Securitization bubbles: Structured finance with disagreement about default risk

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Abstract

An additional reason for the structured finance boom of the 2000s may have been disagreement about default risk of collateral assets. When risk-neutral investors disagree about average default probabilities, structuring collateral cash flow raises prices by concentrating optimists' demand on risky tranches. With disagreement about default correlation, low-correlation investors believe in diversification and pay high prices for senior tranches they deem riskless. High-correlation investors value junior tranches they expect to pay whenever aggregate conditions are good. Risk version and short-selling through Credit Default Swaps reduce the prices of both pass-through and structured securitizations, but may increase the 'return to tranching'.

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1. Introduction

Structured financial products, which allocate the cash flow from a pool of collateral assets to different tranches in order of pre-specified seniority, have been blamed both for their contribution to the US housing boom of the early 2000s, and for the large losses of financial institutions during the crisis that followed. In fact, structured assets, such as residential mortgage-backed securities (RMBSs) or collateralized debt obligations (CDOs), experienced an enormous rise in popularity during the early and mid-2000s, with issuance of private-label mortgage-backed securities alone amounting to USD 5.8 trillion between 1998 and 2007 (Cordell, Huang, and Williams, 2011) before issuance all but stopped by 2008. This boom in structured finance has been attributed to efficiency-enhancing risk-management benefits, through diversification and the creation of information-insensitive senior securities less prone to adverse collateral selection or issuer moral hazard and serving the safe, or AAA-rated, segment of credit markets. However, it has also been ascribed to a number of inefficient distortions, such as regulatory arbitrage (Acharya, Schnabl, and Suarez, 2013; Brunnermeier, 2009), or rating bias (Griffin and Tang, 2011, 2012), as well as to investors' disregard of certain unlikely risks (Gennaioli, Shleifer, and Vishny, 2012) or of their highly systemic nature (Coval, Jurek, and Stafford, 2009a).

This paper points out an additional factor that may have fuelled the boom in structured finance and collateral assets such as mortgage loans, and that may help us understand securitization demand more generally: disagreement about default risk. We show how disagreement about both first and second moments of the default distribution in collateral pools may increase the equilibrium prices of securitizations and the return to structuring the cash flow in tranches rather than selling it as shares in one single pass-through securitization. When disagreement is mainly about the average rate of default in the collateral pool, structuring increases the market value of collateral by selling supersenior tranches to pessimists without a discount, thus concentrating optimists' funds on the risky tranches they value relatively highly. When some risky tranches need to be sold to pessimists, however, all tranche prices are below optimistic valuations as optimists must perceive at least the same return from buying 'their' tranches as from buying those sold to pessimists at low prices. Increasing disagreement about mean default rates may decrease prices in this case.

Disagreement about default correlations, in contrast, always raises prices. This is due to the fact that for investors who believe in low default correlation, diversification is powerful, default rates are stable, and losses are thus concentrated in junior tranches, making more senior tranches essentially risk-free. For investors who believe defaults to be more strongly correlated, in contrast, risk is more reflective of aggregate conditions. Thus, they self-select into buying risky tranches, which they believe will pay whenever times are good, while even more senior tranches retain some risk of not paying fully in bad times. The prices of both junior and senior risky tranches thus reflect the maximum payoffs expected by different investors, increasing the price of the structured securitization above the maximum expectation of its total cash flow. Because both investors buy the risky tranches that they value highly, this *return to tranching* depends less on the distribution of investor resources than with disagreement about mean default rates.

The benchmark environment features risk-neutral investors that cannot engage in short-selling. Risk-aversion makes disagreement about the distribution of high default rates (which occur in states of low consumption that risk-averse investors value more) relatively more important. As we show, this may increase or decrease the return to tranching. The introduction of trade in credit default swaps (CDSs) allows investors to effectively short-sell securitized assets, and thus reduces the prices of both the pass-through securitization and of tranches. As an Online Appendix shows in more detail, CDS trade tends to reduce the return to tranching more when investors disagree about default correlation (where it may even become negative) than when they disagree about unconditional default probabilities (where it may actually increase).

Contrary to other theories, structuring creates value from disagreement by inflating the price of risky tranches in particular, and not mainly through the creation of super-senior information-insensitive tranches that serve a safe segment of credit markets. Since disagreement acts via the perceived distribution of defaults, the theory also predicts its effect to be stronger in sectors with many assets (where the effect of correlation is not muted by idiosyncratic noise), fine granularity of tranches (even of the same ratings, which capture more of the perceived differences in default distributions) and large uncertainty about default risk (providing more room for disagreement). The paper shows how the existing empirical evidence provides some support for these predictions. It also argues that disagreement may have contributed to the bust in securitization after the crisis 'reset' perceptions of default risk and bankrupted investors with extreme beliefs who had concentrated their investments in risky tranches.

The analysis links the literature on CDO mis-pricing to that on investor disagreement, which has focused on first moments, or mean payoffs, and is thus most related to our analysis of disagreement about mean default probabilities. Asset prices are high when few optimists can buy the whole supply (or in our framework the entire securitization) at their high valuation (Miller, 1977). By increasing optimists' resources, selling riskless collateralized debt to pessimists (or, here, structuring into senior and junior tranches) can strongly increase prices (Geanakoplos, 2003, 2010). When disagreement is mainly about downside risk and optimists lack the resources to buy the entire risky part of an asset's payoff distribution, leverage may dampen the effect of belief disagreements on prices because optimists have to sell debt to pessimists at a price they perceive as low (Simsek, 2013). In our framework, disagreement is always strongest on the upside (or about the payoff from junior tranches). The effect of disagreement about mean default rates is contained, however, when some risky tranches are sold to pessimists as optimists arbitrage between 'their' tranches and those sold to pessimists. In more recent related work, Bianchi and Jehiel (2016) also consider disagreement about average default probabilities, which they assume arises because investors identify the average default probability of a loan pool with that of a single, randomly drawn loan they can observe.[1] Pooling heterogeneous loans in a pass-through securitization thus creates disagreement through sampling variation. As in Miller (1977), optimists bid up the price of the securitization as long as total investor wealth is large enough. This paper, in contrast, focuses on the return to tranching the cash flow from homogeneous collateral pools (rather than that of pooling heterogeneous collateral assets) with exogenous disagreement about both the probability and correlation of defaults.

The paper assumes that investors agree on most features of the economy but agree to disagree about one or two model parameters. This simplifying assumption seems appealing in the case of structured financial assets for a number of reasons: first, contrary to most assets that may be valued using different models with potentially many dimensions of disagreement, structured finance products came with a unique "market standard" model (Morini, 2011, p. 127) - the Gaussian copula with homogeneous correlations - with only three main parameters: the average default probability, the homogeneous correlation of defaults, and the value of loss given default. Second, their short history provided little information about the stochastic properties of collateral assets such as subprime RMBS tranches, or, as a matter of fact, their constituent non-prime mortgages. Moreover, there is no consensus about how to best estimate correlation patterns as an input to credit risk models (Luo, Tang, and Wang, 2009). Importantly, investors seem to have ignored the implied parameter uncertainty (Coval, Jurek, and Stafford, 2009b; Morini, 2011), making heterogeneous point-beliefs an ap-

[1] I thank the anonymous referee for drawing my attention to their study after the revised version of this paper was completed.

pealing, if simplifying, assumption.[2] Finally, the scope for learning about fundamentals from market prices may be limited by over-the-counter trades and several unobserved dimensions of investor heterogeneity (most of which the analysis abstracts from), including a suspected lack of understanding of the complexity of the assets traded by some participants.

Although the market for structured finance products was indeed a rated market, the investors in our analysis rely on their own beliefs. This is in line with evidence from surveys that suggests that ratings were only one of many elements in the assessment of credit risks by investors (Fender and Mitchell, 2005). Moreover, the junior tranches where, as it turns out, disagreement leads to the strongest differences in value, were usually held by specialist investors who are likely to rely less heavily on ratings in their judgments (Fender and Mitchell, 2005, p.70). In fact, the prices of both non-AAA tranches of RMBSs (Adelino, 2009) and ABS-CDOs (Mählmann, 2012) did incorporate information over and above credit ratings.[3]

2. Securitization with disagreement about default risk

Consider a static, one-period economy populated by risk-neutral investor types indexed by $k \in \mathbb{K}$. At the beginning of the period, investor k receives a non-storable consumption endowment a^k, which she can keep for consumption at the end of the period or invest in assets collateralized by a pool of N mortgages of equal face value that pay an uncertain amount at the end of the period. The whole pool is sold to investors in the form of mortgage-backed securities by an originator who plays no other role in the analysis. Investors

[2]See Gorton and Metrick (2013, p. 112) for a description of the valuation of structured products by investors who use "vendor-provided packages that model the structure of structured products, but the valuation is based on (point estimate) assumptions that are input by the user".

[3]Firla-Cuchra (2005) finds similar evidence for European securitization data.

disagree about default risk in the mortgage pool along two dimensions: first, they have heterogenenous views about the average likelihood that any given loan defaults. Second, they disagree about whether defaults are driven by common or loan-specific factors, and thus about the correlation of defaults. I compare two ways in which the loan pool is sold to investors: as shares in a pass-through securitization that pays all investors their share in the total cash flow that the collateral generates; or structured as an RMBS by splitting the cash flow into tranches that receive payments in strict order of their pre-specified seniority.

The rest of this section shows, by means of a simple example, how the market price of structured and non-structured securitizations is affected by disagreement about default risk. For this, I consider an economy populated by two risk-neutral investor types (so $u(c) = c$ and $\mathbb{K} = \{1, 2\}$), each of unit mass, with a pool of two mortgages of face value 1.5 each. Disagreement about the probability of default is captured by the perceived unconditional default probability $\overline{\pi}^k$, which is equal for both mortgages. Heterogeneous views about how correlated are defaults are captured by π^k_{DD}, the conditional probability of one mortgage defaulting given default of the other.[4] Finally, assume that investors agree on a recovery value V_{rec} in case of default equal to 0.5.

Investors value the cash flow of the mortgage pool by its expected payoff

$$V^k_{sec} = 1 + 2(1 - \overline{\pi}^k). \qquad (1)$$

Given the set of possible payoffs $\{1, 2, 3\}$ in case both, one, or no mortgage defaults, the natural structure for this loan pool consists of three tranches of

[4]Note that this maps into the default correlation, calculated by assigning the numerical value zero to the outcome of default and one to its complement, equal to $\rho^k_D = \frac{\overline{\pi}^k \pi^k_{DD} - (\overline{\pi}^k)^2}{\overline{\pi}^k (1 - \overline{\pi}^k)}$. Note also that the conditional probability of the second loan being repaid, given payoff by the first, which equals $\pi^k_{NN} = 1 - \frac{\overline{\pi}^k}{1 - \overline{\pi}^k}(1 - \pi^k_{DD})$, is increasing in π^k_{DD}, as is the variance of default rates $Var^k_d = 2[\overline{\pi}^k(1 - 2\overline{\pi}^k) + \overline{\pi}^k \pi^k_{DD}]$.

unit-face value, which I call senior, mezzanine, and junior. The senior tranche pays off with certainty, as it is protected by the recovery value, implying a unit-valuation by both investor types. The value of the mezzanine tranche is reduced by the probability of joint default $\overline{\pi}^k \pi^k_{DD}$

$$V^k_{mezz} = 1 - \overline{\pi}^k \pi^k_{DD}. \tag{2}$$

The junior tranche does, in turn, only pay when both mortgages pay, so

$$V^k_{jun} = 1 - 2\overline{\pi}^k + \overline{\pi}^k \pi^k_{DD}. \tag{3}$$

While the valuation of the non-structured securitization, as well as those of both risky tranches, i.e. of the mezzanine and junior tranche, is declining in the default probability $\overline{\pi}^k$, the default correlation parameter π^k_{DD} leaves that of the mortgage pool as a whole unaffected, but affects the valuation of risky tranches in opposing directions. Particularly, a rise in π^k_{DD} increases the value of the junior tranche, which gains from a rise in the probability of the extreme case of both mortgages paying off. The mezzanine tranche, in contrast, loses value as the likelihood of both mortgages defaulting increases.

Denote the equilibrium price of the entire pass-through securitization as P_{sec}, and the tranche prices as $P_{sen}, P_{mezz}, P_{jun}$, respectively. Given one of the two securitization possibilities - structured or pass-through - an equilibrium is defined as a vector of prices and investor portfolios such that investors maximize expected consumption and demand for all assets equals the exogenous supply. Investor optimality implies that the expected return from all positive investments in their portfolio must be the same, at least equal to 1, and not lower than that from assets that they do not hold. Finally, define the price of the structured, or tranched, securitization by arbitrage as $P_{sec|tr} = P_{sen} + P_{mezz} + P_{jun}$.

The rest of this section studies how disagreement about the default risk parameters $\overline{\pi}^k$ and π^k_{DD} affects the prices of non-structured and structured securitizations. In particular, we are interested in the return to tranching, which we define as the difference in the equilibrium prices of the structured and non-structured mortgage pools as a fraction of the latter. With disagreement, these prices typically depend on the resources of different investors relative to the asset supply. For example, with disagreement about mean default probabilities $\overline{\pi}^k$, equilibrium prices crucially hinge on the resources of optimists, who value all risky tranches strictly more than do pessimists. For simplicity, the analysis leaves the face value of the mortgage pool unchanged at three, but considers different distributions of endowments across investors. Specifically, I assume total endowments to be large enough for all investor types together to afford the securitization even at its highest price.[5] However, the endowment distribution among types varies subject to the condition that disagreement still matters, which I define as a minimum endowment level such that investors of both types can afford at least the entirety of any single of the two risky tranches at their own valuation (implying $a_k \geq V^k_{mezz}$, $k = 1, 2$).

2.1. *Disagreement about default probability* $\overline{\pi}^k$

This section studies a version of the economy where investors disagree about the probability of default $\overline{\pi}^k$, but share the same belief about their correlation $\pi^1_{DD} = \pi^2_{DD} = \pi_{DD}$. This is thus the canonical case of disagreement about mean payoffs where investors are divided into optimists and pessimists. As we will see, different assumptions about the size of optimist endowments imply equilibria that share certain features of classical cases analyzed in the literature on investor disagreement.[6]

[5] This implies a lower bound on total endowments at least equal to the maximum price of the securitization with, respectively, disagreement about $\overline{\pi}$ and π_{DD}: $A \geq \underline{A} = 1 + \max\{2(1 - \min_k \overline{\pi}^k), 2(1 - \overline{\pi}) + \overline{\pi}(\pi^1_{DD} - \pi^2_{DD})\}$.

[6] The analysis in this section is related to Bianchi and Jehiel (2016), who also point out that tranching can increase the price of a pass-through securitization when investors disagree about the mean default probability of a collateral pool.

Let $\overline{\pi}^1 > \overline{\pi}^2$ and call investors of type two 'optimists', and investors of type one 'pessimists'. The following proposition shows how structuring increases the market value of collateral when optimists cannot afford to buy the entire collateral pool at their own, optimistic valuation by concentrating their investments on the risky tranches they value comparatively highly.

Proposition 1. *If $a^2 \geq V^2_{sec} = 1 + 2(1 - \overline{\pi}^2)$, such that optimists can afford the entire mortgage pool at their own valuation, the prices of both the pass-through and structured securitization equal the optimistic valuation: $P_{sec} = P_{sec|tr} = V^2_{sec}$. So the return to tranching is zero.*
If $a^2 < V^2_{sec}$, the return to tranching is strictly positive. The price of the pass-through securitization equals the maximum of optimists' resources and the pessimists' valuation: $P_{sec} = max\{a^2, V^1_{sec}\} < V^2_{sec}$. The price of the structured securitization equals the optimistic valuation V^2_{sec} if $V^2_{sec} > a^2 \geq V^2_{mezz} + V^2_{jun} = 2(1 - \overline{\pi}^2)$, such that optimists can afford both risky tranches at their own valuation. If $a^2 < V^2_{mezz} + V^2_{jun}$, its price equals $P_{sec|tr} = 1 + \max\{a^2, V^1_{mezz}[1 + \frac{V^2_{jun}}{V^2_{mezz}}]\} > P_{sec}$.

Proposition 1, whose proof is contained in an appendix, first states the well-known property that, in the absence of short-selling, disagreement about mean payoffs increases an asset's market price above its mean valuation when a fraction of optimists can purchase the whole supply (Miller, 1977). Since this applies to both the pass-through and structured securitization, originators have no incentive to tranche the collateral cash flow in this case. In contrast, when optimists lack funds to buy the whole collateral pool, structuring can raise prices by concentrating optimists' resources on the risky tranches that they value comparatively highly. This is most powerful when optimists can afford both risky tranches at their high valuation but not the entire collateral pool. Structuring then allows originators to sell the senior tranche to pessimists at a price equal to its face value and the risky tranches to optimists at their high valuation. This is similar to Geanakoplos (2003) and Geanakoplos (2010), where optimists raise funds for investment through leverage, by selling the riskless part of an asset's payoff distribution to pessimists. When optimists cannot afford both risky tranches at their high valuation, however, their absolute advantage in buying default risk can no more be fully realized. Pessimists then buy (part of the) mezzanine tranche, for which their valuation compared to that of optimists is higher than for the junior tranche, at a price equal to their lower valuation. The junior tranche continues to be sold to optimists, but at a discount (relative to its optimistic valuation) that makes them indifferent to buying the mezzanine tranche at its low price.

[Figure 1 about here.]

Figure 1 summarizes the results of this section for an illustrative parameterization, with $\overline{\pi}^1 = 0.2, \overline{\pi}^2 = 0.4, \pi_{DD} = 0.6$, and total endowment $a^1 + a^2 = 3$. The prices of both pass-through and structured securitization are increasing in optimists' endowment a^2 (depicted along the bottom axis). The return to tranching is zero for high levels of a^2, high for intermediate a^2, and lower for low levels. Interestingly, at low levels of a^2 the price of the structured securitization is below the mean collateral valuation (corresponding to the expectation of the collateral cash flow evaluated at the mean default probability $\frac{\overline{\pi}^1+\overline{\pi}^2}{2}$). This illustrates how, when optimist endowments are low, rising disagreement can, in fact, reduce the equilibrium price of the structured securitization. This result, discussed more formally in the Online Appendix, is similar, although not identical, to that in Simsek (2013) where disagreement can discipline asset prices when optimists leverage asset purchases by selling risky collateralized debt to pessimists. This happens when disagreement is concentrated in low payoff states and the price at which pessimists are willing to buy collateralized debt is below the present value of payments that optimists expect to make. Leverage thus becomes less attractive, and the positive effect of disagreement on asset prices is reduced. In the present paper, in contrast, disagreement is, by assumption, strongest about upside risk, embodied in the valuation of junior tranches. Nevertheless, when optimists lack funds and part of the mezzanine tranche needs to be sold to pessimists, the increase in the price of the junior tranche bought by optimists is contained by an arbitrage condition with the mezzanine tranche.

2.2. Disagreement about default correlation π^k_{DD}

Previous studies of investor disagreement have entirely focused on the first moment of the payoff distribution.[7] While risk-neutral investors are indeed indifferent about the payoff dispersion of an asset that they consider buying with cash, the dispersion of default rates turns out to be an important determinant of risk-neutral valuations of structured financial assets. To show this, this section studies a version of the economy where investors agree about mean payoffs, or the expected default rate in the mortgage pool, but have heterogeneous views about whether defaults are driven by common or loan-specific factors. Specifically, suppose agents agree about the probability of default $\overline{\pi}^1 = \overline{\pi}^2 = \overline{\pi}$, but disagree about the default correlation, with $\pi^1_{DD} > \pi^2_{DD}$.

Proposition 2. *The equilibrium price of the pass-through securitization equals the common expectation of the collateral cash flow:* $P_{sec} = 1 + 2(1 - \overline{\pi})$*. The equilibrium price of the structured securitization equals the sum of the highest expected tranche payoffs across investors, so* $P_{sec|tr} = 1 + 2(1 - \overline{\pi}) + \overline{\pi}\Delta\pi_{DD} > P_{sec}$*, for the disagreement measure* $\Delta\pi_{DD} = \pi^1_{DD} - \pi^2_{DD}$*. So the return to tranching is strictly positive and independent of the endowment values* $\{a^1, a^2\}$*.*

Proof of Proposition 2. The first part of the statement follows from the assumption that together, investors can always afford to buy the entire securitization at their common valuation. For tranche prices $P_{jun} = V^1_{jun} > V^2_{jun}$,

[7] An exception is Broer and Kero (2014), who consider disagreement about the payoff dispersion of a single asset in the presence of leverage.

$P_{mezz} = V^2_{mezz} > V^1_{mezz}$ and $P_{sen} = V^2_{sen} = V^1_{sen} = 1$, each investor type is indifferent between keeping her consumption endowment until the end of the period, buying the risky tranche that best she values most highly, and buying (part of) the senior tranche, but expects to make losses on the remaining risky tranche. By assumption, at this price vector, investors can afford to buy, individually, any singly risky tranche in its entirety and, together, the entire securitization. Finally, any vector with higher (lower) prices implies zero (excess) demand for at least one tranche. □

Note that the result is independent of the distribution of endowments that we consider, but relies on the maintained assumption that both investor types can afford to buy any single risky tranche at their own valuation, and that their endowments, together, suffice to buy the whole securitization. It implies that an increase in the disagreement measure $\Delta\pi_{DD}$ leaves the price of the pass-through securitization unchanged but always strictly increases the price of the structured securitization and thus the return to tranching.

For simplicity, the analysis has treated the recovery value V_{rec} as a constant that investors agree about. In fact, disagreement about V_{rec} affects valuations very similarly to disagreement about the default probability $\overline{\pi}$ - both are forms of disagreement about mean payoffs. Importantly, however, by reducing the market value of both forms of securitization by the same amount (as long as the attachment range of the senior tranche is adjusted accordingly), lower recovery values imply higher (relative) returns to tranching. Disagreement may thus be expected to be of greater importance for loan pools with low recovery values, or for CDOs of junior or mezzanine securitization tranches, which have no recovery value at all. Finally, in sectors where recovery values comove negatively with defaults, for example reflecting longer time-until-foreclosure or lower resale values in recessions, a given level of disagreement about default risk implies stronger disagreement about the cash flow distribution, and thus a stronger effect on prices.

Disagreement about default correlation thus raises the market price of the structured securitization above the common expected value of its cash flow, as investors self-select into buying the risky tranche they perceive as most profitable. An increase in disagreement therefore raises the market value of the mortgage collateral. Disagreement about default probabilities, in contrast, makes all tranches most profitable for optimists, who may, however, lack the resources to buy them. This implies a high return to tranching at intermediate levels of optimist resources. At low levels, however, prices have to be lower to lure pessimists into buying risky tranches, and may fall as disagreement increases.

3. Extensions of the simple model

The benchmark analysis in the previous section was kept intentionally simple by looking at a two-mortgage securitization in an economy with only two risk-neutral investor types. Before studying in detail how risk-averse preferences change the benchmark results, this section briefly summarizes several other extensions of the analysis contained in an Online Appendix.

3.1. General disagreement, many investors and mortgages, and short-selling

Section B.2 of the Online Appendix shows how, when investors disagree about both dimensions of default risk at the same time, maximum valuations are always higher than in the benchmark analysis with one-dimensional disagreement, but the return to tranching may, in fact, be lower. This does, for example, happen when additional disagreement about the average default probability makes marginal investors more optimistic and thus reduces the scope of any given disagreement about default correlation to affect valuations.

With a continuum of investors (Section B.3), disagreement affects prices through essentially the same mechanism as in the simple two-type economy.

Tranching always increases the market value of collateral when investors disagree about the default correlation, as self-selection makes investors at both extremes of the π_{DD} distribution buy the junior and mezzanine tranches, respectively, thus raising their market prices. When investors disagree about the default probability, it is once more the ability to sell the senior tranche to pessimists at the bottom of the $\overline{\pi}$ distribution that permits originators to better target their sale of the risky tranches to optimists. And as in Proposition 1, the junior tranche sells at a discount relative to its valuation by the marginal investor.

The analysis of securitizations with many, rather than two, mortgages (Section B.4) where perceptions of correlation are captured by the relative importance of an aggregate factor in determining creditors' asset values yields results very similar to those of the benchmark model. In this case, however, a more granular tranche structure allows originators to extract a larger share of the surplus when there are more than just two investor groups.

The Online Appendix also considers how the effect of disagreement on securitization prices changes when investors can effectively short-sell a securitization, or its tranches, by purchasing protection against their credit risk in the form of CDSs (Section B.5). Since the protection seller has the same exposure to default risk as the buyer of the debt security, this effectively increases the asset supply. It is thus likely to reduce the price of both pass-through and structured securitizations. The Appendix considers trade in fully cash-collateralised CDSs, which implies a simple arbitrage pricing relationship between the price of tranches and the premium paid for protection against their default risk. The introduction of CDS trade reduces the return to tranching most strongly in the case of disagreement about default correlation, where it leaves the price of the pass-through securitization unchanged at its common valuation, but may strongly decrease tranche prices. In fact, the return to tranching may turn negative in this case. With disagreement about mean probabilities, in contrast, it may actually increase, as CDS trade lowers the price of the pass-through securitization more than that of tranches.

We also discuss several frictions that may limit CDS trade and its depressing effect on securitization prices in reality, such as the over-the-counter nature of CDS trade that made it inaccessible to retail investors and whose implied counterparty risk might have made institutional investors reluctant to invest a large part of their funds. Interestingly, since several financial innovations in the mid-2000s made the trading of credit derivatives for securitized debt products substantially easier, disagreement about default risk may have contributed both to the boom in securitization and to its bust after 2007.

Even when financial innovation reduces transaction costs, CDS trade may, however, not completely undo a boom in securitization demand fuelled by optimists or disagreement about correlation. This is due to the fact that the certain short-term losses from premium payments in anticipation of an uncertain future gain may be unattractive for investors without sufficient capital, and particularly for fund managers who fear that their investors might disagree with their assessment of default risk or misinterpret low returns as resulting from a mistaken investment strategy, and thus withdraw funds early.

3.2. Risk aversion

Risk-neutral investors disregard the comovement between asset payoffs and other consumption sources and are happy to concentrate their consumption in a limited number of states with high perceived payoffs. Risk aversion, in contrast, makes variation in consumption across states costly. Risk-averse investors thus prefer diversified portfolios and assets that are good hedges against other sources of consumption risk. This makes concentrated investments in the risky tranches, which only pay in a limited number of high cash flow states, relatively unattractive and lowers the return to tranching.

Risk aversion does, however, increase the importance of two other advantages of structuring. First, the flexibility with which investors can diversify their investments across states of nature and choose tranches they perceive as less risky than their peers. Second, the ability to exploit disagreement about payoffs in states of low consumption, which the falling marginal utility embodied in most risk-averse preferences makes more important for portfolio decisions. As we show, first, analytically, and afterwards in a numerical illustration, this ability to flexibly sell to investors those tranches that they, but not necessarily their peers, regard as safe can make structuring profitable even at high risk aversion.

3.2.1. Safe assets and the rising importance of disagreement about low payoffs

Consider a version of the economy in Section 2 with a common default probability $\overline{\pi} = 0.5$, endowments $n^1 = n^2 = 1.25$ and infinite risk-aversion

$$u(c) = \min(c). \tag{4}$$

Consider, first, an extreme case of disagreement where type one and two view all default risk as aggregate ($\pi^1_{DD} = 1$) and idiosyncratic ($\pi^2_{DD} = 0$), respectively. Type one thus perceives a fifty percent probability of payoffs equal to one (both mortgages default) and three (no default), while type two perceives a sure payoff of two. According to Proposition 2, with risk-neutral preferences, the equilibrium prices of pass-through and structured securitizations equal 2 and 2.5, respectively. With preferences given by (4), in contrast, investors put all their resources in any asset that yields the highest minimum return greater or equal to one. Type two thus values the pass-through securitization more but cannot afford the entire supply at her reservation price of two. The equilibrium price thus equals her endowment of 1.25 (at which type one simply consumes her endowment in the first period).

Structuring, in contrast, allows type one and two to buy, respectively, the senior and mezzanine tranche at their reservation valuations of one, implying an equilibrium price of the structured securitization equal to $P_{sec|tr} = 2$. Risk aversion thus lowers both prices, but increases the return to tranching from $\frac{P_{sec|tr}-P_{sec}}{P_{sec}} = \frac{2.5-2}{2} = 0.25$ to $\frac{2-1.25}{1.25} = 0.6$. Whenever, in contrast, investors disagree about the payoff distribution but agree about its lower bound, high risk aversion can reduce the return to tranching from its positive risk-neutral level to zero. For example, if $1 > \pi^1_{DD} > \pi^2_{DD} > 0$, the return to tranching is strictly positive with risk-neutral investors according to Proposition 2, but drops to zero with the risk-averse preferences in (4) (as both agents share the same perceived minimum payoff of one from the collateral pool).

With high risk aversion, the effect of structuring on the price of securitizations thus crucially depends on the assets that investors perceive as safe. Similarly to Proposition 1, when investors who peceive the securitization to have a large safe component cannot purchase its entire supply, tranching has a high return by channeling their funds to the mezzanine tranches.

3.2.2. Securitization prices with risk aversion - a numerical illustration

This section briefly illustrates how risk aversion can affect the prices of securitizations in the simple two-investor economy with equal endowments ($n_1 = n_2$) and constant relative risk aversion γ:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}. \tag{5}$$

The benchmark environment with two mortgages does not convincingly accomodate other sources of consumption potentially correlated with securitization payoffs. Therefore, I consider a large pool of mortgages whose default is determined by aggregate and loan-specific factors through a copula model of credit risk, according to which mortgage n defaults whenever the following condition is met:

$$x_n = \rho^k \cdot M + (1 - \rho^k) \cdot M_n < \widetilde{\pi}^k. \tag{6}$$

x_n, interpreted as the value of creditor n's assets, equals the weighted average of an aggregate factor M, capturing economy-wide conditions, and a loan- or borrower-specific factor M_n. Investor k believes that loan n defaults whenever x_n falls below a threshold equal to $\widetilde{\pi}^k$. High $\widetilde{\pi}^k$ investors thus think that defaults are more likely on average, while those with a high perceived correlation parameter ρ^k believe that aggregate conditions are more important in determining loan defaults. For simplicity, in this illustrative example, M and M_n are assumed to follow standard uniform distributions. Section B.4 of the Online Appendix provides a more detailed account of this model, while the working paper version of this article (Broer, 2016) includes a quantitative analysis of the standard Gaussian copula.

I consider three tranches with detachment points such that the senior tranche promises all cash flow that is perceived as riskless by both types (reflecting a recovery value of 25 % and a maximum default rate below 100 %), and the attachment point of the junior tranche equals the minimum of the two expected payoffs (calculated as $1 - \widetilde{\pi}^k(1 - V_{rec})$). Arbitrage with the option to store endowments until consumption at the end of the period prices the senior tranche at $P_{sen} = 1$. Finally, I assume that there is an additional source of consumption risk in the form of non-diversifiable income y^k that is proportional to the aggregate factor M in (6) and paid at the end of the period. Rather than to represent any particular economy I choose parameters to illustrate the additional mechanisms that arise with risk-aversion. For this purpose, I set $n = 0.6$ and $y^k = 0.4$, implying that no group can buy the entire securitization at their own valuation, and that the expectation of second period risky income equals on average about half the securitization payoffs across the beliefs considered below.

[Figure 2 about here.]

Consider, first, a case where agents agree about the correlation parameter $\rho = 0.325$, but optimists and pessimists perceive $\widetilde{\pi}^1 = 0.2$ and $\widetilde{\pi}^2 = 0.3$ respectively, implying expected cash flows of about $V_{sec}^2 = 0.9$ and $V_{sec}^1 = 0.8$, respectively. The top panel of Figure 2 depicts, as a function of the coefficient of relative risk aversion γ along the bottom axis, the prices of the pass-through and structured securitizations (on the right-hand axis), and the return to tranching in percent (on the left-hand axis). Since optimists are unable to buy the pass-through securitization at their own valuation, Proposition 1 implies a strictly positive risk-neutral return to tranching, equal to about 11 %. Importantly, optimists perceive the mezzanine tranche, whose attachment point is below their perceived minimum cash flow, as riskless. The only risk in their risk-neutral portfolios is thus from holding the entire junior tranche, which, however, quickly becomes less attractive to them as γ increases. $P_{sec|tr}$ thus falls as γ rises. P_{sec}, the price of the pass-through securitization, in contrast, initially responds less to rising risk aversion as optimists can only afford about 70 % of it and thus have a less risky portfolio. For moderate levels of risk-aversion less than ten, optimist investors continue to hold the entire junior tranche. The return to tranching in Figure 2 thus simply falls in line with their declining relative valuation of the structured securitization. As risk aversion rises further and P_{jun} continues to fall, however, pessimists eventually start investing in the junior tranche (at levels of γ not shown in the figure). From this point, rising risk aversion depresses the price of the pass-through securitization, whose allocation across investors is determined by the optimist budget constraint, more than that of the structured securitization, which allocates risk more flexibly across investors. The return to tranching thus rises. Finally, as pointed out in Section 3.2.1, as risk aversion becomes sufficiently large, the difference in perceptions of minimum payoffs between pessimists (who only perceive the senior tranche as riskless) and optimists (who also regard the mezzanine tranche as riskless) eventually dominates all other dimensions of disagreement. Since optimist resources are less than their minimum perceived payoff (equal to about 0.8), the 'asymptotic' return to tranching is positive, equal to the percentage difference between the sum of the face value of the senior and mezzanine tranches and the minimum securitization cash flow as perceived by pessimists, or about 15 %. In other words, the return to tranching at extreme levels of risk aversion exceeds the risk-neutral return in this example.

The bottom panel of Figure 2 considers equilibrium prices when agents agree about an expected default rate of $\widetilde{\pi} = 0.25$ in the copula model (6) but type one expects defaults to be substantially more correlated ($\rho^1 = 0.6$) than type two ($\rho^2 = 0.25$). In line with Proposition 2, at $\gamma = 0$, type one (two) investors buy the entire junior (mezzanine) tranche and the return to tranching is strictly positive, equal to about 4.5 %. As risk aversion γ rises, type one investors become reluctant to hold the whole junior tranche, whose price drops quickly. As γ increases beyond five, type two shares an increasing part of the junior risk, which slows down the decline in the price of the structured securitization. With a pass-through securitization, investors hold all parts of the cash flow distribution in equal shares. So type one investors hold too much mezzanine risk, while type two investors hold excessive junior risk. Rising risk aversion thus decreases the price more than with disagreement about mean default probabilities. The return to tranching in Figure 2 thus follows a U-shape at moderate levels of risk aversion. As γ grows large (not shown in the figure), the return to tranching again exceeds its risk-neutral level, entirely determined by the perceptions of safe cash flows and the ability of type two investors to pay for the pass-through securitization, whose minimum payoff they perceive as higher.

4. Further predictions, evidence, and challenges

The main prediction of the analysis in this paper is that structured securitization raises the market value of collateral when investors disagree about default risk. Several alternative theories also predict structured securitizations to be more attractive to investors than either sales of single loans or simple pooling in a pass-through securitization. For example, with information asymmetries between issuers and investors, structuring may alleviate the consequences of adverse selection of collateral assets sold to the market (DeMarzo, 2005) or issuer moral hazard by creating information-insensitive senior tranches. Gorton and Metrick (2013) argue that such securities were in huge demand as collateral for repo transactions in a US financial system increasingly based on anonymous wholesale funding chains. Moreover, structuring may realize the diversification and risk transfer benefits of securitization at the same time as it accomodates investor preferences or investment mandates for specific risk attributes. Finally, structured financial assets may have been overpriced before the crisis as investors demanded returns in line with corporate bonds of equal rating, instead of appropriate compensation for the highly systemic risk of securitized loan pools (Brennan, Hein, and Poon, 2009; Coval et al., 2009a; Hamerle, Liebig, and Schropp, 2009; Wojtowicz, 2014). Supply-based explanations of the rise in securitization such as regulatory arbitrage, in contrast, do not share this prediction of a boom in prices.

This section briefly discusses some more general empirical predictions, including variation across structures, tranches, sectors and time, and summarizes some empirical evidence, including several challenges to the theory.

4.1. Further predictions and empirical evidence

Structuring creates value

According to our theory, structuring creates value by increasing the market price of risky securitization tranches. This is in line with the results in Coval et al. (2009a) who find CDX index tranches to be overpriced relative to the (non-tranched) index, although the authors attribute them to investors' disregard for the highly systemic nature of tranche risk. An important caveat, however, is that overvaluation is observed throughout the structure (and even smaller for the most junior 3-7% tranche), contrary to the effect of disagreement, which is predicted to be stronger in the riskier tranches. Moreover, Collin-Dufresne, Goldstein, and Yang (2012) argue that when attributing a lower and more appropriate share of total risk to systemic factors,f spreads are close to but *lower* than those observed in the data, thus illustrating the lack of agreement about the correlation of corporate asset values.

Deng, Gabriel, and Sanders (2011) provide evidence from a time-series analysis of yield spreads and issuance data that re-tranching of subordinate tranches into CDOs significantly contributed to the observed fall in spreads for US subprime RMBSs before the crisis. Nadauld and Weisbach (2012) find that US corporate loan facilities whose attributes make securitization more likely have lower spreads, and argue that this is most likely due to investor demand for securitized corporate loan obligations. Similarly, Xudong, Yongheng, and Gabriel (2009) show that conduit commercial mortgage loans, which are originated exclusively for securitization, have an 11 to 16 basis points lower yield than portfolio loans that are not intended for securitization at the time of origination.

Prices rise with disagreement

The driving factor that distinguishes the analysis in this paper from other theories of stuctured finance, investor disagreement, is usually unobserved. A rare exception that shows how even large, sophisticated institutional investors disagree about the valuation of RMBS tranches is Bernardo and Cornell (1997). They document a strong variance of bids during an auction that liquidated a large portfolio of agency MBS securities, reflecting heterogeneity in valuations (although not in perceptions of default risk). Typically, however, the strength of disagreement must be inferred from observed portfolios, trading patterns and volumes, and, plausibly, from certain collateral characteristics.

Using uncertainty about cash flow distributions as a proxy for disagreement, sectors whose collateral has a shorter history, such as US subprime mortgage loans, and particularly innovative, and often more complex, forms of securitization (such as CDOs and CDOs 'squared') should be more prone to disagreement.[8] This is in line with evidence reviewed by Bianchi and Jehiel (2016) that more complex assets tend to fetch higher prices. Trading patterns might also indicate disagreement. The dramatic increase in 2005 and 2006 of short positions in CDO tranches backed by US subprime RMBS collateral and in 'correlation' trades that comprise simultaneous long and short positions (Financial Crisis Inquiry Commission, 2011, p. 191-96) points to an increase in disagreement before the crisis. Similarly, the increase in the share of synthetic CDOs in that sector, whose collateral consists of credit default swaps referencing RMBS tranches, can be interpreted as an expression of increasing disagreement. Disagreement about default risk may thus have contributed to the strong boom in these sectors. More generally, the increasing heterogeneity and internationalization of investors in US structured financial assets should be expected to have increased disagreement about default risk before the crisis. This is, for example, in line with the observed decline in spreads for US RMBS tranches during that period (Deng et al., 2011).

[8]In the absence of data, correlations among CDO collateral were mainly based on judgement (Financial Crisis Inquiry Commission, 2011, p. 146-8). In fact, FitchRatings (2004, p. 10) admitted that "Due to the lack of structured finance default data, correlation assumptions between structured finance products were established using Fitch's expertise and knowledge base across structured finance sectors."

Disagreement implies heterogeneous portfolios

Disagreement affects prices as investors concentrate their wealth in assets they value highly. This implies portfolio heterogeneity, and a positive effect of ownership concentration on prices similar to that documented for US stocks, for example (Chen, Hong, and Stein, 2002). The large short positions by some hedge funds before the crisis, and some famous 'correlation trades' that seemingly reflected opposing views about the importance of aggregate factors in defaults, provide anecdotal evidence for portfolio heterogeneity that is in line with investor disagreement. Section B.5 of the Online Appendix provides two examples of correlation trades before the crisis. How widespread these investment patterns were, is not clear, however. In contrast, the fact that issuers kept both junior and senior default risk on their books is not easily explained by our simple disagreement-based theory alone that would predict the sale of the entire securitization unless issuers themselves were very optimistic about default rates, as argued in Foote, Gerardi, and Willen (2012).

Price effects are stronger with higher default rates and lower or more volatile recovery values

Investor disagreement impacts prices of tranches, and thus the market value of collateral, via disagreement about the perceived distribution of defaults and, ultimately, collateral cash flow. The effect of correlation on the marginal distribution of defaults is larger in sectors whose collateral has high default probabilities.[9] And the distribution of cash flow is more sensitive to variation in default rates when collateral assets have a low or no recovery value or when recovery values are procyclical. This may have contributed to the boom in US subprime RMBSs, whose collateral has a high default probability and procyclical recovery values, and in CDOs backed by junior or mezzanine

[9] Again, this is intuitively because asset correlations between zero and one map into default variances between 0 and $\overline{\pi}(1-\overline{\pi})$. So for values less than 50 %, an increase in the default probability $\overline{\pi}$ increases the range of the variance spanned by the given range of correlations.

RMBS tranches, whose recovery value is zero.

Finer structures increase the return to tranching

Finer tranche structures do more fully exploit the differences in the perceived default distribution, as shown in more detail in Section B.4 of the Online Appendix. Importantly, since the disagreement effect relies on investors' own assessment of tranche risk, as opposed to ratings, value will be created even when granularity increases through tranches of equal rating. This is in contrast to other theories of securitization, such as Brennan et al. (2009), who predict full use of the rating scale, but not multiple tranches of equal rating. Firla-Cuchra and Jenkinson (2005) analyze the universe of European securitizations between 1987 and 2003 and show how the number of tranches per securitization has increased over time. Importantly, this increase happened partly within the same rating class, in line with the prediction of this paper. The authors also find some evidence that additional tranches decrease yield spreads, at least for securitizations which their model predicts to have a higher optimal number of tranches.

4.2. Challenges to the theory

4.2.1. Optimal security design with disagreement

As Allen and Gale (1988) have pointed out, when a firm sells its output to heterogeneous investors, it is optimal to design claims that allocate the cash flow in any state to those investors that value it most highly. When risk-neutral investors only differ in their perceived probability distributions, as in this paper, it is thus optimal to sell payoffs state-by-state to those whose perceived probability of a state is highest. According to this criterion, the debt-like tranches of structured securitizations are clearly not optimal: apart from the most junior ones, all tranches pay their face value whenever cash flow is above a threshold, and thus group together states with heterogeneous perceived probabilities across investors. To see this in our benchmark environment with disagreement about default correlation π^k_{DD}, note that when endowments are sufficiently large, the price of the securitization P^{max}_{sec} when sold in state-contingent tranches simply equals the sum of state-payoffs weighted by the maximum of state-probabilities

$$\begin{aligned} P^{max}_{sec} &= 1 \times \overline{\pi}\pi^1_{DD} + 2 \times 2\overline{\pi}(1-\pi^2_{DD}) + 3 \times (1 - \overline{\pi}(2 - \pi^1_{DD})) \\ &= 1 + 2(1-\overline{\pi}) + 4\overline{\pi}\Delta\pi_{DD}. \end{aligned} \quad (7)$$

The originator could thus earn profits that are four times as high from a complete state-contingent structure with 'vertical' tranches, rather than structuring the cash flow 'horizontally' into debt-like securities of different seniority. Moreover, the number of assets is the same in both cases, and the required information about realizations of cash flow that determines payoffs is identical.

The debt-like structure of securitizations is thus likely to be the result of other factors mentioned above, such as asymmetric information, regulatory arbitrage or investor preferences for tranches of different seniority because of regulation, investment mandates, etc. In addition, the fact that tranche payoffs are continuous around detachment points of debt-like structures makes their payments less sensitive to small errors in cash flow measurement, gives less incentives for collusion between servicers and investors to misrepresent cash flow, and thus reduces ex post litigation relative to state-contingent claims with their all-or-nothing payments.

4.2.2. Securitization before the boom, and after the crisis

This paper has argued that disagreement about default risk may have contributed to the demand for securitization that fuelled its boom during the early and mid-2000s. But does the theory proposed in this paper have anything to say about the timing of that boom and, more importantly, the massive decline in securitization activity with the financial crisis?

Several supply and demand-based factors have been argued, sometimes controversially, to have driven the securitization boom of the early 2000s.[10] Although our theory does not predict the specific timing of the boom and bust in structured finance, disagreement may have become a more important determinant of investment decisions in the late 1990s and early 2000s, for several reasons: first, there is evidence of growing disagreement about the nature of the Great Moderation in macoeconomic volatility (Broer and Kero, 2014), and thus the relative importance of economy-wide vs idiosyncratic economic shocks that implies stronger disagreement about their relative importance for consumer and mortgage defaults, and thus default correlation. Second, the general internationalization of financial markets had made the investor pool for US securitization tranches more heterogeneous. And finally, the low interest rate environment that started in the early 2000s may have made the perceived return differences pointed out by our theory relatively more important.

After reaching the amount of 1.1 trillion in 2006, the total issuance of US RMBS fell sharply with the beginning of the global financial crisis, and was still a mere USD 18.7 billion in 2015 (Securities Industry and Financial Markets Association, 2016, 2014a). Moreover, global securitization issuance in 2013 (excluding residential mortgages) was still only about 25 % of its level in 2006 (The Economist, 11 January 2014). This persistent bust in securitization has, for example, been attributed to rising concerns about issuer moral hazard and asymmetric information, or the loss of the 'safe asset' label of senior securitization tranches in the face of ratings downgrades and

[10]Among these are the disintermediation of the US financial system (boosting the demand for safe repo collateral), changes in banking regulation (reducing the relative capital requirements for investments in senior securitization tranches), US affordable housing policy, technological innovation in underwriting procedures (Gates, Perry, and Zorn, 2002), the low interest rate environment of the early 2000s, or the boost to the private-label RMBS market through the sale of mortgage portfolios by the US government after the savings and loans crisis of the late 1980s / early 1990s. For a discussion, see for example Part II of Financial Crisis Inquiry Commission (2011).

rising default rates, which strongly reduced their suitability as collateral in repo transactions, for example (Gorton and Metrick, 2013). We believe that the analysis in this paper suggests some additional factors that may have contributed to the depth and persistence of this bust. For example, the unprecedented synchronicity of ratings downgrades and of the increase in default rates across sectors and regions at the beginning of the crisis may have essentially 'reset' default risk perceptions to more likely and more correlated defaults. Unless investor updating rules are themselves strongly heterogeneous, this common signal does not only change mean perceptions as in Gennaioli et al. (2012), but also strongly reduces disagreement and its boosting effect on demand, as the common new information dominates pre-existing heterogeneous priors.

Another contributing factor to the securitization bust may have been the loss of funds by investors with extreme views about default distributions. As our theory predicts both optimists and high-correlation investors to take positions in junior securitization tranches, losses on these tranches during the crisis may have reduced the resources they are able, or willing, to invest and thus have contributed to lower prices and demand thereafter.

Finally, any remaining disagreement about default risk may have contributed to the resilience of securitization in some sectors. For example, issuance of US asset-backed securities (whose collateral consists of auto, credit card, student and other loans) remained above USD 100 billion per year throughout the crisis and rose to USD 220 billion in 2014, about 30 % of the issuance in 2006 (Securities Industry and Financial Markets Association, 2016, 2014b).

5. Conclusion

This paper proposes a new, additional reason for the popularity of structured assets prior to the financial crisis: disagreement about default risk.

With disagreement about default probabilities, structuring drives up the market price of collateral whenever optimist resources suffice to purchase the risky tranches, as riskless tranches can be sold to pessimists without a discount. This raises the price of the structured securitization above that of the collateral cash flow sold in simple shares, as a pass-through securitization. When pessimists need to buy some risky tranches, however, all tranche prices are low. With disagreement about default correlation, high-correlation investors like junior tranches, which they think pay whenever aggregate times are good, while they expect senior assets to incur losses in bad times. Low-correlation investors think that the risk to senior assets is low, while junior assets are essentially junk. This preference for different assets makes investors who disagree about correlation pay high prices for tranches of different seniority. Importantly, this effect is less dependent on the distribution of investor resources than with disagreement about the average probability of defaults. We think that this return to tranching arising from investor disagreement provides an additional explanation for the boom in structured finance and, particularly, for the popular originate-to-distribute model of US mortgage finance prior to the crisis. Moreover, the results may help us understand the fall in securitization activity after the crisis, which may have 'reset' perceptions of default risk and reduced the ability, or willingness, of investors with extreme views to take positions.

Future research should supplement the empirical evidence reviewed in this paper by testing the implications of disagreement against those from alternative theories based, for example, on information asymmetries, market segmentation or pricing-by-analogy to corporate bonds of equal rating. Moreover, it would be interesting to evaluate the quantitative importance of disagreement about default risk for market prices of securitizations and their collateral in a fully calibrated economic environment with risk aversion, additional sources of risk, and perhaps other dimensions of investor heterogeneity.

Do the results in this paper imply an opportunity for policy intervention? Potentially yes. The results predict that average losses for investors are larger than their average expected losses, equivalent to a transfer of resources from investors to issuers, originators or debtors. Importantly, optimists, or investors who perceive default correlation to be low, may suffer losses that they did not expect when tranches they regarded as safe default. As in Gennaioli et al. (2012), where investors react strongly when they are reminded of the possibility of extreme events by bad outcomes, this may lead to abrupt updating of beliefs and investment behavior with potentially destabilizing consequences. Moreover, since these losses coincide with bad aggregate conditions, they may imply high welfare costs, and could contribute to the instability of financial institutions.

With heterogeneous beliefs, there is no obvious benchmark efficiency criterion for policymakers.[11] In the presence of additional dimensions of inequality or heterogeneity, policymakers may, in fact, welcome the additional resources that investors transfer through higher asset prices, in which case they might want to aim at ensuring competitive behavior by issuers and originators to increase the benefits to debtors. On the other hand, to the extent that a boom in structured finance may have ultimately destabilizing consequences, and create negative externalities by causing unexpected losses in bad times, policymakers may want to tighten the regulation of the sector, for example by requiring high enough risk weights for structured financial assets in bank portfolios. Finally, the importance of ratings as a benchmark for beliefs, combined with evidence that ratings agencies may have produced biased ratings potentially due to conflicts of interests (Griffin and Tang, 2011), provides an additional argument for their supervision and regulation.

[11] On efficiency criteria for economics with heterogeneous beliefs, see Brunnermeier, Simsek, and Xiong (2014) and Gayer, Gilboa, Samuelson, and Schmeidler (2014).

Appendix A. Proof of Proposition 1

A.1. Proof of Proposition 1

That $P_{sec} = P_{sec|tr} = V^2_{sec}$ when $a^2 \geq V^2_{sec} = 1 + 2(1 - \overline{\pi}^2)$ follows trivially because optimist demand exceeds supply whenever any asset price is below their valuation, but drops to zero at any price above.

To see that $P_{sec} = max\{a^2, V^1_{sec}\} < V^2_{sec}$ when $a^2 < V^2_{sec}$ note that, at a price of the pass-through securitization below their valuation, investors invest their entire endowment. By assumption investors can, together, always afford the entire securitization at the pessimist valuation. So the latter is the minimum equilibrium price. It leads to excess demand, however, when optimists can afford to purchase the exogenous supply at a strictly higher price (below their valuation). The equilibrium price thus equals the maximum of the optimist endowment and the pessimist valuation. Any higher price would make demand drop below the market value of the exogenous supply; any lower price would decrease the market value below investor demand.

When $V^2_{sec} > a^2 \geq V^2_{mezz} + V^2_{jun} = 2(1 - \overline{\pi}^2)$, with a structured securitization, optimists can afford to buy the junior and mezzanine tranches at their own valuation by the assumption that $a^2 \geq 2(1 - \overline{\pi}^2)$. When $P_{sen} = 1$, they are willing to do so as they expect to make the same (zero net) profits from any of the three tranches. Equally, pessimists are happy to buy the senior tranche at that price and, by assumption, their endowment together with the remaining funds of optimists suffices to buy its whole supply. A higher price for any tranche would make demand for it drop to zero. By the assumptions on endowments, any vector with lower prices implies excess demand for the three tranches together (if optimists continue to perceive all tranches as equally profitable), or for the tranche, or the two tranches, that optimists perceive most profitable.

When $a^2 < V^2_{mezz} + V^2_{jun}$, note that pessimists have higher relative valuations of more senior tranches: $\frac{V^1_{sen}}{V^1_{mezz}} > \frac{V^2_{sen}}{V^2_{mezz}}$ and $\frac{V^1_{mezz}}{V^1_{jun}} > \frac{V^2_{mezz}}{V^2_{jun}}$. So whenever pessimists invest, they must buy the senior tranche, optimists the junior tranche (which they can afford by assumption), and either, or both, the mezzanine tranche. Since together investors can always afford the entire securitization at the pessimist valuation, the price of the senior tranche equals $P_{sen} = 1$. Since optimists cannot afford to buy the supply of both risky tranches ($a^2 < V^2_{mezz} + V^2_{jun}$), the equilibrium features proportionally lower prices for risky tranches ($\frac{P_{jun}}{V^2_{jun}} = \frac{P_{mezz}}{V^2_{mezz}}$) that either exhaust optimists' endowment ($P_{jun} + P_{mezz} = a^2$) or make pessimists happy to buy part of the mezzanine tranche (implying $P_{mezz} = V^1_{mezz}$ and $P_{jun} = \frac{P_{mezz}}{V^2_{mezz}} V^2_{jun} < V^2_{jun}$), whichever higher. The equilibrium price of the structured securitization equals the sum of tranche prices

$$P_{sec|tr} = P_{sen} + P_{mezz} + P_{jun} = 1 + \max\{a^2, V^1_{mezz}[1 + \frac{V^2_{jun}}{V^2_{mezz}}]\} \quad (8)$$

$$= 1 + \max\{a^2, (1 - \overline{\pi}^1 \pi_{DD})[1 + \frac{1 - 2\overline{\pi}^2 + \overline{\pi}^2 \pi_{DD}}{1 - \overline{\pi}^2 \pi_{DD}}]\}. \quad (9)$$

To see how this is the unique equilibrium price vector, note that both optimists and pessimists are indifferent between all of their investments and together can afford to buy the entire securitization. Also, a higher price on any tranche implies excess supply (as either demand for the senior tranche drops to zero, or demand for one of the risky tranches, or both together, falls below the value of supply). Moreover, it is easy to see that any vector with lower prices implies excess demand for at least one tranche.

Appendix B. For Online Publication Only

This appendix contains further proofs, and extensions of the benchmark model of Section 2.

B.1. *Increasing disagreement about $\overline{\pi}$ and the return to tranching*

Corollary 1 shows the conditions under which an increase in disagreement about default probability $\overline{\pi}$ can lower the price of the structured securitization, as illustrated in figure 1. This is most likely when there is a strong positive correlation in defaults, corresponding to high values of π_{DD}, which increases the negative effect of rising $\overline{\pi}^1$ on the valuation of the mezzanine tranche, and thus its price.

Define the mean probability $\overline{\pi}^m$ and disagreement indicator $\epsilon > 0$ such that $\overline{\pi}^m = \overline{\pi}^1 - \epsilon = \overline{\pi}^2 + \epsilon$ and call 'mean-valuation' $V_i^m, i = \{sec, sen, mezz, jun\}$ the payoff of, respectively, the pass-through securitization and the three tranches expected by a hypothetical investor with mean belief $\overline{\pi}^m$.

Corollary 1. *When $a^2 > V_{mezz}^2 + V_{jun}^2$, a symmetric increase in disagreement $d\epsilon > 0$ increases the equilibrium price of the structured securitization $P_{sec|tr}$, which always exceeds the mean valuation V_{sec}^m. When $a^2 < V_{mezz}^2 + V_{jun}^2$, in contrast, an increase in disagreement decreases $P_{sec|tr}$ whenever $\pi_{DD} > \frac{\widetilde{V}}{1-\overline{\pi}^2+\widetilde{V}}$ for $\widetilde{V} = \frac{V_{mezz}^1}{V_{mezz}^2}$. Moreover, $P_{sec|tr} < V_{sec}^m$ holds whenever $\epsilon > \frac{1}{\pi_{DD}} - 2 + \overline{\pi}^m$.*

Proof. When $a^2 > V_{mezz}^2 + V_{jun}^2$, $P_{sec|tr}$ equals V_{sec}^2, which is strictly decreasing in $\overline{\pi}^2$ and thus increasing in ϵ. When a^2 is such that pessimists invest in (part of the) mezzanine tranche, $P_{sec|tr} < V_{sec}^m$ whenever

$$P_{sec|tr} = 1 + \frac{1-\overline{\pi}^1\pi_{DD}}{1-\overline{\pi}^2\pi_{DD}} 2(1-\overline{\pi}^2) \quad < \quad V_{sec}^m = 1 + 2(1-\overline{\pi}^m) \tag{10}$$

$$\Rightarrow (1-\overline{\pi}^m\pi_{DD} - \epsilon\pi_{DD})(2(1-\overline{\pi}^m)+2\epsilon) \quad < \quad 2(1-\overline{\pi}^m)(1-\overline{\pi}^m\pi_{DD}+\epsilon\pi_{DD}) \tag{11}$$

$$\Rightarrow (1-\overline{\pi}^m\pi_{DD})2\epsilon - \epsilon\pi_{DD}(2(1-\overline{\pi}^m)+2\epsilon) \quad < \quad 2(1-\overline{\pi}^m)\epsilon\pi_{DD} \tag{12}$$

$$\Rightarrow (1-\overline{\pi}^m\pi_{DD}) - \pi_{DD}((1-\overline{\pi}^m)+\epsilon) \quad < \quad (1-\overline{\pi}^m)\pi_{DD} \tag{13}$$

$$\Rightarrow 1-\overline{\pi}^m\pi_{DD} - 2\pi_{DD}(1-\overline{\pi}^m) \quad < \quad \pi_{DD}\epsilon \tag{14}$$

$$\Rightarrow \epsilon \quad > \quad \frac{1}{\pi_{DD}} + \overline{\pi}^m - 2. \tag{15}$$

where we assume at least a small degree of disagreement ($\epsilon > 0$). To see that $P_{sec|tr}$ is downward-sloping whenever $\pi_{DD} > \frac{\widetilde{V}}{1-\overline{\pi}^2+\widetilde{V}}$, differentiate the former, to get, for $\widetilde{V} = \frac{V^1_{mezz}}{V^2_{mezz}}$,

$$\frac{dP_{sec|tr}}{d\epsilon} = \frac{2}{1-\overline{\pi}^2\pi_{DD}}[-\pi_{DD}(1-\overline{\pi}^2) + (1-\pi_{DD})\widetilde{V}] \tag{16}$$

$$= \frac{2}{1-\overline{\pi}^2\pi_{DD}}[-\pi_{DD}(1-\overline{\pi}^2+\widetilde{V}) + \widetilde{V}], \tag{17}$$

which yields the expression in the corollary. □

To give an example, $P_{sec|tr}$ is below the mean valuation of the cash flow V^m_{sec} whenever $\pi_{DD} = 1$. $\overline{\pi}^m = 0.35$, $\epsilon = 0.25$ and $\pi_{DD} = 0.7$ is an alternative example.

B.2. Disagreement about both dimensions of default risk

This section briefly discusses the case where investors disagree about both dimensions of default risk - probability $\overline{\pi}^k$ and correlation parameter π^k_{DD} - at the same time. We show how an additional dimension of disagreement always increases maximum valuations. It still decreases the return to tranching, however, if it makes investors in risky tranches more optimistic (and thus reduces the scope of any given disagreement about default correlation to affect valuations), or by reducing valuations of the marginal investor in risky tranches when endowments are low. In a richer framework with more than two mortgages, however, originators may exploit additional dimensions of disagreement through a finer tranche structure to increase the surplus extracted from investors.

Suppose there are four groups of equal mass and endowments with different combinations of high and low perceptions of the two default risk parameters: $\overline{\pi}^k \in \{\overline{\pi}^{low}, \overline{\pi}^{high}\}$, $\pi^k_{DD} \in \{\pi^{low}_{DD}, \pi^{high}_{DD}\}$. Denote as $\overline{\pi}^m$ and π^m_{DD} the intermediate perceptions when agents agree.

Clearly, since the valuation of both risky tranches in (2) and (3) is monotonic in both $\overline{\pi}$ and π_{DD}, adding a second dimension of disagreement always increases maximum valuations. Trivially, when endowments are sufficiently large, the prices of risky tranches thus rise in line with maximum valuations, increasing the market price of the structured securitization. As is easy to see, the return to tranching then equals

$$\frac{P_{sec|tr} - P_{sec}}{P_{sec}} = \frac{\overline{\pi}^{low} \Delta\pi_{DD}}{P_{sec}}, \tag{18}$$

where again the $\Delta\pi_{DD} = \pi_{DD}^{high} - \pi_{DD}^{low}$ measures disagreement about correlations and leaves P_{sec} unaffected. Clearly, as $\Delta\pi_{DD}$ rises the return to tranching thus always increases relative to the case of disagreement about mean default probabilities considered in Section 2.1. An increase in disagreement about $\overline{\pi}$, in contrast, raises the prices of both kinds of securitization but *decreases* the return to tranching when investor endowments are sufficiently high, as the right-hand side of equation (18) strictly decreases as $\overline{\pi}^{low}$ falls.[12] The reason for this is simple: disagreement about correlations (or conditional default probabilities π_{DD}) matters more when defaults are likely. So the return to tranching falls as those that purchase risky tranches become more optimistic.

At lower endowments, additional dimensions of heterogeneity may more generally reduce both the price of the structured securitization and the return to tranching as disagreement about either default risk parameter increases. Intuitively, although the highest valuation rises as investors in a particular asset become more heterogeneous, additional heterogeneity also makes the groups of investors that share the same beliefs smaller. So the highest-valuation group may not be able to buy the whole supply and the valuation of tranches by the marginal investor, and thus the return to tranching, may

[12] To see this, note that the numerator is increasing, and the denominator decreasing in $\overline{\pi}^{low}$ when endowments are sufficiently high.

fall.

Importantly, the simple example considered here does not allow originators to respond to additional dimensions of heterogeneity by changing the structure of the securitization, whose cash flow has only three support points. Section B.4 in the Online Appendix, however, shows how, with a larger pool of mortgages, originators may respond to multi-dimensional disagreement through a finer tranche structure, which extracts more surplus.

B.3. More than two investor types

For concreteness, the analysis in Section 2 looks at only two investors types. To see how the benchmark results generalize to the case of many investor types, consider a version of the simple two-mortgage economy with unit endowments ($a^k = 1, \forall k$) and a continuum of investors located on the unit interval ($k \in \mathbb{K} = [0, 1]$), according to the measure $\mu : \Omega([0, 1]) \longrightarrow [0, A]$ where Ω denotes the Borel algebra. The total mass of investors, equal to their total endowment, equals $A \geq 3$, such that they can collectively purchase the entire securitization at the highest valuation.

Suppose that investors share their perceived default probability ($\overline{\pi}(k) = \overline{\pi} > 0, \forall k$) but have heterogeneous perceptions of the default correlation $\pi_{DD}(k) : 0 \leq \pi_{DD}^{min} \leq \pi_{DD}(k) \leq \pi_{DD}^{max} \leq 1$ where $\pi_{DD}(.)$ is a measurable function that takes values between a minimum value π_{DD}^{min} and a maximum value π_{DD}^{max}. Denote as F the cumulative distribution function of π_{DD} induced by μ, equal to $F(\pi_{DD}) = \mu(\{k : \pi_{DD}(k) \in [\pi_{DD}^{min}, \pi_{DD}]\})$ and assume, for simplicity, that $\pi_{DD}(k)$ and μ are such that F is strictly increasing and has no mass points (in the sense that $\lim_{a \longrightarrow b} F(a) = F(b) \forall b \in [\pi_{DD}^{min}, \pi_{DD}^{max}]$). Remember from Section 2.2 that the valuation of the pass-through securitization is independent of π_{DD} and thus the same for all investors, equal to its price in equilibrium. Remember also that the valuations of the junior, mezzanine, and senior tranches of the structured securitization are, respectively, increasing, decreasing, and constant functions of π_{DD}. The price of the senior tranche thus equals its sure payoff of one. The market-clearing price for the mezzanine tranche is such that there is an investor type with $\pi_{DD}(k) = \pi_{DD}^{low}$ who is indifferent between buying the mezzanine tranche or the senior tranche (or, in fact, storing consumption until the end of the period) and demand from his peers with $\pi_{DD}(k) \leq \pi_{DD}^{low}$ exhausts supply (equal to its face value of one). Specifically, π_{DD}^{low} and P_{mezz} are defined by

$$F(\pi_{DD}^{low}) = P_{mezz} \tag{19}$$

$$1 - \overline{\pi}\pi_{DD}^{low} = P_{mezz}, \tag{20}$$

which are the market-clearing and indifference conditions, respectively. According to (19) and (20), π_{DD}^{low} is continuous and, respectively, strictly decreasing and strictly increasing in P_{mezz}, implying a unique pair of values satisfying these conditions.

Similarly, the market-clearing price for the junior tranche is such that a marginal investor with $\pi_{DD}(k) = \pi_{DD}^{high}$ is indifferent with respect to buying the senior tranche, and demand from investors with $\pi_{DD}(k) \geq \pi_{DD}^{high}$ exhausts supply. Total investor endowments are no smaller than three and thus exceed the expenditure on the risky tranches (equal to $P_{mezz} + P_{jun} = 1 - \overline{\pi}\pi_{DD}^{low} + 1 - 2\overline{\pi} + \overline{\pi}\pi_{DD}^{high} < 2$) by an amount greater than the expenditure on the senior tranche (equal to one). Thus, total demand exhausts supply and $\pi_{DD}^{low} < \pi_{DD}^{high}$ with strict inequality. As in Section 2.2, this implies a strictly positive return to tranching since $P_{sec|tr} = 1 + 2(1 - \overline{\pi}) + \overline{\pi}(\pi_{DD}^{high} - \pi_{DD}^{low}) > P_{sec}$. Again, tranching increases the market price of the securitization by exploiting the self-selection of investors to different subsections of the payoff distribution they value comparatively highly.

The top panel of Figure 3 depicts the equilibrium tranche prices in the continuum economy as the intersections of two lines of the same color: the investor demand curve for mezzanine and junior tranches, respectively $F(\pi_{DD})$ and $A - F(\pi_{DD}(k))$, and their valuation, respectively $V_{mezz} = 1 - \overline{\pi}\pi_{DD}(k)$

and $V_{jun} = 1-\overline{\pi}(2-\pi_{DD}(k))$. For any given π_{DD}, the two valuations sum to the constant valuation of the two risky tranches, whose price equals that of the pass-through securitization minus that of the senior tranche, or $P_{sec}-1$.

[Figure 3 about here.]

Suppose, in contrast, that investors share their perceived default correlation ($\pi_{DD}(k) = \pi_{DD} > 0, \forall k$) but have heterogeneous perceptions of the default probability $\overline{\pi}(k) : \overline{\pi}^{min} \leq \overline{\pi}(k) \leq \overline{\pi}^{max} \leq 1$ for some measurable function $\overline{\pi}(k)$. Denote as G the cumulative distribution function of $\overline{\pi}$ induced by μ and assume, again, that G is strictly increasing on the interval $[\overline{\pi}^{min}, \overline{\pi}^{max}]$ and has no mass points. The valuation of the pass-through securitization is strictly decreasing in $\overline{\pi}(k)$. The market-clearing price P_{sec} is thus such that a marginal investor with $\overline{\pi}(k) = \overline{\pi}^{sec}$ is indifferent between investing and consuming, and the demand by more optimistic investors clears the market.[13] In the case of a structured securitization, remember that both the valuation of the mezzanine tranche and the relative valuation $\frac{V_{jun}(\overline{\pi})}{V_{mezz}(\overline{\pi})}$ are strictly decreasing in $\overline{\pi}$. The most optimistic investors (with $\overline{\pi}(k)$ below a cutoff value $\overline{\pi}^{jun}$) thus buy the junior tranche and those with medium default probabilities (i.e. with $\overline{\pi}^{jun} < \overline{\pi}(k) \leq \overline{\pi}^{mezz}$) the mezzanine tranche. Again, the remaining investors have sufficient mass to buy the senior tranche at its common valuation of one. Equilibrium is thus given by $P_{sec} = 1$, and a pair of prices P_{mezz}, P_{jun}, together with two cutoff values $\overline{\pi}^{jun} < \overline{\pi}^{mezz}$ for the risky tranches such that

$$G(\overline{\pi}^{jun}) = P_{jun} \tag{21}$$

$$G(\overline{\pi}^{mezz}) - G(\overline{\pi}^{jun}) = P_{mezz} \tag{22}$$

$$V_{mezz}(\overline{\pi}^{mezz}) = 1 - \overline{\pi}^{mezz}\pi_{DD} = P_{mezz} \tag{23}$$

$$\frac{V_{jun}(\overline{\pi}^{jun})}{V_{mezz}(\overline{\pi}^{jun})} = \frac{P_{jun}}{P_{mezz}}, \tag{24}$$

[13] The conditions are, respectively $G(\overline{\pi}^{sec}) = P_{sec}$ and $1 + 2(1-\overline{\pi}^{sec}) = P_{sec}$.

where 21 and 22 are the market-clearing conditions, 23 and 24 the indifference conditions for the marginal investors, and $V_{jun}(\overline{\pi})$ ($V_{mezz}(\overline{\pi})$) denotes the valuation of the junior (mezzanine) tranche by an investor with perceived default probability $\overline{\pi}$.

The bottom panel of Figure 3 depicts the equilibrium tranche prices in the continuum economy with disagreement about $\overline{\pi}^k$, again as the intersections of two lines of the same color: the investor demand curve for junior and mezzanine tranches, respectively $G(\bar{\pi}(k))$ and $G(\bar{\pi}(k)) - G(\bar{\pi}^{jun})$, and their valuation, respectively $V_{mezz} = 1 - \overline{\pi}(k)\pi_{DD}$ and $V_{jun} = 1 - \overline{\pi}(k)(2 - \pi_{DD})$. For any $\pi(k)$, the two valuations sum to that of the pass-through securitization minus one, $V_{sec}(k) - 1$, depicted as the black line. The price of the pass-through securitization is given by the intersection with the black dashed line.

From the figures, or equivalently equations (21) to (24), it is easy to show that $P_{sec} < P_{sec|tr}$: as in Section 2.1, structuring permits the originator to sell the risky tranches to more optimistic investors, which increases equilibrium prices.[14] Also as in that section, the price of the junior tranche is low, in the sense that even its marginal investor values it strictly higher than its price, since combining (23) and (24) yields $P_{jun} < V_{jun}(\overline{\pi}^{jun})$. Again, this is to keep her from buying the mezzanine tranche at its low price determined by indifference of the less optimistic investors with higher perceived default probability $\overline{\pi}^{mezz}$. In contrast to Section 2.1, however, the assumption that G has no mass points implies that the return to tranching is always positive. As a sufficient mass of optimists becomes concentrated at the top of the distribution, however, the return to tranching can become arbitrarily small.

The effect of an increase in disagreement in the continuum economy typ-

[14] To see this note that $P_{sec|tr} = 1 + P_{jun} + P_{mezz} = 1 + 2(1 - \overline{\pi}^{mezz}) + V_{mezz}(\overline{\pi}^{mezz})[\frac{V_{jun}(\overline{\pi}^{jun})}{V_{mezz}(\overline{\pi}^{jun})} - \frac{V_{jun}(\overline{\pi}^{mezz})}{V_{mezz}(\overline{\pi}^{mezz})}]) > V_{sec}(\overline{\pi}^{mezz}) > V_{sec}(\overline{\pi}^{sec}) = P_{sec}$, where the second equality follows from (24), the first inequality follows since $\frac{V_{jun}(\overline{\pi}^{jun})}{V_{mezz}(\overline{\pi}^{jun})} > \frac{V_{jun}(\overline{\pi}^{mezz})}{V_{mezz}(\overline{\pi}^{mezz})}$, and the second from $\overline{\pi}^{sec} \geq \overline{\pi}^{mezz}$, as proven in footnote 15.

ically depends on both the initial distribution of investor beliefs and the specification of the increase in their dispersion. Consider a simple case where dispersion increases by relocating mass from median beliefs (defined by respectively $F(\pi_{DD})^m = \frac{A}{2}$ and $G(\overline{\pi}^m) = \frac{A}{2}$) in equal parts to the extremes of the distributions ($\pi_{DD}^{min}, \pi_{DD}^{max}$ and $\overline{\pi}^{min}, \overline{\pi}^{max}$, respectively). Since $\pi_{DD}^{low} < \pi_{DD}^{m}$ and $\pi_{DD}^{high} > \pi_{DD}^{m}$, it is easy to see how, in the case of disagreement about default correlation $\pi_{DD}(k)$, this always increases the demand for the mezzanine and junior tranches, as both $F(\pi_{DD}^{low})$ and $A - F(\pi_{DD}^{high})$ rise. Prices of both tranches thus increase as the cutoff values π_{DD}^{low} and π_{DD}^{high} move outwards to balance supply and demand. Since the price of the pass-through securitization is unaffected by beliefs about π_{DD}, the return to tranching also increases in response to the increase in investor disagreement.

The effect of an increase in disagreement about default probability $\overline{\pi}(k)$ depends, again, on the relative sizes of investor demand and supply. Since $\overline{\pi}^{sec} > \overline{\pi}^{mezz} > \overline{\pi}^{jun}$[15], it is easy to see that, if total investor funds A are large enough such that $\overline{\pi}^{sec} < \overline{\pi}^{m}$, the increase in disagreement raises the price of the pass-through securitization and of both risky tranches, as it raises the mass below all three initial threshold values and thus asset demand at unchanged prices. If, in contrast, A is small enough such that $\overline{\pi}^{mezz} > \overline{\pi}^{m}$, the price of both securitizations falls in response to an increase in disagreement, as the mass of investors that buy at the original prices is reduced. For intermediate values of demand, or A such that $\overline{\pi}^{mezz} < \overline{\pi}^{m} < \overline{\pi}^{sec}$, prices move in opposite directions as disagreement increases. That of the pass-through securitization falls as mass is reduced below the threshold $\overline{\pi}^{sec}$ and demand falls. The prices of both the mezzanine and junior tranche, in contrast, rise, as the mass above both cutoff values $\overline{\pi}^{jun}$ and $\overline{\pi}^{mezz}$ increases.

With a continuum of investors, prices are thus driven by the same mech-

[15] $\overline{\pi}^{sec} > \overline{\pi}^{mezz}$ holds since, according to (24), $P_{jun} < V_{jun}(\overline{\pi}^{jun})$: the price of the junior tranche is discounted relative to the valuation of the marginal investor. This implies $G(\overline{\pi}^{mezz}) = P_{jun} + P_{mezz} < 2(1 - \overline{\pi}^{mezz}) + \pi_{DD}(\overline{\pi}^{jun} - \overline{\pi}^{mezz}) < 1 + 2(1 - \overline{\pi}^{mezz})$. Since $G(.)$ is strictly increasing, $\overline{\pi}^{mezz} < \overline{\pi}^{sec}$ follows.

anisms as in Section 2: tranching always increases prices of the collateral when agents disagree about correlations, and self-selection leads to higher prices for risky tranches. A rise in disagreement always raises the return to tranching in this case. When investors disagree about default probabilities it is the ability to sell the senior tranche to pessimists that permits originators to better target their sale of the risky tranches to optimists. And again, the junior tranche sells at a discount relative to its valuation. The effect of rising disagreement about default probabilities on prices and the return to tranching depends on the relative size of demand and supply in a way similar to that in the simple two-investor example in Section 2.

B.4. Disagreement about default risk in large collateral pools

This section considers securitizations with large collateral pools with a potentially more granular tranche structure, such as those common for US subprime RMBSs, or CDOs backed by their tranches, before the crisis.[16] While the main results from the two-mortgage example continue to hold, a more granular tranche structure allows originators to extract a larger share of the surplus when there are more than just two investor groups.

In the model with two investor types $\mathbb{K} = \{1, 2\}$ of Section 2, consider a loan pool consisting of a large number of N, rather than just two, mortgages, each of unit face value, that together have unit mass. To capture the correlation structure of defaults parsimoniously, both investors use the "market standard" (Morini (2011), p. 127) copula model of credit risk, according to which mortgage n defaults whenever the following condition is met

$$x_n = \rho^k \cdot M + (1 - \rho^k) \cdot M_n < \widetilde{\pi}^k. \tag{25}$$

x_n can be interpreted as the value of creditor n's assets, equal to the weighted

[16]US subprime RMBSs typically allocated cash flow from several thousand mortgages to sometimes more than ten tranches. See also Coval et al. (2009b).

average of an aggregate factor M, capturing economy-wide conditions, and a loan- or borrower-specific factor M_n. Investor k believes that loan n defaults whenever x_n falls below a threshold equal to $\widetilde{\pi}^k$. Investors who perceive a high $\widetilde{\pi}^k$ thus think defaults are more likely. In addition, investors may disagree about the importance of aggregate conditions in determining loan defaults, as summarized by the correlation parameter ρ^k. Specifically, $(\rho^k)^2$ equals the covariance between two individual creditors' asset values perceived by investor k. Typically, M and M_n are assumed to follow standard normal distributions and a simulation approach is used to value, or rate, securitization tranches. This section instead illustrates the role of disagreement about default risk when all factors have standardized uniform distributions $\mathbb{U}[0, 1]$, which yields qualitatively similar results but allows an analytical characterization of the pool's payoff distribution.[17]

[Figure 4 about here.]

ρ^k and $\widetilde{\pi}^k$ completely determine the distribution of the default rate d as perceived by investor k. Together with the recovery value in case of default V_{rec}, which we continue to assume is common for both investors as its effect on the payoff distribution is similar to $\widetilde{\pi}^k$, they define the distribution of the cash flow $C = 1-d(1-V_{rec})$ from the mortgage pool. The top panel of Figure 4 depicts C as a function of the aggregate factor M for different values of ρ^k and common $\widetilde{\pi}^k = \widetilde{\pi}, \forall k$. When $\rho^k = 0$ (the thin solid line), loan-default is entirely idiosyncratic. In a sufficiently large pool of mortgages, the default rate is thus constant and equals its expectation $\widetilde{\pi}$, implying a certain cash flow of $\overline{C} = 1 - \widetilde{\pi}(1 - V_{rec})$. When ρ^k equals 1, in contrast, (the thin dashed line) default is driven entirely by aggregate conditions: either all loans default ($M < \widetilde{\pi}$), or none ($M \geq \widetilde{\pi}$), implying a binary distribution of cash flow C with support $\{V_{rec}, 1\}$ and probabilities $\{\widetilde{\pi}, 1 - \widetilde{\pi}\}$. For intermediate values $1 > \rho^h > \rho^l > 0$ (the bold lines), C is a linear function of M with gradient

[17] For a numerical analysis of disagreement in the Gaussian copula model, see the working paper version of this article (Broer, 2016).

$\frac{\rho^k}{1-\rho^k}$ and intercept $\underline{C}_k = 1 - \frac{\widetilde{\pi}^k}{1-\rho^k}(1 - V_{rec})$, bounded by a full payoff of one and the recovery value V_{rec}. C therefore inherits the uniform distribution of M, between two mass-points at V_{rec} and one.

Conveniently, with M distributed uniformly on the unit interval, the bold lines in the figure coincide with the cumulative distributions of cash flow C. They thus map any value of cash flow $\widetilde{C}$ on the vertical axes to the probability of observing cash flow smaller or equal to $\widetilde{C}$ on the bottom axis (and are thus rotated around the 45 degree line relative to their standard representation). For $\rho = 0$, the CDF is a step function that jumps from zero to one at $C = 1 - \widetilde{\pi}(1 - V_{rec})$. For $\rho = 1$, the CDF takes two steps equal to $\widetilde{\pi}$ and $1 - \widetilde{\pi}$ at $C = 0$ and $C = 1$, respectively. Importantly, for intermediate values, an increase in ρ flattens out the CDF by raising the probability of observing both high and low payoffs as cash flow becomes more sensitive to aggregate conditions. This pushes mass from the interior to the extremes of the uniform distribution.

As in the simple example in Section 2, for a given probability of loan default, changes in ρ^k do not affect the expected cash flow from the securitization, equal to $\overline{C}$, and thus leave the market price of the unstructured, pass-through securitization unchanged. When cash flow is allocated to tranches in order of pre-specified seniority, however, different values of ρ^k again imply different perceived probabilities of default, or of a payment lower than the face value. Again, under the simplifying assumption of standardized uniform distributions, for any tranche with a given detachment point $\widetilde{C}$ on the vertical axis, the bold lines in Figure 4 indicate the tranche's default probability $P(C \leq \widetilde{C})$ on the bottom axis. Exactly as in the simple example of Section 2.2, high-ρ investors thus have an absolute advantage in buying junior tranches, with attachment points at or above the cash flow expectation $\overline{C}$, whose probability of payment they think is high. Low-ρ investors, in contrast, have a higher valuation of mezzanine tranches, defined as those that are perceived as risky by at least one investor but pay fully as long as cash flow is above its expectation (implying attachment range $[\min_k\{\underline{C}_k\}, \overline{C}]$), whose probability of default they perceive as low. In contrast to the simple example, investors disagree about the amount of riskless cash flow the securitization generates, equal to $\underline{C}_k$. But they agree that senior tranches, with attachment ranges below $\underline{C}_1$, are riskless.

The middle panel of Figure 4 depicts the cash flow distribution as a function of M for common $\rho^k = \rho, \forall k$ when $\widetilde{\pi}^l < \widetilde{\pi}^h$, corresponding, respectively, to optimists and pessimists.[18] As in the simple example, the cash flow distribution perceived by optimists first-order stochastically dominates that of pessimists. Specifically, their perceived probability of observing cash flow C equal to any particular value $\widetilde{C} \geq \underline{C}^h$ or above is higher than that of pessimists by a constant amount $\frac{\widetilde{\pi}^h - \widetilde{\pi}^l}{\rho}$. Since that probability is decreasing in $\widetilde{C}$, the ratio of tranche valuations by low and high-$\widetilde{\pi}$ investors increases for more junior tranches, just as in the simple example. In other words, optimists again have a comparative advantage in junior tranches.

As should be clear from the discussion of the figures, versions of Propositions 1 to 4 still hold in the economy with many loans and, potentially, many tranches. With disagreement about default probabilities (or, equivalently, the recovery value V_{rec}), structuring has the strongest effect on prices when optimists can buy all risky tranches, but not the entire securitization. When optimist resources are small, they buy the most junior tranches, whose price, however, is low by arbitrage with the mezzanine tranches that are priced at their pessimistic valuation. With disagreement about default correlation ρ, in contrast, tranching always raises prices above that of the pass-through

[18]For $\rho = 1$ and $\rho = 0$, x_n follows a standardized uniform distribution $U([0, 1])$, so $\widetilde{\pi}^k$ equals the probability of default perceived by investor k. For values $0 < \rho \leq \frac{1}{2}$, and similarly $1 > \rho \geq \frac{1}{2}$, x_n follows a symmetric trapezoidal distribution whose mass increases linearly between 0 and ρ, is constant between ρ and $1-\rho$, and falls linearly to zero between $1-\rho$ and one. Thus, for given ρ, the probability of default is monotonically increasing in, but not equal to, $\widetilde{\pi}^k$. Figure 4 is for simplicity drawn for a common threshold value $\widetilde{\pi}$. The corresponding figure for a common default probability is very similar. I thank Paolo Frumento for pointing this out.

securitization, as long as, individually, low-ρ investors can afford the mezzanine tranches and high-ρ investors the junior tranches, and, collectively, they can also afford to buy the senior tranches at their common valuation.

An important difference to the simple model in Section 2 is shown in the bottom panel of Figure 4, which depicts the perceived cash flow distributions of three types of agents: group one and two have $\rho^h > \rho^l$ as before, and share an intermediate value of the default probability parameter $\widetilde{\pi}^m$. Group three consists of optimists with a low value $\widetilde{\pi}^l$ and a correlation parameter ρ^m between those of the other two types. The optimists expect higher cash flow than the two other investor types whenever M is around the cut-off value $\widetilde{\pi}^2$, but not for more extreme realizations. Importantly, the originator can exploit this additional dimension of heterogeneity through a finer tranche structure: a senior mezzanine tranche Mez_1 is valued most by $\rho^l, \widetilde{\pi}^m$ investors, a junior mezzanine tranche Mez_2 by their $\rho^m, \widetilde{\pi}^l$ peers, and the junior tranche by the $\rho^h, \widetilde{\pi}^m$ group as before. Prices thus simply equal the highest valuation of tranches across investors.

B.5. Short Sales, CDS trade and synthetic CDOs

Like most of the literature on investor disagreement following Miller (1977), the analysis so far ruled out short-sales, or negative investments in securitizations or their tranches and thus prevented additional supply from, say, short-selling pessimists to offset high demand by optimists.[19] While loan pools of securitizations can typically not be borrowed and sold, investors can replicate a short-sale by a derivative contract. Thus, a credit default swap (CDS) pays the buyer of protection against default risk the difference between a debt security's face value and its market value after a default event in exchange for a regular premium payment. While the buyer of protection

[19] In fact, Lintner (1969) shows how, without short-selling constraints, equilibrium asset prices in an economy with risk-averse heterogeneous investors are the same as those with a representative investor whose perceptions are a weighted average across all investors.

gains from higher default rates, the seller of protection has the same exposure to default risk as the buyer of the debt security. The additional supply of equivalent assets is thus likely to reduce the price of both pass-through and structured securitizations. As we will show, CDS trade tends to reduce the return to tranching less with disagreement about mean probabilities, where it may actually increase, than in the case of disagreement about default correlation, where it may turn negative. We also discuss several frictions that may limit CDS trade and its depressing effect on securitization prices in reality. Interestingly, to the extent that financial innovation reduced these frictions during the late 2000s, disagreement about default risk may have contributed both to the boom in securitization and to its bust after 2007.

B.5.1. CDS trade can increase and decrease the return to tranching

Suppose agents in the benchmark environment can also trade cash-collaterized CDS contracts. These pay the difference between the face value of a claim j and its end-of-period cash flow in exchange of a premium p_j paid at the beginning of the period. For tractability, we assume that CDS contracts have to be fully funded by cash collateral equal to the maximum protection payment. Their payments, including the returned collateral, thus equal state-by-state those of the underlying asset (excluding, in the case of shares in the pass-through securitization, its riskless payment), since both generate an end-of-period cash flow equal to the face value minus any default losses. By arbitrage, the collateral payment for the CDS net of the premium received $p_j, j = \{sec, jun, mezz\}$, then equals the underlying asset's price: $3-p_{sec}=P_{sec}$ for the pass-through securitization, and $1-p_{mezz}=P_{mezz}$ and $1-p_{jun}=P_{jun}$, for the mezzanine and junior tranches respectively.[20]

[20] To see this, for example, in the case of the pass-through securitization note that sellers of protection receive an end-of-period cash flow (including the collateral returned) of zero, one or two with probability equal to, respectively, $\overline{\pi}^k\pi^k_{DD}$, $1-2\overline{\pi}^k(1-\pi^k_{DD})$ and $1-2\overline{\pi}^k+\overline{\pi}\pi^k_{DD}$ in exchange for a net collateral payment of $2-p_{sec}$ in period one. By arbitrage, this net payment by the protection seller must equal $P_{sec}-1$, since cash

The focus on fully cash-collateralized CDSs has two immediate implications. First, P_{sec} is unaffected by CDS trade when investors disagree about the default correlation parameter π_{DD}: since both investor types share identical expected losses from default, they also share identical expected payouts from a CDS, which they therefore have no incentive to trade at the price prevailing in the benchmark equilibrium. Second, in the absence of CDS trade the valuation of a rich investor type with a large relative endowments is a lower bound for asset prices (as the first part of Proposition 1 states). When, in contrast, CDS trade allows her to invest profitably in CDS contracts when she perceives prices to be high, her valuation is also an upper bound for prices. Since this holds for both the pass-through and structured securitizations, the introduction of CDS trade thus reduces any positive return to tranching to zero whenever endowments are sufficiently asymmetric. To see this, note that prices of all assets, including the premia and net collateral requirements for CDSs on risky tranches or the pass-through securitization, must be strictly positive in equilibrium. The portfolio of an investor type j with a given finite endowment n^j is thus confined to a compact set. Whenever an asset's price differs from the valuation of a rich second type i, the latter perceives a profitable investment opportunity either from buying the asset (or equivalently issuing a CDS), or from buying CDS protection on it. She therefore invests all her funds in the most profitable such opportunity (that is not necessarily unique). Given the fixed supply of exogenous assets, and the compact set of portfolios type j can afford, for any deviation of prices from the rich type i's valuation there is necessarily a finite value of her endowment n^i such that her demand for claims exceeds what type j can afford to accomodate (independently of her willingness to do so). Prices thus have to equal i's valuation in equilibrium. Since this holds for both the pass-through and structured securitizations[21], whenever endowments are suf-

flows of the CDS equal those of the pass-through securitization minus one, with identical probabilities, implying $p_{sec} = 3 - P_{sec}$.

[21]Since both types perceive the senior tranche as riskless, we abstract from trade in

ficiently asymmetric the introduction of CDS trade thus reduces any positive return to tranching to zero.[22]

The return to tranching may be negative with CDS trade

Whenever endowments are more equally distributed, equilibrium asset prices with CDS trade are a more complicated function of parameters. CDS trade is more likely to depress the return to tranching with disagreement about the default correlation parameter π_{DD}, where it leaves the price of the pass-through securitization unchanged but often lowers that of the structured securitization. The return to tranching may, in fact, even turn negative in this case. This is because whenever the average default probability is low, tranches are expensive but protection is cheap. Because buying, say, the junior tranche realizes the same difference in perceived payoff probabilities as buying mezzanine protection, investors require lower tranche prices than in the absence of CDS trade to buy their exogenous supply. To see this in a numerical example, consider two investor types with identical endowments of three and identical perceived default probabilities of $\overline{\pi} = 0.3$, but different perceived default correlation, equal to $\pi^1_{DD} = 0.4$, and $\pi^2_{DD} = 0.3$. Since type one perceives extreme payoffs as more likely, she is the natural buyer of the junior tranche and of mezzanine protection, both of which she perceives to pay with a probability that is three percentage points higher than that perceived by type two. Importantly, since $\overline{\pi} < \frac{1}{2}$, at the prices in the benchmark equilibrium CDS protection on the mezzanine tranche would be cheaper than the junior tranche and thus more profitable than any long investment.

CDS on senior tranches, which yield no gains from trade.

[22]More explicitly, the price of the pass-through securitization equals an optimist's valuation whenever her endowment exceeds the exogenous asset supply (equal to one) plus the net collateral required to satisfy the CDS demand by pessimists (equal to $\frac{a^1}{p_{sec}}$) evaluated at $P_{sec} = V^2_{sec}$ and $p_{sec} = 3 - P_{sec}$, i.e. whenever $a^2 \geq 1 + 2(1 - \overline{\pi}^2)(1 + \frac{a^1}{2\overline{\pi}^2})$. Conversely, the price equals the pessimist's valuation whenever the sum of her endowment invested in CDS protection plus the exogenous asset supply, both evaluated at pessimist valuations, exhaust optimist endowments, or $a^2 \leq 1 + 2(1 - \overline{\pi}^1)(1 + \frac{a^1}{2\overline{\pi}^2})$. In other words, the range of optimist endowments such that P_{sec} exceeds the pessimist valuation is reduced by CDS trade as both threshold values in Figure 1 move to the right.

Similarly, type two finds CDS protection on the junior tranche a more attractive way to exploit her higher perceived probability of less extreme payoffs than buying the mezzanine tranche. In order to sell the exogenous supply of tranches to investors when CDSs are also available, the prices of both the risky and the senior tranches therefore have to fall, implying higher premia for CDS protection. Indeed, our example results in a typical 'correlation trade': high (low) correlation investors buy CDS protection on mezzanine (junior) losses, hold the entire junior (mezzanine) risk (including some through CDS issuance) and part of the senior tranche.[23] The introduction of CDS trade depresses the price of the structured securitization from 2.43 to 2.36, below the pass-through securitization's unchanged price of 2.4. In other words, CDS trade results in a negative return to tranching because low default probabilities make the purchase of cheap default protection a more efficient way of exploiting disagreement than buying expensive tranches. This depressing effect of CDS trade hinges crucially on a low default probability $\overline{\pi}^k$, and a large recovery value (which boosts the effect of the fall in the senior tranche's price): when, for example, V_{rec} equals 0.2, rather than its benchmark value of one, and default probabilities are on average higher, with $\overline{\pi} = \pi^2_{DD} = 0.65$, and $\pi^1_{DD} = 0.75$, CDS trade reduces the return to tranching from 6.5 to a positive value of 1.5 %.

The return to tranching may be higher with CDS trade

With disagreement about default probability $\overline{\pi}$, CDS trade typically reduces the market prices of both the pass-through and structured securitizations, and may thus decrease or increase the return to tranching. Importantly, whenever its negative effect on the price of the pass-through securitization P_{sec} is strong relative to that on individual tranche prices, CDS trade may raise the return to tranching. The former is likely to be true, for example, when optimist endowments are close, or equal, to the market value of the

[23] Note that there is a range of portfolio shares that are consistent with this equilibrium, because the two budget constraints and the market clearing conditions in the three assets leave one portfolio share undetermined.

pass-through securitization in the absence of CDS trade. As suggested by Figure 1, this implies that any rise in the asset supply from pessimists' demand for CDS insurance would decrease prices along the optimists' budget constraint. The effect of CDS trade on tranche prices is muted whenever parameters are such that pessimists require only a small price discount to continue buying the senior tranche even after the introduction of CDS trade gives them profitable opportunities to buy protection on risky tranches.

As an example, consider optimists with a perceived default probability $\overline{\pi}^2 = 0.2$ and endowments $n^2 = 2.5$, pessimists with $\overline{\pi} = 0.3$ and a smaller endowment $n^1 = 1$, and a common $\pi_{DD} = 0.3$. Without CDS trade P_{sec} equals 2.5, the endowment of optimists divided by the asset supply (equal to one), implying a return to tranching of 4 %. Introducing CDS trade makes P_{sec} drop to the pessimist's valuation of 2.4, at which optimists buy the whole exogenous supply of assets and issue CDS protection to pessimists (who are happy and able to buy it) such that the sum of the two exhausts their endowment. According to Proposition 1, the price of the structured securitization $P_{sec|tr}$ without CDS trade equals the optimist valuation of 2.6, as pessimists buy the senior tranche at their valuation of one and optimists have enough funds to buy the risky tranches at theirs. Introducing CDSs at an unchanged price would make pessimists switch to buying CDS protection on mezzanine tranches, perceived to be the most, and strictly, profitable investment. Since optimists cannot afford the whole supply even absent this additional CDS demand, tranche prices would have to fall proportionally for them to continue investing in all tranches. At low enough P_{sen} pessimists are again happy to buy the senior tranche, which they value comparatively highly and perceive to be as profitable as mezzanine protection (whose premium rises with falling tranche prices). For the given parameters, this is indeed an equilibrium, with $P_{sen} = 0.97$, $P_{mezz} = 0.91$, and $P_{jun} = 0.64$, implying $P_{sec|tr} = 2.52$ and an increased return to tranching of five percent. Optimists and pessimists perceive an identical return on all their investments, equal to three percent.

Pessimists invest 90 % of their endowment in the senior tranche and the rest in mezzanine protection. Optimists exhaust their endowment exactly by buying the remaining exogenous asset supply and issuing the mezzanine protection demanded by pessimists.

B.5.2. Frictions in CDS trade, financial innovation, and the crisis

The previous results raise the question how difficult or costly it is to buy protection against loan defaults in collateral pools through CDSs in reality. Traditionally, CDSs are arranged in bilateral, over-the-counter trades between specialist investors making them unaccessible to, for example, retail investors. Moreover, the implied counterparty risk might make institutional investors reluctant to invest a large part of their funds.[24] Importantly, however, the introduction of standardized CDS contracts for mortgage-backed securities (in 2005), and CDOs (in 2006), as well as the creation of the ABX.HE credit derivative index for subprime RMBS tranches in January of 2007, made the trading of credit derivatives for these securitized debt products substantially easier. This may have contributed to an enormous boom in 'synthetic' CDO issuance, which most simply speaking pay CDS premia to investors rather than collateral asset cash flow, in 2005 and 2006.[25]

Fostel and Geanakoplos (2012), and similarly Scheinkman (2013) and Simsek (2013), argue that the additional supply of asset-backed securities through credit derivatives precipitated the end of the securitization boom

[24]On the consequences of counterparty risk in over-the-counter trades, see Acharya and Bisin (2014).

[25]The Financial Crisis Inquiry Commission estimates that the share of derivatives such as CDS in CDO collateral increased from seven percent in 2004 to 27 % in 2006 (Financial Crisis Inquiry Commission (2011), p. 191). Diverging investor views about the outlook for default rates were, however, only one reason for this boom. In fact, for the more general purpose of transfering credit risk, issuing a cash securitization (backed by loans directly and brought to the market through a formally bankruptcy-remote special purpose vehicle) or a synthetic one is often equivalent, but the latter may be administratvely easier and thus less costly to arrange. For a discussion, see Augustin, Subrahmanyam, Tang, and Wang (2014) pp. 93-98.

in 2007.[26][27] Our results give, prima facie, support to this argument, showing how disagreement about default risk may have contributed both to the boom in securitization, and to its bust once credit derivatives became more common. This is particularly true for disagreement about default correlation, which we showed raises a structured securitization's price above the expected collateral cash flow without CDS trade, but may depress it below expected cash flow once CDS trade is introduced.

Even when financial innovation reduces transaction costs, CDS trade may, however, not undo completely a boom in securitization demand fuelled by optimists or disagreement about correlation. This is because the certain short-term losses from premium payments in anticipation of an uncertain future gain may be unattractive for investors without sufficient capital, and particularly for fund managers who have to answer to their investors. Thus Shleifer and Vishny (1997) argue that arbitrage can be limited when it has to be performance-based. Fund investors partly attribute temporary losses from market noise to unobserved lack of fund manager ability and thus withdraw their investments before arbitrage opportunities are realized. Fund managers may thus dislike short-term losses from CDS premium payments, to avoid investor pressure to close their position before default events occur.[28] Importantly, disagreement about default risk provides an additional reason for limited arbitrage: even when investors have gained sufficient information to trust their fund managers' ability, they may withdraw funds in response to

[26]Importantly, our environment, where markets are exogenously incomplete, differs from that in Fostel and Geanakoplos (2012) or Simsek (2013) where assets are completely state-contingent, or 'vertical' tranches of the probability distribution, while typical CDSs, like those in this paper, are triggered whenever cash flow drops below a level inducing default, and thus 'horizontal' tranches. Fostel and Geanakoplos (2012) show how the introduction of CDS-like assets reduces the price to its Arrow-Debreu level. We show how the effect on the relative price of two assets of limited state-contingency may be positive or negative, depending on the source of disagreement and other parameters.

[27]Gorton (2009) stresses the role of the new index in aggregating negative views about mortgage-backed securities.

[28]Gromb and Vayanos (2010) survey theories explaining limits of arbitrage.

an investment strategy whose profitability they perceive as low. Whenever fund managers find it difficult to attract new investors of a more similar perception, for example because their ability is only revealed to investors over time, they are thus more likely to avoid investment strategies that their current investors might disagree with.[29]

To avoid the problems implied by short-term losses, fund managers had designed self-funding short positions on subprime-CDOs prior to the crisis, whereby premia for protection bought against default of some tranches were paid by premia received for protection sold against that of others. Since the long and short positions usually differed in seniority, these trading strategies were often 'correlation trades'. This is because buying senior CDO tranches (or offering protection against their default) and insuring against default by the most risky ones is profitable when default correlations are low and default rates concentrated around their mean. Paying for protection against senior losses by insuring risky tranches, in contrast, is profitable when defaults are mainly driven by aggregate conditions and tranches are expected to default together. In fact, partly thanks to Michael Lewis' bestseller "The Big Short" (Lewis, 2010), the former trading strategy became closely associated with the name of Howie Hubler, the head of Morgan Stanley's Global Proprietary Credit Group that invested in credit default swaps on junior CDOs, and financed the premia by investments in senior CDOs, causing one of the biggest ever trading losses in financial history as the subprime crisis unfolded. The 'Magnetar Trade', in contrast, made the US hedge fund that simultaneously invested in CDO equity tranches and in credit default swaps referencing more senior tranches huge profits (Mählmann, 2013). The popularity of correlation trades can thus be interpreted as evidence for both disagreement about default correlation among investors and the difficulty to sustain the short-term trading losses from short-selling securitization tranches through

[29]The cinematic rendering of Michael Lewis' bestseller "The Big Short" provides a vivid depiction of how difficult it may be for pessimist fund managers to keep less pessimistic clients on board.

CDSs.

Appendix C. Figures

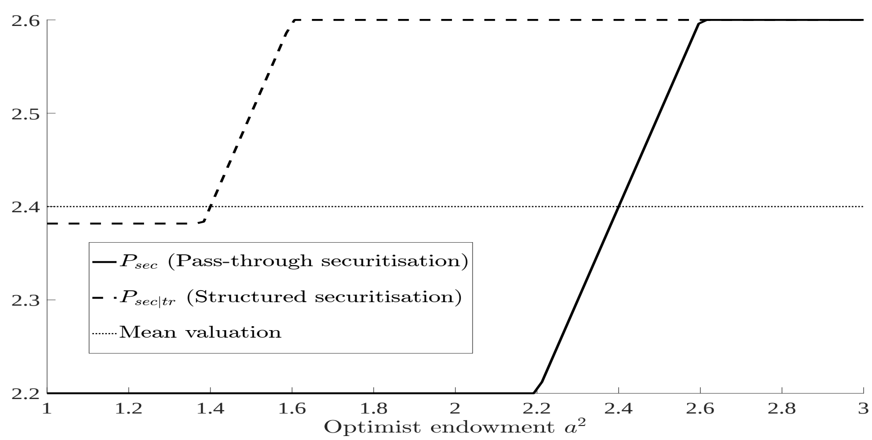

Fig. 1: Prices of securitizations with disagreement about default probability $\overline{\pi}$. As a function of optimist endowments a^2 along the bottom axis, the figure plots the prices of pass-through and structured securitizations, for $\overline{\pi}^1 = 0.2, \overline{\pi}^2 = 0.4, \pi_{DD} = 0.6$, and total endowment $a^1 + a^2 = 3$. It also depicts the mean valuation of the collateral mortgage pool, corresponding to the expectation of the collateral cash flow evaluated at the mean default probability $\frac{\overline{\pi}^1+\overline{\pi}^2}{2} = 0.3$.

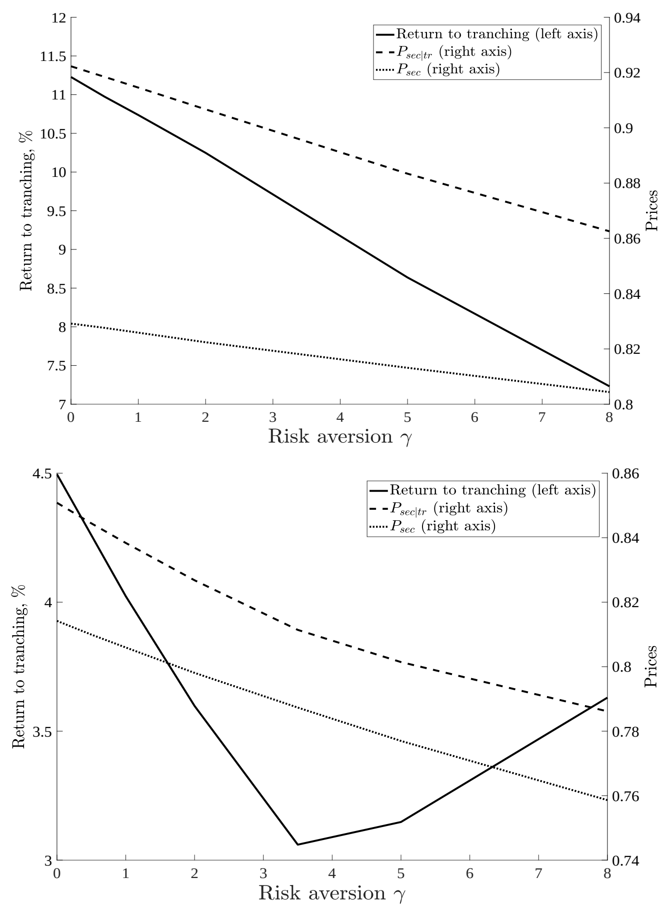

Fig. 2: Prices of securitizations and return to tranching with risk aversion. As a function of the coefficient of relative risk aversion γ along the bottom axis, the figure plots the prices of pass-through and structured securitizations (on the right-hand axis), and the return to tranching (defined as the difference between the prices of the structured and pass-through securitizations as a percentage of the latter, on the left-hand axis), when investors disagree about default risk in the copula model (6), namely the default threshold $\widetilde{\pi}^k$ (in the top panel), and the correlation parameter ρ^k (in the bottom panel).

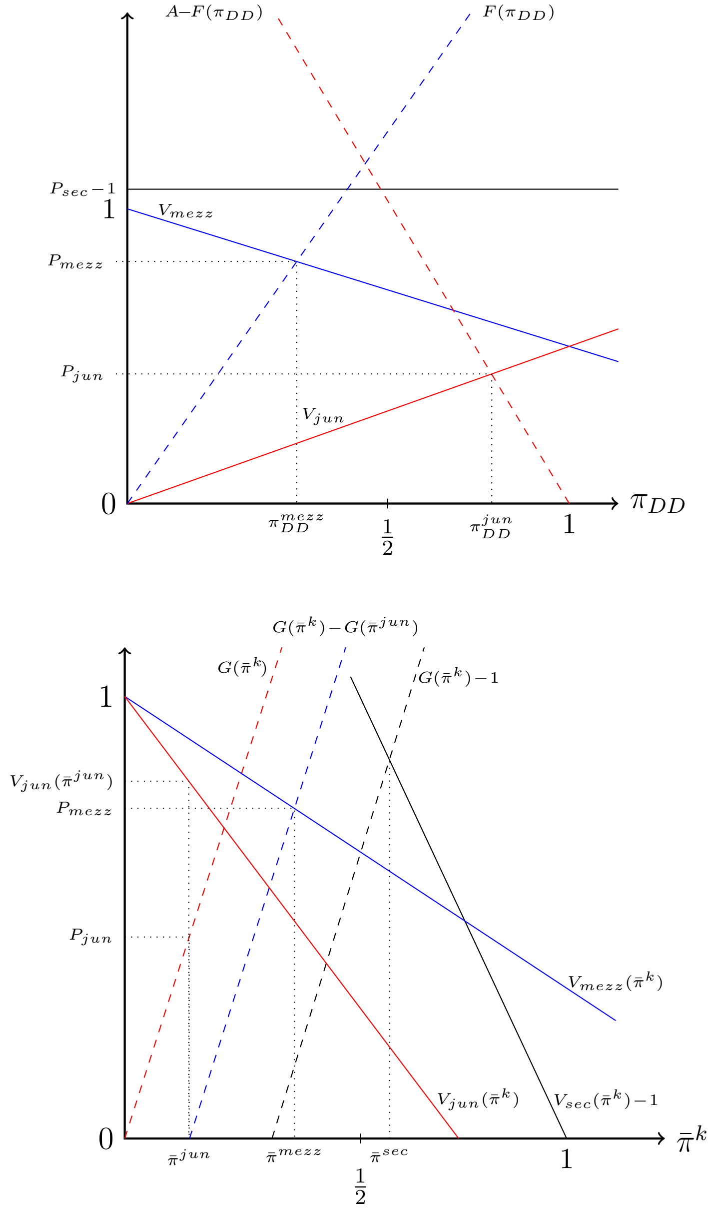

Fig. 3: The continuum economy with disagreement about correlation parameter π_{DD} (top panel) and about default probability $\overline{\pi}$ (bottom panel).

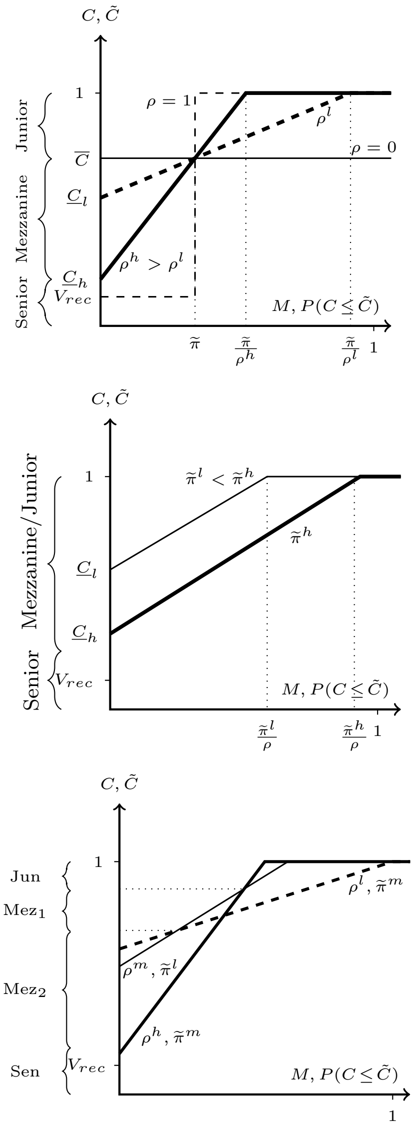

Fig. 4: The figures depict the cash flow from a mortgage pool with a large number of mortgages as a function of the aggregate factor M according to the copula model (25) for disagreement about the correlation parameter ρ^k (in the top panel), about the default threshold $\widetilde{\pi}$ (in the middle panel), and with three types who disagree about both (in the bottom panel). $\overline{C}$ denotes expected cash flow $1 - \widetilde{\pi}(1 - V_{rec})$ and $\underline{C}_k$ the minimum cash flow from the pool as perceived by investor k equal to $1 - \min\{1, \frac{\widetilde{\pi}_2}{1-\rho}\}(1 - V_{rec})$.

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