International Macroeconomics - Session III
International Real Business Cycles with multiple goods

Tobias Broer
IIES
Stockholm Doctoral Program in Economics
Recap: Last session

- "International Real Business cycles": Propagation of shocks in standard model with capital accumulation, intertemporal consumption smoothing and flexible labour supply
- Market structure important for propagation of productivity shocks
- Complete markets
  - Risk sharing implies strong positive consumption correlation across countries
  - Efficiency implies countercyclical NX due to investment dynamics, negative correlation of investment and employment
Recap: Last session

- Incomplete Markets
  - CM and IM almost identical for transitory shocks that spill over to the other country.
  - Only with near-permanent shocks and no spillovers, the two differ, as wealth effects are important.
  - Wealth effects increase consumption at home by more, breaking $c, c^*$ link, and making imports more procyclical, so $NX$ countercyclical.
  - Comovement puzzle for labour inputs and investment remains.
Recap: Complete vs. Incomplete markets I

- Complete markets
  1. 1st WT holds
  2. Equilibrium allocation solves planners problem.
  3. Problem: SPP has infinite no of solutions indexed by relative weights on agents. Also, doesn’t give us equilibrium prices.
  4. Solution: Negishi approach - use lagrange multipliers as prices, and solve for optimal allocation that is ”affordable” at these.

- Incomplete markets
  1. Need to consider individual maximisation given prices
Recap: CM vs. IM II: Budget constraints (with 0 initial wealth)

- Complete markets: Trade only in period 1, s.t.

\[
\sum_{t} \sum_{s_t} \pi(s_t) \frac{q(s_t)}{\pi(s_t)} [y(s_t) - c(s_t)] \geq 0
\]  \hspace{1cm} (1)

\[
E \sum_{t} \frac{q(s_t)}{\pi(s_t)} [y(s_t) - c(s_t)] \geq 0
\]  \hspace{1cm} (2)

- Incomplete markets: Trade every period

\[
c(s_t) + b_{t+1}(s_t) \leq y(s_t) + b_t
\]  \hspace{1cm} (3)

\[
\sum_{0}^{\infty} q_t [y(s_t) - c(s_t)] \geq 0 \forall s_t q_t = \prod_{s=0}^{t} \frac{1}{R_s}
\]  \hspace{1cm} (4)

- So BC holds in expectation with CM, but state-wise with IM
This session

- Add to intertemporal trade intratemporal trade in different goods. Allows to look at RER and ToT movements.
- Transmission of shocks not just by financial markets but also via RER movements, and fluctuations in ToT?
  1. Traded vs. non-traded goods.
  2. Country specific traded goods.
Learning points

• Relative price movements are important shock transmitters. Allow insurance across countries in principle.
• But standard model doesn’t solve IBC puzzles.
• The elasticity of substitution is crucial for equilibrium dynamics. Can lead to NEGATIVE transmission of productivity shocks.
Roadmap: Multigood models

1. Key papers
2. Multigood models: General Issues
3. A reminder on RER facts
4. Complete markets
5. Bond economy and financial autarky.
7. The importance of the elasticity of substitution: Corsetti, Dedola and Leduc (2008)
Intratemporal Trade models: Key Papers

2. BKK 1994: Traded goods in 2-good CM model.
Multigood models - general issues

1. So far: Focus on intertemporal trade. One good for c,i,g, same in both countries. LoOP: relative price 1.

2. Multiple goods
   2.1 How do they feature in c,i,g, and c*,i*,g*?
   2.2 What are the different production functions?
   2.3 What goods are traded?
The biggest problem with multiple goods

- Notation
- Here: $P_h, P_f$ are price indices $p_h, p_f$ goods prices, a $\star$ denotes foreign quantities.
A simple multigood setup

- 2 traded goods $H$ $F$, with price $p_h, p_f$ in some common numeraire.
- Technology standard, but country-specific: country $H$ produces $H$ good, $F$ $F$ good.
- $c, x, g$ are identical CES "Armington aggregates" of 2 goods

$$z = \left( \omega_1^\sigma z_h^{\sigma-1} + \omega_2^\sigma z_f^{\sigma-1} \right)^{\sigma-1} \text{ for } z = c, x, g$$ (5)

$$z^* = \left( \omega_1^\sigma z_f^{*\sigma-1} + \omega_2^\sigma z_h^{*\sigma-1} \right)^{\sigma-1} \text{ for } z^* = c^*, x^*, g^*$$ (6)

for $\omega_1 = (1 - \omega_2)$ the weight of domestic goods in consumption. Assume $\omega_1 > \frac{1}{2}$ (home bias in $c$).
A simple multigood setup

- Preferences, Technology as before, but $c, x, g$ are now CES "Armington aggregates" of 2 goods

$$c = \left( \omega_1^\sigma c_h^\frac{\sigma-1}{\sigma} + \omega_2^\sigma c_f^\frac{\sigma-1}{\sigma} \right)^{\frac{\sigma}{\sigma-1}}$$

- Note: Homogeneity of degree 1 implies budget shares independent of level of consumption.
- Also: Utility is time-separable $\rightarrow$ Can do 2-stage budgeting:
  1. Given period resources $Z$, consume optimal basket of goods.
  2. Allocate resources across periods.
The price index

- For 2 stage budgeting, need price of consumption in period $t$ as function of individual prices.
- Define $P_i$, $i \in \{H, F\}$ as ”minimum expenditure per unit of aggregate consumption”
The price index

Take home country:

\[
\min_{c_h, c_f} \; p_h c_h + p_f c_f \tag{8}
\]

s.t. \( \left( \omega_1^{\frac{1}{\sigma}} c_h^{\frac{\sigma - 1}{\sigma}} + \omega_2^{\frac{1}{\sigma}} c_f^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}} = 1 \tag{9} \)

Solution:

\[
c_h = \omega_1 \left( \frac{p_h}{P_h} \right)^{-\sigma} c \tag{10}
\]

\[
= \omega_1 \left( \frac{p_h}{P_h} \right)^{1-\sigma} Z \tag{11}
\]

Where \( Z = \frac{P_h c}{p_h} \) is expenditure in home goods, for the home consumption price index

\[
P_h = \left[ \omega_1 p_h^{1-\sigma} + \omega_2 p_f^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{12}
\]

\(\Rightarrow\) Symmetric for \( P_f \).
Roadmap: Multigood models

1. Key papers
2. Multigood models: General Issues
3. A reminder on RER facts
Real Exchange rates and relative consumption

Table 3. Unconditional Correlations

<table>
<thead>
<tr>
<th>Country</th>
<th>With real US$ rate</th>
<th>With real effective exch. rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e^u$</td>
<td>$p^u/p^i$</td>
</tr>
<tr>
<td>I. First differences, full sample</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>0.99</td>
<td>0.03</td>
</tr>
<tr>
<td>Canada</td>
<td>0.97</td>
<td>0.18</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.99</td>
<td>-0.07</td>
</tr>
<tr>
<td>France</td>
<td>0.99</td>
<td>-0.12</td>
</tr>
<tr>
<td>Italy</td>
<td>0.99</td>
<td>-0.13</td>
</tr>
<tr>
<td>Japan</td>
<td>0.98</td>
<td>0.12</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.99</td>
<td>0.00</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.98</td>
<td>0.23</td>
</tr>
<tr>
<td>Norway</td>
<td>0.98</td>
<td>0.17</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.99</td>
<td>-0.06</td>
</tr>
<tr>
<td>UK</td>
<td>0.99</td>
<td>0.04</td>
</tr>
<tr>
<td>US</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II. First differences, 1960-1972.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>0.81</td>
<td>0.77</td>
</tr>
<tr>
<td>Canada</td>
<td>0.95</td>
<td>0.15</td>
</tr>
<tr>
<td>Japan</td>
<td>0.82</td>
<td>0.37</td>
</tr>
<tr>
<td>UK</td>
<td>0.97</td>
<td>0.18</td>
</tr>
<tr>
<td>US</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>III. First differences, 1973.1-1998.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>0.99</td>
<td>0.00</td>
</tr>
<tr>
<td>Canada</td>
<td>0.97</td>
<td>0.19</td>
</tr>
<tr>
<td>Japan</td>
<td>0.99</td>
<td>0.10</td>
</tr>
<tr>
<td>UK</td>
<td>0.99</td>
<td>0.10</td>
</tr>
<tr>
<td>US</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Source: Ravn 2001
Roadmap

1. Key papers
2. Multigood models: General Issues
3. A reminder on RER facts
4. Complete markets
Intratemporal trade I: Complete markets
Planer’s Problem

\[
\begin{align*}
\max \{c_{it}^{(*)}, x_t^{(*)}, l_t^{(*)}\} & \quad E_0 \left[ \sum_{t=0}^{\infty} \mu^{(*)} \sum_{t=0}^{\infty} \beta^t u(c_{it}^{(*)}, 1 - n_t^{(*)}) \right] \text{ subject to} \\
\lambda_t : & \quad z_t = \left( \frac{1}{\omega_1^\sigma} z_{ht}^\sigma + \frac{1}{\omega_2^\sigma} z_{ft}^\sigma \right)^{\sigma-1} \text{ for } z = x, g, c \quad (13) \\
\lambda_t^* : & \quad z_t^* = \left( \frac{1}{\omega_1^\sigma} z_{ft}^\sigma x + \frac{1}{\omega_2^\sigma} z_{ht}^\sigma x \right)^{\sigma-1} \text{ for } z^* = x^*, g^*, c^* \quad (14) \\
\nu_{ht} : & \quad c_{ht} + c_{ht}^* + x_{ht} + x_{ht}^* + g_{ht} + g_{ht}^* \leq \exp(A_t) F(k_t, n_t) \quad (15) \\
\nu_{ft} : & \quad c_{ft} + c_{ft}^* + x_{ft} + x_{ft}^* + g_{ft} + g_{ft}^* \leq \exp(A_t^*) F(k_t^*, n_t^*) \quad (16) \\
k_{t+1}^{(*)} : & \quad (1 - \delta) k_t^{(*)} + x_t^{(*)} \quad (17) \\
k_0^{(*)} & \quad \text{given} \quad (18)
\end{align*}
\]
Meaning of multipliers

- $\lambda_t^{(*)}$: Impact of change in H (F) country consumption on planner’s value - shadow price of consumption.
- $\nu_{it}$: Impact of relaxing good i resource constraint on planner’s value - shadow price of good i.
- So get
  - Terms of trade as the relative price of goods $p_t = \frac{\nu_{ft}}{\nu_{ht}}$
  - RER as the relative price of consumption $q_t = \frac{\lambda_t^{(*)}}{\lambda_t}$
  - Note: Law of one price holds, but RER fluctuates with relative costs of consumption baskets.
First order conditions I: Risk-sharing

1. \( c_t^{(\star)} : \mu^{(\star)} u_c(c_t^{(\star)}, 1 - n_t^{(\star)}) = \lambda_t^{(\star)} \)

Risk sharing now takes into account the relative cost of consumption in \( h \) and \( f \), so relative MU fluctuates.

2. \( \rightarrow \) "Backus-Smith Condition"

\[
\frac{u_c(c_t, 1 - n_t)}{u_c(c_t^{\star}, 1 - n_t^{\star})} = \frac{\mu^* \lambda_t}{\mu \lambda_t^*} = \frac{\mu^* 1}{\mu q_t} \tag{19}
\]

3. So real exchange rate movements drive wedge between marginal utilities - might solve consumption correlation puzzle.
First order conditions II: Efficiency in production

1. \[ n_t : \frac{u_n(c_t,1-n_t)}{u_n(c^*_t,1-n^*_t)} = \frac{\nu_{ht}\exp(A_t)F_n(k_t,n_t)}{\nu_{ft}\exp(A^*_t)F_n(k^*_t,n^*_t)} = \frac{1}{p_t} \frac{\exp(A_t)F_n(k_t,n_t)}{\exp(A^*_t)F_n(k^*_t,n^*_t)} \]

2. Efficient provision of labour now takes account of relative value of good \( i \). This might help solve the comovement puzzle, but depends on the impact of productivity shocks on ToT.

3. \[ \frac{E_{t-1}[\nu_{ht} \exp(A_t)F_k(k_t,n_t)+\lambda_t(1-\delta)]}{E_{t-1}[\nu_{ft} \exp(A^*_t)F_k(k^*_t,n^*_t)\mu_t^*+\lambda_t^*(1-\delta)]} = \frac{\lambda_{t-1}^*}{\lambda_{t-1}} = \frac{1}{q_{t-1}} \]

4. Similarly, the productivity of \( h \) capital depends on the relative price of \( h \) goods. The cost of investments depends on the relative price of \( h \) consumption basket. Might solve comovement, but again depends on ToT and RER.
Interpretation

1. The relative price of home and foreign goods breaks the link between the efficiency conditions for labour and investment at home and abroad: Higher home good supply raises demand for foreign goods in consumption and investment, and so makes production of the latter more valuable. This might explain why labour and investment comove positively.

2. The different consumption shares $\omega_1, \omega_2$ introduce fluctuations in the relative cost of consumption (RER) in response to supply shocks. This can break risk-sharing link between the levels of consumption.

3. But Backus-Smith condition predicts strong comovements between RER and relative consumption.

4. Effect of productivity shocks on ToT and ReR is crucial.
Excursion: The ReR and the ToT

1. We can write the real exchange rate as

\[ q_t = \left[ \frac{1 + \frac{\omega_2}{\omega_1} p_t^{1-\sigma}}{p_t^{1-\sigma} + \frac{\omega_2}{\omega_1}} \right] \frac{1}{\sigma-1} \]  

(20)

2. So the real exchange rate is independent of the ToT only if \( \omega_2 = \omega_1 \). For home bias in consumption \( \omega_1 > \omega_2 \), the RER responds positively to the ToT: when home products become cheaper, home consumption becomes cheaper, relative to foreign consumption.
Excursion: The ReR and the ToT

1. Also, in equilibrium, the TOT have to equal the MRS between h and f goods in consumption

\[ p_t = \frac{\omega_2}{\omega_1} \left[ \frac{c_h}{c_f} \right]^{\frac{1}{\sigma}} \text{ (equiv. for } F) \]  \hspace{1cm} (21)

2. So elasticity of substitution \( \sigma \) is key in determining the ToT (and thus RER) response to supply shocks.

3. Low \( \sigma \)
   
   3.1 Small change in quantity has large price effects. So productivity shocks spill over strongly to foreign country.
   
   3.2 The RER becomes more responsive to changes in prices / ToT (less substitution to counteract price movements).
Roadmap

1. Key papers
2. Multigood models: General Issues
3. A reminder on RER facts
4. Complete markets
5. Bond economy and financial autarky.
Intratemporal Trade II: Bond economy

1. Countries trade uncontingent bond that pays off in numeraire.
2. As before, income shocks have wealth effects.
3. Shocks get transmitted via relative price of goods and interest rate.
First order conditions

1. Euler equation now takes into account changes in consumption price - or "consumption based real interest rate"

\[
\frac{U'(c)}{P_H} = E[R\beta \frac{U'(c')}{P'_H}] \quad (22)
\]

So RER movements drive wedge between EXPECTED MRS.

2. FOC for labour and investment unchanged, subject to lack of risk sharing.
Intratemporal Trade III: Financial autarky

1. Balanced trade:

\[ p_h c_h^* = p_f c_f \]  \hspace{1cm} (23)

2. Budget constraint

\[ P_h c = p_h Y, \quad P_f c^* = p_f Y_F \]  \hspace{1cm} (24)

3. Can deliver higher volatility in relative prices.
Roadmap

1. Key papers
2. Multigood models: General Issues
3. A reminder on RER facts
4. Complete markets
5. Bond economy and financial autarky.
A step back: Questions

1. Can trade in goods solve puzzles (Cons corellation, Comovement) and deliver countercyclical NX?

2. Do the models deliver realistic volatility, and comovement, of ToT and RER?
A quantitative model: Heathcote et al 2000

1. Symmetric world (US plus clone), apart from shocks
2. Elasticity of substitution $\sigma = 0.9$
3. Shocks: SD 0.0073, correlation 0.29, and AR(1) parameter matrix $R$

\[
\begin{pmatrix}
0.97 & 0.025 \\
0.025 & 0.97
\end{pmatrix}
\]
Results Heathcote et al 2000: Benchmark calibration

B) Correlations with output

<table>
<thead>
<tr>
<th>Economy</th>
<th>c,y</th>
<th>x,y</th>
<th>n,y</th>
<th>ex,y</th>
<th>im,y</th>
<th>nx,y</th>
<th>p,y</th>
<th>rx,y</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Data</td>
<td>0.86</td>
<td>0.95</td>
<td>0.87</td>
<td>0.32</td>
<td>0.81</td>
<td>-0.49</td>
<td>-0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>Complete markets</td>
<td>0.96</td>
<td>0.96</td>
<td>0.97</td>
<td>0.55</td>
<td>0.89</td>
<td>-0.64</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>Bond economy</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
<td>0.59</td>
<td>0.86</td>
<td>-0.65</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>Financial autarky</td>
<td>0.92</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
<td>0.15</td>
<td>0.00</td>
<td>0.65</td>
<td>0.65</td>
</tr>
</tbody>
</table>

P = terms of trade
rx = real exchange rate

C) Cross country correlations and international relative price volatility

<table>
<thead>
<tr>
<th>Economy</th>
<th>y1,y2</th>
<th>C1,c2</th>
<th>x1,x2</th>
<th>N1,n2</th>
<th>p</th>
<th>Rx</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.58</td>
<td>0.36</td>
<td>0.30</td>
<td>0.42</td>
<td>2.99</td>
<td>3.73</td>
</tr>
<tr>
<td>Complete markets</td>
<td>0.18</td>
<td>0.65</td>
<td>-0.29</td>
<td>-0.14</td>
<td>0.78</td>
<td>0.55</td>
</tr>
<tr>
<td>Bond economy</td>
<td>0.17</td>
<td>0.68</td>
<td>-0.29</td>
<td>-0.17</td>
<td>0.84</td>
<td>0.59</td>
</tr>
<tr>
<td>Financial autarky</td>
<td>0.24</td>
<td>0.85</td>
<td>0.35</td>
<td>0.14</td>
<td>1.68</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Source: Heathcote et al 2000
Discussion

1. Disappointing!

2. Cons-Correlations puzzle remains, so does Comovement puzzle in CM and with bond trade.

3. ToT and RER volatility far too small with CM and bond trade. Correlation \( \frac{c}{c^*} \) with RER far too high.

4. Again, Bond trade and CM similar overall.

5. But: Financial autarky fares better on comovement and price volatility! (but worse on cons-correlation).

6. And: Results are sensitive to value of \( \sigma \) and to persistence of shock.
## Results Heathcote et al 2000: Sensitivity

Table 3: Varying shock persistence and degree of substitutability – no spill-overs

<table>
<thead>
<tr>
<th></th>
<th>Low persistence shocks</th>
<th>Unit root shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho = 0.95$</td>
<td>$\rho = 1.0$</td>
</tr>
<tr>
<td>$\sigma = 0.5$</td>
<td>$\sigma = 1.0$</td>
<td>$\sigma = 1.5$</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 1.0$</td>
<td>$\sigma = 1.5$</td>
</tr>
</tbody>
</table>

### A) $corr(y_1,y_2)-corr(c_1,c_2)$

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>0.22</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete markets</td>
<td>0.13</td>
<td>-0.13</td>
</tr>
<tr>
<td>Bond economy</td>
<td>-0.37</td>
<td>-0.14</td>
</tr>
<tr>
<td>Financial autarky</td>
<td>-0.08</td>
<td>-0.29</td>
</tr>
</tbody>
</table>

### B) $corr(x_1,x_2)$

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete markets</td>
<td>0.29</td>
<td>0.14</td>
</tr>
<tr>
<td>Bond economy</td>
<td>0.46</td>
<td>0.14</td>
</tr>
<tr>
<td>Financial autarky</td>
<td>0.66</td>
<td>0.61</td>
</tr>
</tbody>
</table>

### C) % std. dev terms of trade ($p$)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>2.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete markets</td>
<td>1.05</td>
<td>0.75</td>
</tr>
<tr>
<td>Bond economy</td>
<td>2.22</td>
<td>0.76</td>
</tr>
<tr>
<td>Financial autarky</td>
<td>5.74</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Source: Heathcote et al 2000
Solving the puzzles: The importance of the elasticity of substitution

1. Corsetti, Dedola and Leduc 2008
Recap

- Efficiency: \( c_i \) high when \( P_i \) low.
- Bond economy: holds, but in expectations, via the expected path of consumption based real interest rate.
- But data: positive relation between \( c/c^* \) and \( p_h/p_f \) or \( P_h/P_f \).
Recap

• Solution in 1 good world: strong wealth effects.
• Here: change in LEVEL of consumption $\rightarrow$ change in demand for $H$ vs $F$ goods. Could drive a wedge in Backus-Smith correlation.
• Wealth effects: from rise in PDV of $Y_h$, or $p_h/p_f$, or both.
• But with supply shocks, $Y_h$ and $p_h/p_f$ usually negatively linked - back to Backus-Smith puzzle!
• 2 solutions:
  1. Make wealth effect from $Y_h$ big, reaction in $p_h/p_f$ small, and supply sluggish. Short-term excess demand for $H$ goods yields positive correlation $c/c^*\ p_h/p_f$.
  2. Create positive comovement $Y_h$ and $p_h/p_f$. 
Income and substitution effects: the role of $\sigma$

1. In financial autarky, fall in price of home goods affect home consumption according to

$$c_h = \omega_1 \left(\frac{p_h}{P_h}\right)^{-\sigma} C = \omega_1 \left(\frac{p_h}{P_h}\right)^{-\sigma} \frac{p_h Y_H}{P_h} = \frac{\omega_1}{\omega_1 + \omega_2 p^{1-\sigma}} Y_H \quad (25)$$

2. Given $Y_H$ a deterioration in ToT $p$ has negative income effect, but positive substitution effect for domestic $h$ good demand.

3. Net effect positive (negative) when $\sigma > 1$ ($< 1$). No net effect for CD preferences (cf. Cole and Obstfeld (1991)).

4. Effect of lower $p_h$ on foreign demand always positive.
Wealth effects and $\sigma$

1. For very high $\sigma$, strong positive wealth effect from movements in $Y_H$ (as in one-good model). With sluggish supply, get also right movement in ToT.

2. For very low $\sigma$, a deterioration in terms of trade leads to a large fall in domestic demand. If home bias is high, can the small home good share of foreign consumption make up for this to clear market for h goods? Not always!

3. Corsetti et al (2008): For very low $\sigma$, the ToT (and thus the real exchange rate) IMPROVE (appreciates) from a positive shock to $Y_H$. Transmission is negative!
Negative transmission and $\sigma$

1. Can write relation of relative supply and ToT $p = \frac{p_f}{p_h}$ as

$$\frac{Y_H}{Y_F} = \frac{p(\omega_2 + \omega_1 p^{\sigma - 1})}{(\omega_2 + \omega_1 p^{1-\sigma})}$$ (26)

2. Log-linearization yields:

$$\hat{p} = \frac{\hat{Y}_H - \hat{Y}_F}{1 - \omega_1 (1 - \sigma)}$$ (27)

- So for $\sigma < \frac{2\omega_1 - 1}{2\omega_1}$, get
  - Negative transmission ("Immiserizing growth"): positive shocks to $Y_H$ depress foreign consumption.
  - Positive Relation $c/c^*, p_h/p_f$. 
Corsetti et al (2008): Key elements

1. Bond economy

2. 2 country-specific goods, CES aggregator

3. Non-traded goods enter in consumption, but also in ”distribution sector”

4. Look at 2 different scenarios:
   4.1 Low $\sigma$ (0.85) and low persistence.
   4.2 High $\sigma$ (8) and high persistence.
Corsetti et al (2008): Results

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Baseline economy</th>
<th>Baseline with taste shocks</th>
<th>Baseline with high elasticity</th>
<th>Persistent tradable shocks</th>
<th>Persistent aggregate shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bond economy $\omega = 0.85$</td>
<td>Arrows–Debreu $\omega = 0.85$</td>
<td>Bond economy $\omega = 0.82$</td>
<td>Bond economy $\omega = 0.8$</td>
<td>Bond economy $\omega = 0.8$</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>3.90</td>
<td>2.99</td>
<td>0.73</td>
<td>2.94</td>
<td>0.99</td>
<td>1.17</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>1.68</td>
<td>2.42</td>
<td>0.83</td>
<td>2.45</td>
<td>1.07</td>
<td>0.48</td>
</tr>
<tr>
<td>Relative price of non-tradables</td>
<td>0.86</td>
<td>0.77</td>
<td>0.51</td>
<td>0.76</td>
<td>0.48</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Cross-correlations

(a) Exchange rates and prices in the theoretical economies

<table>
<thead>
<tr>
<th>S.D. relative to GDP</th>
<th></th>
<th></th>
<th></th>
<th>Bond economy $\omega = 0.8$</th>
<th>Bond economy $\omega = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative GDPs</td>
<td>0.19</td>
<td>-0.54</td>
<td>0.21</td>
<td>-0.55</td>
<td>-0.28</td>
</tr>
<tr>
<td>Relative consumptions</td>
<td>-0.71</td>
<td>-0.24</td>
<td>0.98</td>
<td>-0.30</td>
<td>-0.29</td>
</tr>
<tr>
<td>Real net exports</td>
<td>0.60</td>
<td>0.96</td>
<td>-0.62</td>
<td>0.93</td>
<td>0.57</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>0.52</td>
<td>0.99</td>
<td>-0.16</td>
<td>0.99</td>
<td>0.59</td>
</tr>
</tbody>
</table>

(b) Business cycle statistics in the theoretical economies

<table>
<thead>
<tr>
<th>S.D. relative to GDP</th>
<th></th>
<th></th>
<th></th>
<th>Bond economy $\omega = 0.8$</th>
<th>Bond economy $\omega = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.94</td>
<td>0.48</td>
<td>0.48</td>
<td>0.53</td>
<td>0.67</td>
</tr>
<tr>
<td>Investment</td>
<td>4.33</td>
<td>3.20</td>
<td>3.21</td>
<td>3.13</td>
<td>2.91</td>
</tr>
<tr>
<td>Employment</td>
<td>1.19</td>
<td>0.53</td>
<td>0.52</td>
<td>0.59</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Absolute (in per cent)

| Import ratio                            | 4.94  | 1.62          | 0.55          | 1.63                         | 0.81                         | 3.06                         | 2.91                         | 1.25                        |
| Real net exports over GDP               | 0.64  | 0.17          | 0.03          | 0.18                         | 0.13                         | 0.20                         | 0.16                         | 0.12                        |

Cross-correlations

<table>
<thead>
<tr>
<th>Between foreign and domestic</th>
<th></th>
<th></th>
<th></th>
<th>Bond economy $\omega = 0.8$</th>
<th>Bond economy $\omega = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.68</td>
<td>0.38</td>
<td>0.39</td>
<td>0.38</td>
<td>0.33</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.60</td>
<td>0.30</td>
<td>0.37</td>
<td>0.16</td>
<td>-0.01</td>
</tr>
<tr>
<td>Investment</td>
<td>0.25</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.44</td>
</tr>
<tr>
<td>Employment</td>
<td>0.54</td>
<td>0.45</td>
<td>0.49</td>
<td>0.35</td>
<td>0.16</td>
</tr>
<tr>
<td>Between real net exports and GDP</td>
<td>-0.48</td>
<td>-0.38</td>
<td>-0.21</td>
<td>-0.39</td>
<td>-0.28</td>
</tr>
</tbody>
</table>
Summary

- Fluctuations in relative price of traded goods, and RER, can potentially solve comovement and cons-corr puzzle.
- But for intermediate values of elasticity of substitution, and shocks with spill-overs, the puzzles remain.
- But with extreme values of elasticity, model can account for facts:
  - Extremely low $\sigma$: negative transmission and procyclical relative prices of home goods in short and long-run
  - Extremely high $\sigma$ and sluggish capital adjustment: long-run transmission is positive, but wealth effects cause short run excess demand and procyclical relative price of home goods
International Macroeconomics - Session III

International Real Business Cycles with multiple goods

Tobias Broer

IIES

Stockholm Doctoral Program in Economics