v$ . Consider a solution to the problem that ignores the incentive constraints (8a). The principal assigns the lowest possible lifetime utility to a low state since, by Assumption 2, the ratio of probabilities satisfies  $\frac{\pi_{21}}{\pi_{11}} < \frac{\pi_{22}}{\pi_{12}}$ . Thus  $u_1 + \beta w_1^1 = \frac{v}{\pi_{11}}$ ,  $u_2 + \beta w_2^2 = 0$ , and the upper bound is then

$$\bar{V}(v) = \pi_{21} (u_1 + \beta w_1^1) = \frac{\pi_{21}}{\pi_{11}}v.$$

It is easy to verify that the incentive constraint (8a) holds along this upper bound. The incentive constraint (8b) is violated along this upper bound, but adding (8b) can only decrease the upper bound.

To show that  $\underline{V}(V_1^{\text{AUT}}) = V_2^{\text{AUT}}$ , consider the following allocation:  $u_1 = u_2 = 0$  and  $w_1^1 = w_2^2 = \frac{v}{\beta}$ . This allocation is trivially incentive compatible since it is independent of the report. It is the only allocation that delivers  $v_2 = v_1$ , and so it must be on (or below) the lower bound. Since  $w_1^1 = \beta^{-1}V_1^{\text{AUT}} < V_1^{\text{AUT}}$ , it violates (9a) at  $v_1 = V_1^{\text{AUT}}$  and so (9a) must bind, i.e.  $w_1^1 = V_1^{\text{AUT}}$ . By Lemma 1 the incentive constraint (8a) always binds along the lower bound, and by Lemma 2  $w_1^2 = \underline{V}(V_1^{\text{AUT}})$ . The objective function and the promise keeping constraint (11-1) can be written as

$$\begin{aligned} \underline{V}(V_1^{\text{AUT}}) &= \pi_{21}(u_1 + \beta V_1^{\text{AUT}}) + \pi_{22}[\psi(u_1, \delta) + \beta \underline{V}(V_1^{\text{AUT}})] \\ V_1^{\text{AUT}} &= \pi_{11}(u_1 + \beta V_1^{\text{AUT}}) + \pi_{12}[\psi(u_1, \delta) + \beta \underline{V}(V_1^{\text{AUT}})]. \end{aligned}$$

This is a system of two equations in two unknowns  $\underline{V}(V_1^{\text{AUT}})$  and  $u_1$ . Noting that  $\psi(U(y^1), \delta) = U(y^2)$ , the system of equations is solved by  $u_1 = U(y^1)$  and  $\underline{V}(V_1^{\text{AUT}}) = V_2^{\text{AUT}}$ . The allocation is trivially incentive compatible as it involves no transfers. In addition, the constraint (9b) is satisfied automatically since  $w_2^2 = w_1^2 = V_2^{\text{AUT}}$ , and hence does not bind.

■

Table 4: Moments used in estimation

	Sample moments	Smoothed moments	Theoretical moments	Number of obs.
$\Gamma_0$	0.1690	0.1690	0.1686	59550
$\Gamma_1$	0.1197	0.1126	0.1135	37828
$\Gamma_2$	0.1064	0.1055	0.1060	36791
$\Gamma_3$	0.0958	0.0988	0.0990	27202
$\Gamma_4$	0.0909	0.0926	0.0925	27178
$\Gamma_5$	0.0844	0.0868	0.0865	22712
$\Gamma_6$	0.0799	0.0813	0.0810	22074
$\Gamma_7$	0.0745	0.0762	0.0759	19492
$\Gamma_8$	0.0716	0.0715	0.0712	18061
$\Gamma_9$	0.0675	0.0670	0.0668	15972
$\Gamma_{10}$	0.0641	0.0628	0.0628	14861
$\Gamma_{11}$	0.0601	0.0589	0.0591	12106
$\Delta_{0,0}$	-0.0369	-0.0369	-0.0369	59550
$\Delta_{0,1}$	-0.0181	-0.0181	-0.0181	37828
$\Delta_{1,1}$	-0.0167	-0.0167	-0.0167	37828

## Appendix B: Estimating the Earnings Process

The parameters that characterize the income process are the variances of the shocks  $\sigma_\varepsilon^2$ ,  $\sigma_x^2$  and  $\sigma_\alpha^2$ , the autocorrelation of the persistent shock  $\rho$  and the third central moments of  $\alpha_i$ ,  $x_{i,t}$  and  $\varepsilon_{i,t}$ . We estimate these parameters by GMM where the moments are the autocovariances  $\Gamma_k = \mathbb{E}[\tilde{y}_{i,t}\tilde{y}_{i,t+k}]$ , where  $\tilde{y}_{i,t}$  is the logarithm of earnings, the  $k$ th covariance is computed as the average over all possible products  $\tilde{y}_{i,t}\tilde{y}_{i,t+k}$  for which data is available and  $k = 0, 1, \dots, 11$  as well as the following third moments:

$$\begin{aligned}\Delta_{0,0} &:= \mathbb{E}[\tilde{y}_{i,t}^3], \\ \Delta_{0,1} &:= \mathbb{E}[\tilde{y}_{i,t}^2\tilde{y}_{i,t+1}], \\ \Delta_{1,1} &:= \mathbb{E}[y_{i,t}\tilde{y}_{i,t+1}^2].\end{aligned}$$

The autocovariances are geometrically smoothed as described in Table 4.

Our GMM procedure simply involves matching these empirical moments to the

theoretical moments implied by our statistical model. Our parameter estimates are  $\hat{\sigma}_\varepsilon^2 = 0.0157$ ,  $\hat{\sigma}_x^2 = 0.0469$  and  $\hat{\sigma}_\alpha^2 = 0.0132$ , and  $\hat{\rho} = 0.9247$ . These results are very similar to those of Klein and Telyukova (2013), implying that about 64 percent of the total variance of residual log earnings is accounted for by the persistent component. For the CEX, the earnings estimation requires more elaborate techniques, because of the overlapping observations. This is discussed in Gervais and Klein (2010).

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