

# Great Moderation or Great Mistake: Can rising confidence in low macro-risk explain the boom in asset prices?

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## Abstract

The fall in US macroeconomic volatility from the mid-1980s coincided with a strong rise in asset prices. Recently, this rise, and the crash that followed, have been attributed to overconfidence in a benign macroeconomic environment of low volatility. This paper introduces uncertainty and learning about the persistence of volatility regimes in a standard asset pricing model. With full information about the highly persistent, but not permanent, nature of volatility regimes, the observed fall in US macroeconomic volatility leads to a relatively small increase in asset prices. Uncertainty about persistence increases prices during the Great Moderation as investors are willing to pay for a small probability of very persistent low volatility. When agents also update the posterior distribution of persistence values during the Great Moderation using Bayes' rule, prices rise by up to 80% and fall strongly upon return to high volatility. A simple learning mechanism that highlights increasing confidence in a permanent, rather than transitory, fall in volatility predicts the magnitude and timing of the boom better than standard learning about transition probabilities as in Cogley and Sargent (2008a,b).

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*“From the Great Moderation to the Great Conflagration: The decline in volatility led the financial institutions to underestimate the amount of risk they faced [...], thus essentially (though unintentionally) reintroducing a large measure of volatility into the market.”*

Thomas F. Cooley, Forbes.com, 11 December 2008

*“The stress-tests required by the authorities over the past few years were too heavily influenced by behavior during the Golden Decade. [...] The sample in question was, with hindsight, most unusual from a macroeconomic perspective. The distribution of outcomes for both macroeconomic and financial variables during the Golden Decade differed very materially from historical distributions.”*

Andrew Haldane, Bank of England, 13 February 2009

*“But what matters is how market participants responded to these benign conditions. They are faced with what is, in essence, a complex signal-extraction problem. But whereas many such problems in economics involve learning about first moments of a distribution, this involves making inferences about higher moments. The longer such a period of low volatility lasts, the more reasonable it is to assume that it is permanent. But as tail events are necessarily rarely observed, there is always going to be a danger of underestimating tail risks.”*

Charles Bean, European Economic Association, 25 August 2009

*“The remaining question is whether the relaxation in financial prudence could have been triggered by false expectations of a perennially smooth economic environment that policymakers could have avoided in words and deeds.”*

Jean-Claude Trichet, European Central Bank, 5 September 2008

## 1 Introduction

Previous studies have found that a fall in macroeconomic volatility of the magnitude observed in the United States between the late 1980s and the early 1990s would essentially have to be permanent to explain a significant proportion of the subsequent boom in equity prices (Lettau et al 2008). Both academics and market participants, however, seem to have been highly uncertain about the likely persistence of this “Great Moderation” in macro-volatility.<sup>1</sup> Moreover, after the economic

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<sup>1</sup>While some authors have attributed the Great Moderation to structural changes in developed economies that are indeed very persistent, or potentially permanent, such as central bank independence, the increase in world trade, or the development of new financial products to diversify risk, others have pointed to its transitory origins, such as an unusually long period of small exogenous shocks (“good luck”) that hit Western economies during this period. See section II for more detail and statements by market participants.

crisis that started in 2007, both policymakers and academics have attributed the boom in asset prices and their subsequent crash to the overconfidence of investors in a benign macroeconomic environment of low volatility (Bean 2009, Cooley 2008, Haldane 2009, Trichet 2008).

This paper looks at the behaviour of asset prices in an environment where investors are uncertain about the persistence of observed volatility regimes and form beliefs based on empirical evidence. Specifically, we interpret the economic experience of the US economy after World War II as consisting of realisations of high and low volatility regimes, whose transition probabilities are unknown to investors. We show how uncertainty per se increases prices during low volatility periods. This is because small posterior probabilities of high persistence have a strong effect on the asset price due to a 'Jensen's inequality' effect that results from the convexity of full-information prices in persistence values implied by previous results (Lettau et al 2008). When investors also update their priors about transition probabilities in line with observed realisations of high and low volatility regimes according to Bayes' rule, a standard asset pricing model predicts a strong and gradual boom during the Great Moderation, and a bust in asset prices at its end. We compare the size and shape of this boom-bust cycle across several learning mechanisms that turn out to have very different implications. A standard specification where agents have a beta prior and thus attach positive probabilities to the whole range of persistence values (Cogley and Sargent 2008a,b) delivers a boom and bust in asset prices of between 30 and 45 percent. But since agents update their priors most quickly at the beginning of the Great Moderation, this is where prices rise most quickly. The implied concave relation of prices with time is not, however, found in the data. With a two-point prior that captures the debate about the nature of the Great Moderation as either an unusually long sequence of small shocks ("good luck") or permanent structural change ("good policy"), both boom and bust are about twice as large. Moreover, the increase in prices follows an S-shaped pattern with time that is more in line with the evidence. Finally, non-optimal, "adaptive" learning schemes, where investors use simple statistical rules to update their inference about volatility on the basis of observed data, predict strong overvaluation of assets, relative to the prices implied by full information about the data generating process, and a very gradual fall in prices after the end of the Great Moderation (which we identify with the beginning of the economic crisis in 2007).

Within the extensive literature on asset prices with parameter uncertainty and learning, which

we partly review in section 6, this paper is most related to Lettau et al. (2008), who also study the asset price effect of changes in macro volatility-regimes under limited information about the environment. Particularly, while knowing all parameters of the data-generating process, including the persistence of volatility regimes, agents in their model ignore whether the economy is currently in a high or low volatility regime.<sup>2</sup> The authors then calculate asset prices given the sequence of posterior state probabilities implied by an econometric regime-switching model estimated on post-war consumption data for the US. This setup makes the quantitative results highly dependent on the parameter that describes the persistence of the low volatility state. Only when low volatility is, essentially, permanent (in the sense of a probability of remaining at low volatility in the following quarter of more than 99.9 percent), does the Great Moderation imply a strong increase in asset prices. Our contribution is to point out how uncertainty and learning about persistence affects asset prices in a simplified version of the Lettau et al. (2008) model. Specifically, evidence from the academic literature and the business press suggests that both academics and market participants were certain about a change in aggregate volatility from the mid-1990s onwards but unsure about how long it was likely to last. This motivates the main assumption of this paper is that agents observe volatility regimes but are uncertain about their persistence. We point out, first, that uncertainty about transition probabilities per se increases asset prices in the Lettau et al. (2008) model, as an expression of Jensen’s inequality and the strong convexity of prices in the persistence parameter. Our second contribution is to show how, as as the Great Moderation persists and eventually ends in 2007, learning about regime persistence can lead to a strong boom, and subsequent bust, in asset prices that is up to 5 times larger than in the full information economy, but still smaller than that observed in US data. In this sense, we are the first to systematically model the intuition, found in policy statements and the popular press, that increasing confidence in the persistent, or permanent, nature of the Great Moderation contributed to the boom and bust in asset prices in the 1990s and 2000s. Finally, we identify the differences in the maximum size and the time path of asset price booms and busts across different kinds of learning mechanisms.

The rest of the paper is organised as follows. To motivate our approach in more detail, section

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<sup>2</sup>Lettau et al (2008) also have two states of different mean growth, leaving four states of the economy in total.

### Moments of US GDP growth

<i>Date</i>	<i>Mean (%)</i>	<i>StDev (%)</i>
1952Q2 : 1983Q4	0.53	1.1
1984Q1 : 2006Q4	0.51	0.51
2007Q1 : 2012Q3	-0.01	0.76

Table 1: The table reports the mean and standard deviation of the real GDP growth rate. Output is defined in real per-capita terms. GDP and the population data are taken from Bureau of Economic Analysis. The data are quarterly and span the period 1952Q2 – 2012Q3.

II reviews the main empirical facts on the Great Moderation as well as the debate about its causes among academics and market participants. Section III presents the model. Section IV presents the result for a standard Bayesian model of learning about transition probabilities between volatility regimes. Section V compares this to the results under a different prior assumption designed to capture the suspicion about a possibly permanent Great Moderation, and with non-Bayesian learning schemes. Section VI compares the results to those in the literature

## 2 The Great Moderation, its Uncertain Cause and Persistence, and the Boom in Asset Prices

### 2.1 Asset Prices and the Great Moderation: Stylized Facts

Table 1 and 2 reproduce the well-known fact of a decline in the volatilities of both consumption and GDP growth in the US in the 1990s and 2000s. The standard deviations of both approximately halve, although the fall occurred somewhat later for consumption growth. Mean growth rates do not change much.<sup>3</sup> In the third sub-sample that covers the recent crisis, we observe a sharp decrease in mean growth for both GDP and consumption and a strong rise in volatility.

**Enter Figure 1 and 2 about here**

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<sup>3</sup>McConnell and Perez-Quiros (2000) provide evidence that 1984Q1 was the break date for the GDP growth series and Lettau et al. (2008) provide evidence that 1992Q1 was the break date for the aggregate consumption growth series. See the Data Appendix for a more detailed description of the data series.

### Moments of US Consumption Growth

<i>Date</i>	<i>Mean (%)</i>	<i>StDev (%)</i>
1952Q2 : 1991Q4	0.57	0.82
1992Q1 : 2006Q4	0.61	0.36
2007Q1 : 2012Q3	0.04	0.49

Table 2: The table reports the mean and standard deviation of the real consumption growth rate. Consumption is defined in real per-capita terms. Consumption and population data are taken from BEA. The data are quarterly and span the period 1952Q2 – 2012Q3.

Figure 1 shows how the decline in macroeconomic volatility coincided with a strong rise in asset prices and a fall in the US dividend-price ratio for the *S&P* 500. Importantly, this fall was much less abrupt than the decline in volatility itself. The exact magnitude of the rise in US stock market valuation depends on the measure that is used to quantify payouts to shareholders. Specifically, the boom in asset prices between the early 1980s and the early 2000s is around half as strong when dividends are adjusted to include share repurchases, which increased steadily in magnitude since the 1980s.<sup>4</sup> In correspondance to the previous tables, table 3 shows average stock price valuation measures for three subperiods, choosing 1995Q1, the period identified by Lettau et al (2008) as a structural break in the price-dividend ratio, as the start of the second subperiod. The price-dividend ratio more than doubled across the first two periods, while the price-earnings ratio increased more than 90 percent. The rise in the adjusted price-dividend ratio, for which data end in 2003, is with 60 percent about half as strong as that in the unadjusted measure. Both price-dividend and price-earnings ratios fell back to levels seen in the 1960s and 1970s with the start of the recent crisis. We do not choose a preferred valuation ratio among the three measures discussed in this section. But figure 2 suggests that a reasonable lower bound for the magnitude of the asset price boom should be around 100 percent.

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<sup>4</sup>Payouts to shareholders through repurchases of stocks are attractive to firms because of the preferential tax treatment of capital gains relative to high incomes in the US. The importance of repurchases has increased steadily after 1982, when SEC rule 10b-18 clarified the conditions under which firms could avoid an SEC investigation for market manipulation after a share repurchase, to reach a magnitude similar to dividend payments around the turn of the century. See Grullon and Michaely (2002) for details. Note that the share repurchase data are only available between 1971 and 2003.

### US Equity Prices

<i>Date</i>	<i>Mean</i> $\frac{p}{d}$	<i>Mean</i> $\frac{p}{e}$	<i>Mean</i> $\frac{p}{d^{adj}}$
1952Q2 : 1994Q4	27.49	15.54	22.24
1995Q1 : 2006Q4	62.25	30.03	35.90
2007Q1 : 2012Q3	47.9	21.26	

Table 3: The table reports means of the price-dividend ratio  $\frac{p}{d}$  and the price-earnings ratio  $\frac{p}{e}$  for the S&P 500.  $\frac{p}{d^{adj}}$  is the price-dividend ratio adjusted for share repurchases using the data by Boudoukh et al (2007). Their last available data relate to the year 2003, and the calculations are based on the assumption that repurchases are zero prior to 1971, as suggested by figure 2. As the sample of US firms in Boudoukh et al (2007) is slightly broader than that underlying the measures for PD and PE ratios, which are taken from Robert Shiller’s homepage, the adjusted PD ratio is calculated as  $PD^{adj} = PD \frac{PD^{adj\star}}{PD^\star}$ , where a  $\star$  denotes the measures presented in their paper.

## 2.2 Uncertainty about Origin and Persistence of the Great Moderation

By the second half of the 1990s, both the academic literature (Kim and Nelson (1999), McConnell and Perez-Quiros (1997, 2000)) and the business press had noticed a break in the volatility properties of US output growth around the middle of the preceding decade. Somewhat later, a similar decline in volatility was documented for a broader set of US macro-economic variables (Blanchard and Simon (2001), Stock and Watson (2005)), as well as for other industrial countries (Stock and Watson 2003). However, although the Great Moderation itself had become a stylised fact, there was no consensus about its causes. While some authors explained the phenomenon by changes in the structure of industrial economies, such as financial innovation (Dynan et al 2006), improved inventory management, or financial and trade liberalisation (see Wachter (2006) for a brief summary), the two perhaps most prominent hypotheses competed under the heading of “Good Policy or Good Luck?”. Specifically, following the seminal article by Stock et al (2003), several studies<sup>5</sup> used time-varying VAR models to find that a string of unusually small shocks, rather than changes in their transmission to main macroeconomic variables or in the conduct of monetary policy, were at the root of the decline in macro-volatility. Against this, both academics (Benati et al 2008) and policymakers (Tucker 2005, Bernanke 2004) argued that reduced-form models were likely to mistake effects of improved monetary policy, such as more stable but unobserved inflation expecta-

<sup>5</sup>Primiceri (2005), Sims and Zha (2006), and Canova, Gambetti, and Pappa (2007).

tions, for changes in the variance-covariance-properties of exogenous economic shocks. For example, Bernanke (2004) argued that “some of the benefits of improved monetary policy may easily be confused with changes in the underlying environment”. Importantly, the lack of consensus about the causes of the observed fall in macro-volatility left it unclear whether the phenomenon was likely to be permanent, as suggested by structural change or possibly improved policy environments, or transitory, in line with the “good luck” hypothesis.

How did market participants perceive the Great Moderation and its effect on prices? Investment analysts explicitly attributed part of the observed fall in the equity risk premium since the late 1980s to the decline in macro-volatility. For example, it was noted in Goldman Sachs research (2002) that an estimated 8 percentage point fall in the risk premium since the 1970s was “underpinned by dramatic improvements in the economic environment. Inflation fell sharply, and the volatility of GDP growth, inflation and interest rates all declined significantly.” (p. 2). But while investors acknowledged the effect of the Great Moderation on asset prices, they were also aware of the uncertain persistence of this low-volatility environment and thus, of the decline in equity premia. For example, regarding risk premia in fixed income securities, Unicredit analysts (2006) argued that “the ongoing deterioration in surprise risk should be seen as one of the arguments behind the declining risk premium. Whether this is due to a more effective central bank policy, a major improvement in the forecast ability of economic observers around the globe, sheer luck or maybe a mix of all three factors can’t finally be answered.” (p. 10). Researchers at JP Morgan (2005), on the other hand, attribute most of the fall in volatility to a changed orientation of policymakers towards a “Stability Culture” which, however, they see as uncertain to persist.

We draw two conclusions from this evidence: first, the fall of macro-volatility since the mid-1980s was accepted as a stylised fact, and widely seen as a contributing factor to higher asset prices during the 1990s and 2000s. Second, investors were very uncertain about the persistence of the Great Moderation. At the same time, this persistence is the key determinant of prices in a model with switches in volatility regimes, such as Lettau et al (2008). Therefore, this paper puts learning about the persistence of volatility changes at the center of its analysis.



## 3 The model

### 3.1 Preferences

We consider an economy with an infinitely-lived representative agent who maximises utility defined recursively as in Epstein and Zin (1989, 1991) or Weil (1989)

$$U_t = [(1 - \beta)C_t^{\frac{1-\gamma}{\alpha}} + \beta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\alpha}}]^{\frac{\alpha}{1-\gamma}}$$

where  $C_t$  is aggregate consumption,  $E_t$  is the mathematical expectation with respect to the agent's subjective probability distribution conditional on period  $t$  information,  $\alpha = \frac{1-\gamma}{1-\frac{1}{\psi}}$ ,  $\gamma$  is the coefficient of relative risk aversion, and  $\psi$  the elasticity of intertemporal substitution.

The first-order condition associated to holdings of a risky asset paying random dividends  $D_t$  is

$$P_t = E_t^s[M_{t+1}(P_{t+1} + D_{t+1})] \quad (1)$$

where  $P_t$  is the asset price in period  $t$  and  $M_{t+1}$  is the stochastic discount factor, which, with Epstein-Zin preferences, equals

$$M_{t+1} = (\beta(\frac{C_{t+1}}{C_t})^{-\frac{1}{\psi}})^{\alpha} R_{w,t+1}^{\alpha-1}$$

Here,  $R_{w,t+1}^{\alpha-1}$  is the return on the aggregate wealth portfolio of the representative agent, equal to the aggregate consumption flow.

### 3.2 The Processes for Consumption and Dividend Growth

We choose a simple and transparent way of modelling an economy that goes through periods of low and high macro-volatility by assuming that log consumption follows an exogenous random walk with drift

$$g_t = \Delta \ln C_t = \bar{g} + \varepsilon_t$$

where  $\bar{g}$  is constant mean consumption growth.<sup>6</sup> Shocks  $\varepsilon_t$  are independently normally distributed, and their variance follows a two-state Markov process  $\varepsilon_t \sim N(0, \sigma_t^2)$ ,  $\sigma_t^2 \in \{\sigma_l^2, \sigma_h^2\}$ .

The transition probabilities for the Markov process are

$$\begin{aligned}\Pr(\sigma_{t+1}^2 = \sigma_l^2 \mid \sigma_t^2 = \sigma_l^2) &= F_{ll} \\ \Pr(\sigma_{t+1}^2 = \sigma_h^2 \mid \sigma_t^2 = \sigma_h^2) &= F_{hh}\end{aligned}$$

which yields the transition probability matrix as

$$\mathbf{F} = \begin{bmatrix} F_{ll} & 1 - F_{ll} \\ 1 - F_{hh} & F_{hh} \end{bmatrix}$$

Following Mehra and Prescott (1985), and in line with the endowment nature of the economy, it is common to assume that dividend flows equal consumption flows. To capture the higher empirical volatility of dividends, we follow Campbell (1986), Abel (1999), Bansal and Yaron (2004) or Lettau et al (2008), and use a generalised version of the standard model where shocks to dividend growth are a multiple of those to consumption

$$\Delta \ln D_t = \bar{g} + \lambda \varepsilon_t \quad \lambda \geq 1$$

Dividends thus follow the same volatility pattern as consumption, but are on average more volatile.

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<sup>6</sup>Previous studies like Lettau et al (2008) have also looked at the time-variation in  $\bar{g}$ , although Boguth and Kuehn (2012) find no significant effect on asset prices when they add estimated revisions in beliefs about mean growth from the Lettau et al framework to a factor model of asset prices. Positive revisions to the anticipated volatility of consumption growth, however, are significantly negatively priced. Here, we assume  $\bar{g}$  to be constant over time, and instead concentrate on changes over time in the variance of shocks  $\varepsilon_t$ .

### 3.3 Full Information Price-Dividend Ratios

We can use the first-order condition for share holdings to express the price-dividend ratio  $p_t = \frac{P_t^D}{D_t}$  as

$$p_t = (\rho_t)^{1-\alpha} E_t[\beta^\alpha (\frac{C_{t+1}}{C_t})^{-\frac{\alpha}{\psi} + \alpha - 1} (\rho_{t+1} + 1)^{\alpha-1} (p_{t+1} + 1) \frac{D_{t+1}}{D_t}] \quad (2)$$

where  $\rho_t = \frac{P_t^C}{C_t}$  is the price of a claim to aggregate consumption relative to its flow return and  $\frac{P_t^C}{C_t}$  equals  $\frac{P_t^D}{D_t}$  whenever  $\lambda = 1$ . When the agent knows the true structure of uncertainty, given the random walk nature of consumption and dividends, the price-dividend and price-consumption ratios are functions only of the volatility state,  $p_t = p(\sigma_t^2)$ , and thus non-random conditional on  $\sigma_t^2$ . Thus, we can simplify (2) by taking expectations across realisations of log-normal consumption and dividend growth conditional on  $\sigma_{t+1}^2$ , which gives a recursive expression for the price-dividend and ratios.<sup>7</sup>

Note that in the special case when  $\lambda = 1$ , both consumption and dividend growth follow the same log-normal distribution. With  $\psi = \frac{1}{\gamma}$  (CRRA preferences), this yields an analytical solution to the vector of price-dividend ratios  $p$  as

$$p = \beta F(1 + p) e^{(-\gamma+1)\bar{g}} (e^{(-\gamma+1)^2 2\sigma^2}) \quad (7)$$

$$= \mathbb{F} \beta F e^{(-\gamma+1)\bar{g}} (e^{(-\gamma+1)^2 2\sigma^2}) \quad (8)$$

where  $\sigma^2 = [\sigma_l^2; \sigma_h^2]$  is the vector of volatilities, and  $\mathbb{F} = [I - \beta F e^{(-\gamma+1)\bar{g}} (e^{(-\gamma+1)^2 2\sigma^2})]^{-1}$ .

<sup>7</sup>Specifically, for  $F_{ij} = 1 - F_{ii}$  and  $i, j \in \{h, l\}, i \neq j$ ,  $p(\sigma_i^2)$  is defined by

$$p(\sigma_i^2) = \sigma_i^2 = \rho_i^{1-\alpha} \beta^\alpha e^{(-\frac{\alpha}{\psi} + \alpha)\bar{g}} \quad (3)$$

$$\left( F_{ii} e^{\frac{(-\frac{\alpha}{\psi} + \alpha - 1 + \lambda)^2}{2}} \sigma_i^2 (1 + \rho_i)^{\alpha-1} (1 + p_i) + F_{ij} e^{\frac{(-\frac{\alpha}{\psi} + \alpha - 1 + \lambda)^2}{2}} \sigma_j^2 (1 + \rho_j)^\alpha (1 + p_j) \right) \quad (4)$$

where  $\rho = \frac{P_t^C}{C_t}$  follows

$$\rho^\alpha(\sigma_t^2) = \sigma_t^2 = \rho_i^{1-\alpha} \beta^\alpha e^{(-\frac{\alpha}{\psi} + \alpha)\bar{g}} \quad (5)$$

$$\left( F_{ii} e^{\frac{(-\frac{\alpha}{\psi} + \alpha)^2}{2}} \sigma_i^2 (1 + \rho_i)^{\alpha-1} + F_{ij} e^{\frac{(-\frac{\alpha}{\psi} + \alpha)^2}{2}} \sigma_j^2 (1 + \rho_j)^\alpha \right) \quad (6)$$

### 3.4 Learning and Subjective Beliefs

To study whether a long spell of  $\sigma_l$  can lead to a boom in asset prices by increasing the confidence in the persistence of a low-volatility environment, we assume that the representative agent does not know the full probabilistic structure of the economy. Specifically, the agent knows that log-changes of dividends are normally distributed with mean  $\bar{g}$  but learns about the transition probabilities between volatility states  $F_{hh}$  and  $F_{ll}$  from observed transitions between high and low volatility. The agent thus knows the structure of the model and all parameter values except the true transition probabilities,  $F_{hh}$  and  $F_{ll}$ , which she aims to infer on the basis of the history of volatility-states  $\Sigma^t = \{\sigma_t^2, \sigma_{t-1}^2, \dots, \sigma_2^2, \sigma_1^2\}$ . Thus, we assume that every period, the agent observes a dividend realization and the distribution that this specific realization is drawn from, parameterised by  $\sigma_t^2$ . We believe that this assumption captures well the evidence, contained in Section II, that market participants were certain of a change in the volatility around the mid-1980s, but uncertain about its persistence. It is similar to that in studies of growth regimes, such as Lam et al (2000), where agents are assumed to observe the growth state of the economy but have mis-specified beliefs about transition probabilities.

Our benchmark version of the model follows Cogley and Sargent (2008a) and assumes that the agent has independent beta-binomial prior distributions about  $F_{hh}$  and  $F_{ll}$

$$f_0(F_{hh}, F_{ll}) \propto f_0(F_{hh})f_0(F_{ll})$$

with

$$\begin{aligned} f_0(F_{hh}) &= f(F_{hh} | \Sigma^0) = \text{beta}(n_0^{hh}, n_0^{hl}) \propto F_{hh}^{n_0^{hh}-1} (1 - F_{hh})^{n_0^{hl}-1} \\ f_0(F_{ll}) &= f(F_{ll} | \Sigma^0) = \text{beta}(n_0^{ll}, n_0^{lh}) \propto F_{ll}^{n_0^{ll}-1} (1 - F_{ll})^{n_0^{lh}-1}. \end{aligned}$$

where  $\Sigma^0$  denotes a prior belief about frequencies  $n_0^{ij}$  of transitions from state  $i$  to state  $j$ .

The agent updates this prior on the basis of the likelihood function  $L$  for the history of volatility states  $\Sigma^t$  conditional on  $F_{hh}$  and  $F_{ll}$ , which is the product of two independent binomial density

functions, thus

$$L(\Sigma^t | F_{hh}, F_{ll}) \propto L(\Sigma^t | F_{hh})L(\Sigma^t | F_{ll})$$

where

$$\begin{aligned} L(\Sigma^t | F_{hh}) &= \text{binomial}(F_{hh}, F_{hl}) \propto F_{hh}^{n_t^{hh} - n_0^{hh}} (1 - F_{hh})^{n_t^{hl} - n_0^{hl}} \\ L(\Sigma^t | F_{ll}) &= \text{binomial}(F_{ll}, F_{lh}) \propto F_{ll}^{n_t^{ll} - n_0^{ll}} (1 - F_{ll})^{n_t^{lh} - n_0^{lh}} \end{aligned}$$

Here,  $n_t^{ij}$  is a ‘‘counter’’ that equals the number of transitions from state  $i$  to state  $j$  up to time  $t$  plus the prior frequencies  $n_0^{ij}$ . The posterior kernel is the product of the beta prior and the binomial likelihood function,

$$f(F_{hh}, F_{ll} | \Sigma^t) \propto \underbrace{L(\sigma_t^2 | F_{hh}, F_{ll})}_{\text{Likelihood}} \cdot \underbrace{f(F_{hh}, F_{ll} | \Sigma^{t-1})}_{\text{Prior}}$$

which after normalizing by  $M(\Sigma^t) = \int \int F_{hh}^{n_t^{hh}-1} (1 - F_{hh})^{n_t^{hl}-1} F_{ll}^{n_t^{ll}-1} (1 - F_{ll})^{n_t^{lh}-1} dF_{hh} F_{ll}$  yields the posterior density function as the product of independent *beta* distributions

$$\begin{aligned} f(F_{hh} | \Sigma^t) &= \text{beta}(n_t^{hh}, n_t^{hl}) \propto F_{hh}^{n_t^{hh}-1} (1 - F_{hh})^{n_t^{hl}-1} \\ f(F_{ll} | \Sigma^t) &= \text{beta}(n_t^{ll}, n_t^{lh}) \propto F_{ll}^{n_t^{ll}-1} (1 - F_{ll})^{n_t^{lh}-1} \end{aligned}$$

Note that in this context, the counters  $n_t^{ij}$  are sufficient statistics for the posterior.

So far, we have described how agents update their priors about transition probabilities on the basis of observed data. Even with posterior distributions for  $F_{hh}$  and  $F_{ll}$  in hand, however, the uncertainty about future transition paths fundamentally changes the problem of the representative agent. This is because her willingness to pay for the asset today depends on transition probabilities in future periods, whose distribution is a function of the realised sequence of regimes via the updating rule described above. A fully Bayesian agent thus needs to take expectations across not only dividend realisations and the distribution of  $F_{hh}$  and  $F_{ll}$ , but also across different beta distributions indexed by the counters  $n_t^{ij}$ ,  $i, j \in \{l, h\}$  that in turn depend on future regime sequences.

Like Boz and Mendoza (2010), or Johannes, Lochstoer and Mou (2011), in this paper we circumvent the curse of dimensionality that this implies by adopting the “anticipated utility” learning scheme proposed by Cogley and Sargent (2008a). They assume that the agent updates her prior in a Bayesian fashion as a function of observed data, but neglects the dependence of future transition probabilities on the future realised regime paths. Rather, she takes today’s posterior distributions  $f(F_{ii} | \Sigma^t), i \in \{h, l\}$  as a time-invariant description of uncertainty about transition probabilities forever in the future. Note that this assumption can be motivated either as an approximation to fully Bayesian behaviour, or as a summary of behaviour by agents who learn in a Bayesian fashion on the basis of observed data, but use a simplified updating for future periods.<sup>8</sup> The following section will look at an alternative learning scheme where the agent takes into account the effect of future regime realisations on her inference about transition probabilities.

Using Cogley and Sargent (2008a)’s anticipated utility approach, the equilibrium asset price can be calculated using a fixed point method similar to that under full information. Let  $p(\sigma_t^2, F)$  denote the price-dividend ratio when the transition probability matrix is  $F$ . Following Cogley and Sargent (2008a),  $p_t^{BL}$ , the vector of price-dividend ratios under Bayesian learning about transition probabilities can then be written as

$$p_t^{BL} = \int p(\sigma_t^2, F) f(F, \Sigma^t) dF \quad (9)$$

where  $f(F, \Sigma^t)$  is the posterior distribution of  $F$ <sup>9</sup>. Note that for given  $F_{hh}, F_{ll}$ ,  $p(\sigma_t^2, F)$  is described by the same pair of equations as under full information ((6), (4)). And the law of iterated expectations implies that we can compute  $p(\sigma_t^2, F)$  as a fixed point of these two equations.  $p_t^{BL}$  can then easily be calculated by numerical integration across the independent beta posteriors for  $F_{hh}, F_{ll}$ .

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<sup>8</sup>Cogley and Sargent (2008a) compare the anticipated utility approach to fully Bayesian decision making in a finite horizon savings problem. They find that the differences are small especially for moderate risk-aversion and at the beginning of the agent’s life, when consumption in the following periods can still be smoothed well by changing savings behaviour. How good the approximation is in an infinite-horizon setting, and with the preferences assumed here, is an open issue.

<sup>9</sup>For a derivation of equation (9) see Appendix B.

## 4 Quantitative Results for the Benchmark Economy

### 4.1 The exercise

This section presents the results of numerical simulations to answer the two main questions of this paper: Can learning about the persistence of the Great Moderation explain the observed boom and bust in US asset prices? And can increasing confidence in this persistence lead to an overvaluation of assets, and a larger fall in prices at the end of the low-volatility period, relative to the case of full information? To answer these questions, we analyse a scenario that is similar to the economic experience of the US after World War II. In particular, we interpret this experience as a long realisation of high volatility followed by the Great Moderation that ends with the recent crisis. Our data generation process thus consists of three sequences of shocks corresponding to three subperiods of different consumption growth volatility  $\sigma_t^2$ . Specifically, our analysis starts with a high volatility regime in 1952Q2<sup>10</sup>. Since in our highly stylised model, there is no distinction between consumption and GDP, we use a starting date for the Great Moderation at the beginning of 1984, as suggested by the fall in GDP volatility, but also look at later dates as suggested by the consumption growth series. In line with the observed rise in volatility in figure 1, we locate the end of the Great Moderation at the beginning of 2007, the starting year of the crisis. To compute the fall in asset prices around this end of the Great Moderation we also make the stronger assumption that the economy returned to the high volatility environment observed before the Great Moderation. This assumption is largely heuristical. It allows us to isolate the crash in asset prices implied by the end of the Great Moderation from other factors that this paper abstracts from.

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<sup>10</sup>This particular date is chosen to equal the initial data period. The model results for the behaviour of asset prices during the Great Moderation, however, are not sensitive to the exact choice of the start date for the high volatility regime.

## 4.2 Parameter choice

### 4.2.1 Preferences

As shown by Bansal and Yaron (2004), for a fall in consumption volatility to increase asset prices with Epstein-Zin preferences, the intertemporal elasticity of substitution  $\psi$  has to be greater than unity. Thus, we follow Lettau et al (2008) and set  $\psi = 1.5$ . For our statements about the size of boom and bust to be interesting, the model has to deliver a level of asset prices that is approximately equal to the data in the period before the Great Moderation. Rather than changing parameters across different learning rules to target asset prices exactly, however, we choose  $\beta = 0.9935$  to target an interest rate of 2 percent p.a. (which varies very little across different specifications), and set  $\gamma = 30$  which yields equity prices that are, on average across the versions of the model we analyse, close to US data, but not exactly equal to it for any particular specification.

### 4.2.2 The Process for Consumption and Dividends

Apart from the transition matrix  $\mathbf{F}$ , the consumption process in this model is characterised by three parameters: constant mean growth  $\bar{g}$ , and the standard deviations in the two subperiods  $\sigma_h, \sigma_l$ , which we estimate directly from quarterly data on US personal real per capita consumption expenditure, using the subperiods from table 1. This yields mean growth of 0.6 percent per quarter and standard deviations of 0.82 and 0.37 percent respectively.

When agents learn about the persistence of regimes from the observed transitions between low and high volatility periods, the matrix  $\mathbf{F}$  that defines the underlying data generating process has no relevance for the equilibrium asset prices in the economy. However, to obtain benchmark values of asset prices in the absence of uncertainty about transition probabilities and without learning, we use a particularly simple ex-post estimate of  $\mathbf{F}$ , which we denote as the “full-information” transition probability matrix  $\mathbf{F}^{FI}$ . Specifically, we choose  $F_{ll}^{FI}, F_{hh}^{FI}$  such that the expected durations of high and low volatility regimes equal the subperiods identified from US data. So  $F_{ii}^{FI} = 1 - \frac{1}{T_i}$ , where  $T_l, T_h$  are the durations of the Great Moderation and the high-volatility period preceding it, which



<b>Parameter Values for the Benchmark Model</b>		
<b>Preferences</b>		
$\beta$	0.9935	Discount Factor
$\gamma$	30	Relative Risk Aversion
$\psi$	1.5	Elasticity of Intertemporal Substitution
<b>Endowment Process</b>		
$\bar{g}$	0.0059	Mean of Consumption Growth
$\sigma_l$	0.0037	Low Standard Deviation of Consumption Growth
$\sigma_h$	0.0082	High Standard Deviation of Consumption Growth
$\lambda$	4.5	Leverage

Table 4: Parameter values in the benchmark model.

yields

$$\mathbf{F}^{FI} = \begin{bmatrix} 0.989 & 1 - 0.989 \\ 1 - 0.992 & 0.992 \end{bmatrix}$$

It is interesting to note that these transition probabilities are almost identical to those in Lettau et al (2008), based on a more sophisticated estimated Markov process on the same data.<sup>11</sup>

Unless otherwise mentioned, we set  $\lambda = 4.5$  as suggested by Lettau et al (2008) on the basis of the relative volatility of US consumption and dividends. Table 4 summarises the parameters of preferences and the endowment process for the benchmark model.

### 4.2.3 Learning Parameters

When agents learn about transition probabilities, the only remaining free parameters are those describing their beta prior distribution  $f_0(F_{hh}, F_{ll})$ . To be as agnostic as possible about the information agents have at the beginning of the scenario we analyse, we choose as a benchmark an uninformative prior distribution with initial parameters  $n_0^{ij} = 1, \forall i, j$ , for which the beta distribution coincides with the uniform distribution on  $[0, 1]$ . This implies that, as regimes persist, agents

<sup>11</sup>Their point estimates are

$$\mathbf{F} = \begin{bmatrix} 0.991 & 1 - 0.991 \\ 1 - 0.994 & 0.994 \end{bmatrix}$$

Their process is more complex, however, as they also include uncertainty about mean growth.

update relatively quickly their posterior beliefs in the direction of higher persistence. Particularly, at the beginning of the Great Moderation, agents believe the high-volatility regime that was uninterrupted for 32 years to be strongly persistent. The independence of  $F_{hh}, F_{ll}$ , however, implies that this increase in the persistence estimate during the course of the high volatility regime does not lead agents to assume any prior persistence for the low volatility regime at the beginning of the Great Moderation. In other words, although agents have significantly changed their views about the durability of one of the two regimes, which they estimate to be highly persistent by the early 1980s, they continue to expect that any move to low volatility is, essentially, a short-lived outlier. To see to which degree the results depend on this, we also investigate the implications of an alternative assumption, that agents have a moderately persistent prior for the low volatility regime. Specifically, in this alternative case, we assume that  $n_0^l = 1.5, n_0^h = 0.5$ , so agents have the same amount of information as in the benchmark case, but with a moderate mean persistence of 0.75.<sup>12</sup>

### 4.3 Asset Price Dynamics with Learning

#### 4.3.1 Rising Posterior Mean Persistence Gradually Increases Asset Prices

Figure 3 presents the time path of the PD ratio in US data in the upper panel. The bottom panel depicts both the PD ratio with learning from an uninformed prior (solid lines) and when agents take as certain the ex-post, full-information transition probability matrix  $F^{FI}$ . As a result of the calibration, and independently of learning, the model delivers realistic levels of asset prices, and thus a realistic equity premium, before the beginning of the Great Moderation. The model with full information delivers a small jump in prices of around 15% in 1985, but no sustained asset price boom during the Great Moderation. With learning, however, although the model is not able to replicate the hump-shape in PD ratios or their almost three-fold rise until 2007, we see a strong and gradual rise in prices of more than thirty percent as agents increase their persistence estimate of the Great Moderation. And importantly, when the low volatility period comes to an end at the end of 2007, the model predicts a strong fall in prices of 22%. Again, this is larger than in the full-

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<sup>12</sup>This is equivalent to giving equal weight to an uninformative prior and a distribution with the same amount of information and a mean equal to the posterior mean persistence for the high volatility regime.

information case, as the move back to the high-volatility regime does not only lead to a “switch” in the conditional probabilities (from the top to the bottom rows of the matrix  $\mathbf{F}$ ), but also reduces the estimated persistence of the low-volatility regime through the addition of an observed regime change.

**Enter Figure 3 and 4 about here**

Figure 4 presents the same results for a moderately persistent prior at the beginning of the Great Moderation. The time path of price-dividend ratios has a shape very similar to that in figure 4, but the magnitudes are larger, with a boom of 42%, and a fall in asset prices at the end of the Great Moderation of 32 percent. Table 5 summarises the results.

In summary, while the model predicts a strong gradual boom in prices during the Great Moderation and a sharp fall at its end, neither of the two calibrations replicates the full magnitude, or the exact shape, of the observed rise in US PD ratios. Particularly, during the Great Moderation, US asset prices rose most steeply at the end of the 1990s, fell at the beginning of the 2000s and then remained flat until the crisis. In contrast, in the model, the marginal effect of an additional observation on posterior beliefs about regime persistence is strongest when priors are flat. This leads to the steepest increase in prices at the beginning of the Great Moderation. The failure to predict the hump-shape observed in the data, on the other hand, results from the fact that the model concentrates on changes in volatility over time around constant mean growth. It is thus unable to capture phenomena like the dotcom boom of the late 1990s and early 2000s, which has been attributed to a rise and fall in average productivity expectations. In comparison, in the Lettau et al (2008) model, when the probability of remaining in the low volatility state is fixed exogenously at 0.999, the model predicts 50 percent of the observed rise in asset prices between 1990 and 2002. But while in the data the steepest increase in prices occurred between 1995 and 2000, all of the rise in prices in their model is located in the years 1991 to 1996, during which period their econometric model estimates a rise in the posterior probability of a low-volatility state from 16 to 98 percent.

To illustrate the learning dynamics underlying the path of asset prices, Figure 5 depicts, for the case of an uninformative prior, the evolution of posterior probability distributions during the Great Moderation. Starting from the initial uniform distribution, probability mass becomes more

<b>Asset Prices - Benchmark Model</b>			
	Boom	Overvaluation	Bust
Full Info	15%	0	15%
Uninformed prior	32%	11%	22%
Moderately persistent prior	42%	20%	32%

Table 5: “Boom” denotes the increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of five years). “Overvaluation” is the overvaluation at the end of the Great Moderation relative to the prices under full information. And “Bust” is the fall in prices in the first period after the Great Moderation.

and more concentrated at values close to 1, leading to the rise in asset valuation in figure 3 as agents predict low volatility to last longer on average.

**Enter Figure 5 about here**

### 4.3.2 Uncertainty about Transition Probabilities Increases Price Levels above their Full-Information Value

By construction of our ex-post estimate  $\mathbf{F}^{FI}$ , mean persistence at the end of the Great Moderation under learning is with 0.989 almost exactly equal to the persistence in the full-information case. In other words, the increase in mean persistence alone cannot explain the boom in prices under learning, where price-dividend-ratios rise significantly above the full information value at the end of the low-volatility period. The reason for this is an additional variance effect on prices that arises from the uncertainty about transition probabilities and is absent under full information. To understand this effect, it is important to note that certainty PD ratios are a strongly convex function of persistence at high values of  $F_{ll}$ . Under learning, where persistence values are dispersed around their mean value, this convexity implies a strong positive Jensen’s inequality effect on PD ratios. For the simplified case of identical conditional distributions ( $f(F_{hh}) = f(F_{ll})$ ), figure 6 illustrates this by showing how asset prices change as a function of both the mean and variance of the beta-distributed transition probabilities. The solid lines depict the value of PD ratios at high and low volatility in the absence of uncertainty, as a function of persistence  $F_{hh} = F_{ll}$ . As persistence rises, high-volatility prices fall, since agents are less willing to pay for assets whose payoffs they anticipate

to remain volatile with a larger probability. Interestingly, low-volatility prices initially fall slightly, but rise strongly for high values of persistence above 0.995. The remaining lines in figure 6 show that this non-linearity of the certainty price leads to an increase in the level of prices as priors about persistence become looser.

**Enter Figure 6 and 7 about here**

Importantly, this convexity effect is different from that in Veronesi (1999), or Lettau et al (2008). There, with uncertainty about the current regime, but not its persistence, an increase in posterior probability of the good state always implies an “improvement” in the outlook of future consumption growth (or risk in Lettau et al 2008) on average. But while uncertainty about the current and future state is increased by a rise in the posterior from a low level, it is reduced by the same rise at high levels, as agents’ posterior moves closer to one. This inverse-U shaped relationship of posteriors and uncertainty implies a convex reaction of prices to the posterior of being in a good state. In our model, higher uncertainty increases prices due to the convexity of certainty prices. Moreover, this convexity of certainty prices in regime persistence does not arise from the inverse U-shaped relationship between the variance of a binary process and its parameter as in Veronesi (1999). Rather, this convexity results from the fact that the infinite sum of a random variable that follows a markov process has a non-linear relationship with the persistence of that process. Figure 7 illustrates this by plotting the diagonal and off-diagonal elements of the present discounted value matrix  $V = \sum_{i=0}^{\infty} \beta^i \mathbf{F}^i = [I - \beta \mathbf{F}]^{-1}$  as a function of persistence  $F_{hh} = F_{ll}$ . As the figure shows, for other than very high persistence, the geometrically declining probability of remaining in the same state for  $1, 2, \dots, n$  periods leads to entries in  $V$  that are close to  $\frac{1}{2}$ , and thus, asset prices that differ little between regimes. Thus, it seems to be the geometric nature of present discounted probabilities that leads to the highly non-linear relationship between asset prices and persistence in figure 6, not the inverse U-shape of the variance of binary processes.

### 4.3.3 Sensitivity of the Results to Alternative Parameter Choices

The assumption of high risk-aversion was made to target price-dividend ratios that are close to those observed in the period before the Great Moderation. With lower risk aversion ( $\gamma = 20$ ),

<b>Asset Prices - Benchmark Model</b>			
	Boom	Overvaluation	Bust
<b>Benchmark</b>			
Uninformed prior	32%	11%	22%
<b>Sensitivity</b>			
$\gamma = 20$	30%	9%	18%
$\lambda = 2.5$	17%	6%	11%
Start of GM 1992Q1	26%	9%	19%

Table 6: “Boom” denotes the increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of 5 years). “Overvaluation” is the overvaluation at the end of the Great Moderation relative to the prices under full information. And “Bust” is the fall in prices in the first period after the Great Moderation.

table 6 shows that the boom and bust in asset prices are only marginally reduced relative to the benchmark case, but the level of asset prices is about a quarter higher. When the relative volatility of log-dividend growth is reduced to  $\lambda = 2.5$ , both the boom and the bust are only about half as strong as in the benchmark calibration of  $\lambda = 4.5$  that followed Lettau et al (2008) and their estimates of the relative volatility of dividends observed in post-war US data. With a later beginning of the Great Moderation in 1992, as suggested by the data for US consumption growth volatility, the boom is, with 25%, 7 percentage points smaller than in the benchmark case, although the bust is almost of the same magnitude. Finally, the results for the behaviour of asset prices during the Great Moderation are not much affected by the exact starting period of the scenario (here 1952 Q2), which is why we do not report a corresponding sensitivity analysis here.

## 5 Asset Prices under Alternative Learning Schemes

This section compares the benchmark results from the last section to two alternative learning mechanisms. First, we replace the continuous beta prior with a discrete prior distribution that puts strictly positive mass on a persistence value of 1. This is in order to investigate a key-feature of the Great Moderation: as low-volatility persisted, market participants were increasingly suspicious of whether “this time is different”, and low volatility might in fact be a permanent feature of the post-1980s economy, rather than just a temporary regime. As a second alternative, we investigate

asset price dynamics under ad hoc, or “recursive” learning schemes, where agents ignore the two-regime nature of the world and compute a best guess for average volatility using different statistics of the observed history.

## 5.1 “This Time is Different”: Learning when Low-Volatility is Suspected to be Permanent

In this section, we propose an alternative learning scheme that tries to explicitly capture the uncertainty about the “permanent vs. transitory” character of the Great Moderation. This has the advantage that we can dispense with the simplifying “anticipated utility” assumption, that agents use current posteriors to evaluate all future transitions, in favour of a fully Bayesian mechanism that takes into account how future posteriors change as a function of realised states of the economy. For this, we assume, as previously, that the agent updates a prior about transition probabilities between two volatility regimes on the basis of the observed history of volatility-states  $\Sigma^t = \{\sigma_t^2, \sigma_{t-1}^2, \dots, \sigma_2^2, \sigma_1^2\}$ . However, in contrast to the previous section, we assume that the prior about transition probabilities is a two point distribution. Specifically, agents attach a small prior probability of  $\hat{p}$  to  $F_{ll} = 1$ , or a permanent Great Moderation, and probability  $1 - \hat{p}$  to a persistent value in “normal times” of  $F_{ll} = F_{ll}^0 < 1$ . Note that, after a switch to low volatility, the conditional probability of observing  $T$  consecutive low-volatility periods declines with  $T$  if  $F_{ll} = F_{ll}^0$ , but equals 1 if  $F_{ll} = 1$ . More specifically, after a switch to low volatility, the likelihood of observing  $\sigma_l^N$ , a sequence of  $N$  additional low-variance periods, when  $F_{ll} = F_{ll}^0$  is simply

$$L(\sigma_l^N | \sigma_t^2 = \sigma_l^2, F_{ll} = F_{ll}^0) = P(\sigma_{t+1}^2 = \sigma_l, \sigma_{t+2}^2 = \sigma_l, \dots, \sigma_{t+N}^2 = \sigma_l | \sigma_t^2 = \sigma_l^2, F_{ll} = F_{ll}^0) = (F_{ll}^0)^N$$

where  $P(A|B, F_{ll})$  denotes the probability of event A conditional on event B and persistence  $F_{ll}$ . During a sequence of low-volatility observations, the posterior probability of a permanent great

moderation, denoted  $P(F_{ll} = 1|\sigma_i^N)$ , thus increases with  $N$  according to Bayes' Rule

$$\begin{aligned} P(F_{ll} = 1|\sigma_i^N) &= \frac{P(F_{ll} = 1 \wedge \sigma_i^N)}{P(F_{ll} = 1 \wedge \sigma_i^N) + P(F_{ll} = F_{ll}^0 \wedge \sigma_i^N)} \\ &= \frac{\hat{p}}{\hat{p} + F_{ll}^0 N(1 - \hat{p})} \end{aligned} \quad (10)$$

Importantly, any observed switch from low to high volatility fully reveals that the great moderation was not permanent, thus “resetting”  $P(F_{ll} = 1)$  back to its unconditional prior.

We focus on the same scenario as in the previous section, designed to capture the experience of the US economy after World War II. Moreover, for simplicity, we abstract from uncertainty about transition probabilities in high-volatility times, and set  $F_{hh}$  equal to its ex-post estimate with probability 1.<sup>13</sup> The vector of price dividend ratios under Bayesian learning about a ‘permanent vs. transitory’ Great Moderation, denoted  $p_t^{PT}$ , is then described by equations similar to (6) and (4). With  $\lambda = 1$ , this yields

$$p_{it}^{PT} = \beta e^{\left(-\frac{a}{\psi} + a\right)\bar{g}} \left( \mathbf{P}_{ii,t} e^{\frac{\left(-\frac{a}{\psi} + a\right)^2}{2}\sigma_i^2} (1 + p_{i,t+1}^{PT})^a + \mathbf{P}_{ij,t} e^{\frac{\left(-\frac{a}{\psi} + a\right)^2}{2}\sigma_j^2} (1 + p_{j,t+1}^{PT})^a \right)^{1/a} \quad (11)$$

where once more  $i, j \in \{h, l\}$ ,  $\mathbf{P}_{hj,t} = F_{hj}$ ,  $j = h, l$  and  $\mathbf{P}_{lj,t}$  is the probability of moving from low volatility to regime  $j$  given the period  $t$  posterior probability of the change to low volatility being permanent in equation (10).

### 5.1.1 Solution Algorithm

Price-dividend ratios under this learning schemes are not simply fixed points to equation (11). Rather, the representative agent anticipates that, should low volatility persist in the next period, the probability of a permanent change increases, as does the price-dividend ratio. Thus, we have to compute the whole path of price-dividend ratios jointly. This is done as follows: first, calculate the price dividend ratio for permanently low volatility  $p(F_{ll} = 1)$  as a fixed point to (11) for  $F_{ll} = 1$ . Second, calculate the path of posterior probabilities  $P(F_{ll} = 1|\sigma_i^N)$  as  $N$  rises, and the associated

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<sup>13</sup>As illustrated by figure 6, the effect of rising persistence of the current regime on asset prices during high-volatility regimes is much weaker, and more linear, than in low-volatility times. So the effect of uncertainty and learning on high-volatility asset prices is small, which justifies our simplification.



transition probabilities  $\mathbf{P}_{Li,t}$ . Once  $P(F_{ll} = 1 | \sigma_l^N)$  is close enough to 1, say after  $\bar{N}$  low volatility periods, we know that  $p_{l,t+\bar{N}}^{PT} = p(F = \mathbf{1})$ . Third, we know that the high-volatility price-dividend ratio  $p_{ht}^{PT}$  is constant through time. This follows from the assumption of perfect knowledge of  $F_{hh}$  and the fact that any switch to high volatility reveals the temporary nature of the preceding low-volatility-sequence, and thus “resets” the probability of any future switch to low-volatility being permanent back to the prior  $\hat{p}$ . Choose a value for  $p_{hs}^{PT}, s = t, t + 1, \dots$  and calculate the sequence of price-dividend ratios at low volatility  $p_{ls}^{PT}$  for  $s = \bar{N} - 1, \bar{N} - 2, \dots, t$  by backward induction using (11), given  $p_{l,t+\bar{N}}^{PT} = p(F = \mathbf{1})$  and the sequence  $\mathbf{P}_{Li,t+s}, s = 1, \dots, \bar{N} - 1$ . Fourth, calculate  $p_{ht-1}^{PT}$ , the price-dividend ratio at high volatility in period  $t - 1$ , from  $F_{hh}$  and the values of  $p_{lt}^{PT}$  and  $p_{ht}^{PT}$  using (11). If  $p_{ht-1}^{PT} = p_{ht}^{PT}$ , we have found an equilibrium price sequence. If not, set  $p_{hs}^{PT} = p_{ht-1}^{PT}, s = t, t + 1, \dots$ , and iterate.

### 5.1.2 Parameter Choice

As mentioned previously, we set  $F_{hh}$  to its ex-post estimate with probability 1, thus implicitly abstracting from any learning during the high-volatility period that preceded the Great Moderation. In addition, the model requires us to specify the transition probability in “normal times”  $F_{ll}^0$ , as well as  $\hat{p}$ , the prior probability that any observed change to a low-volatility regime be permanent. We set the latter to  $\hat{p} = 1\%$  and perform a robustness analysis with other values. For the transition probability  $F_{ll}^0$  in “normal times”, there is equally no obvious target. However, one requirement that disciplines its value is that a long sequence of low-volatility observations be sufficiently unlikely in normal times to warrant the debate about the character of the Great Moderation among both academics and market-participants during the late 1990s. The ex-post estimate  $\mathbf{F}_{ll}^{FI}$  therefore seems a bad choice not only because it conditions, implicitly, on the ex-post length of the Great Moderation before it happened, but also because the latter becomes, by definition, a normal event. Instead, we set  $F_{ll}^0$  such that the likelihood of 48 consecutive observations (12 years) of low-volatility is with 10 % sufficiently low to explain the debate about a potentially permanent Great Moderation in the second half of the 1990s. This results in a value of  $F_{ll}^0 = 0.87$ . Again, we also document the robustness of the results to this parameter choice.

### 5.1.3 Results

**Enter Figure 8 about here**

Figure 8 shows the time path that results from this learning scheme, as compared to US data. The rising posterior probability of a permanent moderation in macro-volatility over the course of the Great Moderation leads to an S-shaped increase in prices. In contrast to the benchmark learning mechanism taken from Cogley and Sargent (2008), this alternative learning scheme thus features an acceleration of price increases during the great moderation, although slightly later than that observed in the data. Again, by construction, the price path increases monotonically until the end of the Great Moderation, and thus does not replicate the hump-shape of observed PD ratios. Importantly, however, the magnitude of the boom in table 7 is with 77% more than twice as large as in the benchmark learning scheme above. Moreover, the observed end of the Great Moderation comes with a strong bust in asset prices of 84%, as agents update the probability of being in a permanently more benign macroeconomic environment to zero. The reason for this stronger effect, relative to learning with a standard beta prior, is that the effect of additional information on the posterior probability is concentrated at the maximum persistence value of 1. Given the non-linearity of asset prices in figure 6, this leads to an effect that is stronger than the one resulting from a model where the increase in persistence affects the probabilities attached to all values in  $[0, 1]$ .

**Enter Figure 9 about here**

Figure 9 illustrates how the results depend on the assumptions about the conditional prior probability  $\hat{p}$  and the value of  $F_{ll}^0$ . Specifically, when the prior probability  $\hat{p}$  equals 0.1% (second panel of figure 9), the rise in prices is delayed, and the Great Moderation comes to an end before the posterior converges to 1. So the rise in prices is “cut off”, and the resulting boom is with 56% somewhat smaller. Unsurprisingly, as the transition probabilities of the Great Moderation in normal times become highly persistent (third and fourth panel), the boom in prices all but vanishes because a long sequence of low volatility periods provides less reason to suspect a permanent change in the data generating process. Finally, the effect of lower risk aversion  $\gamma$  or leverage  $\lambda$  is very similar

Asset Prices - Learning about a Transitory vs. Permanent GM			
	Boom	Overvaluation	Bust
No Learning	3%	0	3%
Learning	77%	79%	84%

Table 7: “Boom” denotes the increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of five years). “Overvaluation” is the overvaluation at the end of the Great Moderation relative to the prices without learning. And “Bust” is the fall in prices in the first period after the Great Moderation (which can be higher than the “Boom”, as the latter is calculated as the difference in averages over 20 quarters).

under this learning scheme to that in the benchmark economy<sup>14</sup>.

## 5.2 Ad hoc learning

It has been argued by Haldane (2009), for example, that overconfidence in a low volatility environment may arise when agents base their inferences about the future predominantly on recent observations of small shocks. This over-reliance on the recent past is not captured by the optimal nature of the learning schemes considered so far, but in line with a large number of studies where agents follow ad hoc learning rules that map observations into estimates of parameters of interest (see for example Evans and Honkapohja (1999)). To see whether non-optimal learning rules can deliver a boom and bust in asset prices similar to those observed in US data, we assume that the representative agent knows the mean dividend growth  $\bar{g}$  and observes the history of shocks  $\Omega_t = \{\varepsilon_s\}_{s=0}^t$ . But she ignores, or chooses to ignore, the two-stage nature of the data generating Markov process in her estimate about future macro-volatility. Rather, she uses simple ad hoc rules that map observed histories into estimates  $\widehat{\sigma}_{t+1}^2$  of the variance of future shocks  $\sigma^2$

$$\widehat{\sigma}_{t+1}^2 = G(\Omega_t)$$

<sup>14</sup>The results are available from the authors upon request.

where  $G : R^t \rightarrow R^+$ . Specifically, we consider three simple mappings  $G$

$$G^{OLS} = \frac{1}{N} \sum_{s=0}^t (\varepsilon_s)^2 \quad (12)$$

$$G^{CG} = \xi(\varepsilon_t)^2 + (1 - \xi)G_{t-1}^{CG} = \sum_{s=0}^t \xi(1 - \xi)^{t-s} \varepsilon_t^2, \quad 0 < \xi < 1 \quad (13)$$

$$G^{CW} = \frac{1}{n} \sum_{s=t-n}^t (\varepsilon_s)^2 \quad (14)$$

Thus, under  $G^{OLS}$  agents simply compute their best guess of the future variance as an average over the entire history of shocks.  $G^{CG}$  describes a simple “constant-gain” learning rule: the agent computes the variance as a weighted average of his best guess in the previous period and the squared shock today. Relative to  $G^{OLS}$ , this overweighs more recent observations, as the weight on more distant observations decays geometrically at the rate  $1 - \xi$ . Finally,  $G^{CW}$  uses windows of the  $n$  most recent observations to compute the variance.

To implement the three ad-hoc learning rules quantitatively, we choose a window length of 20 years ( $n = 80$ ), and a constant gain parameter of 3 percent. Figure 10 presents, for each of the three rules, averages over 120 realisations of the time path of asset prices, together with full-information prices. With OLS learning, the fall in the variance estimate for consumption growth is relatively slow. Moreover, since each estimate weighs all past periods equally, the variance estimate remains an average across high and low volatility periods, resulting in a relatively small rise in prices. Nevertheless, the price rises above that under full information, as agents are more willing to pay for an asset with average volatility, compared to one whose payoff transits between periods of high and low volatility with moderate persistence.

**Enter Figure 10 about here**

With constant gain learning, the contribution of past periods to the variance estimate falls geometrically over time. This implies that the estimate of consumption variability during the Great Moderation falls faster, and further, than with OLS learning. The boom in prices is thus steeper and stronger, amounting to around 70 % at the end of the Great Moderation, far above

<b>Asset Prices - Ad hoc Learning</b>			
	Boom	Overvaluation	Bust
Full Info	15%	0	15%
OLS	22%	4%	0%
Constant Gain	73%	48%	2%
Constant Window	82%	57%	1%

Table 8: “Boom” denotes the relative increase in prices until the end of the Great Moderation relative to the high-volatility regime preceding it (computed over windows of 5 years). “Overvaluation” is the overvaluation at the end of the Great Moderation relative to the prices under full information. And “Bust” is the fall in prices in the first period after the Great Moderation.

that implied by full information. When agents compute their estimate of the consumption growth variance as an average across a window of constant length, asset prices reach an even higher plateau than under constant gain learning, although their path is slightly more convex, as estimates adjust more slowly at the beginning of a new regime.

Under all three ad-hoc learning rules, the fall in prices at the end of the Great Moderation is relatively slow: only as information about a change in volatility accumulates do agents adjust their estimates. Contrary to their Bayesian counterparts, recursive, ad hoc learning rules are thus not able to deliver sudden crashes in prices. Table 8 summarises the results for the benchmark case.

## 6 Relation of this Study to the Literature

This paper is most related to the literatures on asset pricing with time-varying volatility, and with learning about features of the economic environment. After earlier papers on the effect of changes in economic volatility for asset prices in stationary environments (Bonomo and Garcia (1994, 1996) and Drifil and Sola (1998), more recently Bansal and Lundblad (2002)), Lettau et al (2008) ask whether a persistent change to a low macro-volatility regime can help explain the boom in US asset prices of the 1990s and early 2000s. They find that the low volatility environment would essentially have to be permanent to explain the data.<sup>15</sup>

<sup>15</sup>Lower macro-volatility is only one item on a long list of potential reasons behind the asset price boom of the 1990s and 2000s. Others are a lower equity premium (Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2000), Fama and French (2002)), higher long-run growth (Jagannathan, McGrattan, and Scherbina (2000), Fama and French (2002), Campbell and Shiller (2004), although Siegel (1999) finds no

Most papers that incorporate learning into asset pricing models look at environments where agents learn about the mean growth rate of output or consumption. For example, Zeira (1999) looks at asset price behaviour when investors continuously update their priors about the length of high productivity regimes. Cogley and Sargent (2008b) assume that after the Great Depression, investors had pessimistic priors about the probability of transitions from a high to a low-growth state. Using a learning mechanism that is identical to one of those analysed in our study, they show how this may explain a sustained fall over time from an initially high equity premium, as learning leads to rising confidence in high growth. More recently, Adam and Marcet (2010) show how learning about an unknown process for cum-dividend equity returns introduces a self-referential element in equity prices that leads to persistent bubbles and occasional crashes. Another related paper, that does not look at learning but at misspecified beliefs about the persistence of growth regimes, is Lam et al (2000), who show that a standard asset pricing model where investors have less persistent beliefs than the truth can explain a number of asset pricing puzzles. More related to this paper is a growing number of contributions that study learning about risk. Branch and Evans (2010) employ self-referential adaptive learning about asset prices and return volatility in order to explain high frequency booms and busts in asset prices. Weitzman (2007) adopts a consumption-based asset-pricing model and replaces rational expectations with Bayesian learning about consumption growth rate volatility, which allows him to solve a number of asset pricing puzzles. Finally, a recent paper by Johannes, Lochstoer and Mou (2011) look at Bayesian learning when agents are uncertain not only about current volatility and growth regimes and the parameters governing their transitions, but also attach positive prior probability to more than 1 model. They show how this leads to non-stationary revisions of beliefs that are correlated with realised asset returns. Moreover, with standard preferences, this setup implies a realistic equity premium and return predictability.

Most relevant for this paper are two studies that link the asset price boom and bust of 1990s and 2000s to learning about regime changes in key parameters of the economic environment. Boz and Mendoza (2010) study a partial equilibrium model where investors face an exogenous leverage

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evidence for this), stronger intangible investment in the 1990s (Hall (2000)) saving during the 1990s by the baby boom generation (Abel (2003)), redistribution of rents towards owners of capital (Jovanovic and Rousseau (2003)) or reduced costs of stock market participation and diversification (Heaton and Lucas (2000), Siegel (1999), Calvet, Gonzalez-Eiras, and Sodini (2003)).

constraint that follows a two-state Markov process with unknown transition probabilities. Assuming Bayesian learning as in Cogley and Sargent (2008b), the authors show that with little prior information, the observation of a string of high leverage periods can lead to overoptimism about their persistence and thus a boom in asset prices, leverage and consumption which crashes abruptly once the economy switches back to a tighter constraint. While one of our learning mechanisms also follows Cogley and Sargent (2008a), we analyse exogenous changes in macro-volatility, rather than in regimes of financial regulation. This focus is similar to that of Lettau et al. (2008) who also study the asset price effect of changes in macro volatility-regimes under limited information about the environment. Particularly, while knowing all parameters of the data-generating process, including the persistence of volatility regimes, agents in their model ignore whether the economy is currently in a high or low volatility regime.<sup>16</sup> The authors then calculate asset prices given the sequence of posterior state probabilities implied by an econometric regime-switching model estimated on post-war consumption data for the US. This setup implies, first, that model-implied prices are always lower than those that would prevail in the most benign low-volatility regime with full information. Moreover, this setup makes the quantitative results highly dependent on the parameter that describes the persistence of the low volatility state. Only when low volatility is, essentially, permanent (in the sense of a probability of remaining at low volatility in the following quarter of more than 99.9 percent), does the Great Moderation imply a strong increase in asset prices.

Our results complement this literature in several ways. First, we point out that uncertainty about the persistence of the Great Moderation directly increases prices, due to a Jensen's inequality effect that results from the convexity of prices in the persistence parameter under full-information (Lettau et al 2008). Second, we are the first to systematically model the intuition, found in policy statements and the popular press, that increasing confidence in the persistent, or permanent, nature of the Great Moderation contributed to the boom and bust in asset prices in the 1990s and 2000s. And finally, we show how Bayesian learning schemes that capture this intuition in a standard asset pricing model imply a boom in asset prices, and a subsequent bust if the economy returns to a regime of high volatility, of between 35 and 80 percent, and compare the size and shape of these

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<sup>16</sup>Lettau et al (2008) also have two states of different mean growth, leaving four states of the economy in total.

boom and bust cycles across different learning models.

## 7 Conclusion

From a review of both academic and investment research, we conclude that, first, the “Great Moderation” in macro-volatility was perceived to be an important factor behind the asset price boom of the 1990s and 2000s, and, second, that academics and investors alike were uncertain about the origins and persistence of the new low-volatility environment. Using different learning mechanisms, we modelled this uncertainty explicitly in an asset pricing model with time-varying volatility. The results confirmed the intuition of policymakers (Bean 2009, Haldane 2009) that increasing confidence in a benign macroeconomic environment may have led to a strong and gradual increase in asset prices above values that are consistent with ex-post estimates of the persistence of volatility regimes. In particular, we find that Bayesian learning can lead to an asset price boom of around 30 to 80 percent. This increase results from both an increase in posterior mean persistence and a pure Jensen’s inequality effect that increases asset prices with uncertain transition probabilities above certainty levels. A learning scheme with a two-point prior distribution that highlights the uncertainty about a permanent vs. transitory Great Moderation leads to a stronger boom in asset prices and an S-shaped time pattern of price increases that is more in line with the data than the concave price path under our benchmark learning mechanism based on Cogley and Sargent (2008a). Finally, Bayesian learning predicts that the end of the low-volatility period, which we identified with the beginning of the recent crisis, leads to a strong crash in prices. Neither of the two Bayesian learning schemes, however, replicates the exact shape of asset prices observed in the data, particularly the strong rise and fall around the so-called dotcom boom. Ad hoc, or statistical, learning rules, also predict a strong boom in prices, but do not predict a strong crash at the end of the Great Moderation period, as they react much more slowly to information than Bayesian learning schemes.

Future research could extend this study in several directions. For example, although mean growth during the Great Moderation was essentially the same as during the preceding period, it should be interesting to include time variation in the mean growth of the economy. Also, one could analyse



an alternative scenario where agents directly form expectations about future prices, rather than the distribution of dividends as in the model studied here. Adam and Marcet (2010) show how this can lead to self-fulfilling bubbles and crashes in asset prices, as a rise in prices is sustained by generating expectations of rises in the future. When learning about volatility, this self-referential mechanism is less clear, as higher expected volatility primarily feeds into the level of prices, and not into their second moment. But when volatility is itself stochastic, as for example in Bacchetta et al (2010), there is potential for self-referential learning about asset price volatility. These issues, however, are left for future research.

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## 8 Appendix

### 8.1 Data Appendix

**Consumption** is quantified as the *Total Real Personal Consumption Expenditures* measured in quantity index [index numbers, 2005 = 100]. The data are quarterly, seasonally adjusted and their source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

**GDP** is quantified as the *Real Gross Domestic Product*, measured in 2005-chained dollars. The data are quarterly, seasonally adjusted and their source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

**Population** is quantified as the *Midperiod Population* of each quarter. The data source is the National Economic Accounts of the Bureau of Economic Analysis (BEA).

**Asset Price** is quantified as the average *S&P 500 Stock Price Index* of each quarter. The data source is Robert Shiller's homepage. The original data are monthly averages of daily closing prices.

**Dividend** is quantified as the original quarterly *Dividend Payment* reported in the Robert Shiller's homepage.

**Price-Earning Ratio** is quantified as the *Cyclically Adjusted Price Earnings Ratio* (P/E10), known also as the *CAPE*. The data source is the Robert Shiller's homepage.



## 8.2 Appendix B

### 8.2.1 Solving for the equilibrium price numerically with Bayesian learning about Transition Probabilities

The Bayesian agent enters each period with a prior. He observes the realization of the exogenous process and he updates the counters

$$\begin{aligned}n_{t+1}^{ij} &= n_t^{ij} + 1 \text{ if } s_{t+1} = j \text{ and } s_t = i \\n_{t+1}^{ij} &= n_t^{ij} \quad \text{if } \textit{otherwise}.\end{aligned}$$

The posterior density function is

$$f(F_{hh}, F_{ll} | \Sigma^t) = \textit{beta}(n_t^{hh}, n_t^{hl}) * \textit{beta}(n_t^{ll}, n_t^{lh}). \quad (15)$$

We would like to calculate

$$p_t = \int p(S_t, F) f(F | \Sigma^t) dF$$

which can be also expressed as

$$\int p(S_t, F) f(F | \Sigma^t) dF = E_{\Sigma^t}[p(F)]. \quad (16)$$

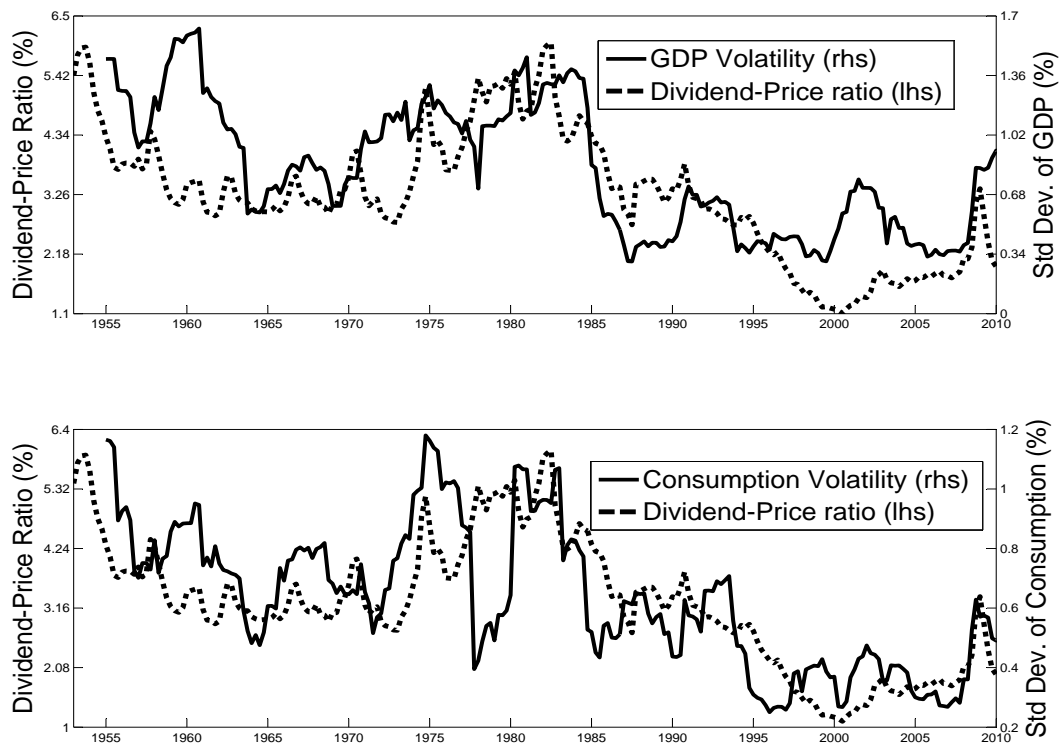
Therefore, equation (2) can be approximated as

$$E_{\Sigma^t}[p(F)] \approx \frac{\sum_{i=1}^n p(S_t, F_i)}{n} \quad (17)$$

In order to compute equation (17) at each time  $t$  we generate a sample of  $n = 3000$  transition probability matrixes,  $F$ , as random observations from equation (15). Then, we approximate the price function by its sample average, so

$$p_t \approx \frac{\sum_{i=1}^n p(S_t, F_i)}{n}$$

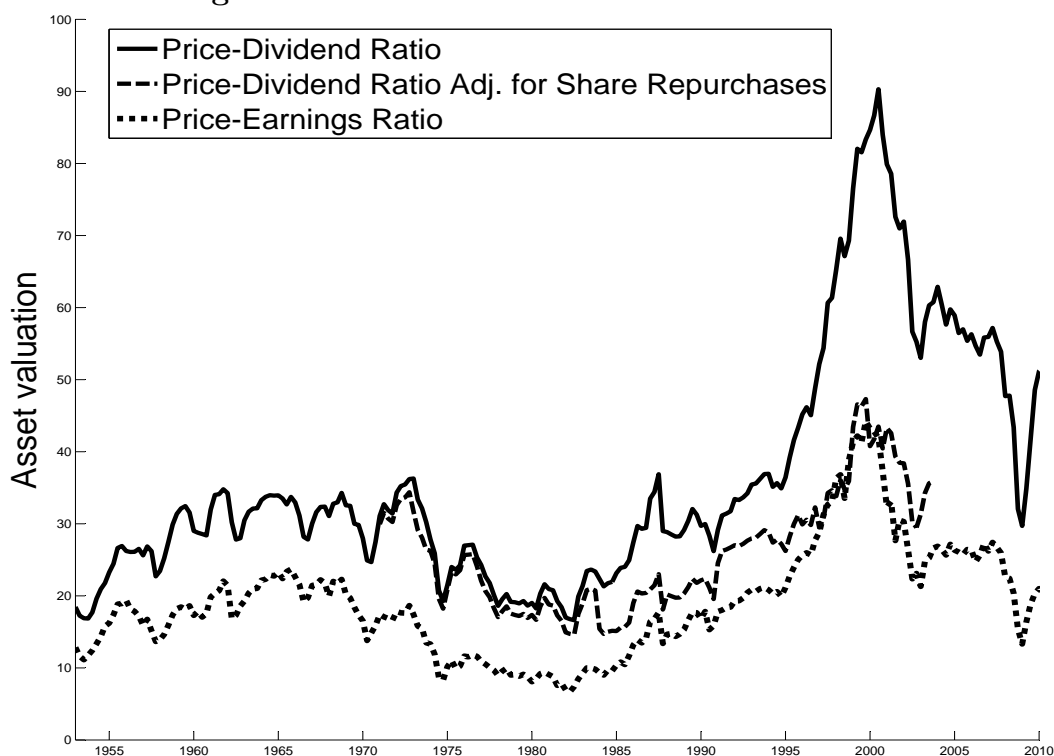
Figure 1: Macro-Volatility and Asset Prices



The figure plots the dividend price ratio together with the standard deviation of the real GDP growth rate (first subplot) and the standard deviation of the real consumption growth rate (second subplot), estimated in 10-quarter rolling windows. GDP and consumption are defined as in figures 1 and 2. The financial data are taken from the Robert Shiller's homepage and the rest of the data from the BEA.

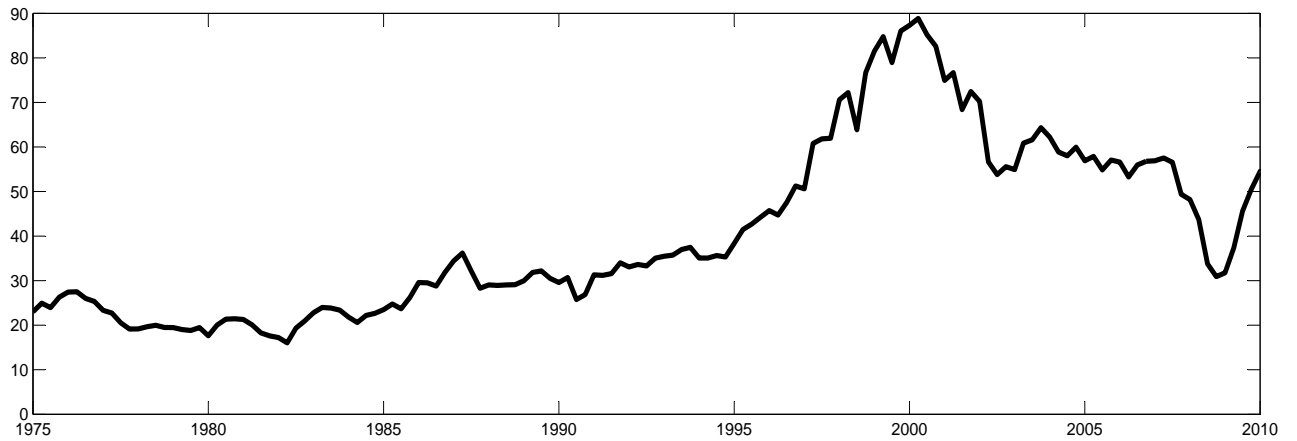
## 9 Figures

Figure 2: Stock Market Valuation Measures

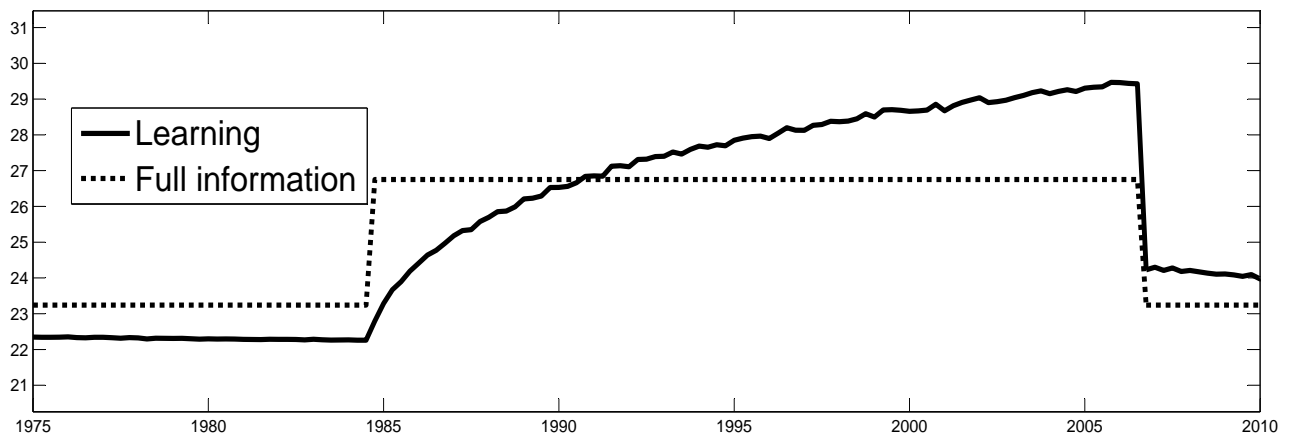


The figure plots the price-dividend and the price-earnings ratio for the *S&P* 500, as well as the price-dividend ratio adjusted for share repurchases using the data by Boudoukh et al (2007), which is available between 1971 and 2003. As their sample of US firms is slightly broader than that underlying the measures for PD and PE ratios, which are taken from Robert Shiller’s homepage, the adjusted PD ratio is calculated as  $PD^{adj} = PD \frac{PD^{adj*}}{PD^*}$ , where a  $\star$  denotes the measures presented in Boudoukh et al (2007). The data on prices are monthly, those on dividends and on the price-earnings ratio are quarterly, and those on the adjusted price-dividend ratio is yearly. We calculate quarterly estimates for the prices by taking quarterly averages over the monthly data, and using the yearly observation for the adjusted series in all 4 quarters.

Figure 3: Price-Dividend Ratios: Benchmark Model  
US Data

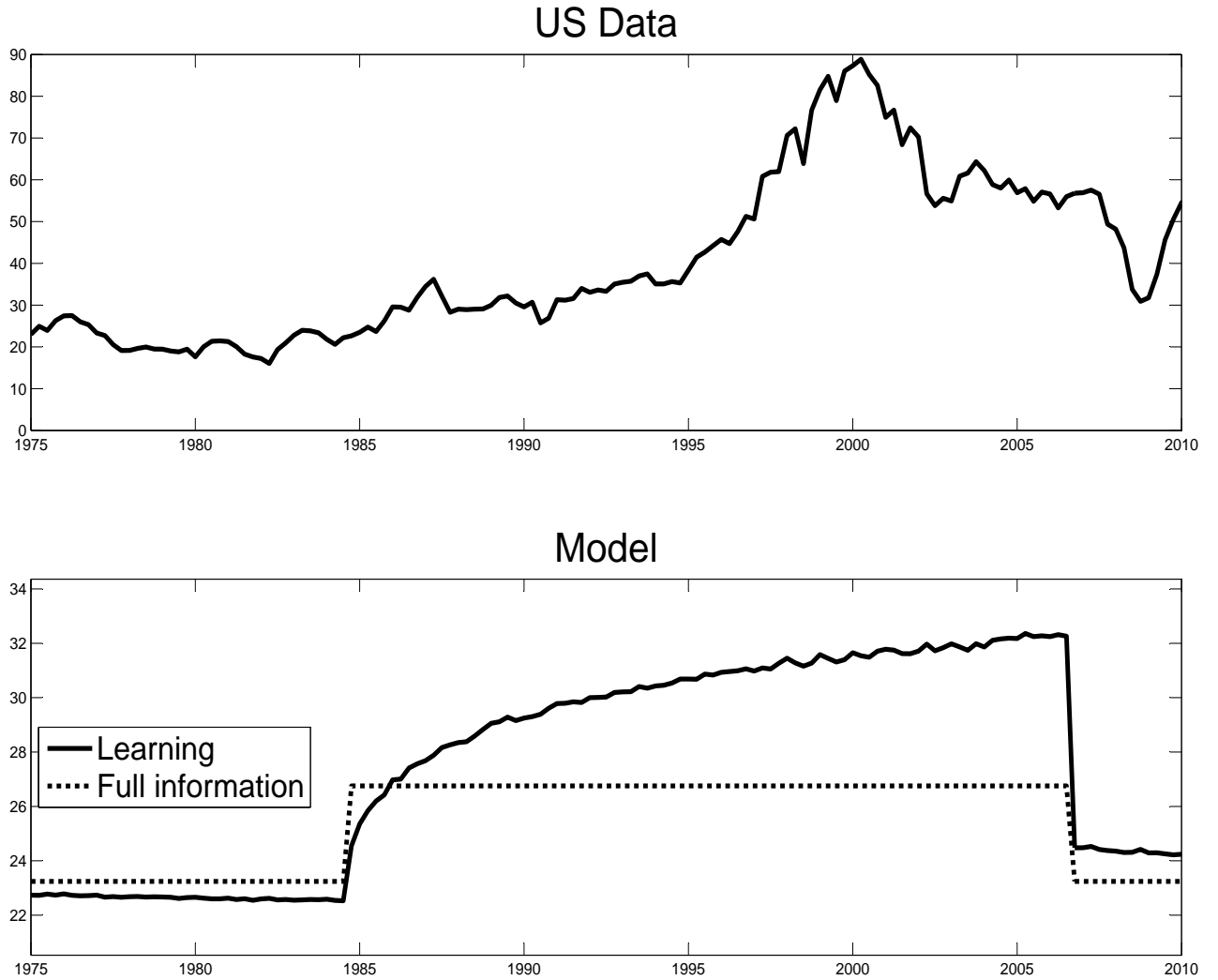


Model



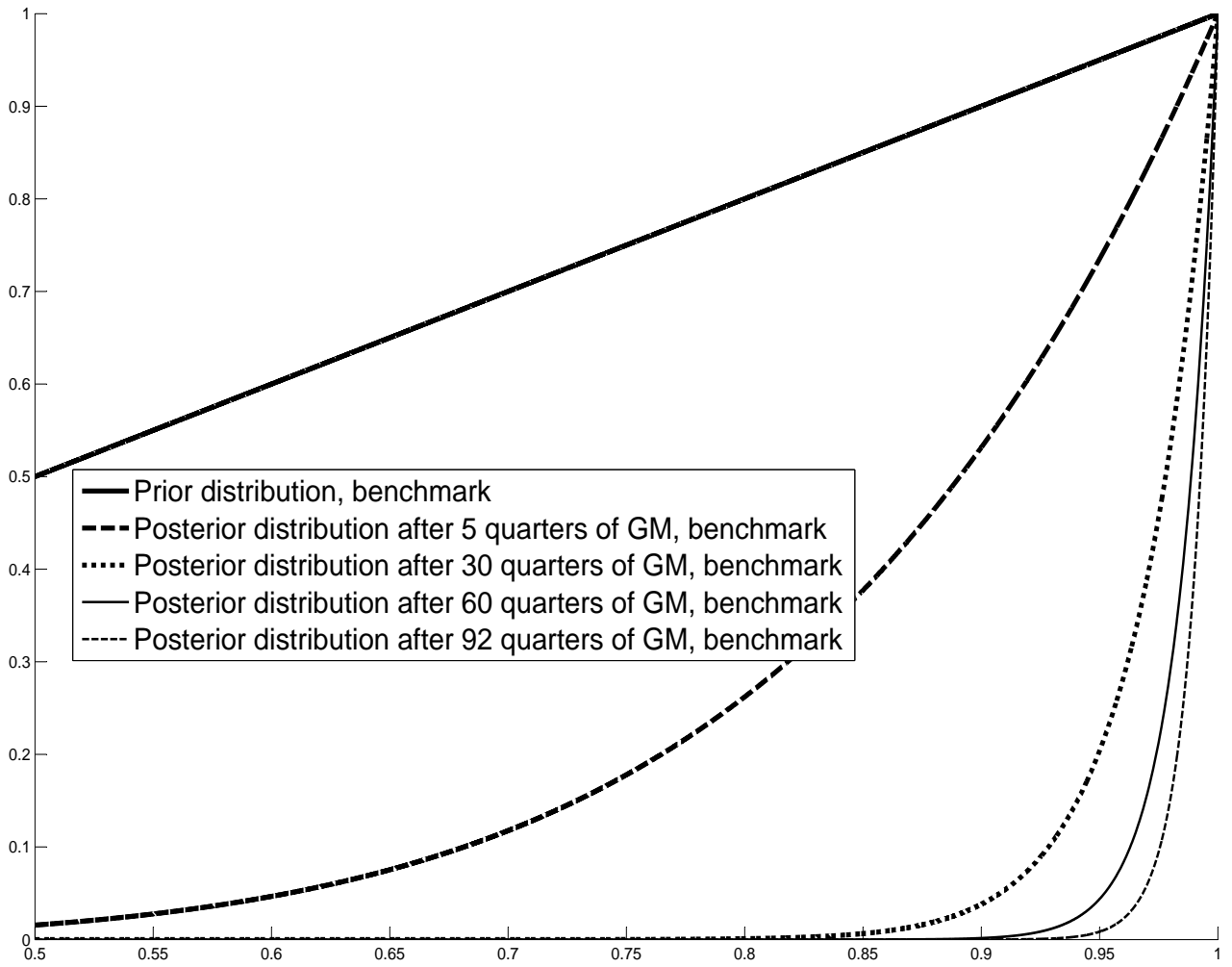
The figure plots the price dividend ratio in US data (upper Panel), and in the model (lower panel), for the benchmark calibration of the model.

Figure 4: Price-Dividend Ratios with Moderately Persistent Prior



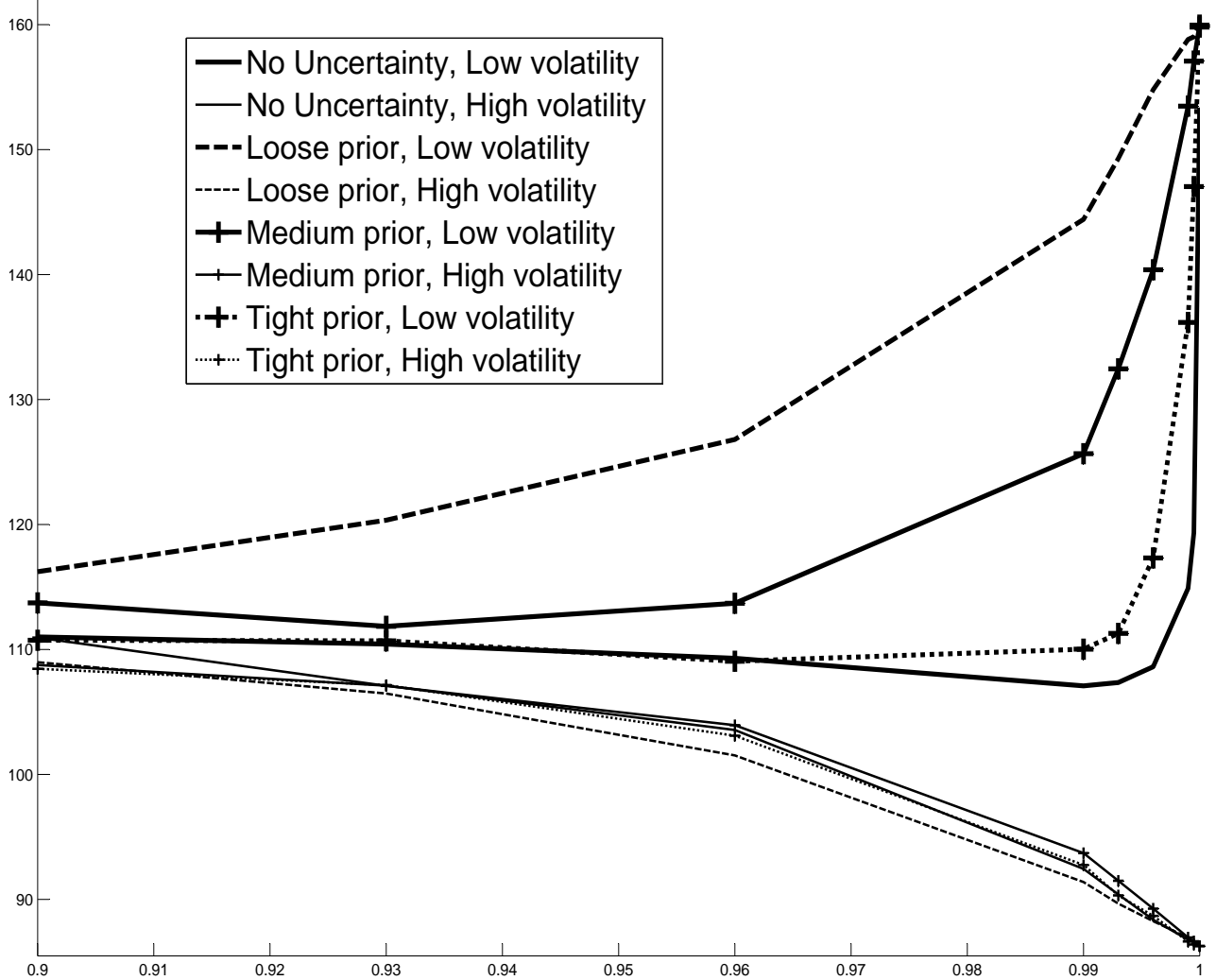
The figure plots the price dividend ratio in US data (upper Panel), and under learning about transition probabilities with beta priors (lower panel), for a moderately persistent prior with mean persistence of 0.75.

Figure 5: Posterior Distributions



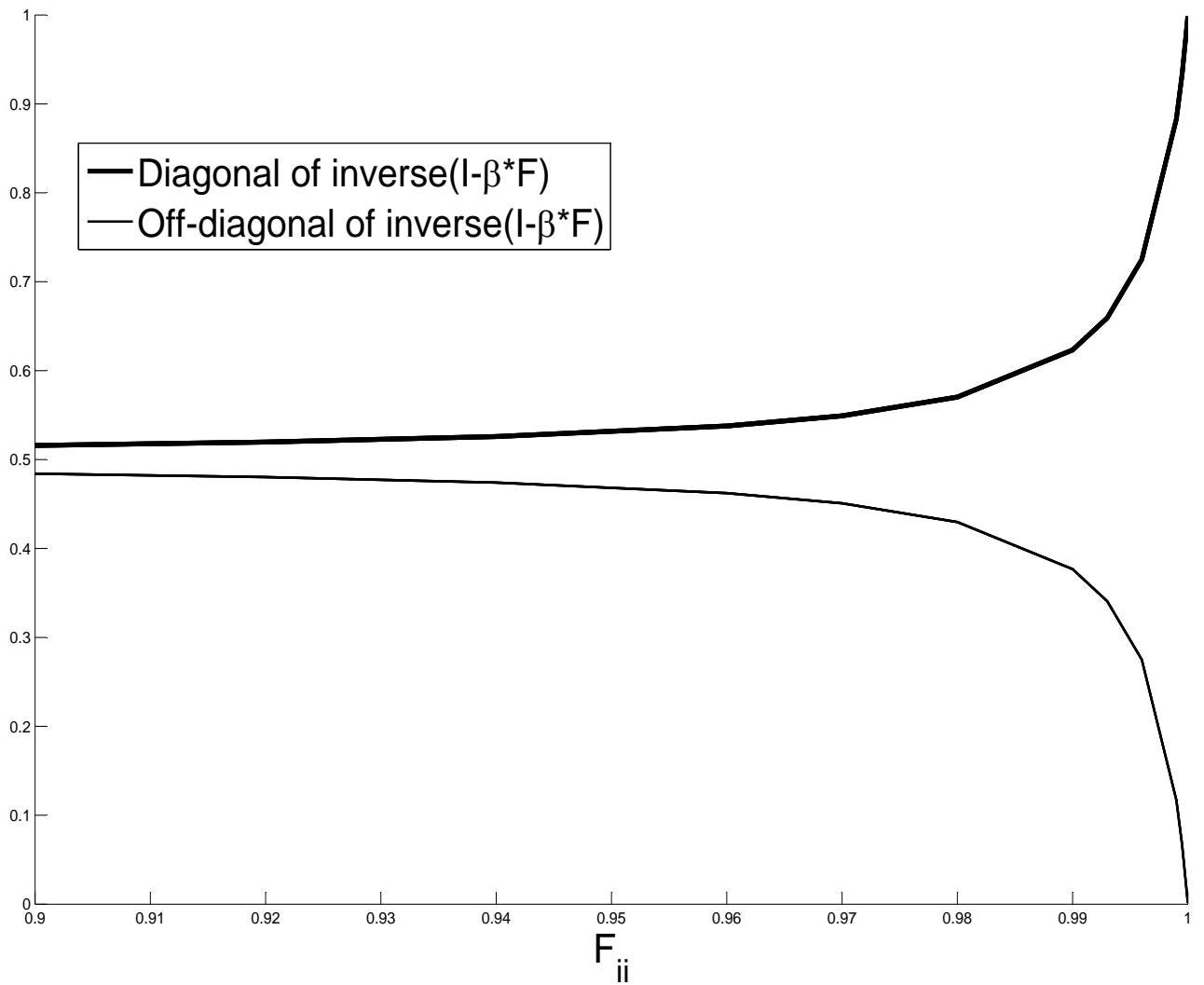
The figure plots the cumulative posterior distributions for the transition probability  $F_U$  after an increasing number of observations on the Great Moderation, starting from an uninformative uniform prior.

Figure 6: Price-Dividend Ratios as a Function of Persistence and Prior Tightness



For the simplified case of symmetric transition probabilities ( $F_{ll} = F_{hh}$ ), the figure plots the price dividend ratio as a function of persistence for different values of the tightness of priors for the benchmark calibration of the model.

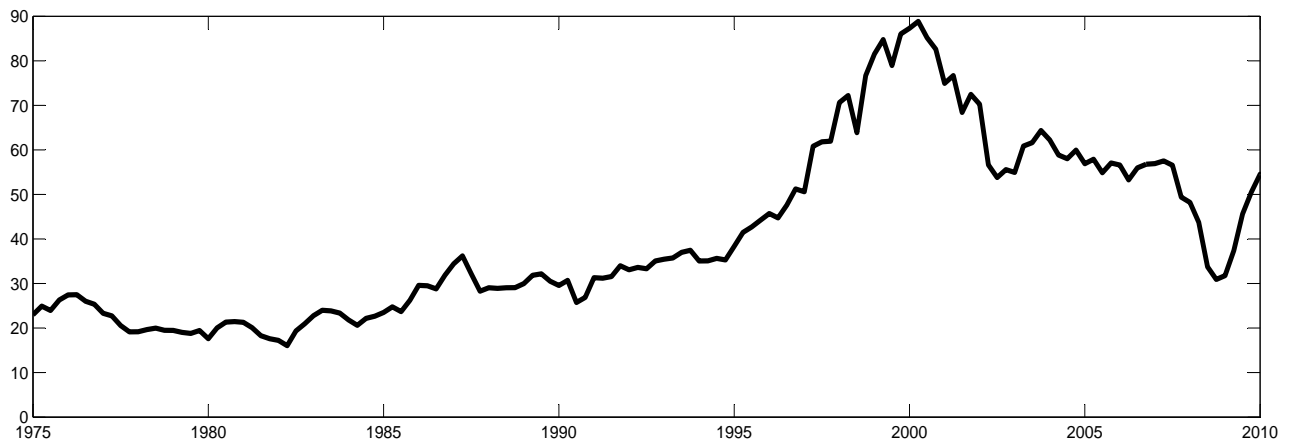
Figure 7: Behind the Non-Linear Asset Price-Persistence Relation



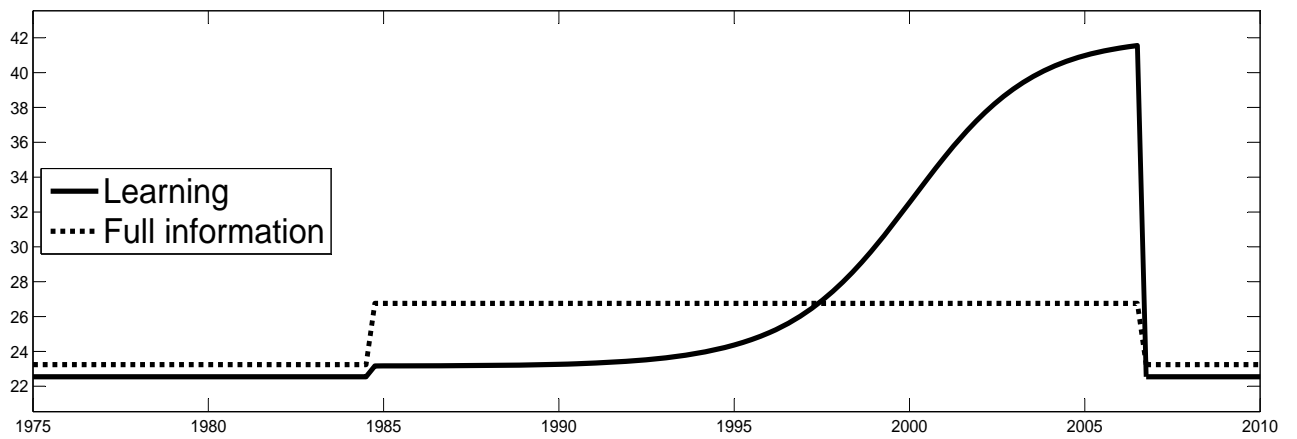
For the case of symmetric transition probabilities  $F_{hh} = F_{ll}$ , the figure depicts the diagonal and non-diagonal elements of the present discounted value matrix  $(I - \beta F)^{-1}$ .



Figure 8: Price-  
Dividend Ratios: Learning about a Permanent vs. Transitory Great Moderation  
US Data

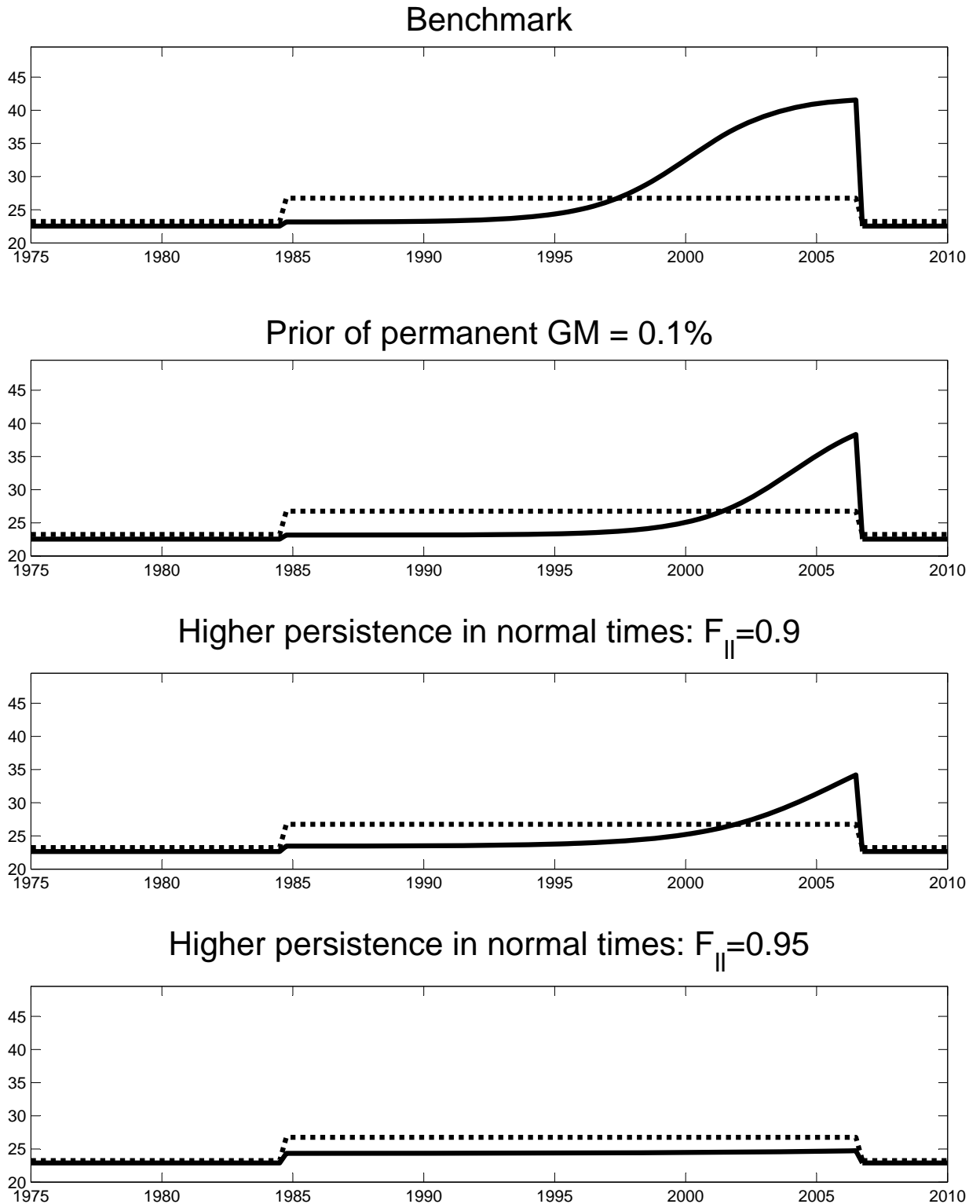


Model



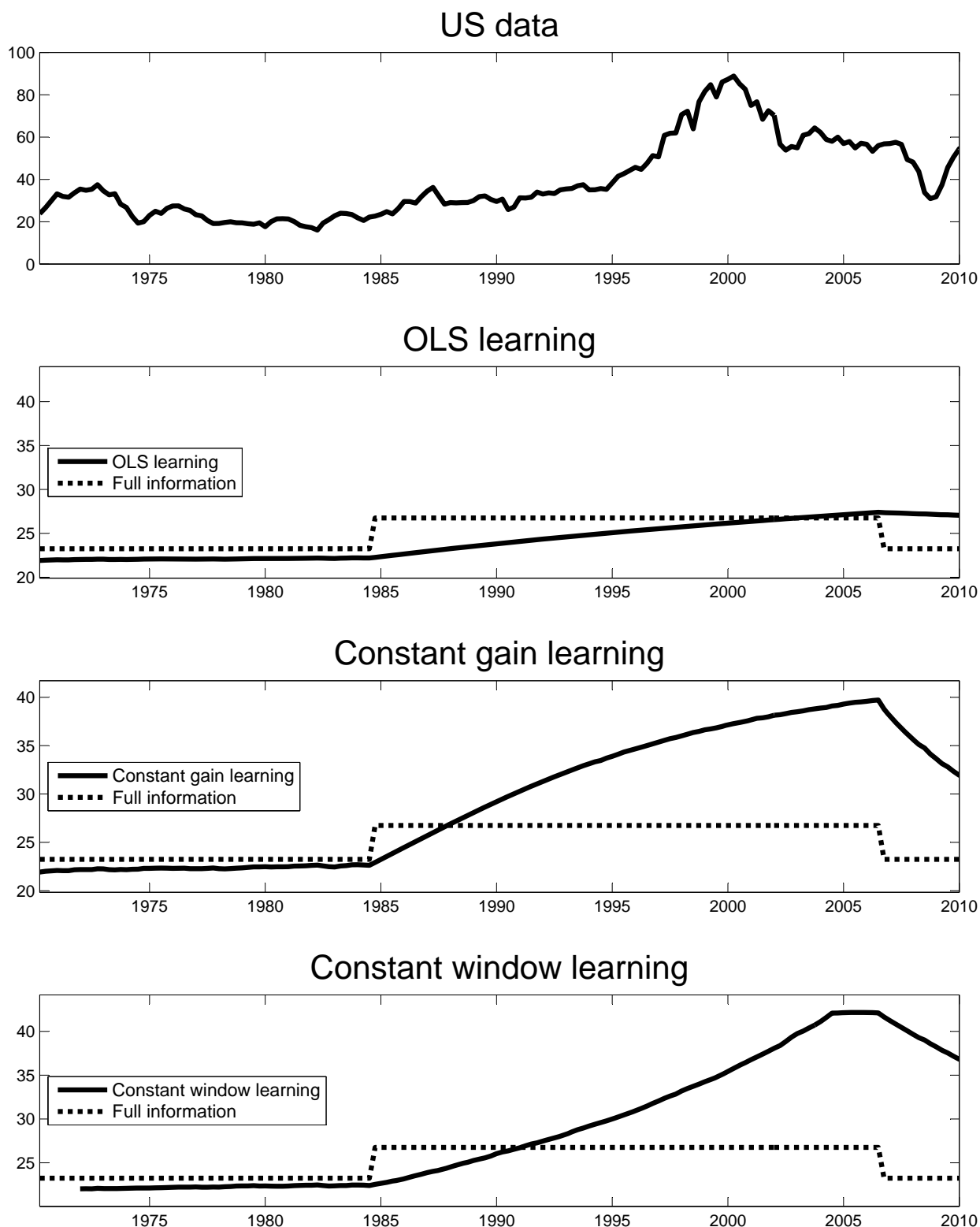
The figure plots the price dividend ratio in US data (upper panel), and under learning about a permanent vs. transitory Great Moderation (lower panel), for the benchmark calibration of the model.

Figure 9: Learning about a Permanent vs. Transitory Great Moderation with Different Priors



The figure shows the time-path of dividends with learning about a permanent vs. transitory Great Moderation with different prior probabilities.

Figure 10: Price-Dividend Ratios - Ad Hoc Learning



The figure plots the price dividend ratio in US data (upper Panel), and under three ad hoc learning rules: OLS (second panel), constant gain (third panel), and constant window (bottom panel), for the benchmark calibration of the model. The full information prices correspond to the case of high persistence.