Collateralized lending and asset prices when investors disagree about risk

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Abstract

Survey respondents disagree strongly about return volatility and, increasingly, macroeconomic uncertainty. This may have contributed to higher asset prices through increased use of collateralized debt products, which allow investors with different risk perceptions to realize perceived gains from trade. Collateralization splits cash flow into senior debt, which investors with low perceived volatility value as riskless, and junior debt or equity claims, whose upside potential is appreciated by those who expect high volatility. This self-selection may have contributed significantly to the boom in structured securitizations as investors disagreed about the volatility of aggregate economic conditions and their importance for default rates in collateral pools. Disagreement about mean payoffs, in contrast, inflates prices without collateralization, which may even discipline prices as risky loans are sold to pessimists with lower collateral valuations.

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1 Introduction

From the mid-1990s to the beginning of the Great Recession, the world economy has seen an unprecedented wave of financial innovation, partly in the form of new collateralized debt products. At the same time, prices of collateral assets, such as real estate, but also stocks, experienced an unprecedented increase. This paper links these two phenomena to a third, less documented one: disagreement among investors about economic risk. We provide evidence for this argument from several US surveys. We first show how the data analyzed by Amromin and Sharpe (2008) and Ben-David et al. (2013) imply strong disagreement among both retail investors and finance professionals about the dispersion of stock returns. Second, to analyze a longer time horizon covering the Great Moderation period, we document that, since 1980, near-term GDP forecasts from the Survey of Professional Forecasters show an increasing disagreement between forecasters about the dispersion of GDP growth, while disagreement about mean growth has fallen.

We show how these heterogeneous risk perceptions, when combined with financial innovation in the form of collateralized debt products, can create asset price bubbles. In the absence of collateralization, risk-neutral investors trade assets at their common fundamental value even if they disagree about payoff risk. The introduction of collateralized debt increases asset prices above this common fundamental value by unleashing perceived gains from trade. This is because an investor who believes asset returns to be more dispersed than another perceives more upside potential at the same time as more downside risk than her counterpart. Collateralization allows them to trade those risks by splitting cash flow into senior debt and junior debt or equity claims. Investors who perceive low volatility are happy to pay high prices for senior debt, which they regard as riskless. Those who think volatility is high, in contrast, value the upside potential in junior claims, which they leverage by selling cheap debt to their counterparts. Disagreement about risk thus raises the equilibrium price of collateral assets as investors self-select into buying the claims they value most highly. We show how this may have been an important driver of the boom in 'Structured Finance' assets, such as collateral debt obligations (CDOs), whose senior tranches are attractive to investors who believe in diversification and thus think default rates of collateral pools are stable. Those, in contrast, who think default rates are more reflective of aggregate conditions, and thus more volatile, think senior tranches may still fail in bad times, but are happy to pay for junior and equity tranches, which they expect to pay when conditions are sufficiently good.

Importantly, this effect of disagreement about risk on prices is different to that with disagreement about means. Our analysis is thus complementary to the large previous literature on investor disagreement, where 'optimists' expect higher payoffs than 'pessimists'.[1] In the absence of short-selling, prices are thus determined by optimists and can exceed mean-valuations even without collateralisation (Miller, 1977). Leverage through riskless collateralized loans may raise prices further by increasing investment funds of optimists (Geanakoplos, 2003). When collateralized debt is risky, for example due to low minimum asset payoffs, and beliefs satisfy the common assumption of first-order stochastic dominance, optimists perceive both senior debt to bear less risk and junior debt (or residual claims from leveraged asset holdings) to be more profitable. They thus face a trade-off between raising funds for investments and selling downside risk at unfavorable prices to pessimists. Only if optimism is about upside risk do pessimists have a relative, but not absolute, advantage in buying collateralized debt. When optimism is about downside risk, in contrast, collateralized contracts can discipline asset prices (Simsek, 2013). Generally, equilibrium prices do not exceed the maximum asset valuation across investors who disagree about mean payoffs even with collateralized debt products (although they may with collateralized Arrow securities (Fostel and Geanakoplos, 2012)).[2]

With disagreement about payoff dispersion, in contrast, we show how the effect of collateralization is fundamentally different. First, in the absence of collateralized contracts, such as in Miller (1977)'s original setup, asset prices equal their fundamental value that all investors agree on. In other words, a departure of prices from their fundamental requires financial in-

[1] See (Xiong, 2013) for a survey of the literature on disagreement.

[2] Harrison and Kreps (1978) show how in a dynamic framework equilibrium prices may exceed an asset's present discounted expected cash flow, but not the expectation of future returns, when current investors speculate on higher prices that pertain when optimists enter the market in the future.

novation, e.g. in the form of collateralized debt. Second, collateralization allows investors to realize perceived gains from trade only when debt is risky. Then, by channeling upside and downside risk to those that value them most highly, it can raise asset prices above the maximum valuation across investors. This implies, third, that there is no trade-off, and no disciplining effect of collateralization: issuing risky collateralized debt claims realizes pure perceived gains from trade. Finally, an increase in disagreement makes collateralized loans and leveraged assets more valuable to those that hold them, and thus always raises asset prices, while the opposite may be true with disagreement about mean payoffs.[3]

Our results partly build on the well-known insight (Rothschild and Stiglitz, 1970; Stiglitz and Weiss, 1981) that a rise in dispersion increases expected profits when the latter are a convex function of fundamentals. We point out that, when risky collateralized debt issuance splits payoffs into a convex (equity) and a concave (debt) payoff function, self-selection raises equilibrium asset prices when risk-neutral investors disagree about risk. The proof of this result, and its comparative statics extension, are, however, made difficult by the endogeneity of the payoff functions, determined by investor choices on debt issuance. Collateralization is, effectively, a substitute for trade in simple options whenever these are not available or not used. Our benchmark results therefore apply most immediately to collateral assets that are not usually referenced by options, such as a household's or company's real estate, the profits of private companies, individual stocks of smaller enterprises, etc. We show, however, in an extension of our results that even with trade in (cash-collateralized) call and put options disagreement continues to imply a premium in collateral asset prices.

The first contribution of this paper is to document, in Section 2, the (increasing) disagreement about the dispersion of asset returns and GDP growth in US surveys. The second contribution is to point out how this raises equilibrium asset prices when investors trade risky collateralized debt products. For simplicity, our benchmark theoretical results in Section 3 are

[3] Note that the effect we point out is also different to Phelan (2015), where, in the absence of disagreement, the general equilibrium response of collateralized debt contracts to an increase in risk may raise asset prices.

derived in a simple two-period environment with risk-neutral investors who trade only debt and an exogenous collateral asset. We show, however, that the main result, that disagreement about risk raises the price of collateral assets, continues to hold in richer environments with several debt 'tranches' (in Section 4), with risk aversion (in Section 5.1), and with trade in (cash-collateralized) options (in Section 5.2). Contrary to the benchmark analysis, however, where the systemic nature of risk is of little importance, the quantitative effect of disagreement on asset prices with risk aversion is smaller when investors disagree about more aggregate risk, associated to assets whose supply is large relative to total consumption.

Our third contribution is to quantify the effect of investor disagreement about risk on the prices of 'Structured Finance' assets, which were blamed for their role in the US housing boom and the financial crisis that followed after their issuance had experienced a spectacular rise in the early 2000s. Specifically, Section 4 studies a model calibrated to capture the main features of US subprime residential mortgage-backed securities (RMBSs) and RMBS-backed collateralized debt obligations (CDOs). It shows how modest disagreement about the variability of default rates, due to diverging views about the importance of aggregate risk in determining defaults, can raise the market value of structured loan pools significantly above the expectation of collateral cash flow (that we assume is shared by all investors). This 'return-to-tranching' is with between 50 and 110 basis points sizeable for RMBSs, but an order of magnitude larger for RMBS-backed CDOs, whose payoff distributions are not bounded below by a minimum recovery value and thus more sensitive to changing perceptions of risk. Disagreement about risk may thus be one factor behind the boom in Structured Finance in the early 2000s. More specifically, our theory provides an additional reason both for strong housing demand (by households who perceive a high upside potential to the housing market) and increasing supply of mortgage finance as financial liberalization draws a larger and more diverse pool of (international) investors into a growing market of non-agency mortgage securitizations.

Although our theory does not predict the specific timing of the boom and bust in Structured Finance in particular, nor of collateralised lending more generally, it suggests that three developments made such a boom more likely from the late 1990s onwards: first, the advent of a large pool of high-risk collateral accessible to a wide variety of investors in the form of US subprime or Alt-A mortgages, whose growth has been attributed to the technological innovation in underwriting procedures and, more controversially, affordable housing policies of the 1990s[4]; second, the increase in disagreement about macroeconomic risks during the 1990s, as suggested by the evidence from the SPF presented in Section 2; and third, the low interest rate environment that started in the early 2000s, which may have made the perceived return differences pointed out by our theory relatively more important.

Structured Finance is not the only asset class where our theory may be important. For example, our results have implications for the theory of firm financing: contrary to Modigliani and Miller (1958), they call for a mix of debt and equity finance that depends on the heterogeneity of risk perceptions in the investor pool. Specifically, firms optimally issue debt to investors who perceive risk to be low, and sell equity to those who perceive higher risk and thus stronger upside potential to shares in the firm.

2 Data: Disagreement about risk in US surveys

This section shows evidence from US surveys that documents the extent to which investors, or forecasters, disagree about risk, or the dispersion of outcomes around their expectations. For this we use three data sources: first, the forecasts for S&P 500 returns by a sample of Chief Financial Officers (CFOs) reported in Ben-David et al. (2013). Second, supplementary questions to the Michigan Survey of Consumer Attitudes that, between 2001 and 2005, asked stock market investors for the stock market returns they expect on average and the uncertainty around them in the medium and long-run. And third, a longer history of GDP forecasts elicited in the Survey

[4]Gorton (2009) argues that technological innovation was the main driver of the rise in subprime mortgages. For a discussion of the resulting 'automated underwriting' of mortgages, see also Gates et al. (2002). The role of affordable housing policy in the subprime boom, in contrast, is controversial. On both points see e.g. the Financial Crisis Inquiry Commission's Report (Financial Crisis Inquiry Commission, 2011), p. 68-80. The FCIC concludes that the Community Reinvestment Act did not play a large role in the subprime crisis. For a contrasting view see (Financial Crisis Inquiry Commission, 2011), p. 219.

of Professional Forecasters (SPF) that contains a finer, fully specified histogram of near-term GDP growth, which is interesting as one of the main macroeconomic determinants of investment returns, if not a perfect predictor.

All three surveys describe perceived aggregate, or market, risks. While our benchmark theory in Section 3 applies to disagreement about aggregate risk as much as to idiosyncratic, or asset-specific, risks, survey data are typically only available for perceptions of market indices or measures of aggregate output. The extension to risk aversion in Section 5.1 shows how disagreement about idiosyncratic risks may have a larger impact on prices than disagreement about aggregate risks.

2.1 Disagreement about US stock market returns

This section uses information from two US surveys to show how investors strongly disagree not only about expected returns, but also about return risks. Table 2 reports summary statistics of the supplementary questions in the Michigan Survey of Consumer Sentiments, covering 22 surveys in the years 2000 to 2005, taken from Amromin and Sharpe (2008).[5]

The first row of Table 2 shows that expected annual returns, averaged across respondents and surveys, equal 9 percent, which coincides almost exactly with the average 10 year annual returns on the S&P total returns index in the period before the last survey in 2005. Disagreement about future mean returns, however, is strong, with 10 percent of respondents expecting an average return of or below 5, and another 10 percent expecting above 16 percent. The perceived riskiness of stock investments, however, also varies strongly across investors: while 10 percent of respondents believe realized returns to fall within 2 percentage points of their expectation with a probability of at least 80 percent, another 10 percent expect returns to fall <u>outside</u> this range with at least 80 percent probability. Using a normality assumption to transform these assessments into standard deviations, the 90-10 percentile difference of standard deviations equals 6.3, compared

[5]The authors eliminate incomplete responses, those deemed by the interviewer to have a low level of understanding or a poor attitude towards the survey, and those that answered "50 percent" to all probability questions.

to 11 for expected returns.

Table 2: Return Expectations in the Michigan Survey 2000-2005

	N	Mean	10th pct	25th pct	Median	75th pct	90th pct
Expected return R_e	3,046	10.4	5	7	10	12	16
Prob $\|R - R_e\| < 2pp$	3,015	43.3	20	25	50	50	80
Implied σ_{10-20} (in percent)	2,854	4.56	1.56	1.73	2.96	2.96	7.88

The first row reports the distribution of investors' answer to the question about the "annual rate of return that you would expect a broadly diversified portfolio of U.S. stocks to earn, on average". The second row reports the probability "that the average return over the next 10 to 20 years will be within two percentage points of your guess", and the third one shows the corresponding standard deviation assuming normally distributed beliefs about stock market returns.

Ben-David et al. (2013) present similar survey evidence for a sample of senior finance executives, mainly Chief Financial Officers. They show how their respondents' forecasts of US S&P 500 returns are 'miscalibrated', in the sense that respondents underestimate the uncertainty around their expected returns both relative to history and relative to subsequent outcomes. Interestingly for the present study, they also show how respondents strongly disagree in their individual volatility estimates. For example, the individual standard deviations of 1-year-return forecasts implied by their survey responses have a distribution whose 95-5 percentile difference equals 15 percentage points (both for the whole sample from 2001 to 2011, and the 2011Q1 cross section). Interestingly, the disagreement about expected returns is similar to that about expected volatility both in terms of the standard deviation across respondents (which equals 5.3 for expected returns and 4.3 for standard deviations) and the 95-5 percentile difference (equal to 15 percentage points also for means).

2.2 Disagreement about US Macro Risk 1980-2010

The Survey of Professional Forecasters SPF is a quarterly survey that asks forecasters to indicate, among other measures, their probability distribution for GDP growth in the current calendar year.[6] Specifically, forecasters report the probability that short-term growth falls in any of 6

[6] Since 1992, the survey also asks for the same distribution for the following year. We don't use this measure because of the short history.

brackets.[7] This allows us to study the evolution of disagreement between forecasters about short-term US growth prospects. Particularly, using a normal approximation of the distributions, as in Giordani and Söderlind (2003) we can look at the distribution across forecasters of forecaster-specific means μ_{it} and standard deviations σ_{it} for every quarter since 1980 (when the survey changed from nominal to real GDP projections). Based on this cross-sectional distribution, we look at two measures of disagreement about the mean and volatility of output growth across forecasters: first, the standard deviations of $\widehat{\mu_{it}}, \widehat{\sigma_{it}}$ defined as[8]

$$\begin{aligned} \mu_{it} &= \widehat{\mu_{it}} + \mu_t \\ \sigma_{it} &= \widehat{\sigma_{it}}\sigma_t. \end{aligned} \tag{1}$$

The second disagreement measure is based on the integral of absolute differences of any two forecaster-specific normal densities, averaged across forecasters.

$$d = \frac{1}{N_t^2} \sum_i \sum_j \int |f_i(g_y) - f_j(g_y)| dg_y, \tag{2}$$

where N_t is the time-varying number of forecasters in the sample.[9] We calculate the contribution of the heterogeneity in standard deviations to this average disagreement using the formula in (2) with the mean of the two normal distributions held constant ($\mu_{it} = \mu_{jt}$), and define the remaining difference with overall disagreement as the contribution of heterogeneous means.

Figure 2.2 shows how the dispersion of means and standard deviations of short-term growth forecasts has evolved over time in the survey. In the early 1980s, the standard deviations of means (in the left panel) was about twice that of standard deviations (in the right panel). But while mean forecasts converged - with their standard deviation falling to less than half their initial value before rising abruptly at the beginning of the recent 'Great Recession' - the

[7] The brackets have changed slightly in 1990.

[8] For the positive variable σ_{it} we use the normalized standard deviation to prevent it from falling to zero mechanically as the mean of σ_{it} falls.

[9] This measure equals zero for any two identical distributions and is bounded above by 2 (for two disjoint distributions).

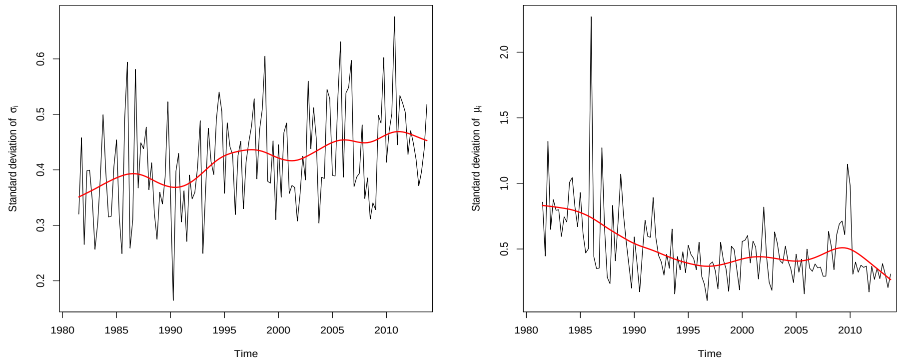


Figure 1: The left panel plots the time series of the standard deviations of $\widehat{\sigma_{it}}$. The right panel plots the corresponding measure for $\widehat{\mu_{it}}$ and as defined in equation (1). The red line shows the trend from an HP filter with smoothing parameter 1600.

dispersion of forecast standard deviations has increased strongly, amid noticeable cyclical swings. Figure 2.2 shows the contributions to the overall disagreement measure d of heterogeneity in forecaster-specific means (in the left panel) and standard deviations (in the right panel).[10] While overall disagreement (not shown) does not follow any trend over the sample, the (smoothed) contribution of heterogeneous standard deviations increases by about 1/3 until the beginning of the Great Recession. The contribution of mean growth dispersion, of about the same magnitude at the beginning of the sample, falls by about 1/3 until the recession. Therefore the evidence from the SPF suggests that the contribution of heterogeneous perceptions of growth dispersion has risen strongly since the early 1980s, while disagreement about mean growth has become less important.

Both the evidence from the Michigan Survey and the SPF thus suggest that there is strong belief heterogeneity about the riskiness of stock market returns among US investors and about macroeconomic risk among professional forecasters. Finally, given that there is presumably little private information about future stock returns or GDP growth, we believe that the evidence above reflects indeed agree-to-disagree type differences as opposed to informational differences.

[10] We only use the first quarter of every year to keep the forecast horizon constant and equal to the remainder of the current year.

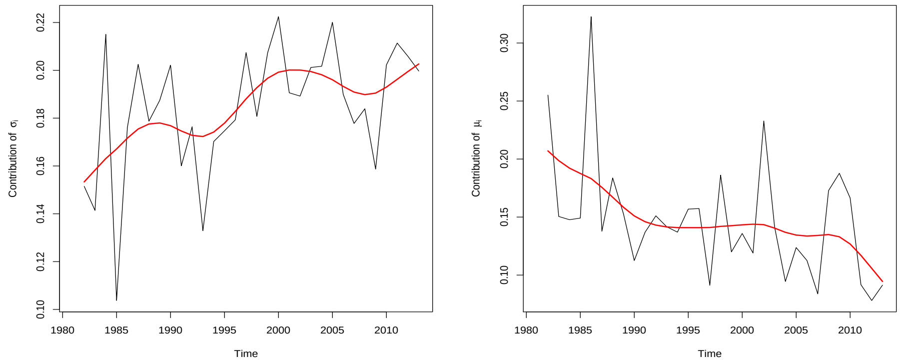


Figure 2: The left panel plots the contribution of heterogeneous standard deviations to overall disagreement d about current year GDP growth in the SPF as defined in equation (2). The right panel plots the corresponding contribution of heterogenous means. The red line shows the trend from an HP filter with smoothing parameter 25 (to adjust for the annual frequency, see Ravn and Uhlig (2002).

3 Theory: Leveraged asset trade with disagreement about risk

This section studies a simple equilibrium economy to show how disagreement about payoff risk implies a 'bubble' in asset prices, defined as a situation where equilibrium asset prices exceed the fundamental valuation (that is shared by all investors). We also show how a further increase in disagreement inflates the bubble. To make the economic mechanism most transparent, we choose a particularly simple environment with two investor types that differ in their risk perceptions and trade only simple collateralized debt. Later sections consider additional and more complex assets. A previous working paper version of this article (Broer and Kero, 2014) presents results for the general case with a continuum of types.

3.1 The general environment

3.1.1 Preferences and beliefs

We study an economy that exists for two periods $t \in \{0, 1\}$. There are two types of agents $i = H, L$, both of unit mass. In period 0, agents of type i receive an endowment $n_i > 0$ of the unique perishable consumption good and 1 unit of a risky asset (a "tree") that pays a stochastic amount $s \in S = [s_{min}, s_{max}]$, $s_{min} > 0$ in period 1. All agents are assumed to be risk-neutral, maximizing the present discounted sum of expected consumption in both periods equal to $U_i = c_i + \frac{1}{R} E_i(c_i')$, where E_i is the mathematical expectation of agent i, c_i (resp. c_i') denotes consumption in period 0 (resp. 1) and $\frac{1}{R} \leq 1$ is the discount factor.

We assume that types differ in their beliefs about the distribution of random payoffs s, summarized by distribution functions $f_i : S \longrightarrow R^+$. Specifically, all agents expect payoffs to be the same on average, but the 'high-risk' type H believes them to be less tightly distributed than the 'low-risk' type L. Specifically, for $\succ^2$ denoting second order stochastic dominance, we assume

Assumption A1 $E_H(s) = E_L(s) \equiv E_s, \ f_L \succ^2 f_H,$

3.1.2 Asset markets

Agents trade in 2 asset markets. In $t = 0$, agent i purchases $a_i - \overline{a}_i$ units of the physical asset in exchange of $p(a_i - \overline{a}_i)$ units of the consumption good. In addition, agents can borrow by pledging part of their future income. However, agents cannot commit to future payments, and therefore have to collateralize their borrowing. For simplicity, in this section we look only at the simplest form of these contracts, namely a debt contract, but consider more complex contracts below. Debt contracts are characterized by a fixed promised face value. The absence of commitment means that agents transfer to their creditor the face value of the loan or the payoff of the assets that serve as collateral, whatever is smaller. We normalize contracts to be secured by 1 unit of the asset as collateral.[11] Thus collateralized loan contracts have unit payoffs equal to $\min\{s, \overline{s}\}$,

[11] Note that one unit of a bond with face value 1 collateralized by x units of the asset is payoff-equivalent to x units of a bond of face value 1/x collateralized by one unit of the asset.

where $\overline{s}$ is the promised face value. In $t = 0$, agents trade these contracts at competitive price $q(\bar{s})$. Given that borrowing is subject to a collateral constraint, each unit of collateralized loans agent i issues must be secured by at least one unit of the risky asset that agent i possesses and can be used as collateral:

$$b_i \geq -a_i. \tag{3}$$

In the special case of a given unique $\overline{s}$, the set of available assets implies that the budget constraints of agent i in $t = 0$ and $t = 1$ respectively are:

$$c_i + pa_i + qb_i \leq n_i + p\bar{a}_i, \tag{4}$$

$$c_i' \leq a_i s + min\{s, \bar{s}\}b_i, \tag{5}$$

where a_i and b_i represent agent i's total holdings of risky assets, including the initial endowment, and of collateralized loans respectively.

3.1.3 Expected profits

At a given vector of prices p, q and a given face value $\overline{s}$, expected profits from buying a quantity b_i of collateralized loans with face value $\overline{s}$ are, for $i = L, H$:

$$\Pi_i^l = b_i[\frac{E_i[min\{s, \bar{s}\}]}{R} - q].$$

The expected profits from buying a_i units of risky assets partly financed through a collateralized loan of equal size are

$$\Pi_i^a = [\frac{E_i(s) - E_i(\min(s, \bar{s}))}{R} - (p - q)]a_i. \tag{6}$$

Finally, buying the asset outright using consumption goods as payment implies expected profits equal to $\Pi_i = [\frac{E_i(s)}{R} - p]a_i$.

Figure 3.1.3 illustrates how gross unit profits in period 1 change as a function of the asset

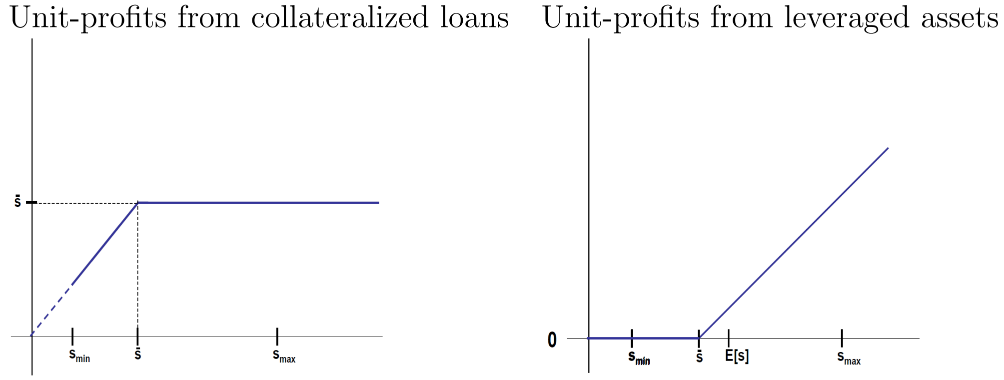


Figure 3: The left panel plots the profits from collateralized loans that are concave in s and the right panel plots those from leverage asset purchases, which are convex in s.

payoff s. The definition of profits implies that returns on collateralized loans are convex in s, while those on leveraged asset purchases are concave in s. Given the second order stochastic dominance relationship of beliefs, this immediately implies that investors with more (less) dispersed beliefs expect to make higher profits from investment in leveraged assets (collateralized loans):

Proposition 1 - *Profits and risk perceptions*

Type L agents (with low risk perception) have higher expected profits from investing in collateralized loans of a given face value $\bar{s}$ than type H agents (with high risk perception). The inverse is true for profits from leveraged asset purchases:

$$\Pi_H^l \leq \Pi_L^l \ \forall \ \overline{s} \in (s_{\min}, s_{\max}), \forall p, q, R,$$

$$\Pi_H^a \geq \Pi_L^a \ \forall \ \overline{s} \in (s_{\min}, s_{\max}), \forall p, q, R.$$

Moreover, there exists $\overline{s} \in (s_{\min}, s_{\max})$ such that both equalities are strict.

According to proposition 1, type L agents are the natural buyers of collateralized loans, and H agents are the natural investors in leveraged assets. In other words, if there is trade in collateralized loans in equilibrium $-b_H = b_L > 0$.

3.2 Equilibrium characterization

Definition 1 *A general equilibrium is a set of prices* $(p,\, q(\bar{s}))$ *and allocations* $\{c_i, c_i', a_i, b_i(\bar{s})\}_{i\in\{L,H\}}$ $\forall \bar{s}$*, such that both agents optimally choose their consumption and investments subject to their budget constraint and the collateral constraint* (3)*, the demand for assets equals the fixed supply, and the collateralized loan market clears,*

$$b_H(\bar{s}) + b_L(\bar{s}) = 0,\ \forall \bar{s}.$$

In the following, we assume that type L agents who have less dispersed beliefs are cash-rich.

Assumption A2 $n_L \geq \frac{E_s}{R}$.

Assumption A2 has two implications: first, the equilibrium asset price p is bounded below by the fundamental value $\frac{E_s}{R}$, since any lower price contradicts goods-market clearing in period 0, as it would give both types at least one investment possibility that they would strictly prefer over current consumption. Second, the total value of type L agents' endowment equals $n_L + p \geq 2\frac{E_s}{R} \geq 2max_{\bar{s}}\frac{E_L[min\{s,\bar{s}\}]}{R}$). So type L agents can afford to buy all collateralized loans at their maximum expected payoff. Since, moreover, they do not expect to make strictly positive profits from any other investment, they bid up the price of any collateralized loan issued by type H agents to their expected discounted value, where they are indifferent between investing and consuming, implying a bond price function

$$q(\bar{s}) = \frac{E_0[\min\{s, \bar{s}\}]}{R}. \tag{7}$$

In turn, this implies that type H agents expect profits Π_H from buying assets outright to be lower than from leveraged asset purchase.

Corollary 1 *With* $q(\bar{s})$ *given by* (7) $\Pi_H^a \geq \Pi_H$ $\forall p, \bar{s}$*, with strict inequality for some* $\bar{s} \in (s_{\min}, s_{\max})$.

So without loss of generality, we can focus on equilibria where type H agents leverage their entire asset holdings.

3.2.1 Type H's simplified problem and the choice of $\overline{s}$

Since under assumption A2 type L agents buy all collateralized loans at their reservation price $q(\overline{s})$, and type H agents leverage their entire asset holdings, the problem of type H agents simplifies to the choice of current consumption, which through the budget constraint determines their investment in leveraged assets, and the choice of the level of leverage $\overline{s}$ given p and the price function $q(\overline{s})$.

$$\max_{c_1,\overline{s}} \quad U_1 \quad = \quad c_1 + \frac{(n_1 + p - c_1)}{R} R_1^a. \tag{8}$$

where $R_1^a(p, \overline{s}) \doteq \frac{[E_s - E_1(\min\{s,\overline{s}\})]}{p - \frac{E_0\{\min\{s,\overline{s}\}\}}{R}}$ is the leveraged gross return of the asset using a loan with riskiness $\overline{s}$. The first order condition for $\overline{s}$ can be written as:

$$\frac{(n_H + p)}{Rp - E_L\{min\{s, \overline{s}\}\}}[(1 - F_H(\overline{s})) - \frac{R_H^a}{R}(1 - F_L(\overline{s}))] = 0. \tag{9}$$

Proposition 2 - *Interior choice of* $\overline{s}$.

Suppose that p *satisfies* $\frac{E_s}{R} = \underline{p} < p < \overline{p} \doteq \frac{E_s + E_L(min(s,\overline{s})) - E_H(min\{s,\overline{s}\})}{R}$ *for some* $\overline{s} \in (s_{min}, s_{max})$*, such that agent 1 expects to make profits for some* $\overline{s}$ *when she buys assets at* p *that exceeds the fundamental value. Then* $R_H^a(p, \overline{s})$ *has an interior maximum at some* $\overline{s}^\star \in (s_{min}, s_{max})$.

Proof of Proposition 2.

Note that $R_H^a(p, s_{max}) = 0$. Also, if $p > \frac{E_s}{R}$, $R_H^a(s_{min}) = \underline{R_H^a} < R$. But if at some $\overline{s}'$, $p < \frac{E_s + E_L(min\{s,\overline{s}'\}) - E_H(min(s,\overline{s}'))}{R}$, then $R_H^a(\overline{s}') > R$. The statement then follows from continuity of R_H^a. ■

3.2.2 Existence and uniqueness of a bubble equilibrium

The following proposition, whose proof is in the appendix, shows that equilibrium is defined by two conditions: first, the optimal choice of leverage $\overline{s}$; and second, the market clearing for leveraged assets, which defines the price such that type H agents either exhaust all their wealth buying assets, or are indifferent between investing and consuming. Intuitively, as agent 1 wealth rises, their increasing demand for assets bids up the price until it reaches indifference level $\overline{p}$.

Proposition 3 - *Existence and uniqueness of equilibrium.*

Denote as $n_1^{\max}(\overline{s}) = n_1 + 2\frac{E_0[\min\{s,\overline{s}\}]}{R}$ the resources available to type H agents for net purchases of assets when they issue collateralized loans backed by the whole asset endowment of the economy. p and $\overline{s}$ are given by the unique solution of the following equations:

$$\mathbb{C} \equiv [E_s - E_1(\min\{s,\overline{s}\})](1-F_L) - (1-F_H)(Rp - E_L[min\{s,\overline{s}\}]) = 0, \tag{10}$$

$$p = max\{\overline{p}, p^\star\}, \tag{11}$$

$$p^\star = n_1^{\max}(\overline{s}), \tag{12}$$

where the left hand side of (12) *are the net purchases of assets and the right-hand side equals the available resources, both weighed by the mass of type H agents.*

As Proposition 3 shows, with heterogeneous risk perceptions, collateralized contracts lead to a bubble in asset prices, in the sense that equilibrium prices exceed the common fundamental value of the asset, shared by all investors. Moreover, it is easy to see from (12) that a rise in resources of type H agents (weakly) increases prices. In addition, as Proposition 2 has shown, there is a unique endogenous choice for leverage $\overline{s}$.[12]

[12] A previous working paper version of this article, Broer and Kero (2014), shows that similarly to the two-type economy, with heterogeneity in perceived risks across a continuum of types, the equilibrium prices of risky assets are necessarily above their common fundamental valuation. Unlike the two-type economy, however, these results require an exogenous upper bound for the face value $\overline{s}$.

3.3 Comparative statics

This section looks at the effect of 'belief divergence', in the sense of a further mean-preserving contraction to f_L, or equivalently a dispersion of f_H. For this, we concentrate on economies where funds of type H agents are large either because their endowments are high, or because they can raise enough funds from issuing collateralized loans. This is stated in assumption A3, which implies that asset prices are at their upper bound, as lemma 1 states. Its proof, like that of corollary 2, is in the appendix.

Assumption A3 $n_H \geq \underline{n_H} = \frac{E_s - E_L(min\{s,s^\star\}) - E_H(min\{s,s^\star\})}{R}$.

Corollary 2 *For any symmetric distributions f_H, f_L, we have $\underline{n_H} < 0$. So Assumption A3 trivially holds.*

Lemma 1 *Assumption A3 implies $p = \overline{p}$.*

To look at belief divergence we assume that the distribution function f_i is parameterized by a variable v such that:

1. f_i is continuous in v for all s, $i = L, H$.
2. $E_{i,v}(s) = E_s, \forall v$, $i = L, H$.
3. $f_i(v)$ second order dominates $f_i(v')$, $i = L, H$ whenever $v > v'$.
4. $F_L(v, s) - F_L(v', s)$ is downward sloping in s whenever $v > v'$ and crosses the zero line once at $s^\star$.

In the following we define 'belief-divergence' as small changes in the beliefs of type L and H agents, f_L, f_H, through a pair of small changes $dv_L \geq 0, dv_H \leq 0$ in their corresponding values of $v_i, i = L, H$, with at least one strict inequality, corresponding to a mean-preserving contraction to f_L and a mean-preserving spread to f_H.

From the pricing equation for bonds (7), it is immediately clear that $dv_L > 0$ increases the valuation of collateralized loans by type L agents, and thus their price.

Lemma 2 *A fall in risk perceived by type L increases prices of collateralized loans* $\frac{\delta q}{\delta v_L} > 0$.

Also, under Assumption A3 we can identify the effect of belief divergence on asset prices.

Proposition 4 - *Belief divergence increases asset prices.*

Proof of Proposition 4.
Assumption A3 implies $p = \overline{p} = \frac{E_s + E_L(min(s,\overline{s})) - E_H(min\{s,\overline{s}\})}{R}$, and $F_L(\overline{s}) = F_H(\overline{s})$ from (10), so $\overline{s} = s^\star$ from single-crossing. Since $s^\star$ does not change in response to dv_i, neither does $\overline{s}$. But $\overline{p}$ rises with a mean preserving spread $\{dv_L \geq 0, dv_H \leq 0\}$ as $\frac{\delta E_L[min\{s,s^\star\}]}{\delta v_L} \geq 0$ and $\frac{\delta E_H[min\{s,s^\star\}]}{\delta v_H} \leq 0$. ■

As Proposition 4 shows, divergence of risk perceptions across investors increases asset prices because a mean-preserving spread to f_H increases the perceived upside potential of leveraged assets payoffs while a mean-preserving contraction in f_L reduces the perceived riskyness of collateralized loans. These results rely on assumption A3, which implies that prices rise along the upper bound $p = \overline{p}$, and that the optimal $\overline{s}$ is unchanged.[13]

4 Application: Disagreement about default correlation and the Structured Finance boom

Both the US housing boom of the early 2000s, and the large losses of financial institutions during the crisis that followed have been blamed on the rise of 'Structured Finance' assets, such as residential mortgage backed securities (RMBSs), which allocate the cash flow from a pool of collateral assets to different 'tranches' in order of seniority. This section shows how disagreement

[13]When Assumption A3 does not hold, in contrast, a mean preserving spread in beliefs has an ambiguous effect on marginal profits and thus the optimal value of riskyness $\overline{s}$. Specifically, while a rise in v_L increases the return at any given riskyness, it can increase or decrease $1 - F_L$, the marginal effect of a change in $\overline{s}$ on profits at given returns. Although intuition suggests that the former effect would dominate, this is difficult to show formally.

about the riskyness of the collateral pool, or about the power of diversification through pooling of many loans, affects the collateral price in a way that is very similar to disagreement about the riskyness of a single asset that collateralizes simple debt contracts, studied in Section 3. Particularly, investors who expect the loan pool's default rate to be tightly distributed around its mean regard senior tranches as riskless, and junior tranches that pay only in the event of lower-than-expected default rates as worthless. Those who believe in volatile default rates, in contrast, expect senior tranches to default in bad times, but junior tranches to pay when times are sufficiently good. Self-selection of investors then raises tranche prices, and, by arbitrage, the collateral price, just like with collateralized loan trade in Section 3. In fact, with only two tranches and complete loss-given-default, the results in that section are easily re-interpreted as applying to a pool of many identical loans with total face value s_{max} whose random default rate equals $\frac{s}{s_{max}}$.

Disagreement about the variability of default rates arises very naturally from different opinions about the importance of idiosyncratic vs. aggregate risk for loan defaults as captured by the default correlation in credit risk models, a parameter that is particularly difficult to estimate for loan categories with a short history. This section quantifies the effect of disagreement about default correlations on the market price of RMBSs, and collateralized debt obligations (CDOs) backed by their tranches. The analysis is made easier by the fact that, contrary to most assets that may be valued using different models with potentially many dimensions of disagreement, Structured Finance products came with a unique "market standard' (Morini (2011), p. 127) model - the Gaussian copula with homogeneous correlations - with only three main parameters: the average default probability, the value of loss given default, and the homogeneous correlation of defaults. We concentrate on disagreement about this default correlation as the parameter that determines the dispersion of the pool-wide default rate.[14] This is partly for simplicity, but also because, as Broer (2016) shows, disagreement about loss-given-default or average default probabilities does not lead to the self-selection at the heart of this paper, since the value of all

[14]More precisely, for a pool containing a large number of identical loans with default probability $\overline{\pi}$, the variance of the fraction of defaulting loans rises from 0 to $\overline{\pi}(1-\overline{\pi})$ as their default correlation increases from 0 to 1.

(senior and junior) tranches is monotonically declining in both parameters. This is important since the rising popularity of Structured Finance assets makes it difficult to argue that prices were determined by only the most optimistic investors. With disagreement about correlations, in contrast, an increasingly heterogeneous pool of investors increases prices as agents self-select into buying their preferred tranches. In addition, the parameter uncertainty implied by their short history and the lack of consensus about how best to estimate correlation patterns[15] seems to have been largely neglected by investors (Coval et al., 2009; Morini, 2011), making heterogeneous 'point' beliefs an appealing, if simplifying, assumption.[16] Finally, the analysis assumes that investors rely entirely on their own beliefs. They thus do not use market prices to adjust their beliefs (in line with the dominance of over-the-counter trades) and do not exclusively use information from rating agencies.[17]

4.1 A Gaussian copula model

Consider a version of the economy in Section 3 with $k = 1, .., K$ types of risk-neutral investors who live for two periods $t = 1, 2$ and discount second period consumption by the common discount factor $\frac{1}{R}$. At the beginning of period 1, investors receive a non-storable consumption endowment a, which they can consume or invest in assets collateralized by a given pool of $n = 1, ..., N$ mortgages of face value and mass 1, which are sold by a single originator. In period 2 a stochastic fraction d of mortgages defaults. Defaulted mortgages pay recovery value $V_{rec} < 1$. Let $\overline{\pi}$ denote the common belief about the homogeneous default probability of mortgages.

[15]Thus, Luo et al. (2009) argue that the inclusion of an unobserved 'frailty' factor would have substantially improved the predictive power of portfolio credit risk models applied to CDO pricing.

[16]See Gorton and Metrick (2013) for a description of the valuation of structured products by investors who use "vendor-provided packages that model the structure of structured products, but the valuation is based on (point estimate) assumptions that are input by the user" (p.112).

[17]While the market for Structured Finance was indeed a 'rated market', evidence from surveys suggests that ratings were only one of many elements in the assessment of credit risks by investors (Fender and Mitchell, 2005). Moreover, the junior tranches where, as it turns out, disagreement leads to the strongest differences in value, were usually held by specialist investors who are likely to rely less heavily on ratings in their judgments (see Fender and Mitchell (2005), p. 70). In fact, prices of both non-AAA tranches of RMBSs (Adelino, 2009) and ABS-CDOs (Mählmann, 2012) did incorporate information over and above credit ratings. See also Cuchra (2004).

Investor k models the credit risk of the loan pool using a standard Gaussian copula model with homogeneous correlation (Li, 2000; Laurent and Gregory, 2005). Specifically, she believes mortgage n to default whenever the following condition is met

$$x_n = \rho_k \cdot M + \sqrt{1-\rho_k^2} \cdot M_n < \overline{x} = \mathbb{N}^{-1}(\overline{\pi}),\ \ M, M_n \propto N(0,1) \tag{13}$$

The index variable x_n can be interpreted as the value of creditor n's assets. It equals the weighted average of an aggregate factor M, capturing economy-wide conditions, and a loan- or borrower-specific factor M_n, which are both distributed according to the standard normal distribution. Investors agree that loan n defaults whenever the index x_n falls below a threshold $\overline{x}$ equal to the inverse normal distribution evaluated at the default probability $\overline{\pi}$, which this section assumes is shared by all investors. Investors disagree, however, about the importance of aggregate conditions in determining loan defaults, as summarized by the parameter ρ_k. Specifically, ρ_k^2 equals the correlation between two individual creditors' asset values perceived by investor k. For example, investors who believe in $\rho_k = 0$ expect the default rate of the pool d, equal to the share of loans whose asset values are less then the threshold, to equal $\overline{\pi}$ with certainty. Investors with higher perceived ρ_k believe individual defaults to comove more strongly, and thus expect d to be less tightly distributed around $\overline{\pi}$. Together with the recovery value in case of default V_{rec}, ρ_k and $\overline{\pi}$ completely determine the distribution of the cash flow from the mortgage pool equal to $\mathbb{C} = 1 - d(1 - V_{rec})\ \forall d$.

The originator maximizes current profits from selling the loan pool to investors in one of two ways: as shares in a 'pass-through' securitization that pays all investors their share in the total cash flow that the collateral generates, equal to $1 - d(1 - V_{rec})\ \forall d$; or structured as an RMBS by splitting the cash flow into 'tranches' that receive payments in strict order of their pre-specified seniority. Specifically, tranche 1 promises to make a total payment of $a_1 < 1$ to its holders in period 2, where a_1 is the 'detachment point' of tranche 1, and receives any cash flow that defaulting and non-defaulting mortgages generate until a total of a_1 is reached. Tranche 2 promises to pay $a_2 - a_1$, where $a_1 < a_2 < 1$, but only receives cash flow once a_1 has been paid to holders of the first tranche, etc.

The Gaussian copula model (13) conveniently allows investors to also value CDOs whose collateral consists of a pool of J RMBS tranches, rather than individual loans. The crucial additional parameter that determines the CDO's payoff distribution is the additional diversification gain from pooling tranches. For this, I assume that investor k perceives the aggregate factor M in (13) to be the sum of a global factor $\mathbb{M}$ and an RMBS-specific factor M_j

$$M = \rho'_k \cdot \mathbb{M} + \sqrt{1 - \rho'^2_k} \cdot M_j, \ \mathbb{M}, M_j \propto N(0,1) \tag{14}$$

The CDO's payoff distribution is thus determined by a 3-factor Copula, with an asset correlation that investor k perceives to equal ρ_k^2 for mortgages in the same RMBS, and $\widehat{\rho}_k^2 = \rho_k^2 \rho'^2_k \leq \rho_k^2$ for mortgages in different RMBS pools.

Given one of the two securitization possibilities - structured or pass-through - an equilibrium is defined as a vector of prices such that the originator maximizes current profits, investors maximize utility, and demand for all assets equals supply. Investor optimality implies that the expected return from all positive investments in their portfolio must be the same, at least equal to R, and, in case it exceeds R, strictly higher than that from assets they do not hold.

4.2 Payoff distributions and valuation of tranches

This section illustrates how investors who perceive higher asset correlation ρ_k^2 (or $\widehat{\rho}_k^2$) expect collateral cash flow to be more widely dispersed around its mean and therefore have a relative taste for the most junior tranches of RMBSs (or CDOs). To capture the characteristics of the market for US subprime mortgage-backed securities prior to the crisis, I look at pools of 5000 mortgages in an RMBS and of 100 RMBS tranches of equal seniority in a CDO. Also, I choose a common perceived default probability $\overline{\pi}$ equal to 12.5 percent, and a recovery value V_{rec} that comoves inversely with the pool's default rate d in a range of $+/-15$ percentage points around its average of $\overline{V}_{rec} = 60$ percent, in order to account for longer time-until-foreclosure and lower resale values when default rates are high.[18] Since both RMBSs and CDOs usually had similarly granular structures, the analysis uses the same 6 tranche structure as Coval et al. (2009) for both, consisting of an equity tranche (100-97 percent), a junior tranche (97-93 percent), mezzanine tranches I and II (93-88 and 88-80 percent respectively) and senior tranches I and II (80 to 65 and 65 to 0 percent).

The upper left panel of Figure 4 shows how, for $\rho_k^2 = 0$, the perceived distribution of payoffs from an RMBS's collateral pool collapses around the expected payoff equal to $1 - \overline{\pi}(1 - \overline{V}_{rec}) = 95$ percent. As ρ_k^2 rises, the distribution fans out at a decreasing rate, but the lowest percentile remains above 75 percent as payoffs are protected by the recovery value.[19]

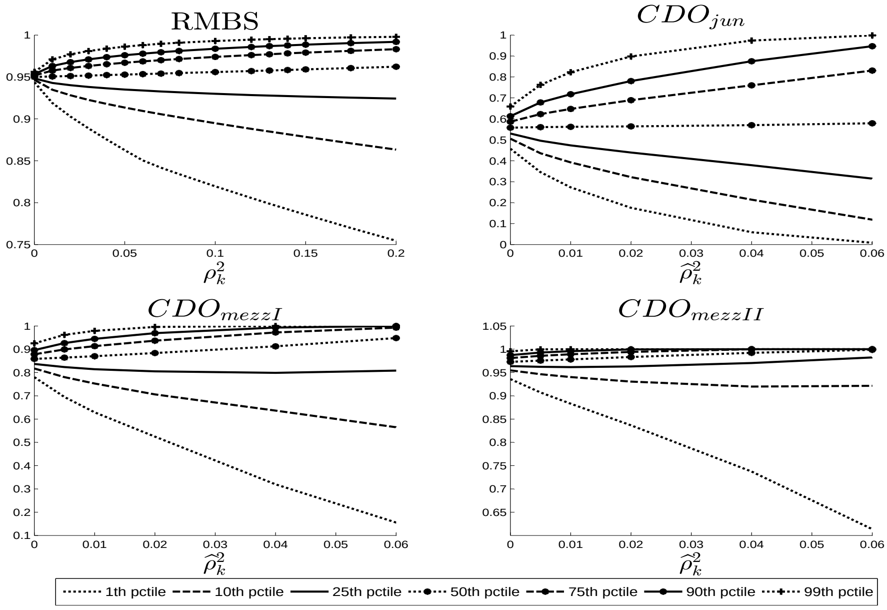
The remaining panels of Figure 4 depict the perceived payoff distributions from pools of 100 junior, mezzanine I and mezzanine II RMBS tranches as a function of $\widehat{\rho}_k^2$, the weight on the global factor in equation (14) that captures the perceived 'remaining' asset correlation, when ρ_k^2 equals 0.1. The lower number of collateral assets implies that payoffs remain uncertain even when the diversification gain is perceived to be perfect ($\widehat{\rho}_k = 0$). More importantly, junior tranches of RMBSs are not protected by the recovery value of the underlying mortgages from the bottom, and, just as the underlying tranche payoffs, are not bounded away from a full payoff at the top. The distributions thus fan out more strongly, implying stronger disagreement about payoff variances. For example, probabilities of a zero payoff are greater than 1 percent for junior RMBS tranches when $\widehat{\rho}_k^2$ exceeds 6 percent, while for mezzanine tranches, the upper percentiles bunch increasingly at 100 percent as $\widehat{\rho}_k^2$ increases above 2 percent.

To illustrate how the heterogeneous perceived payoff distributions depicted in Figure 4 affect the expected payoffs of RMBS and CDO tranches, Figure 5 shows the difference between their payoffs expected by an investor with perceived asset correlation ρ_k^2 or $\widehat{\rho}_k^2$ (depicted along the

[18] The default rates for subprime mortgages differed strongly over time, fluctuating around 10 percent during the years of strong house price growth up to 2006 and increasing to above 40 percent thereafter (see, e.g., Beltran et al. (2013)). The recovery value equals $V_{rec} = 0.6 + (d - \overline{d})$, where $\overline{d}$ is the average default rate equal to $\overline{\pi}$, but is bounded by a minimum of 45 percent.

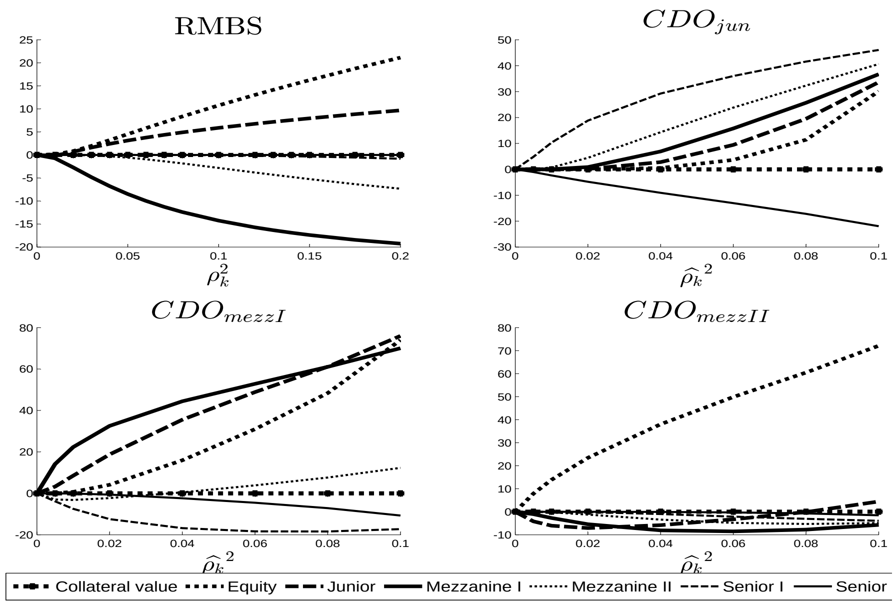
[19] To interpret the magnitudes, note that ρ_k^2 and $\widehat{\rho}_k^2$ do not equal default correlations. In fact, as Figure 3 in Broer (2016) shows, the correlation between default events of any two mortgages in the RMBS is about half as large as the correlation of the underlying asset value x_n.

Figure 4: Distribution of collateral payoffs



The figure shows the distribution of collateral payoffs in an RMBS (upper left panel) and CDOs with homogenous collateral consisisting of junior (upper right panel), mezzanine I (lower left panel), and mezzanine II (lower right panel) RMBS tranches.

Figure 5: Expected value of tranches



For RMBSs (upper left panel) and CDOs with homogenous collateral consisisting of 100 junior (upper right panel), mezzanine I (lower left panel), and mezzanine II (lower right panel) RMBS tranches, the figure shows the difference between their tranches' payoffs expected by an investor with perceived asset correlation ρ_k^2 or $\widehat{\rho}_k^2$ (depicted along the bottom axes) and that expected by a 'zero correlation' investor (whose ρ_k^2 or $\widehat{\rho}_k^2$ equals 0), as a percentage of the underlying collateral's face value (the 'width' of the tranche).

bottom axes) and that expected by a 'zero correlation' investor (whose ρ_k^2 or $\widehat{\rho}_k^2$ equals 0), as a percentage of the underlying collateral's face value (the 'width' of the tranche).

As expected, the collateral value, or total expected payoff from the mortgage pool (the starred dashed line flat at 0), is unaffected by the perceptions of correlation as all investors share the same average default probability. Because the 'zero correlation' investor expects the payoff from the mortgage pool to equal 95 percent with certainty, she deems the junior and equity tranches of the RMBS (in the upper left panel), with attachment points close to or above 95 percent, to be worth nothing or little. High ρ_k^2 investors, in contrast, who perceive both a larger downside and upside risk, think that junior tranches are more likely to pay off, while they expect the mezzanine tranches to default with positive probability. Interestingly, the downside risk is never strong enough for investors to considerably disagree about the valuation of the senior tranches.

In line with the stronger rise in payoff variation of CDO collateral in Figure 4, the disagreement about tranche valuations is more widely spread there, and differences in valuation an order of magnitude larger as a fraction of collateral face value. Thus, an investor who perceives no diversification gain from pooling RMBS tranches ($\rho_k^2 = \widehat{\rho}_k^2 = 0.1$) expects payoffs as a fraction of the face value from the three most junior CDO tranches backed by mezzanine I collateral to be between 70 and 80 percentage points higher than his counterpart who expects diversification gains to be perfect ($\rho_k^2 = 0.1$, $\widehat{\rho}_k^2 = 0$).[20]

4.3 The return to tranching

From figure 5, it is evident that disagreement about default correlations may raise securitization profits if originators can sell tranches backed by collateral cash flow to investors who value them particularly highly, rather than selling the cash flow as a pass-through securitization at its common, lower valuation. The extent to which this is possible, however, depends on supply

[20] A final difference between RMBS and CDO tranche valuations is that the latter do not necessarily rise or fall monotonically as $\widehat{\rho}_k^2$ rises and the middle of the distribution 'thins out'.

relative to the demand by investors with different beliefs. With several assets and general disagreement, it is usually not clear that an equilibrium price vector exists, or that it is unique. In the context of this paper, the equilibrium price vector p_i of tranches $i = 1, ...I$ ordered by seniority has to be such that supply equals demand for every tranche, and that any investor k expects to earn on all positive investments in her portfolio an equal return R^k not smaller than R, and greater than that she expects from any assets she does not hold. Since this implies $(K+1) \cdot I$ equilibrium conditions, solving for any such equilibrium is complex. In the case of RMBSs, however, as Figure 5 suggests, the valuation of tranches is typically monotonic in ρ_k^2: high ρ_k^2 types value junior tranches more than low ρ_k^2 types, while the reverse is true for senior tranches. The analysis exploits this feature to calculate the equilibrium price of all tranches in the example of a subprime RMBS with a small number of ρ_k^2 types. We compare these equilibrium prices to an alternative price vector that we call 'maximum' prices, which simply equal the maximum valuation of investors (and are in fact equilibrium prices if the endowment of any single investor type was large enough relative to the loan supply). The reason for this is to show that the self-selection of investors to their preferred tranches implies equilibrium prices that are close to maximum prices even when many investors participate in equilibrium. Again, this contrasts with disagreement about means, where equilibrium prices are typically smaller than maximum valuations. Given the monotonicity of valuations in ρ_k, we can interpret maximum prices as equilibrium prices in an economy with only two types whose ρ_ks equal the bounds of the support for ρ_k.

4.3.1 Calibrating disagreement

As argued above, the distribution of cash endowments across ρ_k-types is crucial for equilibrium prices. This section does not aim at an exact calibration of this distribution. Rather, we first choose a support for $\{\rho_k\}_{k=1}^K \in [\rho_{min}, \rho_{max}]$ that is meant to be conservative, and then show how the distribution of cash endowments on that support is only of limited importance even for equilibrium prices. The choice of support $\{\rho_k\}_{k=1}^K \in [\rho_{min}, \rho_{max}]$ is informed by three facts:

first, the large uncertainty around estimates of default correlation parameters on short histories of data[21]; second, the ratings of RMBS and CDO tranches by the main ratings agencies; and finally, the default experience in the US subprime mortgage market.[22]

We use two pairs of values $\{\rho^2_{min}, \rho^2_{max}\}$ which I label 'weak' and 'strong' disagreement. In the case of weak disagreement, to capture the intuition that structures were designed to give senior tranches top credit ratings, and that rating agencies often had optimistic assessments of default probabilities (Griffin and Tang, 2011), I choose a ρ^2_{min} equal to the maximum correlation compatible with an AAA rating of both senior RMBS tranches.[23] To calibrate ρ^2_{max}, I assume that the highest-correlation investors perceived a small but significant probability of default rates rising to levels experienced during the crisis. In the case of 'weak' disagreement I interpret this to be a 0.5 percent probability of default rates in the RMBS pool reaching 40 percent or more, as observed in 2007 for US subprime mortgages (see, e.g., Beltran et al. (2013), especially figure 4). This calibration yields values of ρ^2_{min} and ρ^2_{max} equal to 7 and 12 percent, respectively (equivalent to default correlations of roughly 3 and 5 percent). In a 'strong' disagreement specification, I extend the range of values such that the ρ^2_{min}-investor would also just give the mezzanine II tranche an AAA rating, and such that the ρ^2_{max} investor perceives a probability of 1 percent of default rates rising to 40 percent or above. This yields values of ρ^2_{min} and ρ^2_{max} equal to 2 and 16 percent, respectively (corresponding to default correlations of 1 and 6.5 percent).

To calibrate, in a similar fashion, the two extremes of the $\widehat{\rho}^2_k$ distribution, capturing disagreement about the additional diversification gain from pooling RMBS tranches, is more difficult. I choose $\widehat{\rho}^2_{min}$ to yield a default correlation of mezzanine RMBS tranches approximately equal to that calculated by Griffin and Nickerson (2015) for the main rating agencies, when fixing ρ^2 at

[21]Figure 4 in Broer (2016) shows the wide standard errors of these estimates.

[22]I do not consider information from RMBS and CDO tranche prices. This is, first, because their prices are often determined over the counter and thus unobserved. Moreover, the aim of the exercise is to isolate the effect of disagreement about default correlations on equilibrium prices via expected payoffs. Observed prices, on the other hand, were affected by many other factors, such as risk aversion or investment mandates of some investors.

[23]In line with this, Ashcraft et al. (2010), p.13 find that the average fraction of subprime RMBS that received an AAA rating was 82 percent.

a mean value of 10 percent.[24] This yields a diversification gain $1 - \rho_k'^2$ in equation (14) equal to 88 percent, or $\rho_k'^2 = 0.12$. This diversification gain has been criticized as too optimistic[25], partly since most RMBSs were already geographically diversified (see, e.g., Cordell et al. (2011)). I thus use this value as the lower bound of the ρ_k'-distribution $\rho_{min}'^2$, and set $\rho_{max}'^2$ to yield a lower additional diversification of 70 and 50 percent in the weak and strong disagreement cases respectively.

For the support of ρ^2 and $\widehat{\rho}^2$, I choose a particularly simple, uniform distribution on $[\rho_{min}^2, \rho_{max}^2]$ with 5 support points. I then calibrate the supply of mortgages and investors' consumption endowment such that at least three ρ_k^2-types are needed to buy the mortgage pool. Specifically, I normalize the mass at each support point to 1 and set the endowment a equal to $\frac{3}{7}$ for all investors. This implies that 20 percent of the consumption endowment is located at the extremes. As it turns out, this usually suffices for all junior tranches, whose valuation increases with ρ_k^2, to be bought by the ρ_{max}^2 investor.

4.3.2 The return to tranching subprime mortgage pools

This section concentrates on the 'return to tranching' subprime mortgage pools into RMBSs, defined as the difference in market values between the RMBS and that of the collateral when sold as a non-tranched, pass-through securitization. Table 3 shows how, when all prices are at their maximum, this return equals 35 (85) basis points in the weak (strong) disagreement case. Interestingly, although the equilibrium prices of all RMBS tranches are approximately equal to maximum prices, the return to tranching in Table 3 is *higher* with equilibrium prices. This is because the equilibrium price of the non-tranched, pass-through securitization (the denominator of the return-to-tranching), equal to the valuation by the median investor, is lower than its maximum valuation (by the ρ_{min}^2 investor). The reason for this is that loss-given default rises slightly with default rate d. Payoffs, which decline in both, thus have a concave relationship

[24] Griffin and Nickerson (2015) back out an average default correlation of 0.03 for Moody's, and 0.042 for S&P. We choose $\widehat{\rho}_{min}^2$ to yield a default correlation of mezzanine II tranches equal to 3.8 percent.

[25] See e.g. the investor statements reported in the Financial Crisis Inquiry Commission's Report (Financial Crisis Inquiry Commission, 2011), p. 193-4.

with default rates, such that their expectation declines with the variance of defaults, or with the correlation parameter ρ_k^2, due to a Jensen's inequality effect. An online appendix evaluates the importance of this effect for the results.

Equilibrium tranche prices equal their maximum valuation for two reasons. First, ρ^2_{max} investors can afford to buy the two most junior tranches - which they value most highly - because their small attachment range and low payoff probability make them cheap. Similarly, the two mezzanine tranches, whose valuation is declining with ρ^2 in the upper left panel of Figure 5, are affordable to the ρ^2_{min} investors. Second, the valuation of the remaining 2 senior tranches, which are not affordable to any single investor since their attachment range is large and expected loss negligible, is approximately the same across the three investors with the lowest ρ_k^2, who all view them as basically riskless. Thus, their price is, again, equal to their maximum valuation. Moreover, none of the investors has an arbitrage opportunity at these prices, as they are indifferent between their investments and consuming their endowment. In fact, this is a general pattern which in this setting of structured assets dampens the equilibrium effect that reduces asset prices under disagreement in other contexts: tranching makes the valuation of the bulk of senior tranches insensitive to disagreement. Rather, disagreement about valuations is concentrated in the junior tranches, which are cheap and can thus be bought by a small number of specialized investors. This increases the equilibrium return to tranching, as the collateral pool as a whole, or the pass-through securitization, is typically priced at less than its highest valuation.

We do not model the mortgage market in this simple exercise, assuming for simplicity that a single originator reaps all the surplus from a return to tranching of between 45 and 110 basis points in equilibrium. An alternative, more complex environment where the surplus accrues to mortgage borrowers, would predict a corresponding fall in mortgage rates from selling the loan pool in tranches. We think that the magnitude of this fall is sizeable when compared to the low real interest rates of the early 2000s and the expected loss rates on mortgage pools of only 5 percent. As it turns out, the return from tranching pools of RMBS tranches into CDOs,

Table 3: Return to tranching

	Max	**Equ**
Weak disagreement about ρ^2	34	44
Strong disagreement about ρ^2	84	111

The table presents the return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitization, measured in basis points (100th of a percent) of the latter's market price.

Table 4: Return to re-tranching

	$Junior$	$Mezz_I$	$Mezz_{II}$
Weak disagreement about $\widehat{\rho}^2$	496	207	44
Strong disagreement about $\widehat{\rho}^2$	960	400	85

The table presents the return from selling a pool of junior / mezzanine I / mezzanine II RMBS tranches in the form of a CDO, measured in basis points (100th of a percent) of the price of the non-structured pool.

however, is an order of magnitude larger. Unfortunately, it is prohibitively complex to compute the equilibrium prices of tranches and collateral pools in the case of CDOs. The following analysis therefore concentrates on returns to (re-)tranching RMBS tranches into CDOs based on their maximum prices, noting that the previous results and intuition suggest these to be a lower bound for the equilibrium estimates.

4.3.3 CDOs and the 'returns to re-tranching"

Table 4 shows the returns to (re-)tranching RMBS tranches of three different levels of seniority into CDOs when investors disagree about $\widehat{\rho}_k^2$ but agree on ρ^2, the asset correlation within any given RMBS (set equal to 10 percent). The returns for both weak (top row) and strong disagreement (bottom row) are an order of magnitude higher than for RMBSs, explained by the greater sensitivity of the CDOs' cash flow distributions (in figure 4) and tranche valuations (in figure 5) to disagreement about $\widehat{\rho}_k^2$. Particularly, to an originator, selling junior RMBS tranches as a CDO yields a return of between 500 and 1000 basis points, as disagreement increases tranche values strongly and the price of the non-structured RMBS tranches is low due to a high average expected loss (somewhat below 50 percent, corresponding to an average RMBS payoff in the middle of the junior tranche's attachment range).

By arbitrage, the re-tranching of RMBS tranches into CDOs increases the value also of the original mortgage pools. The extra return this implies relative to that of selling the RMBS tranches to investors without re-tranching in Table 3 is sizeable (with values between 15 and 40 basis points), but small relative to the large returns from packaging junior RMBS tranches in CDOs in Table 4. This is because the value of RMBSs is dominated by the senior tranches whose expected payoff is perceived to be, essentially, riskless due to the substantial recovery value of 60 percent on average, and thus unaffected by disagreement.[26]

5 Extensions: risk aversion and options trade

5.1 Risk-averse investors

Like almost all studies of investor disagreement, including those where leverage amplifies payoff risk, such as Geanakoplos (2010), Fostel and Geanakoplos (2012), or Simsek (2013), our benchmark results are derived under the assumption that investors maximize expected profits, and are therefore risk-neutral. In this section we show how the main result, that leveraged asset trade increases asset prices when investors disagree about risk, also holds in a version of the environment with risk aversion.[27] As we will see, however, with risk-averse preferences, the quantitative effect of disagreement about payoff risk on the price of collateral assets is decreasing in the asset supply as agents are more reluctant to leverage aggregate risks that comove strongly with consumption, as opposed to idiosyncratic risks.

Consider the two-type environment of Section 3 but with preferences that have constant

[26]See Broer (2016) for details. An online appendix shows how the quantitative results are affected by alternative assumptions about the average default probability and the specification of the recovery value.

[27]The analysis is related to Dow and Han (2016), who prove existence of equilibrium in a similar environment with heterogeneity in endowments and give numerical examples for asset price overvaluation.

relative risk aversion equal to γ[28]

$$U = u(c) + \frac{1}{R}u(c'),\ u(c) = \frac{c^{(1-\gamma)} - 1}{1-\gamma} \tag{15}$$

We continue to abstract from discounting for simplicity by setting $R = 1$, and normalize the endowment of consumption goods in the first period to $n_i = 1, i = L, H$. Moreover, for tractability, we assume that type $i = L, H$ expects payoffs to follow a uniform distribution on a support $[1 - \epsilon_i, 1 + \epsilon_i]$, with $\epsilon_H > \epsilon_L > 0$.

Both agents solve a version of problem (8) adjusted for the risk-averse preferences (15), and the additional investment opportunities (non-collateralizable assets and storage). A general competitive equilibrium is then a set of prices $\{p^1, p^2, q(\bar{s})\}$, consumption plans c_j and c'_j, storage d_j and financial portfolios $\{a^1_j, a^1_j, b_j(\bar{s})\}$ for both types $j = L, H$, such that all agents solve their problem at given prices subject to their budget set, non-negativity constraints on storage, short-sale constraints on assets, as well as collateral constraints on loans, and the markets for securities and consumption goods clear.[29] Note that the price and quantity of collateralized loans are, again, a function of their face value $\bar{s}$.

The introduction of risk aversion changes the analysis in two important ways: First, risk-neutral agents concentrate all their investments in the highest yielding opportunity, whose equilibrium price equals investor resources per unit of supply, or its expected value discounted at the rate of time preference, whatever higher. With risk aversion, in contrast, and in the absence of second period income, the ability to invest in alternative, particularly (lower-yield) riskless assets, is important for collateralization incentives.[30] We thus assume that agents can transfer resources to period 1 through one of three ways: as before, they can issue an amount b_j in loans by providing collateral (as there is no commitment to promises) and buy the exogenous asset.

[28]We make this assumption partly for simplicity. A well-behaved utility function with a strictly positive third derivative is sufficient for the analytical results below.

[29]See (Geanakoplos and Zame, 2014) for an equivalent definition of competitive equilbrium in a more general economy with collateral constraints, as well as a proof of existence of equilibrium.

[30]Trivially, with CRRA utility, agents will never leverage their whole asset holdings using risky loans when they have no other claims to future consumption.

In addition, however, we add the possibility to invest d_j units in a storage technology with gross return of R, normalized to 1.

A second difference arises because, unlike with risk neutrality, where expected payoffs discounted at the rate of time preference $\frac{E_s}{R}$ are an obvious benchmark value of the asset, with risk aversion there is no such 'fundamental value'. To characterize how disagreement about payoff risk affects asset prices with and without trade in risky collateralized loans, one can either study the relative price of collateralizable and non-collateralizable assets in the equilibrium of an economy where both are traded (which we call the 'within-economy collateral premium', studied, for example, in Dow and Han (2016)), or across equilibria of economies with different collateralization possibilities (the 'cross-economy collateral premium', as in Allen and Gale (2000)'s analysis of price bubbles due to limited liability). Below, we characterize the within-economy premium analytically, and provide quantitative examples for the cross-economy premium. For this we assume that agents are endowed with two kinds of assets 1 and 2 whose quantities satisfy $\overline{a}^1 + \overline{a}^2 = \overline{a}$. Both assets have identical payoffs but only asset 1 can be used as collateral for loans (for example because payoffs from asset 2 are observable only to the owner). We denote as a_j^i type j's holdings of asset i at the end of the first period, as $a_j = a_j^1 + a_j^2$ j's total asset holdings, and as p^i the equilibrium price of asset i.

Figure 6 illustrates how risk aversion changes the incentives to trade collateralized loans. As a function of (uniformly distributed) asset payoff $s \in \{0.1, 1.9\}$, it depicts payoffs of collateralized loans (in the left panel) and leveraged assets (in the right panel) discounted using type H's 'autarky pricing kernel', equal to her stochastic discount factor when optimally choosing storage but keeping her asset endowment $\overline{a}$ unchanged, for $\overline{s} = 1$ and different values of $\overline{a}$ and risk aversion γ.[31] For low asset endowment and low risk aversion, the discounted payoffs approximately equal those under risk-neutrality. Collateralized loan payoffs are thus a concave function of the underlying asset payoff s while leveraged asset payoffs follow a convex function.

[31]Given their identical endowments these 'autarky valuations' are similar for both types (and only differ because of small differences in precautionary storage), but their expectations differ because of disagreement about payoff dispersion.

At high values of risk aversion, and with large asset holdings $\overline{a}$, however, the shape of the pricing kernel, which follows a declining and (with CRRA preferences) convex relation with consumption in the second period, affects more strongly the relationship of discounted payoffs with the underlying payoff s. Apart from increasing precautionary savings (and thus reducing the level of the discounted payoffs, which otherwise would all cross the point $(1, 1)$ in the left panel), the fact that low payoffs become more valuable relative to high payoffs has two effects: First, it reduces the average discounted payoffs of leveraged assets (which pay nothing below $\overline{s}$) relative to those of collateralized loans. Second, discounted payoffs no longer inherit the convexity and concavity properties from the risk-neutral case. The first effect dampens the impact of disagreement on asset prices through leveraged asset trade when risk aversion is high and asset supply large. The second effect makes it, essentially, impossible to derive general analytical results for the environment with risk aversion. With uniform payoffs, we can, however, show that type H agents always buy assets using leverage, and that the within-economy collateral premium is strictly positive for exogenous and extreme levels of leverage.

5.1.1 Equilibrium leverage and the within-economy collateral premium

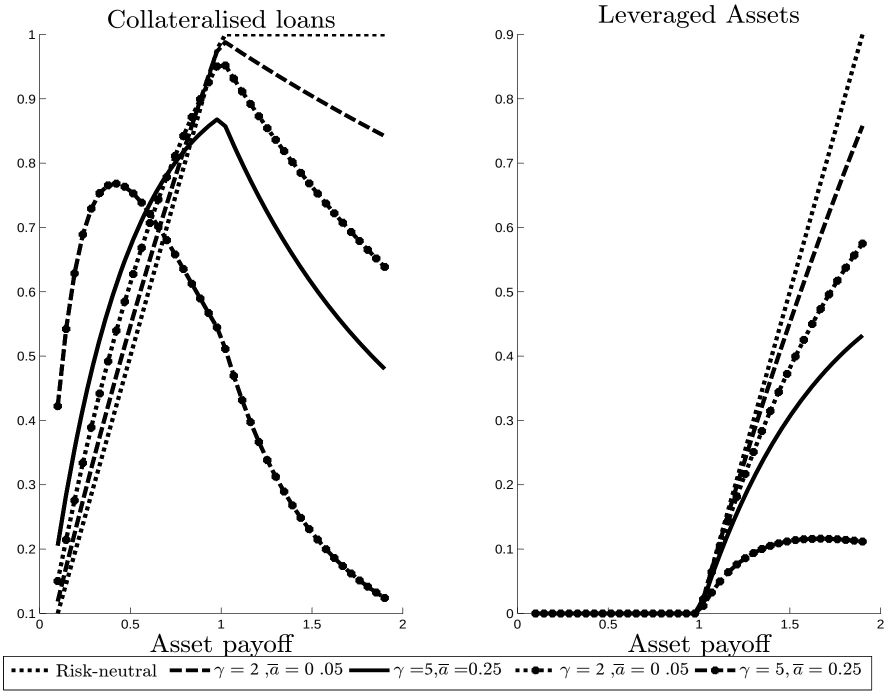
To derive analytical results, we assume that assets are endowed to outside agents who derive utility only from first period consumption, as in Simsek (2013). This eliminates wealth effects of price changes. We return to the standard assumption of asset endowments in the quantitative analysis.

Proposition 5 - *Leveraged asset trade in equilibrium*

Consider an economy where asset 1 *is traded and collateralized loans of face values* $1 - \epsilon_L$ *and* $1 + \epsilon_L$ *are available. In any equilibrium, type* H *always buys a strictly positive amount of asset* 1 *and uses at least part of it as collateral for loans.*

The proof of proposition 6, in the appendix, shows that there is no equilibrium without collateralized loans as, in any such equilibrium, type H would perceive a profitable deviation from

Figure 6: Discounted asset payoffs in autarky with risk aversion



As a function of asset payoffs $s \in [0.1, 1.9]$ along the bottom axis, the figure plots the discounted 'autarky' payoffs of collateralized loans $\frac{u'(c'(s))}{u'(c)} \min\{s, \overline{s}\}$ and leveraged assets $\frac{u'(c'(s))}{u'(c)} \max\{s - \overline{s}, 0\}$ when agents choose their storage optimally but keep their asset endowment $\overline{a}$ unchanged, for $\overline{s} = 1$ and different values of relative risk aversion γ, and two levels of the asset endowment $\overline{a}$ whose expected payoffs equal, respectively, 10 and 40 percent of average per-period consumption.

selling collateralized loans to type L that the latter either perceives as riskless ($\overline{s} = 1 - \epsilon_L$) or payoff-equivalent to the asset itself ($\overline{s} = 1 + \epsilon_L$). The following proposition shows, however, that even with equilibrium leverage there is no within-economy collateral premium unless type H buys the whole supply of collateral assets using leverage.

Proposition 6 - *Within-economy collateral premium I*

Consider the economy where both assets are traded ($\overline{a}^1 > 0, \overline{a}^2 > 0$). There is no within-economy collateral premium unless type H agents buy the whole supply of asset 1 using leverage.

Proof. Note that type L agents would never use leverage to buy asset 1. Thus, they buy asset 1 only at a price that does not exceed that of asset 2. Moreover, even when type H holds the whole supply of asset 1 but does not use all of it as collateral, arbitrage between assets 1 and 2 using type H's equilibrium pricing kernel implies $p^1 \leq p^2$. ■

Since it is difficult to characterize equilibria where $\overline{s}$ is optimally chosen as in the benchmark case, the remainder of this section takes as given a single exogenously given level of leverage $\overline{s}$.

Proposition 7 - *Within-economy collateral premium II*

Consider the economy where both assets are traded. The equilibrium price of asset 1 *strictly exceeds that of asset* 2 *if either of the following conditions holds*

1. $\overline{s} = 1 + \epsilon_L$ *and the endowment of asset* 1 *does not exceed that of asset* 2 ($\overline{a}^1 \leq \overline{a}^2$).

2. $\overline{s} = 1 - \epsilon_L$ *and asset endowments are small in the sense that* $b_L > (1 - \epsilon_L)\overline{a}^1$ *and* $a_H > \overline{a}^1$.

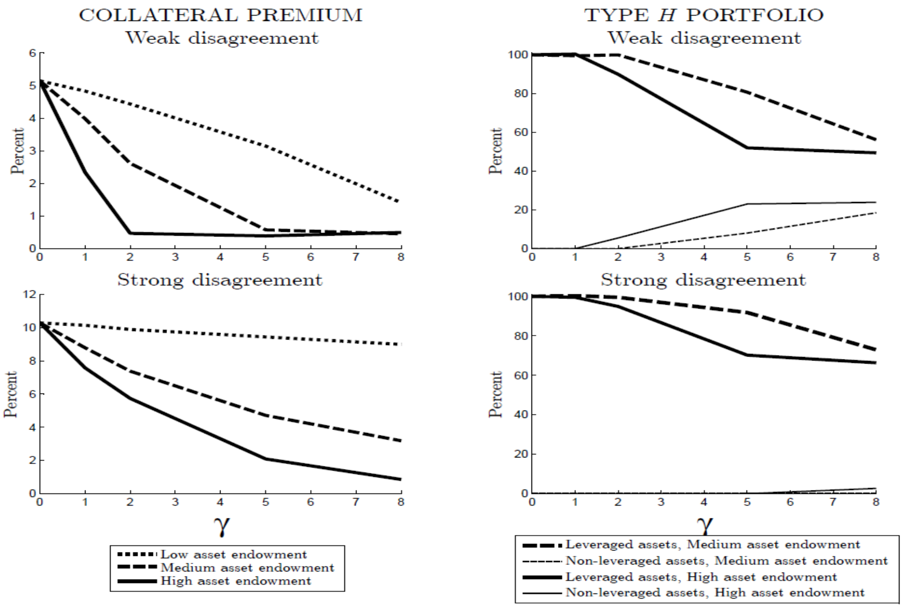
Note that, since storage b_L approaches $\frac{1}{2}$ as $\overline{a}$ approaches 0, and since $a_L > 0$ and $a_H > 0$ whenever $\overline{a} > 0$, there are numbers $\overline{a} > 0$ and $\overline{a}^1 > 0$ that fulfill the condition in 2. The proof is in an appendix and exploits the fact that, for a sufficiently small supply of collateralizable asset 1 and either of the two extreme levels of $\overline{s}$, type L is happy to substitute her investments in either storage or outright asset purchase with the entire collateralized loan supply at an unchanged expected return. At the price that prevailed in the absence of leverage, H's expected return from the marginal unit of leveraged assets thus exceeds its costs, implying an increase in the equilibrium asset price.

Note that all results in this section hold when we relax the assumption of uniform distributions, as long as the support of payoffs perceived by type H has upper and lower bounds that are, respectively, strictly greater and lower than those of type L agents.

5.1.2 A quantitative analysis of the cross-economy collateral premium

The analytical results focused on extreme values of leverage equal to the bounds of type L's perceived payoff distribution and the resulting within-economy collateral premium. This is because it is difficult to theoretically study intermediate leverage levels, or to characterize the cross-economy collateral premium, equal to the relative asset price in identical economies with and without collateralized loan trade. Since it is similarly difficult to compute within-economy collateral premia, requiring the solution to a four-asset general equilibrium portfolio problem, with accuracy, the rest of the section looks at the cross-economy collateral premium in several quantitative examples of risky collateralized lending when leverage equals the intermediate value $\bar{s} = 1$, the optimal face value with risk-neutrality under assumption A3. For this, we set $\epsilon_H = 0.9$ and look at two different values for ϵ_L such that the standard deviation of asset payoffs s perceived by type L is, respectively, 20 percent ('weak disagreement') and 40 percent ('strong disagreement') lower than that of type H. To highlight the importance of the portfolio share of risky assets, we set $\bar{a}$ such that expected asset payoffs approximately equal 2.5, 20 and 40 percent of average per-period consumption, which we call 'low', 'medium' and 'high' asset supply, respectively.[32] For different values of risk aversion γ (along the bottom axis),

Figure 7: Cross-economy collateral premium and portfolios with risk aversion



For different values of risk aversion γ along the bottom axis the figure presents, in its left hand column, the cross-economy collateral premium (calculated as the percentage difference of asset prices in an economy where the whole asset stock can be used as collateral for loans and those in an economy without collateralization) and, in its right hand column, the leveraged and non-leveraged assets held by type H as a percentage of the total asset supply.

[32]We focus on the share of average consumption as it is unaffected by equilibrium prices. The corresponding values of $\bar{a}$ are 0.0125, 0.1, and 0.25 respectively. To compute the equilibria, we use discrete uniform distributions with 7 support points.

Figure 7 depicts in its left hand column the cross-economy collateral premium, calculated as the percentage difference of asset prices in an economy where the whole asset stock can be leveraged ($\overline{a}^1 = \overline{a}$) and those in an economy without collateralization ($\overline{a}^2 = \overline{a}$). With risk-neutral agents, the premium is independent of the economy's asset supply (as all cases we look at fulfill assumptions A2 and A3), and at around 10 percent about twice as large under strong disagreement (in the bottom left panel) compared to weak disagreement (in the top left panel). As suggested by Figure 6, rising risk aversion γ reduces the attractiveness of leveraged assets, and thus the collateral premium. And, as expected, for a large asset supply the premium declines faster (to about a tenth of its risk-neutral value at $\gamma = 8$) than at low endowments (where the premium at $\gamma = 8$ is still 90 percent of its risk-neutral value in the case of strong disagreement). In fact, type H agents are increasingly less willing to hold a large stock of assets using leverage at higher risk aversion. As the right-hand panels of Figure 7 show, when the incentives to diversify their portfolio rise with γ, type H agents eventually invest in assets without leverage. Their leveraged investments decline faster than non-leveraged investments rise, however. This is because type L agents start buying the asset as its price drops with rising risk aversion. Importantly, even when both agents buy a positive quantity of assets without leverage, the cross-economy collateral premium remains positive. This is in contrast to the within-economy collateral premium, which, according to proposition 6, drops to zero in this case. In fact, at high values of γ, a small quantitiy of asset 2 would achieve the same price as asset 1. This price, however, exceeds that in an economy without collateralization: using part of her asset holdings as collateral for loans sold to type L agents makes type H's consumption less sensitive to downside payoff risk and thus increases her valuation even of non-leveraged assets.

5.2 Options trade

In the benchmark environment, the equilibrium price of assets is elevated because of their collateral value: collateralized loans are the only means of exploiting perceived gains from trading upside and downside risk. Collateralization can thus be viewed as a substitute for trade in simple

(European) options, whenever these are not available, too costly or simply not used. This section shows, however, that with disagreement about risk, collateralization continues to be used for speculative purposes and collateral prices continue to include a premium even when options are available and traded. The size of the premium, however, depends crucially on collateral requirements for options.[33]

When there is no other collateral than the exogenous asset, put options, whose payouts are high when assets pay little, cannot be collateralized. Call options, in contrast, pay in high payoff states and can thus be collateralized by the asset or any other claim collateralized by it. In equilibrium, type L agents thus optimally use their loan portfolio as collateral for issuing call options, whose payouts they expect to be low. As is easy to show formally[34], this raises the expected return on collateralized loans, the equilibrium loan price q and ultimately the price of the collateral asset above its value in the absence of options trade. That options trade increases asset prices is, in fact, not surprising: when more complex contracts allow agents to better exploit perceived gains from trade, collateral for financial trade becomes more valuable, implying a higher equilibrium price of collateral assets.

The rest of this section concentrates on trade in options when there is an additional cash technology to collateralize them. Typically, this widening of the collateral pool reduces the price of other collateral assets through collateral arbitrage. In our environment, however, cash is an inefficient form of collateral for call options relative to the asset itself: while, for any given strike price, one unit of the asset always suffices to collateralize a call option, the cash collateral requirement rises one-for-one with the maximum asset payoff. Proposition 8 shows how this implies that, when cash is not too unequally distributed among investor types, the asset price continues to include a premium, similar to the equilibrium without options trade.

[33]To see how trade in collateralized loans and options imply the same payoffs, note that issuing a put option with strike price s^p collateralized by s^p units of riskless debt is equivalent to buying risky debt with face value s^p collateralized by the asset: both have payoffs equal to $min\{s, s^p\}$ in period 1. Similarly, buying the asset using a collateralized loan of face value $\overline{s}$ as leverage has the same stochastic payoff as buying a call option with strike price $s^c = \overline{s}$, or - due to to 'put-call-parity' - as buying the asset plus a put option with equal strike price and issuing riskless debt equal to $\overline{s}$.

[34]Results for this case are contained in a previous draft that is available from the authors upon request. There, we also provide an example where trade in call options doubles the equilibrium collateral premium.

This section sets $s^{min} = 0$ and $R = 1$ to ease notation. In addition to the exogenous asset and collateralized loan of Section 3, agents can also trade simple (European) put and call options, whose issuer receives a fee p_c and p_p in return for payments equal to $max\{s - s_c, 0\}$ and $max\{s_p - s, 0\}$, respectively, in period 1, where s_c and s_p are the strike prices of the options. The equilibrium definition is the same as that in section 3, amended to include two additional assets with associated collateral requirements. Suppose there is a cash asset, or storage technology, which returns $R = 1$ units of consumption in period 1 for 1 unit of consumption invested in period 0 and can be used as collateral for options trade. Importantly, this eliminates the kind of cash-rich equilibria we focused on in the previous section, because the consumption endowment now represents not just asset demand, but also collateral supply. Option prices are thus a function of supply (through type L's endowment) and demand (type H's endowment). Depending on their relative size, the perceived gains from trading them accrue to the issuer, the buyer, or both, implying that the average expected portfolio return $R_i, i = L, H$ of one or both agents exceeds their discount factor $R = 1$. The exogenous asset supply continues to be used as collateral either for call options or loans, which yields higher returns than buying assets outright. Prices are determined by 'collateral arbitrage', such that agents are indifferent between posting cash or the asset as collateral.

We show how the asset price continues to include a premium, as the returns on the exogenous asset $\frac{E_s}{p}$ are lower than those on options as long as both agents share the perceived gains from trade (equivalent to $R_L, R_H > R$). Asset prices may or may not, however, exceed the fundamental value $\frac{E_s}{R}$. Intuitively, the cash required to collateralize options is a function of the maximum payoffs only, equal to either s^p (for put options, since we normalize the lower bound $s_{min} = 0$) or $s^{max} - s^c$ (for call options). The value of the collateral asset, however, depends on its whole payoff distribution, not just the extremes. The higher the probability of low asset payoffs (which reduces the fundamental asset value) and the wider the range of payoffs (which increases the required amount of cash collateral), the cheaper the exogenous asset becomes as a form of collateral relative to cash. The proof of a positive asset price premium even with options trade, in the appendix, is complicated by the fact that strike prices are endogenous and may differ between the four assets (cash-collateralized calls and puts, asset-collateralized loans and calls). We therefore use a 'revealed portfolio preference' argument: whenever cash-collateralized call and put options are traded, their return must not be lower than that on asset-collateralized calls or loans respectively, which - at the same strike prices - would have the same period 1 payoffs. This bounds the price of the asset from below.

Proposition 8 - *Asset prices with option trade and cash collateral*

When cash-collateralized put and call options are traded, and endowments are such that perceived gains from trade are shared (in the sense that both agents expect positive returns $R_L, R_H > R = 1$), the asset price exceeds discounted values expected by either type: $p > \frac{E_s}{R_i}, i = L, H$.

The following proposition shows, by example, how asset prices may be above their fundamental value $\frac{E_s}{R}$ even when cash-collateralized put and call options that reference its payoff distribution are traded.

Proposition 9 - *Asset prices may exceed their fundamental value*

When cash-collateralized put and call options are traded, the asset price may exceed its fundamental level $\frac{E_s}{R}$.

Proof of proposition 9. The proof is by example. Suppose $S = \{0, 1, 2\}$ with $f_L = \{\frac{5}{8}, \frac{2}{8}, \frac{1}{8}\}$, $f_H = \{\frac{3}{4}, 0, \frac{1}{4}\}$, $n_L = \frac{11}{30}$ and $n_H = \frac{19}{30}$. Take expected portfolio returns $R_L = \frac{5}{4}$ and $R_H = \frac{15}{14}$, and an asset price $p = \frac{16}{30} > \frac{E_s}{R} = \frac{1}{2}$. To see how this is indeed an equilibrium, note that type L and H perceive $s^p = s^c = 1$ and $\overline{s} = 1$ as, respectively, optimal strike prices for options and an optimal face value of collateralized loans, because return functions all have a kink at $s = 1$.[35] Puts are then priced by type L agents such that

$$\frac{E_L[min\{1, s\}]}{1 - p^p} = \frac{5}{4} \tag{16}$$

[35]Concentrating on put options and loans, since all assets issued are priced at the assumed portfolio returns, we have price functions $p^p = \frac{14}{15}E_H[max\{s^p - s, 0\}]$ and $q = \frac{4}{5}E_L[min\{s, \overline{s}\}]$. This implies an issuer return of $\frac{E_L[min\{s^p,s\}]}{s^p - \frac{14}{15}E_L[min\{s^p,s\}]} = \frac{\frac{1}{4}min\{s^p,1\}+\frac{1}{8}s^p}{\frac{6}{20}s^p}$ for put options and $\frac{E_H[max\{\overline{s}-s\}]}{p-\frac{4}{5}E_L[min\{\overline{s},s\}]} = \frac{\frac{1}{4}(2-\overline{s})}{p-\frac{4}{5}(\frac{1}{4}min\{\overline{s},1\}+\frac{1}{8}\overline{s})}$ for loans. The former is flat below and declining above $s^p = 1$. The latter is increasing below and decreasing above $\overline{s} = 1$.

implying $p^p = \frac{7}{10}$, and similarly $p^c = q = \frac{3}{10}$. Type H's expected return from buying cash-collateralized put and call options at these prices are $R_H^p = \frac{15}{14}$ and $R_H^p = \frac{10}{12}$ respectively, implying that cash-collateralized call options are not actually traded. Leveraged asset investments by type H agents have an expected payoff equal to $\frac{E_H[max\{s-1,0\}]}{p-q}$ which implies an arbitrage asset price equal to $\frac{16}{30}$.[36] At the assumed endowments (plus the return from selling put options and their asset endowment equal to 1), type H agents can exactly afford to issue 1 unit of put options and buy all 2 units of collateralized loans. Similarly, type H agents can afford to buy all put options and all assets using leverage. ■

Note that the asset returns in the example crucially depend on relative endowments: it is easy to see that when $n_L \geq \frac{3}{5}n_H + \frac{2}{5}$ type H agents' endowment is sufficiently small to lower option and loan prices to type L's reservation level, implying $R_L = 1$, and $R_H = 2$. In other words type H agents harvest all perceived gains from trade. The reverse is true when type L's endowments are small enough, or $n_H \geq 3n_L$, implying $R_H = 1$, and $R_L = 2$. In both cases, the asset price equals its fundamental value: $p = \frac{E_s}{R} = E_s$.

6 Conclusion

Motivated by the strong, and in the case of GDP forecasts rising disagreement about the dispersion of outcomes in US surveys of investors and forecasters, this paper has looked at the role of collateralized asset trade in economies where investors disagree about risk, rather than mean payoffs as in the literature. A simple static model of investor disagreement showed how the introduction of simple collateralized loans allows investors who perceive high payoff dispersion to purchase upside risk by buying assets and using them to collateralize loans that their low-risk counterparts value highly. Extensions of the benchmark analysis showed how similar results hold with risk-averse investors and more sophisticated collateralized contracts. A quantitative

[36] At this asset price and type H's valuation of a call option, type L expect a return from issuing asset-collateralized call options equal to their portfolio return. They are thus indifferent between issuing them or not. We look at an equilibrium where call options are not traded, other than implicitly through leveraged asset trade.

application to the market of US subprime RMBSs and CDOs showed how disagreement about the volatility of default rates, or the importance of aggregate factors for mortgage defaults, raises the price of junior RMBS tranches by between 40 and 110 basis points. For junior CDO tranches, the rise is an order of magnitude larger.

The theory presented in this paper has additional empirical predictions that can be compared to data even without information on the, typically unobserved, risk perceptions of investors.[37] For example, our mechanism requires that investors can issue non-recourse collateralized loans. It thus predicts an effect of heterogeneous risk perceptions on the price of private-label residential mortgage-backed securities (but not of seemingly government-guaranteed agency securitizations), or on house prices in jurisdictions with non-recourse residential mortgages (but not, or less so, in those with recourse mortgages such as some US states and most European countries). Moreover, we would expect larger effects in markets where risk is important (in the sense of substantial default probabilities), and where disagreement about risk is likely to be stronger (such as for assets or contracts with a shorter history). Finally, we would expect the effect to increase over time both because disagreement about risk has seemingly increased (at least in the given sample of forecasters interviewed by the SPF), and because issuers of collateralized assets were increasingly able to draw on a more international and diverse investor pool. We leave formal empirical tests of these predictions to future research.

We also hope that our analysis opens some avenues for further theoretical research. Thus, a dynamic analysis, where risk arises both from future payoffs and price movements, seems particularly interesting,[38] as do concrete applications of the theory to other financial markets. Finally, an investigation into the sources of disagreement, or the determinants of risk perceptions, would be valuable.

[37]A case of observed disagreement is that about ratings, as documented by Norden and Roscovan (2014) in a large sample of US and European firms. It would be interesting to study empirically how credit rating disagreement affects asset prices.

[38]The working paper version of this paper (Broer and Kero, 2014) presents a simple example of a dynamic equilibrium in a scenario with learning that tries to capture the main features of the Great Moderation in the US. As a subset of investors adjusts their posterior estimate of volatility more quickly to the Great Moderation than the rest, increasing divergence of posteriors raises asset prices between 5 and 20 percent.

Our results imply that investors on average make losses relative to their required rate of return. This suggests that there might be welfare-improving policy interventions that would be interesting to study.[39] Moreover, the fact that disagreement about payoff dispersion makes investments more risky and raises leverage in the economy should be of interest for policy makers and regulators.

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7 Appendix: Omitted proofs

Proof of Proposition 3.

Equation (10) is simply the optimality condition for leverage choice. To understand equations (11) and (12), note that for any $p < \frac{E_s}{R}$ all agents would like to buy risky assets, which cannot be an equilibrium. Equivalently, for any $p > \overline{p} \doteq \frac{E_s + E_L(min(s,\overline{s})) - E_H(min\{s,\overline{s}\})}{R}$ both type L and type H agents would like to sell their risky assets, again contradicting equilibrium. Agent 1 optimality implies that they invest all resources in leveraged assets when $\frac{E_s}{R} \leq p < \overline{p}$, but are indifferent between buying leveraged assets and consuming at $p = \overline{p}$. Thus, for $\overline{s}(\overline{p})$ the value of $\overline{s}$ that solves (10) when $p = \overline{p}$, if $n_1^{\max}(\overline{s}(\overline{p})) \geq \overline{p}$, type H's endowment is large enough to buy type L's assets at the maximum price $\overline{p}$ that ensures her participation. There is thus an equilibrium price $\overline{p}$ at which type H agents are happy to consume in period 0 any resources that remain after purchasing all of type L's assets.

If for some price $p : \frac{E_s}{R} \leq p < \overline{p}$ $n_1^{\max}(\overline{s}(p)) < p$, type H agents cannot buy all assets at that price but expect to make strictly positive profits $R_H^a > R$, so invest all their resources to buy type L's assets, implying market clearing condition (12).

Finally, to prove uniqueness, since (12) is trivially strictly upward-sloping, it suffices to show that (10) is downward-sloping. This follows by differentiating (10) totally

$$\frac{dp}{d\overline{s}} = -\frac{\frac{d\mathbb{C}}{d\overline{s}}}{\frac{d\mathbb{C}}{d\overline{p}}} \tag{17}$$

Weak concavity of $R_H^a(\overline{s})$ at the optimum choice of $\overline{s}$ implies that the numerator is weakly negative. Since $\frac{\mathbb{C}}{d\overline{p}} < 0, \forall p, \overline{s}$ the result follows. ∎

Proof of Corollary 2.

Note that for any symmetric distribution $E_s = s^\star = \frac{1}{2}(s_{max} + s_{min})$. So

$$\begin{aligned}\underline{n_H} = \frac{E_s - E_L(min\{s, s^\star\}) - E_H(min\{s, s^\star\})}{R} \\ \leq \frac{E_s - 2s^\star(1 - F_L(s^\star))}{R} \\ = \frac{E_s - 2E_s\frac{1}{2}}{R} = 0,\end{aligned} \tag{18}$$

where the last line follows from $s^\star = E_s$ and $1 - F_L(s^\star) = 1 - F_H(s^\star) = \frac{1}{2}$ due to symmetry. ∎

Proof of Lemma 1.

Under Assumption A3 we have

$$\begin{aligned} n_1^{\max}(s^\star) &\geq \frac{E_s - E_L(min\{s, s^\star\}) - E_H(min\{s, s^\star\}) + 2E_0[\min\{s, s^\star\}]}{R} \\ &\geq \frac{E_s + E_0[\min\{s, \overline{s}\}] - E_H(min\{s, s^\star\})}{R} = \overline{p} \end{aligned} \tag{19}$$

which implies that type H agents have resources larger than the value of assets evaluated at any $p \leq \overline{p}$. So equilibrium requires type H agents to be indifferent between consuming and investing in leveraged assets, implying an equilibrium price equal to $\overline{p}$. ∎

Proof of proposition 6.

In the absence of leverage, both agents are indifferent between assets 1 and 2. Type H always purchases a strictly positive total amount of assets $a_i > 0$ that is strictly smaller than that purchased by type L.[40] When collateralized loans are available, there are deviations from this portfolio that type H perceives as strictly profitable. First, if type L stores a positive amount, type H can issue at least a small positive quantity of collateralized loans with unit face value $\overline{s} = 1 - \epsilon_L$. Type L perceives this loan as riskless and is happy to substitute it at the unit price $1 - \epsilon_L$ for (part of) her storage. Storing the proceeds in a separate account, type H's payoffs strictly dominate those of the original portfolio, as she earns an additional amount

[40] To see this, assume that either type holds 0 assets. Her portfolio is thus riskless, and she values the asset at the fundamental value E_s, which exceeds any price the second type will be happy to pay when holding the whole asset supply, independent of her amount of storage. A similar contradiction can be derived assuming $a_L <= a_H$.

$(1-\epsilon_L) - s^i_H > 0$ whenever payoffs are below $1-\epsilon_L$.

If type L does not store, type H can still offer to buy some of her asset holdings in exchange of a collateralized loan with face value $1+\epsilon_L$. Type L is exactly indifferent between the asset and that loan, which she perceives to have identical payoffs since, for her, $min\{s, 1+\epsilon_L\} = s$. Type H, however, expects to receive a net payment of $s-(1+\epsilon_L)$ for high realizations of $s > 1-\epsilon_L$. There is thus no equilibrium without leverage. ■

Proof of proposition 7.

Ad 1. We show how, in any equilibrium with leverage at $\bar{s} = 1+\epsilon_L$, type L agents hold collateralized loans and non-collateralized assets that they regard as payoff equivalent, implying $q = p^2$. Type H's perceived net profits from leveraged asset purchase then imply $p^1 > p^2$.

In the absence of leverage, both assets trade at the same price p^{nc} and type L agents, who hold a larger quantity of assets than type H, can buy the whole supply of asset 1 since $a^0 > \frac{1}{2}\bar{a} \geq \bar{a}^1$. Suppose instead type H buys the entire supply of asset 1 financed by collateralized loans of face value $1+\epsilon_L$. Type L agents see those loans as payoff-equivalent to both assets. Whenever $min\{q, p^1, p^2\}$ does not exceed p^{nc}, their combined demand of assets and loans thus strictly exceeds the total quantity of collateralized loans (equal to $\bar{a}^1$). Moreover, since demand for non-collateralized assets by type H agents is strictly reduced by their leveraged asset purchases (whose net payoff is perfectly correlated with non-collateralized asset payoffs for $s > 1-\epsilon_L$), we can rule out a price of asset 2 above p^{nc}. Market clearing together with type L's arbitrage condition for assets and loans thus requires $p^2 \geq q$ and $p^1 \geq q$ with at least one equality. Type H agents' optimality condition and market clearing for asset 1 imply

$$p^1 = E_H[\frac{U'(c'_H)max\{s-(1+\epsilon_L)\}}{U'(c_H)}] + q > q \tag{20}$$

implying $p^2 = q$ and $p^1 > p^2$.

Ad 2. When $a_H > \bar{a}^1$ type H agents can buy the entire stock of asset 1 and collateralize it using loans issued at $\bar{s} = 1-\epsilon_L$ to type L agents. The price of this loan is simply $q = 1-\epsilon_L$ from arbitrage with type L's remaining storage investment, which is positive since $b_L > (1-\epsilon_L)\bar{a}^1$.

Type H agents' optimality condition and market clearing thus imply

$$p^1 = E_H[\frac{U'(c'_H)max\{s-(1+\epsilon_L)\}}{U'(c_H)}] + 1 - \epsilon \tag{21}$$

$$= E_H[\frac{U'(c'_H)}{U'(c_H)}s] + E_H[\frac{U'(c'_H)}{U'(c_H)}max\{(1+\epsilon_L)-s,0\}]] > p^2 \tag{22}$$

where the last inequality follows because the payoff of a marginal unit of asset 1 perceived by type H dominates that of asset 2 for every state $i = 1, ..., N$ (and strictly so for states where $s < 1 - \epsilon_L$). Since type H's pricing kernel is strictly positive, this implies a higher equilibrium price of asset 1. ■

Proof of proposition 8.

Suppose cash-collateralized put options are traded. A put option with strike price s^p collateralized by s^p units of cash gives a type L issuer a period 1 payoff equal to $min\{s, s^p\}$ and requires a net injection of cash collateral equal to $s^p - p^p > 0$. Type H can thus always sell type L at price $q = s^p - p^p$ a collateralized loan of face value s^p that yields the same period 1 payoff $min\{s, s^p\}$. The resulting expected return must not be larger than the portfolio return expected from type H's actual investments (which include the put option by assumption): $R_H \geq \frac{E_H[max\{s,s^p\}]}{p-q}$. This puts a lower bound on the asset price

$$\begin{aligned} p &\geq \frac{E_H[max\{s-s^p,0\}]}{R_H} + s^p - p^p \\ &= \frac{E_H[max\{s-s^p,0\}]}{R_H} + s^p - \frac{E_H[max\{s^p-s,0\}]}{R_H} \\ &= \frac{E_s}{R_H} + \frac{R_H-1}{R_H}s^p \end{aligned} \tag{23}$$

where the second line follows since type H agents expect a return from put options equal to their portfolio return $R_H = \frac{E_H[max\{s^p-s,0\}]}{p^p}$ and the third from 'put call parity' at the common strike price s^p. The asset price thus strictly exceeds type H's 'fundamental valuation' $\frac{E_s}{R_H}$ whenever $R_H > 1$.

By a similar reasoning, when cash-collateralized call options are traded at a strike price

s^c, the return from issuing them using $s_{max} - s^c$ units of cash as collateral must equal type L's expected portfolio return: $R_L = \frac{E_L[max\{s_{max}-s, s_{max}-s^c\}]}{s_{max}-s^c-p^c}$. Asset prices must be such that the type L issuers do not strictly prefer to issue asset-collateralized calls whose return equals $\frac{E_L[min\{s,s^c\}]}{p-p^c}$. This again bounds the asset price

$$
\begin{aligned}
p &\geq \frac{E_L[min\{s, s^c\}]}{R_L} + p_c \\
&= \frac{E_L[min\{s, s^c\}]}{R_L} + \frac{E_L[max\{s_{max} - s, s_{max} - s^c\}]}{R_L} + s_{max} - s^c \\
&= \frac{E_s}{R_L} + \frac{R_L - 1}{R_L}(s_{max} - s^c) \qquad (24)
\end{aligned}
$$

where the third line follows since $E_L[min\{s, s^c\} + E_L[max\{s_{max} - s, s_{max} - s^c\} = E_s$. Again, $p > \frac{E_s}{R_L}$ whenever $R_L > 1$. ■

8 For online publication only: Robustness of the results in Section 4

This section shows how the effect of disagreement on the price of the structured collateral cash flow increases with the default probability $\overline{\pi}$, and when recovery values V_{rec} depend negatively on default rates, as in the benchmark analysis, as opposed to a constant V_{rec}.[41] The fact that high default rates and default-rate-sensitive recovery values are characteristics of US subprime mortgage markets suggests that investor disagreement about default correlation may have been of particular importance there.

Alternative assumptions about the default probability Tables 5 and 6 show the returns from structuring the mortgage pool at values of the default probability $\overline{\pi}$ higher and lower than the benchmark of 12.5 percent.[42] Note that asset correlations between 0 and 1

[41] A previous version of this paper also shows how the effect is higher with a larger number of assets in the pool, reducing stochastic 'noise' in default rates, and thus increasing the role of the default parameter ρ in determining default distributions.

[42] The effect of alternative assumptions about the recovery value on the payoff distribution, and thus

Table 5: Return to structuring the mortgage pool for different values of $\overline{\pi}$

	$\overline{\pi} = 7.5\%$	$\overline{\pi} = 20\%$
Weak disagreement about ρ^2	21	61
Strong disagreemen about ρ^2	57	161

The table presents the return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitization, measured in basis points (100th of a percent) of the latter's market price, for different values of the default probability $\overline{\pi}$.

Table 6: Return to re-tranching for different values of $\overline{\pi}$

	$\overline{\pi} = 7.5\%$	$\overline{\pi} = 20\%$
Weak (weak) disagreement about ρ^2 ($\widehat{\rho}^2$)	28	91
Weak (strong) disagreement about ρ^2 ($\widehat{\rho}^2$)	37	123
Strong (weak) disagreement about ρ^2 ($\widehat{\rho}^2$)	61	182
Strong (strong) disagreement about ρ^2 ($\widehat{\rho}^2$)	66	207

The table presents the return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitization, measured in basis points (100th of a percent) of the latter's market price, for different values of the default probability $\overline{\pi}$.

map into default variances between 0 and $\overline{\pi}(1-\overline{\pi})$. At default probabilities below 50 percent, any given disagreement about ρ thus implies a larger disagreement about the distribution of default rates as $\overline{\pi}$ increases. For example, when the probability of default equals 20 percent, the maximum return to tranching is between 35 and 75 basis points higher than in the benchmark case. Similarly, the maximum return when also selling RMBS tranches to investors who disagree about $\widehat{\rho}_k^2$, or the diversification gain of pooling different RMBS tranches, rises to about 210 basis points in this case, as can be seen in Table 6.

Constant recovery value V_{rec}

The benchmark quantitative results of this paper were derived under the assumption that recovery values of mortgages are negatively affected by realized default rates in order to account for longer time-until-foreclosure and lower resale values when default rates are high. Since losses on individual mortgages are then large (small) when many (few) loans default, this increases the payoff variance from the mortgage portfolio for any given $\rho > 0$, and thus amplifies the effect of disagreement about ρ. Indeed Tables 7 and 8 show that returns from structuring the mortgage

on the prices of CDO and RMBS tranches, is very similar to that of alternative default probabilities, and thus omitted here.

Table 7: Return to structuring the mortgage pool with constant recovery value V_{rec}

	Max	**Equ**
Weak disagreement about ρ^2	30	30
Strong disagreement about ρ^2	71	71

The table presents the return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitization, measured in basis points (100th of a percent) of the latter's market price, when the recovery value V_{rec} is constant at 60 percent.

Table 8: Return to re-tranching with constant recovery value V_{rec}

	Weak disagreement about ρ^2	**Strong disagreement about ρ^2**
Weak disagreement about $\widehat{\rho}^2$	45	83
Strong disagreement about $\widehat{\rho}^2$	60	95

The table presents the partial equilibrium return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitization, measured in basis points (100th of a percent) of the latter's market price, when the recovery value V_{rec} is constant at 60 percent.

pool are between 5 and 15 basis points lower than those in Table 3 when the recovery value V_{rec} is constant at 60 percent. Interestingly, both returns in Table 7 are now equal. To understand this, remember that the higher equilibrium return in the benchmark results was entirely due to a lower equilibrium price of the pass-through securitization. This was because, when the recovery value declines with default rate d, payoffs fall faster as d rises, implying a concave relationship between payoffs and default rates. Expected payoffs thus decline with the variance of defaults, or with the correlation parameter ρ^2, due to a Jensen's inequality effect. When the recovery value is constant, in contrast, this effect of ρ on the price of the pass-through securitization is absent. So investors agree about its expected value and there is no difference between maximum valuation and equilibrium price.