Political Economics II

Spring 2019

Lectures 4-5

Part II Partisan Politics and Political Agency

Torsten Persson, IIES

Introduction: Partisan Politics

Aims

continue exploring policy choice in representative democracy

when politicians are "partisan" – like citizens, their preferences are defined over policy outcomes, rather than derived from pure electoral – or rent-seeking – objectives

this will introduce another set of "work-horse" models

Agenda

A. Electoral competition with given citizen candidates

B. Endogenous citizen candidates

C. Agenda setting and legislative bargaining

A. Electoral competition with given citizen candidates

1. Quick rehash of results from Lecture 1

Study one-dimensional size of government example

simple model with Condorcet winner and discrete $y^J \sim F(\cdot)$

voters have no candidate preferences, initially

"Citizen candidates" in Downsian setting

individuals with $y^J = y^C$, $W^C(g) = (y - g)\frac{y^C}{y} + H(g)$

2 candidates $C = L, R$ with *exogenous* ideal points on opposite sides of the median voter's

$$y^L < y^M < y^R, \quad g^L = G(\frac{y^L}{y}) > g^M = G(\frac{y^M}{y}) > g^R = G(\frac{y^R}{y})$$

2. Different equilibrium outcomes

Crucial assumptions

(V1) voters preferences only over policy $W^J(g)$

(V2) add stochastic preferences over candidates

(P1) politicians can commit to electoral platforms (g_L, g_R)

(P2) such commitments cannot be made

Outcomes

policy convergence: under (V1), (P1), $g_L = g_R = g^M$

policy divergence: if replace (V1) by (V2), or (P1) by (P2), $g^R \leq g_R < g < g_L \leq g^L$

But if candidate (party) preferences endogenous, aren't we back to policy convergence through convergence of candidate types?

B. Endogenous citizen candidates

Add entry stage ahead of election

any citizen, with income y^C, can enter as candidate *at cost* ε
stay in size-of-government example ($\mathcal{J}$ a large number)

after entry, no-commitment subgame as in Lecture **1.E.2.b**

Timing: three stages

1. citizens make entry decisions,
   if no entry $\Rightarrow g = \overline{g}$, "status quo" policy
2. plurality election among entering candidates,
   voters cast their ballot *strategically*
3. winning candidate chooses policy

Stage 3

if elected, C with y^C implements $g^C = G(\frac{y^C}{y})$

Stage 2

voter in group J casts ballot for C that maximizes $E[W^J]$, *given* strategy of other voters (meaning of strategic voting)

Stage 1

a member of group J enters only if that raises $E[W^J]$, *given* entry strategy of other candidates

a. One-candidate equilibria

Do such exist?

yes, several equilibria may exist (due to entry cost)
of focal interest: will somebody with y^M run, and win?
y^M beats any other candidate y^C, as g^M Condorcet winner

One equilibrium condition

y^M can run uncontested if

$$W^M(g^M) - W^M(\overline{g}) > \varepsilon,$$

i.e., no other type J finds it profitable to enter,
as she cannot win against y^M and entry is costly

no other member of group M enters either,
as this does not change g and entry is costly

b. Two-candidate equilibria

Do such exist?

yes, several with $C = L, R$ $\qquad y^L < y^M < y^R$

Two equilibrium conditions

$$W^M(G(\frac{y^L}{y})) = W^M(G(\frac{y^R}{y})),$$

i.e., each candidate has equal chance of winning, and

$$\frac{1}{2}[W^L(G(\frac{y^L}{y})) - W^L(G(\frac{y^R}{y})) > \varepsilon$$
$$\frac{1}{2}[W^R(G(\frac{y^R}{y})) - W^R(G(\frac{y^L}{y})) > \varepsilon,$$

i.e., each gains enough expected utility by entering

Third equilibrium condition

3rd candidate does not enter in between y^L and y^R
voters' equilibrium strategies keep entry unprofitable
y^L and y^R balance each other, votes from either side of y^M

Implications

never policy convergence in two-candidate equilibria

"candidate identity matters", but predictions are not so sharp because of multiplicity

Why work-horse model?

intuitively appealing

why can it handle multi-dimensional policy problems?
because it restricts voter choices to candidates' ex-post optimal policies, cycling cannot arise

C. Agenda setting and legislative bargaining

Introduction

Aims

introduce another work-horse model

study legislative bargaining: how do policy-motivated?
politicians set policy after election? who is powerful?

explore general modeling
apply to specific policy examples
discuss lessons

1. General modeling

Two steps in developing generalized agenda-setter model

(i) first: one-dimensional analysis of politician-initiated referenda among voters – readings in syllabus

(ii) second: multi-dimensional analysis of legislative bargaining among incumbent lawmakers – here, and many applications

Incumbent legislators

consider *three* policy-motivated parties (legislators) J
perfect delegates of three groups: each maximizes $W^J(g)$

General introduction, then apply to two generic policy problems

2.a Size of government example, with $J = L, M, R$

2.b Composition of government example, with $J = 1, 2, 3$

Closed-rule, one-round bargaining:

“agenda-setter”, $A \in \{L, M, R\}$ or $\in \{1, 2, 3\}$ makes take-it -or-leave-it proposal for single majority vote

as congressional committee (or government formation)

Timing

1. nature picks A
2. A proposes g_A
3. legislature votes:

   if at least one of $J \neq A$ in favor $\Rightarrow g^b = g_A$

   if not$\Rightarrow g^b = \overline{g}$, “status quo” implemented

Status-quo policy?

$\overline{g} = 0$ “close down government”

$\overline{g} > 0$ “last year’s policy”

Requirement for acceptable proposal at stage 3

$$W^J(g_A) \geq W^J(\overline{g}) \text{ for at least one } J \neq A$$

A maximizes $W^A(g)$ subject to this "participation constraint"

General properties of g^b

(i) A puts together minimum-winning coalition: seeks support only from one $J = X$, if g generates conflict of interests

(ii) X held to status-quo payoff: $W^X(g_A) = W^X(\overline{g})$ costly to overfulfill participation constraint

(iii) $J = N$ non-coalition member screwed: $W^N(g_A) \leq W^N(\overline{g})$

(iv) X is legislator whose vote "cheapest to get" – will mean small (group) size α^J or low status-quo payoff $W^J(\overline{g})$

2. Specific results

a. Size of government example

Three different income groups

one party each $y^L < y^M < y^R, \quad g^J = G(\frac{y^J}{y})$

Equilibrium when $A = M$

$g^b = g^M$ Condorcet winner in legislature

Equilibrium when $A = L$ ($A = R$ case analogous)

$$g^b = \begin{cases} g^L & \text{if } \overline{g} \geq g^L \\ \overline{g} & \text{if } g^L \geq \overline{g} \geq g^M \\ \text{Min}[g^L, \widetilde{g}^M] & \text{if } g^M > \overline{g} \end{cases}$$

where $W^M(\widetilde{g}^M) = W^M(\overline{g})$ with $\widetilde{g}^M > g^M$

Intuition

L seeks support only from closest legislator M
cf. properties (i), (iii) and (iv) in **1**

L never sets g above g^L and need not go below g^M
A is maximizing

L goes to status quo or equivalent, depending on $g^M \gtrless \overline{g}$
cf. property (ii) in **1**

Implications

party representing "center group" M politically powerful:
member of every coalition

A 's power related to the status quo

b. Composition of government example

For instance, three different regions $J = 1, 2, 3$

have one (set of) legislator(s) each

Properties of equilibrium g^b

$$g^{b,N} = 0$$

$$H(g^{b,X}) - \alpha^X g^{b,X} - \alpha^A g^{b,A} = H(\overline{g}^X) - \textstyle\sum_J \alpha^J \overline{g}^J$$

$$H_g(g^{b,A}) = \alpha^A \, \frac{H_g(g^{b,X})}{H_g(g^{b,X}) - \alpha^X}$$

$g^{b,N} = 0 < g^*$ (property (iii) in **1**)

$g^{b,X} \lesseqgtr g^*$ depending on parameters (property (ii) in **1**)

$g^{b,A} > g^*$

under weak conditions, in particular α^X not too large

note that A spends less than if unconstrained, which would mean setting $H_g(g^{b,A}) = \alpha^A$

Intuition

if A spends more on her own group, she must raise τ

then, X is worse off and needs compensation by higher spending equal to $\frac{dg^X}{dg^A} = \frac{\alpha^A}{H_g(g^{b,X})-\alpha^X}$, which costs A $\alpha^X \frac{dg^X}{dg^A}$

total cost of raising g^A is $\alpha^A + \alpha^X \frac{dg^X}{dg^A} = \alpha^A \frac{H_g(g^{b,X})}{H_g(g^{b,X})-\alpha^X}$

Who does A choose as coalition partner?

compute cost for each level of g^A and each prospective majority partner – i.e., solve 2nd condition for each $J \neq A \Rightarrow$

$$g^J = Z(g^A, \overline{g}^J, \alpha^J) \ ,$$

where Z increasing in all arguments

pick $J \neq A$ whose vote is cheapest (property (iv) in **1**) $\Rightarrow$ pick X such that $\overline{g}^X$ and/or α^X are low

Implications

groups with powerful lawmakers – i.e., with $J = A$ – are better off: their representatives often make policy proposals

small, or rather overrepresented – i.e., low α^J– groups are better off: their lawmakers often part of coalition

and so are "weak"– i.e., low $\overline{g}^J$– groups, in apparent contrast with standard (unanimity) bargaining

3. Discussion – three natural extensions

Extend to *open*-rule bargaining

proposals can be amended by other legislator(s)
dilutes power of agenda setter, A

Extend to multi-*round* bargaining

$A_N \neq A_{N-1}$ makes N^{th} round proposal if $g_{A_{N-1}}$ fails
same logic, only A_N has to offer coalition partner
continuation value, rather than status-quo value
dilutes agenda-setter power

Extend to multi-*period* setting with dynamic status quo

$\overline{g_t} = g_{t-1}$
strategic concerns enter the setting of current policy

Why work-horse model?

framework is intuitively appealing

easily handle multi-dimensional policy problems

easily reformulated to represent government formation,
or alternative legislative arrangements – e.g., parliamentary
vs. presidential systems

Introduction: Political Agency

Aims

explore agency problem between voters and elected representatives

how serious is it? does it spill over on policy?

can voters discipline rent seeking by politicians?

theory:

begin by slightly extending size-of-government example

modify to illustrate three different functions of elections

Agenda

A. Electoral competition with rent-seeking

B. Electoral accountability

C. Electoral selection

A. Electoral competition with rent-seeking

1. Policy efficiency

Introduce endogenous rents in size-of-government model

interpret $r \geq 0$ as diversion of funds for personal gain, party finance, or mismanagement of government funds

$$\tau y = g + r \tag{1}$$

$\mathbf{q} = (g, \tau, r)$ denotes policy vector

Candidate objectives

rewrite as

$$E(v_C) = p_C(R + \gamma r) \tag{2}$$

γ "transaction cost"

direct conflict of interest between politicians and voters

Voters

rewrite policy preferences

$$W^J(\mathbf{q}) = [y - (g + r)]\frac{y^J}{y} + H(g)$$

new dimension, r, is a "valence" issue

preferences are again monotonic and well-behaved, despite two dimensions: satisfy condition for "intermediate" preferences $\Rightarrow$ Condorcet winner exists

$$g^M = G(\frac{y^M}{y}), \qquad r^M = 0$$

Benchmark Downsian model

same assumptions as in Lecture **1**

$y^J \sim F(\cdot)$ discrete with many groups

2 candidates make binding commitment to platforms $\mathbf{q}_C$

Probability of winning

as before, p_A is discontinuous in policy

$$p_A = \begin{cases} 0 & \text{if} \quad W^M(\mathbf{q}_A) < W^M(\mathbf{q}_B) \\ \frac{1}{2} & \text{if} \quad W^M(\mathbf{q}_A) = W^M(\mathbf{q}_B) \\ 1 & \text{if} \quad W^M(\mathbf{q}_A) > W^M(\mathbf{q}_B) \end{cases}$$

by monotonicity in y^J

Equilibrium

unique outcome is

$$g_A = g_B = g^M, \qquad r_A = r_B = r^M = 0$$

identical to outcome in Downsian models without rents, and with (i) opportunistic or (ii) policy-motivated citizen candidates

Intuition

competition for exogenous rents R is fierce enough

(p_A discontinuous in policy) to keep endogenous rents r to zero

cf. results on policy convergence for partisan candidates

another type of political agency (relative to majority of voters)

2. Policy inefficiency

Competition may not deliver efficiency when less fierce

Illustrate in probabilistic voting set-up

consider version of model in Lecture **1.3**

$\phi^J = \phi$ all J, timing as in **A.1**

Probability of winning

swing voters in each group

$$\sigma^J = W^J(\mathbf{q}_A) - W^J(\mathbf{q}_B) - \delta \tag{3}$$

same type of calculations as in Lecture **1.3** $\Rightarrow$

$$p_A = \frac{1}{2} + \psi[W(\mathbf{q}_A) - W(\mathbf{q}_B)] \tag{4}$$

Candidate objectives

if purely opportunistic, max $p_C R \Rightarrow$ (4) gives efficiency

but, here max $p_C(R + \gamma r) \Rightarrow$ trade-off between r and p_C

intuition analogous to case with partisan candidates

Equilibrium spending?

candidates converge on policy that maximizes (2), given (4)

$$\frac{\partial E[v_A]}{\partial g_A} = (R + \gamma r_A)\frac{\partial p_A}{\partial g_A} = (R + \gamma r_A)W_g = 0$$

i.e., $g = g^*$, efficient spending

Equilibrium rents?

may not be driven to zero

trade off probability of winning vs. marginal rents

$$\frac{\partial E[v_A]}{\partial r_A} = (R + \gamma r_A)\frac{\partial p_A}{\partial r_A} + p_A\gamma$$
$$= -(R + \gamma r_A)\psi + p_A\gamma \leq 0 \quad [r_A \geq 0]$$

we get ($p_A = \frac{1}{2}$ in eq.), $r = \text{Max}\,[0, \frac{1}{2\psi} - \frac{R}{\gamma}]$

Rents positive if

R small, γ large, or ψ small

Intuition

candidates not perfect substitutes (except for swing voters)

as probability of winning continuous in r, candidates have room to pursue their own agenda – analog to the results on policy divergence for partisan candidates

Positive implications

$r > 0$ means that $\tau > \frac{g^*}{y}$

rents (measured spending) higher if

more illegitimate regimes (low ego-rents): R small

weaker checks and balances: γ large

large electoral uncertainty (weak voter response to r): ψ small

(asymmetric popularity: see Problem 4.1 in P-T, 2000)

B. Electoral accountability

Assumption of binding commitment too strong?

enforcement and information problems

credibility of platform promises becomes a real issue

2nd function of elections

in models, so far: voters behave "prospectively", to choose among policies candidates have committed themselves to

now, instead: behave "retrospectively" to punish bad behavior such accountability shapes policy incentives without commitment

all voters have same utility: $W(\mathbf{q}) = y - (g + r) + H(g)$

Timing

(i) voters set reservation utilities ϖ^i, (ii) incumbent I sets policy $\mathbf{q}_I$, (iii) election is held

Incumbent objective

$$E[v_I] = \gamma r_I + p_I \beta R \tag{5}$$

reflects new timing

Opponent

identical to I in all respects (but see model in **C**)

Voter coordination

all voters coordinate on same strategy $\varpi^i = \varpi$

$$p_I = \begin{cases} 1 & \text{if } W(\mathbf{q}_I) > \varpi \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

useful to trace out best possible outcome for voters

alternative assumption: distribution of reservation utilities, works basically as prior probabilistic voting model (see model in **C**)

Basic incentive constraint

intertemporal trade-off for I

$$\gamma r_I + \beta R \geq \gamma y \tag{7}$$

comply (LHS): hold back to get re-elected and earn future rents
deviate (RHS): maximize current diversion give up re-election

Best feasible policy for voters?

maximize $W(\mathbf{q})$ subject to (7) and (1) $\Rightarrow$

$$\begin{aligned} r^* &= \text{Max}\,[0, y - \frac{\beta R}{\gamma}] \\ g^{**} &= \text{Min}\,[g^*, \frac{\beta R}{\gamma}] \qquad [\tau \leqq 1] \end{aligned} \tag{8}$$

I gets away with some rents, unless
βR high, γ and y low – cf. results in **A.2.**

How can voters implement (8)?

I sets policy according to (8) to earn re-election if voters set ϖ at

$$\varpi^* = y - (g^{**} + r^*) + H(g^{**})$$

Extension: asymmetric information (about cost of g)

more complex case

I earns additional (state-dependent) rents

voters worse off

C. Electoral selection

3rd role of elections

neither select policy, nor reward good behavior,
but rather select able (competent) leader

assume that ability: (i) comes in different types,
(ii) affects performance, and (iii) lasts over time

Simplified two-period model – election at end of period 1

period-t utility of voter i

$$w_t^i = y - \tau_t + \alpha g_t - D_2^I \sigma^i \tag{9}$$

linearity in $g \Rightarrow$ risk neutrality

σ^i taste bias against I_1, which is uniform on $[-\frac{1}{2\phi}, \frac{1}{2\phi}]$

$D_1^I = 0$, $D_2^I > 0$ only applies in period 2, if I_1 re-elected

note: there is no average popularity shock δ, but "ability"
shock η (see below) will play similar role

Government policy

$$g_t = \overline{\tau} - r_t + \eta_t + \nu_t \tag{10}$$

τ_t fixed at $\overline{\tau}$, $r_t \leq \overline{r}$, i.e., upper bound on r_t

η_t any *new* politician's ability is iid $\sim N(\overline{\eta}, \mathrm{Var}(\eta))$
but lasting over time – see below

ν_t productivity shock is iid $\sim N(0, \mathrm{Var}(\nu))$

Incumbent objective

$$E(v_I) = \ln(r_1) + p_I \beta[(R + E(\ln(r_2))] \tag{11}$$

set $\gamma = 1$, add curvature over rents, to get simple solutions

Assumptions about politician ability

I_1 does *not* know η_1 (and ν_1) when sets r_1 (avoid signaling), as in Holmström's career-concern model

I_1 re-elected: $\eta_2^I = \eta_1^I$ (incumbent ability lasts), $E(\eta_2^I) = E(\eta_1^I)$

I_1 ousted: $E(\eta_2^O) = \overline{\eta}$ (opponent expected to have average ability)

Period 2 choice of r

all incumbents set $r_2 = \overline{r}$ (as world ends)
$\Rightarrow$ from (9)-(10) $E(g_2) = \overline{\tau} - \overline{r} + E(\eta_2^C), C = I, O$ and

$$E(w_2^i) = y - \overline{\tau} + \alpha(\overline{\tau} - \overline{r} + E(\eta_2^C)) - D_2^I \sigma^i$$

voters like able politicians better, *ceteris paribus*

Optimal voting strategy

I_1 has $E(\eta_2^I) = E(\eta_1^I)$, opponent has $E(\eta_2^O) = \overline{\eta}$

$\Rightarrow$ vote for I_1 if $\sigma^i < \alpha[E(\eta_1^I) - \overline{\eta}]$ such that

$$\pi_I = \frac{1}{2} + \phi\alpha[E(\eta_1^I) - \overline{\eta}] \tag{12}$$

is vote share of incumbent

Information at $t = 1$ pins down $E(\eta_1^I)$: study two cases

1. informed voters: observe g_1 and $\nu_1 \Rightarrow E(\eta_1^I \mid g_1, \nu_1)$

2. uninformed voters: observe only $g_1 \Rightarrow E(\eta_1^I \mid g_1)$

1. Informed voters

Voters' inference problem

given (10), can perfectly gauge incumbent ability $\Rightarrow$

$$E(\eta_1^I \mid g_1, \nu_1) = \eta_1^I = g_1 - \overline{\tau} + r_1^* - \nu_1, \tag{13}$$

where r_1^* is expected equilibrium rents

Incumbent choice of r

when I_1 sets r_1 uncertain about η_1 (and ν_1) and hence g_1, so has to form an expectation $\mathbb{E}(E(\eta_1^I \mid g_1, \nu_1))$

knows how $E(\eta_1^I \mid g_1, \nu_1)$ is formed and takes r_1^* as *given*

by (10), (12) and (13), his anticipated vote share conditional on η_1 and r_1 becomes

$$\pi_I = \frac{1}{2} + \phi\alpha[\eta_1^I - \overline{\eta} + r_1^* - r_1]$$

and the perceived probability of winning is

$$p_I = \ \mathrm{Prob}_\eta \left[\pi_I \geq \frac{1}{2}\right] = 1 - F(\overline{\eta} - r_1^* + r_1) \tag{14}$$

where F is the c.d.f. of η – clearly, larger r_1 cuts (perceived) p_I

Optimal policy

maximize (11) over r_1 subject to (14), and set $r_2 = \overline{r}$ to get

$$r_1 = \frac{1}{f(\overline{\eta} - r_1^* + r_1)\beta\widetilde{R}}$$

where $\widetilde{R} = R + \ln(\overline{r})$, and f is the p.d.f. of η

Equilibrium

voters expectations are correct, such that $r_1^* = r_1$, and

$$r_1 = \frac{1}{f(\overline{\eta})\beta\widetilde{R}}$$

Interpretation

voters look like they follow retrospective strategy,
rewarding high performance (utility) with re-election

but current performance is an indicator of future ability
and this creates an intertemporal trade-off for I_1

Positive implications

rents higher (cf. results in **A** and **B**) when

electoral reward is small: $\beta \widetilde{R}$ low

electoral uncertainty is large: $f(\overline{\eta})$ low, i.e., $\mathrm{Var}(\eta)$ large
like result in **A.2** about uncertainty over δ (value of ψ)

2. Uninformed voters

Voters' inference problem

can no longer gauge η_1^I perfectly, as ν_1 unobserved

using (10), they can only infer the sum $\Rightarrow$

$$E(\eta_1^I + \nu_1 \mid g_1) = \eta_1^I + \nu_1 = g_1 - \overline{\tau} + r_1^* \ , \qquad (15)$$

let voters form an optimal (OLS) estimate of η_1^I, given that they see $E(\eta_1^I + \nu_1 \mid g_1)$ and have unconditional (prior) mean $\overline{\eta}$

This yields (see Appendix)

$$E(\eta_1^I \mid g_1, \overline{\eta}) = h_\eta \overline{\eta} + h_\nu E(\eta_1^I + \nu_1 \mid g_1) \ , \qquad (16)$$

where $h_\eta = \frac{\mathrm{Var}(\nu)}{\mathrm{Var}(\eta)+\mathrm{Var}(\nu)}$ and $h_\nu = \frac{\mathrm{Var}(\eta)}{\mathrm{Var}(\eta)+\mathrm{Var}(\nu)}$

observation of g_1 is less (more) valuable in inference about η_1^I the more (less) noisy is ν_1

Incumbent expectations

by (10), (12), (15) and (16), I forms an expectation about voters' expectations $\mathbb{E}(E(\eta_1^I \mid g_1, \overline{\eta}))$ and anticipates vote share

$$\pi_I = \frac{1}{2} + \phi\alpha h_\nu[\eta_1^I + \nu_1 - \overline{\eta} + r_1^* - r_1]$$

π_I responds less to rents when voters uninformed

perceived probability of winning is

$$p_I = \text{ Prob}_{(\eta+\nu)}\,[\pi_I \geq \frac{1}{2}] = 1 - G(\overline{\eta} - r_1^* + r_1) \tag{17}$$

where G is the c.d.f. (with p.d.f. g) of random variable $\eta + \nu$ sum of two normals, mean $\overline{\eta} + 0$ and variance Var(η)+Var(ν)

Optimal policy

maximize (11) over r_1 subject to (17) to get

$$r_1 = \frac{1}{g(\overline{\eta} - r_1^* + r_1)\beta\widetilde{R}}$$

In equilibrium ($r_1^* = r_1$)

$$r_1 = \frac{1}{g(\overline{\eta})\beta\widetilde{R}}$$

Compare to the case with informed voters

G, distribution of $\eta + \nu$, has same mean (i.e., $\overline{\eta}$), but larger variance (i.e., Var(η)+Var(ν)) than F, distribution of η

therefore, it must be that $g(\overline{\eta}) < f(\overline{\eta})$

so r_1 is larger with uninformed voters, and more so the larger is Var(ν) – the more difficult is inference about η

3. Discussion – three natural extensions

Informed *and* uninformed voters

combination of **1** and **2**

larger uninformed share (less "media coverage") implies larger rents and smaller voting response to misbehavior

Embed in multi-period model

elections every two periods, and MA process for η $\Rightarrow$

electoral cycle: cut r (raise spending) in election periods, unless there is a term limit

Assume η known by incumbent $\Rightarrow$ incentives to signal

more complex solution, but many results similar

Appendix

Here is probably the simplest way to derive equation (16).

Agents want to estimate η, drawn from a normal distribution with mean $E(\eta) = \overline{\eta}$ and variance $\mathrm{Var}(\eta)$. They observe sum $\eta + \upsilon$, where υ drawn from another (independent) normal distribution with mean $E(\upsilon) = 0$ and variance $\mathrm{Var}(\upsilon)$. Think about their inference problem as optimally choosing the coefficients a and b in the OLS regression

$$\eta = a + b(\eta + \upsilon) \tag{OLS}$$

i.e., minimize the expected squared deviation of the estimate from the data

$$\mathrm{Min}\{E[(\eta - a - b(\eta + \upsilon))(\eta - a - b(\eta + \upsilon)]\}.$$

Taking the expectation and minimizing with respect to a yields the first-order condition

$$E(-2\eta + 2a + 2b(\eta + \upsilon)) = 2E(a - \eta + b\eta) = 0,$$

which implies

$$a = (1-b)E(\eta) = (1-b)\overline{\eta} \tag{A1}$$

Similarly, the first-order condition for b is

$$\begin{aligned} 2E[-\eta^2 - \eta\upsilon + b\eta^2 + b\eta\upsilon + b\upsilon^2] &= \\ 2[-\mathrm{Var}(\eta) - \mathrm{Cov}(\eta\upsilon) + b(\mathrm{Var}(\eta) + \mathrm{Cov}(\eta\upsilon) + \mathrm{Var}(\upsilon))] &= 0. \end{aligned}$$

Because $\mathrm{Cov}(\eta\upsilon) = 0$, this condition implies

$$b = \frac{\mathrm{Var}(\eta)}{\mathrm{Var}(\eta) + \mathrm{Var}(\upsilon)} \tag{A2}$$

Substituting (A2) in (A1) and simplifying we obtain

$$a = \frac{\mathrm{Var}(\upsilon)}{\mathrm{Var}(\eta) + \mathrm{Var}(\upsilon)}\overline{\eta} \tag{A3}$$

Finally, identifying $\eta + \upsilon$ with $E(\eta + \upsilon)$ in (OLS) and using (A2) and (A3), we get expression (16) in the notes.